

**VIVEKANANDHA COLLEGE OF TECHNOLOGY FOR WOMEN
ELAYAMPALAYAM, TIRUCHENGODE – 637205.**

MATRICES AND CALCULUS

UNIT-1 (MATRICES)

(2021-2022)

I. Answer all the questions

$5 \times 2 = 10$

1. Find the Sum and the product of the matrix $A = \begin{pmatrix} 1 & 1 & 5 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$

2. State Cayley Hamilton theorem.

3. Find the eigenvalues of the matrix $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$

4. Find the eigenvalues of the inverse of the matrix $A = \begin{pmatrix} 2 & 1 & 5 \\ 0 & 4 & 4 \\ 0 & 0 & 5 \end{pmatrix}$

5. If 1 and 2 are the eigenvalues of a 2×2 matrix A, what are the eigenvalues of A^2 and A^{-1} .

II. Answer all the questions

$1 \times 8 = 8$ & $1 \times 16 = 16$

6. Find the eigenvalues and eigenvectors of $\begin{pmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{pmatrix}$ (8)

7. Verify Cayley-Hamilton Theorem for the matrix $\begin{pmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$. Hence find its inverse

and A^4 (16)

8. Reduce the Q.F. $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ into a canonical form and find the nature of the Q.F. (16)

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MATRICES AND CALCULUS

UNIT-2 (DIFFERENTIAL CALCULUS)

(2021-2022)

I. Answer all the questions

$5 \times 2 = 10$

- Evaluate $\lim_{x \rightarrow 2^+} \frac{x^2+x-6}{|x-2|}$
- Find $\frac{d}{dx} [(\sin x)^{\cos x}]$
- Find the domain of the function $f(x) = \frac{x+4}{x^2-9}$
- Explain why the function is discontinuous at the given number a $f(x) = \frac{1}{x+2}$, $a = -2$
- Find the critical values of the function $f(x) = 2x^3 - 3x^2 - 36x$

II. Answer all the questions

$5 \times 8 = 40$

- Find an equation of the tangent and normal lines to the given curve at specified point
 $f(x) = \frac{x^2-1}{x^2+x+1}$, $(1,0)$
- (i) Find y' if $y = x^{x^{x^{\dots}}}$ (ii) If $\sin(x+y) = y^2 \cos x$, then find $\frac{dy}{dx}$.
- (i) Prove that equation $x^3 - 15x + c = 0$ has atmost one real root in the interval $[-2, 2]$.
(ii) If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$ how small can $f(4)$ possibly be?
- Find the local maximum and minimum values of function $f(x) = x^5 - 5x + 3$ using both the first and second derivatives tests.
- (i) Suppose f and g are continuous functions such that $g(2) = 6$ and
 $\lim_{x \rightarrow 2} [3f(x) + f(x)g(x)] = 36$. Find $f(2)$
(ii) Show that $f(x) = 3x^2 + 2x - 1$ is continuous at $x=2$.

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MATRICES AND CALCULUS

UNIT-3 (FUNCTIONS OF SEVERAL VARIABLES)

(2021-2022)

I. Answer all the questions

5 × 2 = 10

1. Evaluate $\lim_{\substack{x \rightarrow \infty \\ y \rightarrow 2}} \frac{xy+5}{x^2+2y^2}$
2. Find $\frac{dy}{dx}$ when $x^3+y^3 = 3axy$
3. Find the domain of the function $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$ find $\frac{\partial(u,v)}{\partial(x,y)}$
4. If $u = f\left(\frac{x}{y}, \frac{y}{x}, \frac{z}{x}\right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$
5. Write two properties of jacobians.

II. Answer all the questions

5 × 8 = 40

6. If $u = \log(x^3+y^3+z^3-3xyz)$, Show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$
7. Show that the functions $u = x+y-z$, $v = x-y+z$, $w = x^2+y^2+z^2-2yz$ are dependent. Find the relation between them.
8. Expand the function $\sin(xy)$ in powers of $x-1$ and $y-\frac{1}{2}$ upto second degree terms.
9. Find the maxima and minima of $x^4+y^4-2x^2+4xy-2y^2$
10. Find the maximum volume of the largest rectangular parallelepiped that can be inscribed in an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

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MATRICES AND CALCULUS

UNIT-4 (INTEGRAL CALCULUS)

(2021-2022)

I. Answer all the questions

5 × 2 = 10

1. Evaluate $\int \frac{\log x}{x} dx$
2. Evaluate $\int \frac{\sin 2x}{1 + \cos^2 x} dx$
3. $\int_0^1 \tan^{-1} x dx$
4. Evaluate $\int \frac{1}{\sqrt{a^2 - x^2}} dx$ by using trigonometric substitution.
5. For what values of p in the integral $\int_1^{\infty} \frac{1}{x^p} dx$ convergent?

II. Answer all the questions

5 × 8 = 40

6. Evaluate $\int_0^3 (x^2 - 2x) dx$ by using Riemann sum by taking right end points as the sample points.
7. Find the reduction formula for $\int \sin^n x dx$, n ≥ 2 is an integer and $\int_0^{\frac{\pi}{2}} \sin^n x dx$
8. Evaluate $\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$
9. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$
10. Evaluate $\int (3x - 2)\sqrt{(x^2 + x + 1)} dx$

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MATRICES AND CALCULUS

UNIT-5 (MULTIPLE INTEGRALS)

(2021-2022)

I. Answer all the questions

5 × 2 = 10

1. Evaluate $\int_0^3 \int_0^2 e^{x+y} dy dx$

2. Sketch roughly the region of integration for $\int_0^1 \int_0^x f(x,y) dy dx$

3. Find the domain of the function $\int_0^{\sin} \int_0^r r dr$

4. Change the order of integration of $\int_0^a \int_y^a f(x,y) dx dy$

5. Express the region $x \geq 0, y \geq 0, z \geq 0, x^2 + y^2 + z^2 \leq 1$ by triple integration.

II. Answer all the questions

5 × 8 = 40

6. Change the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dx dy$ and hence evaluate the same.

7. (i) Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = 4 - 4x$ as a double integral and evaluate it.

(ii) Find the area of the cardioid $r = a(1 + \cos \theta)$, using a double integral.

8. (i) Evaluate $\int_0^1 \int_0^{b(1-\frac{x}{a})} \int_0^{c(1-\frac{x}{a}-\frac{y}{b})} x^2 z dz dy dx$

(ii) Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) dx dy dz$

9. Find the volume of that portion of $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which lies in the first octant using triple integration.

10. Evaluate by changing to polar co-ordinates, the integral $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dx dy$
