Reg. No. :

Question Paper Code : 40056

B.E. DEGREE EXAMINATION, APRIL/MAY 2018

Second Semester

Marine Engineering

MA8201 – MATHEMATICS FOR MARINE ENGINEERING – II

(Regulations 2017)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. Obtain the differential equation of the coaxial circles of the system $x^2 + y^2 + 2ax + c^2 = 0$ where c is a constant and a is a variable.

2. Solve $(1 + 2xy\cos x^2 - 2xy)\,dx + (\sin x^2 - x^2)\,dy = 0$.

3. Find the particular integral of $(D^2 + 6D + 9)y = e^{-2x} x^3$.

4. Solve $[(2x + 3)^2 D^2 - 2(2x + 3)D - 12]y = 0$.

5. If $\nabla\phi = yz\vec{i} + xz\vec{j} + xy\vec{k}$, then find ϕ.

6. Find the constants space a, b, c, so that the vector $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational.

7. State any two properties of an analytic function.

8. Find the invariant points of the bilinear transformation $w = \dfrac{2zi + 5}{z - 4i}$.

9. Find L [t cos 3t].

10. Prove that Laplace transform of unit step function is $\dfrac{e^{-as}}{s}$.

PART – B (5×16=80 Marks)

11. a) i) Solve $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$. (8)

ii) Find the orthogonal trajectory of the cardioids $r = a(1 - \cos\theta)$. (8)

(OR)

b) i) Solve $(1 + y^2)\,dx = (\tan^{-1} y - x)dy$. (8)

ii) Solve $\left[x\tan\left(\frac{y}{x}\right) - y\sec^2\left(\frac{y}{x}\right)\right]dx - x\sec^2\left(\frac{y}{x}\right)dy = 0$. (8)

12. a) i) Solve by the method of undetermined coefficients, $(D^2 + 2D + 4)y = 2x^2 + 3e^{-x}$. (8)

ii) Solve $Dx - (D - 2)y = \cos 2t$ and $(D - 2)x + Dy = \sin 2t$. (8)

(OR)

b) i) Solve by the method of variation of parameters, $(D^2 - 2D + 1)y = e^x \log x$. (8)

ii) Solve $(x^2D^2 - xD + 4)y = x^2 \sin(\log x)$. (8)

13. a) i) Find the constants a and b, so that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2y + bz^3 = 4$ may cut orthogonally at the point (1, – 1, 2). (8)

ii) Evaluate $\iint_S \vec{F}.\hat{n}\,dS$, where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and S is the surface bounded by the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$ by using Gauss divergence theorem. (8)

(OR)

b) Verify Stoke's theorem for $\vec{F} = y^2z\vec{i} + z^2x\vec{j} + x^2y\vec{k}$ where S is the open surface of the cube formed by planes $x = \pm a$, $y = \pm a$ and $z = \pm a$, in which the plane $z = -a$ is cut. (16)

14. a) i) Prove that $v = \log[(x-1)^2 + (y-2)^2]$ is harmonic in every region which does not include the point (1, 2). Find the corresponding analytic function $w = u + iv$ and also u. (8)

ii) Find the bilinear transformation that maps the points $1 + i, -i, 2 - i$ of the z – plane into the points 0, 1, i of the w – plane. (8)

(OR)

b) i) If $f(z) = u + iv$ is an analytic function of z, then prove that $\nabla^2\left[\log|f(z)|\right] = 0$. (8)

ii) Find the image of $1 < x < 2$ under the transformation $w = \frac{1}{z}$. (8)

15. a) i) Find the Laplace transform of $L\left[\frac{\sin^2 t}{t}\right]$. (8)

ii) Solve the differential equation, using Laplace transform $y'' - 3y' + 2y = 4t$ given that $y(0) = 1$ and $y'(0) = -1$. (8)

(OR)

b) i) Find the Laplace transform of the function $f(t) = \begin{cases} t, & 0 < t < \pi/2 \\ \pi - t, & \pi/2 < t < \pi \end{cases}$ and $f(\pi + t) = f(t)$. (8)

ii) Using convolution theorem, find $L^{-1}\left[\frac{4}{(s^2 + 2s + 5)^2}\right]$. (8)