Reg. No. : | E | N | G | G | T | R | E | E | . | C | O | M |

Question Paper Code : 70138

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Mechanical Engineering

MA 3351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aeronautical Engineering/Aerospace Engineering/
Automobile Engineering/Biomedical Engineering/
Civil Engineering/Manufacturing Engineering/Marine Engineering/
Materials Science and Engineering/Mechanical Engineering (Sandwich)/
Mechanical and Automation Engineering/Mechatronics Engineering/
Medical Electronics/ Petrochemical Engineering/Production Engineering/
Robotics and Automation/Safety and Fire Engineering/Bio Technology/
Biotechnology and Biochemical Engineering/Food Technology/
Petrochemical Technology/ Petroleum Engineering/Pharmaceutical Technology)

(Regulations 2021)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the complete solution of the PDE $p^2 + q^2 = 4pq$.

2. Find the complementary function of the PDE $(D^3 - 3DD'^2 + 2D'^3)z = e^{2x-y}$.

3. Identify the given $f(x)$ is an even or odd function. Also sketch its graph.

$$f(x) = \begin{cases} \pi + x & -\pi \le x \le -\dfrac{\pi}{2} \\ -x & -\dfrac{\pi}{2} \le x \le 0 \\ x & 0 \le x \le \dfrac{\pi}{2} \\ \pi - x & \dfrac{\pi}{2} \le x \le \pi \end{cases}$$

4. State Parseval's identity in Fourier series.

5. Write down the appropriate solution of the one dimensional heat flow equation. How is it chosen?

6. The ends A and B of a rod 30 cm long, have their temperature kept at 10°C and 100°C respectively. Then obtain the steady state temperature.

7. What are the sufficient conditions for the existence of Fourier transform of a function $f(x)$?

8. Obtain the Fourier cosine transform of $\frac{1}{2^t}$.

9. Find the inverse Z transform of $\frac{z}{(z-1)^2}$.

10. State final value theorem in Z transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve : $(x-2z)p+(2z-y)q = y-x$. (8)

      (ii) Solve : $(D^3 + D^2D' - 4DD'^2 - 4D'^3)z = \cos(2x+y)$. (8)

Or

(b) (i) Solve the PDE $2z + p^2 + qy + 2y^2 = 0$. (8)

     (ii) Solve $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x+2y)$. (8)

12. (a) (i) Obtain the Fourier series of periodicity 2π for $f(x) = e^x$ in the interval $0 < x < 2\pi$. (8)

     (ii) Obtain the half range Fourier cosine series of $f(x) = x(l-x)$ in $(0, l)$. (8)

Or

(b) The following table gives the variations of periodic current over a period.

t sec :	0	T/6	T/3	T/2	2T/3	5T/6	T
A amp :	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. (Harmonic Analysis). (16)

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13. (a) A tightly stretched string of length $2l$ is fastened at both ends. The midpoint of the string is displaced by a distance 'b' transversely and the string is released from rest in this position. Find the displacement y at any distance x from one end at any time t. (16)

Or

(b) An infinitely long metal plate in the form of an area is enclosed between the lines $y = 0$ and $y = \pi$ for positive values of x. The temperature is zero along the edges $y = 0$ and $y = \pi$ and the edge at infinity. If the edge $x = 0$ is kept at temperature 'ky', find the steady state temperature at any point in the plate. (16)

14. (a) (i) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1, & \text{for } |x| \le a \\ 0, & \text{for } |x| > a \end{cases}$. (8)

(ii) Find the Fourier sine transform of $f(x) = e^{-ax}$, $a > 0$ and hence find $F_C(xe^{-ax})$. (8)

Or

(b) (i) Using Parseval's identity for Fourier transforms, evaluate $\int\limits_0^\infty \dfrac{ds}{(a^2 + s^2)(b^2 + s^2)}$. (8)

(ii) Find the Fourier cosine transform of $f(x) = \begin{cases} x & ; \ 0 < x < 1 \\ 2 - x & ; \ 1 < x < 2 \\ 0 & ; \ x > 2 \end{cases}$. (8)

15. (a) Using Z transform, solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0$, $u_1 = 1$. (16)

Or

(b) State and prove convolution theorem in Z transforms and use it to find $Z^{-1}\left\{ \dfrac{z^2}{(z-a)(z-b)} \right\}$. (16)

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