CHENNAI INSTITUTE OF TECHNOLOGY

MA3151 MATRICES AND CALCULUS
QUESTION BANK
UNIT – I
MATRICES

1. **Find the Eigen values and Eigenvectors of the matrix** $A = \begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$
2. **Find the Eigen values and Eigenvectors of the matrix** $A = \begin{pmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{pmatrix}$
3. Find the Eigen values and Eigenvectors of the matrix $A = \begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$
4. Find the Eigen values and Eigenvectors of the matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$
5. Find the Eigen values and Eigenvectors of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$
6. **Using Cayley-Hamilton theorem find A^4 and A^{-1}, if** $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$
7. Using Cayley-Hamilton theorem find A^4 and A^{-1}, if $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$
8. Using Cayley-Hamilton theorem find A^4 and A^{-1}, if $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$
9. Verify Cayley-Hamilton theorem find A^4 and A^{-1}, if $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$
10. **Use Cayley-Hamilton theorem to find the value of the matrix given by** $f(A) = A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ **if** $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$
11. **Reduce the quadratic form** $Q = 6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ **canonical form by an orthogonal reduction. Hence find its nature.**
12. **Reduce the quadratic form** $Q = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$ **canonical form by an orthogonal reduction. Hence find its nature.**
13. Reduce the quadratic form $2xy - 2yz + 2xz$ canonical form by an orthogonal reduction. Hence find its nature.
14. **Reduce the quadratic form** ${x_1}^2 + 5{x_2}^2 + {x_3}^2 + 2x_1x_2 + 2x_2x_3 + 6x_1x_3$ **canonical form by an orthogonal reduction. Hence find its nature.**

15. Reduce the quadratic form $10x_1^2+2x_2^2+5x_3^2-4x_1x_2+6x_2x_3-10x_1x_3$ canonical form by an orthogonal reduction. Hence find its nature.

Unit II Differential Calculus

1. **For what value of the constant C is the function f continuous on $(-\infty,\infty)$,** $f(x)=\begin{cases} cx^2+2x; & x<2 \\ x^3-cx; & x\geq 2 \end{cases}$.
2. **Find the values of a and b that make f continuous on $(-\infty,\infty)$.** $f(x)=\begin{cases} \frac{x^3-8}{x-2} & if\ x<2 \\ ax^2-bx+3 & if\ 2\leq x<3 \\ 2x-a+b & if\ x\geq 3 \end{cases}$
3. **Find $\frac{dy}{dx}$ if $y=x^2e^{2x}(x^2+1)^4$**
4. **Find y'' if $x^4+y^4=16$.**
5. **Find the derivative of $f(x)=cos^{-1}\left(\frac{b+acosx}{a+bcosx}\right)$.**
6. **Find y' for $\cos(xy)=1+siny$.**
7. Find the tangent line to the equation $x^3+y^3=6xy$ at the point (3, 3) and at what point the tangent line horizontal in the first quadrant.
8. Guess the value of the limit (if it exists) for the function $\lim_{n\to\infty}\frac{e^{5x}-1}{x}$ by evaluating the function at the points $x=\pm 0.5,\pm 0.1,\pm 0.01\pm 0.001,\pm 0.0001$ (correct to 6 places).
9. **For the function $f(x)=2+2x^2-x^4$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity.**
10. **For the function $f(x)=2x^3+3x^2-36x$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity.**
11. **Find the local maximum and minimum values of $f(x)=\sqrt{x}-\sqrt[4]{x}$ using both first and second derivatives tests.**
12. For the function $f(x)=sin\,x+cos\,x,\ 0\leq x\leq 2\pi$ (i) find the intervals on which it is increasing or decreasing (ii) find the local maximum and minimum values of *f* (iii) find the intervals of concavity and the inflection points.
13. Find the local maximum and local minimum of $f(x)=x^4-2x^2+3$.
14. Calculate the absolute maximum and minimum of the function $f(x)=3x^4-4x^3-12x^2+1$ in $[-2,3]$.

Unit – III FUNCTIONS OF SEVERAL VARIABLES

1. If $u=(x^2+y^2+z^2)^{-\frac{1}{2}}$, then evaluate the value of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}$.
2. If $u=f\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$ find $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}$.

3. If $z = f(x, y)$ where $x = rcos\theta, y = rsin\theta$, then prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$.
4. **Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_3x_1}{x_2}, y_3 = \frac{x_1x_2}{x_3}$.**
5. **Find the Jacobian $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$ of the transformation $x = rsin\theta cos\emptyset, y = rsin\theta sin\emptyset$ and $z = rcos\theta$.**
6. **A rectangular box open at the top, is to have a volume of 32cc. Find dimensions of box which requires least amount of material for its construction.**
7. **Classify the shortest and the longest, distances from the point (1,2,-1) to the sphere $x^2 + y^2 + z^2 = 24$.**
8. Find the maximum volume of the largest rectangular parallelepiped that can be inscribed in an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
9. Find the dimension of the rectangular box without a top of maximum capacity, whose area is 108squ.cm.
10. Find the minimum distance from the point $(1, 2, 0)$ to the cone $z^2 = x^2 + y^2$.
11. Expand $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x + 2)$ and $(y - 1)$ using Taylor's series upto third degree terms.
12. **Expand $e^x siny$ in powers of x and y using Taylor's series upto third degree terms.**
13. **Expand $e^x cosy$ about $\left(0, \frac{\pi}{2}\right)$ using Taylor's series upto third degree terms**
14. Find Taylor's series expansion of function of $f(x, y) = \sqrt{1 + x^2 + y^2}$ in powers of $(x - 1)$ and y upto second degree terms.
15. **Obtain Taylor's series expansion of $x^3 + y^3 + xy^2$ in terms powers of $(x - 1)$ and $(y - 2)$ upto third degree terms**
16. **Find the maxima and minima of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.**
17. **Find the maxima and minima of $f(x, y) = 3x^2 - y^2 + x^3$.**
18. **Examine $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values.**
19. Find the maxima and minima of $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$.
20. Find the maxima and minima of $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$.

Unit – IV Integral Calculus.

1. **Evaluate $\int \frac{2x+5}{\sqrt{x^2-2x+10}} dx$.**
2. Evaluate $\int \frac{x}{\sqrt{x^2+x+1}} dx$
3. **Find $\int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}} \frac{dx}{x^5\sqrt{9x^2-1}}$**
4. **Evaluate $\int_0^{\frac{\pi}{2}} sin^8 x\, dx$**
5. Find $\int_0^{\frac{\pi}{4}} x\, tan^2 x\, dx$.

6. **Establish a reduction formula for $I_n = \int sin^n x\, dx$. Hence, find $\int_0^{\frac{\pi}{2}} sin^n x\, dx$.**
7. **Establish a reduction formula for $I_n = \int cos^n x\, dx$. Hence, find $\int_0^{\frac{\pi}{2}} cos^n x\, dx$.**
8. **Establish a reduction formula for $I_n = \int sec^n x\, dx$. and $I_n = \int tan^n x\, dx$.**
9. Evaluate $\int_0^{\frac{\pi}{2}} sin^8 x\, dx$
10. **Evaluate $\int_0^{\frac{\pi}{2}} \frac{sinx\, cosx}{\cos^2 x+3cosx+2} dx$**
11. **Evaluate $\int \frac{(ln\, x)^2}{x^2} dx$**
12. Evaluate $\int_0^{\frac{\pi}{2}} cos^5 x\, dx$.
13. **Evaluate $\int \frac{tanx}{secx+tanx} dx$**
14. **Evaluate $\int \frac{tanx}{secx-tanx} dx$.**
15.
16. **Evaluate $\int_0^{\infty} e^{-ax} sinbx\, dx\ (a > 0)$ using integration by parts**
17. **Evaluate $\int_0^{\infty} e^{-ax} cosbx\, dx\ (a > 0)$ using integration by parts**
18. Evaluate $\int \frac{x^2+x+1}{(x-1)^2(x-2)} dx$
19. **Evaluate $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} dx$**
20. *Evaluate* $\int \frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)}$dx by partical fraction method
21. For what value of p is $\int_0^{\infty} \frac{1}{x^p} dx$ convergent?

UNIT – V MULTIPLE INTEGRALS

1. **Change of order of integration for the given integral $\int_0^a \int_0^{2\sqrt{ax}} x^2 dy\, dx$ and evaluate it.**
2. **Change of order of integration for the given integral $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xydy\, dx$ and evaluate it.**
3. **Change of order of integration for the given integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xydy\, dx$ and evaluate it.**
4. Evaluate by change of order of integration for the given integral $\int_0^{\infty} \int_0^y ye^{-\frac{y^2}{x}} dy\, dx$ and evaluate it.
5. Evaluate by change of order of integration for the given integral $\int_1^3 \int_0^{\frac{6}{x}} x^2 dy\, dx$ and evaluate it
6. **Using double integral find the area bounded by $y = x$ and $y = x^2$.**
7. **Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.**

8. **Find by double integration, the area enclosed by the curves $y^2 = 4ax$ and $x^2 = 4ay$.**
9. Evaluate $\iint xy dxdy$ over the positive quadrant of the circle. $x^2 + y^2 = a^2$.
10. **Evaluate by changing to polar coordinates $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$.**
11. Express $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}} dx\, dy$ in polar coordinates and then evaluate it
12. **Find, using a double integral , the area of the cardioid $r = a(1 + cos\theta)$.**
13. Calculate the area which is inside the cardioids $r = 2(1 + cos\theta)$ and outside the circle $r = 2$.
14. **Evaluate $\iiint xyz\, dxdydz$ over the first octant of $x^2 + y^2 + z^2 = a^2$.**
15. **Find the volume of that portion of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which lies in the first octant.**
16. **Compute the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0, y + z = 4$.**
17. Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = x$.
18. Evaluate $\iiint_v dx\, dy\, dz$, where V is the finite region of space (tetrahedron) bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 3y + 4z = 12$.
19. **By changing to polar coordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$.**
20. **Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2) dy\, dx$ by changing into polar coordinates.**

All the Best