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Question Paper Code : 80209

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Second Semester

Civil Engineering

MA 8251 — ENGINEERING MATHEMATICS – II

(Common to All branches (Except Marine Engineering))

(Regulation 2017)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If λ is the eigenvalue of the matrix A , then prove that λ^2 is the eigenvalue of A^2 .
2. If the eigenvalues of the matrix A of order 3×3 are 2, 3 and 1, then find the determinant of A .
3. Find the unit normal vector to the surface $x^2 + y^2 = z$ at $(1, -2, 5)$.
4. State Stoke's theorem.
5. Is the function $f(z) = e^z$ analytic.
6. Find the fixed point of the bilinear transformation $w = \frac{1}{z}$.
7. Evaluate $\int_C \sin z \, dz$, where C is the entire complex plane.
8. Define singularity of a function $f(z)$.
9. Find $L[e^{-t} \sin t]$.
10. State sufficient conditions for the existence of Laplace transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and the eigenvectors of the matrix
- $$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}. \quad (8)$$

- (ii) Using Cayley-Hamilton theorem find A^{-1} , if $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. (8)

Or

- (b) Reduce the quadratic form $2xy - 2yz + 2xz$ into a canonical form by an orthogonal reduction. (16)
12. (a) (i) Verify Gauss divergence theorem for the vector function $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cuboids bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 1$, and $z = 1$. (10)
- (ii) Find the value of n so that the vector $r^n\vec{r}$ is irrotational and solenoidal. (6)

Or

- (b) (i) Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$, where C is the boundary of the area by the x -axis and the upper half of the circle $x^2 + y^2 = a^2$. (8)
- (ii) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} + 2xy\vec{j}$, taken around the rectangle bounded by the lines $x = 0, y = 0, x = 1$ and $y = 1$. (8)

13. (a) (i) Determine the analytic function $f(z) = u + iv$, if $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$. (8)
- (ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto $w = i, 0, -i$. (8)

Or

- (b) (i) Show that the real and imaginary parts of an analytic functions are harmonic. (8)
- (ii) Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$. (8)

14. (a) (i) If $F(\alpha) = \oint_C \frac{(3z^2 + 7z + 1)}{z - \alpha} dz$, where C is $|z| = 2$, then find $F(1-i)$ and $F'(1-i)$. (8)

(ii) Using contour integration, evaluate $\int_0^{\infty} \frac{dx}{(x^2 + 1)^2}$. (8)

Or

(b) (i) Obtain the Laurent's series expansion of $f(z) = \frac{z^2 - 1}{(z + 2)(z + 3)}$ if $2 < |z| < 3$. (8)

(ii) Evaluate by using contour integration $\int_0^{2\pi} \frac{d\theta}{13 + 5 \sin \theta}$. (8)

15. (a) (i) Find the Laplace transform of $f(t)$ with period $2a$, where $f(t) = \begin{cases} t, & \text{for } 0 < t < a \\ 2a - t, & \text{for } a < t < 2a \end{cases}$. (8)

(ii) Using convolution theorem, find $L^{-1} \left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)} \right]$. (8)

Or

(b) (i) Find $L \left[\frac{\cos 2t - \cos 3t}{t} \right]$. (8)

(ii) Solve $\frac{d^2 y}{dt^2} - 3 \frac{dy}{dt} + 2y = e^{3t}$; given that $y(0) = 0, \frac{dy}{dt}(0) = 0$. (8)