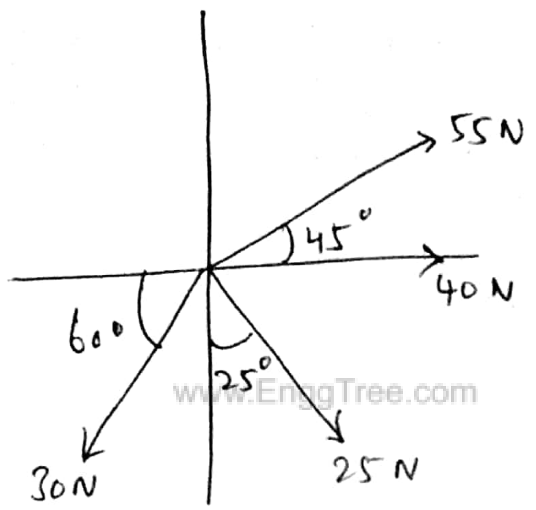
MOHAN S.R
MECH / AP

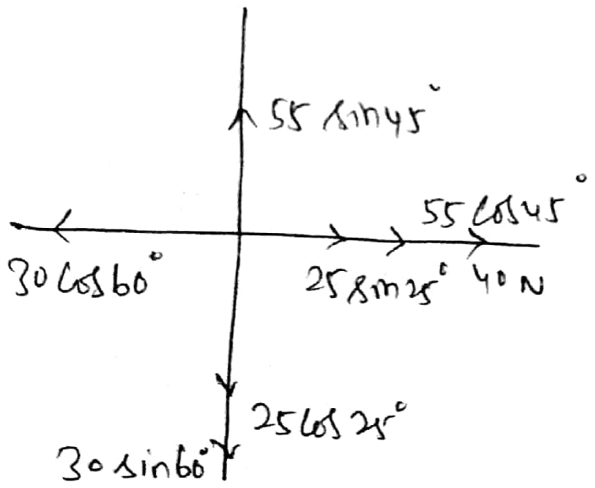
UNIT-1

BASICS & STATICS OF PARTICLES

① Determine the magnitude and the direction of the resultant of a system of concurrent, coplanar forces as shown in fig.



Sol:



Apply force system,

$$\Sigma F_x = 40 + 55\cos 45^\circ + 25\sin 30^\circ - 30\cos 60^\circ$$

$$= 76.39\ N$$

$$\Sigma F_y = 55\sin 45^\circ - 25\cos 30^\circ - 30\sin 60^\circ$$

$$= -8.74\ N$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

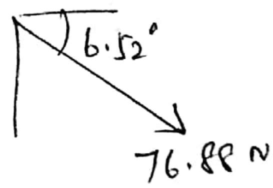
$$= \sqrt{(76.39)^2 + (-8.74)^2}$$

Resultant, $R = 76.88\ N$

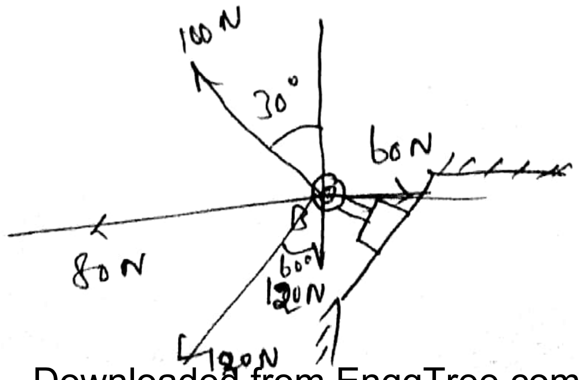
Resultant angle $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

$$= \tan^{-1}\left(\frac{8.74}{76.39}\right)$$

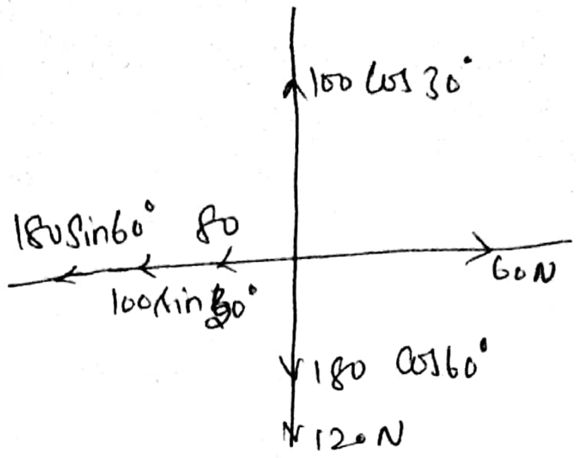
$\theta = 6.52^\circ$



(2) Five forces act on a bolt B shown in fig. Determine the resultant of the forces on the bolt.



Sol:



$$\Sigma F_x = 60 - 100 \sin 30° - 180 \sin 60° - 80$$

$$= -225.88\,N$$

$$\Sigma F_y = 100 \cos 30° - 180 \cos 60° - 120$$

$$= -123.39\,N$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$R = \sqrt{(-225.88)^2 + (-123.39)^2}$$

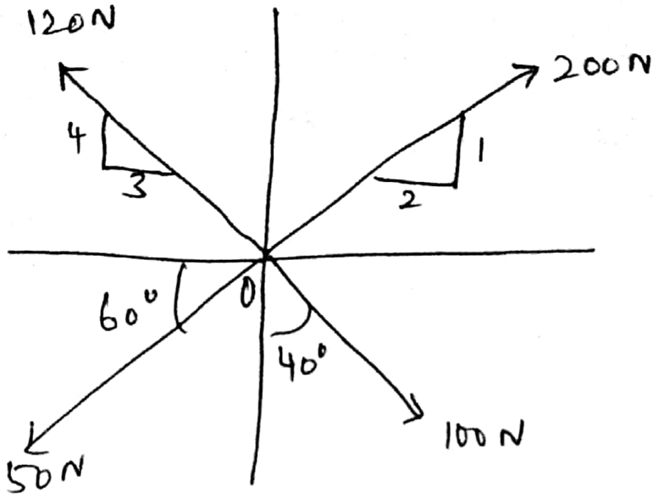
$$R = 257.27\,N$$

Angle, $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

$$\theta = \tan^{-1}\left(\frac{123.39}{225.88}\right)$$

$$\theta = 28.62°$$

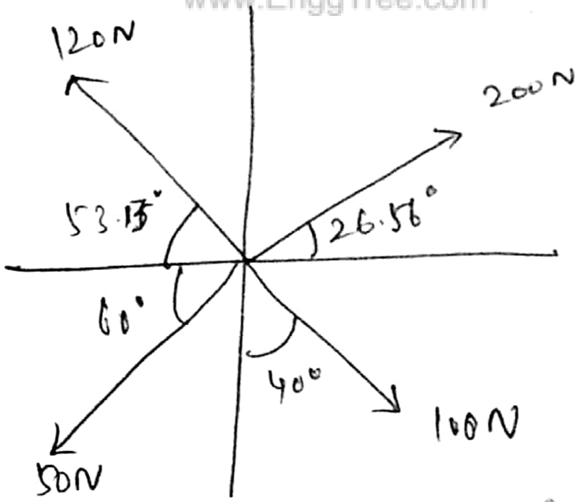
③ A system of four forces acting on a body is shown in fig. Determine its resultant & direction.



Sol:

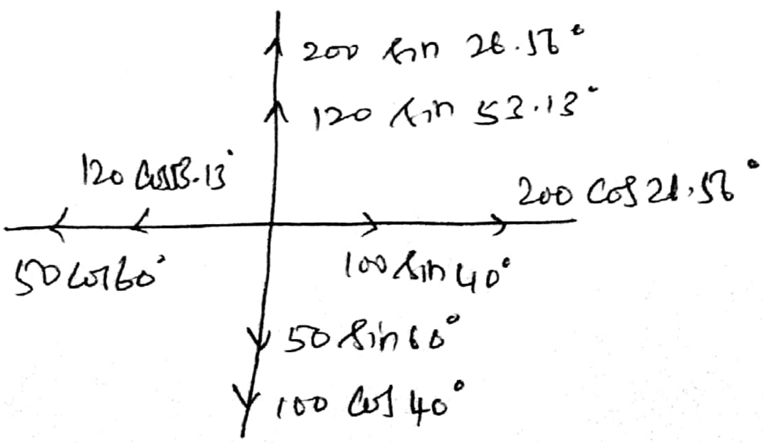
$\theta = \tan^{-1}(4/3)$

$\theta = 53.13°$



$\theta = \tan^{-1}(1/2)$

$\theta = 26.56'$



$$\Sigma F_x = 100 \sin 40^\circ + 200 \cos 26.56^\circ - 120 \cos 53.13^\circ - 50 \cos 60^\circ$$

$$= 146.17 \, N$$

$$\Sigma F_y = 200 \sin 26.56^\circ + 120 \sin 53.13^\circ - 50 \sin 60^\circ - 100 \cos 40^\circ$$

$$= 65.52 \, N$$

Resultant $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$$= \sqrt{(146.17)^2 + (65.52)^2}$$

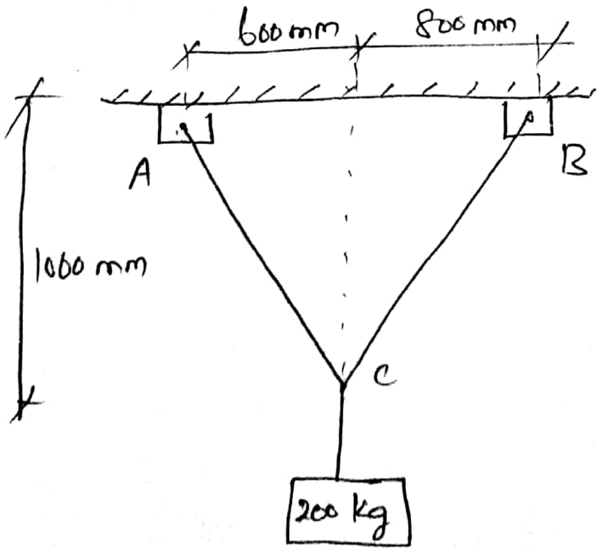
$$R = 160.18 \, N$$

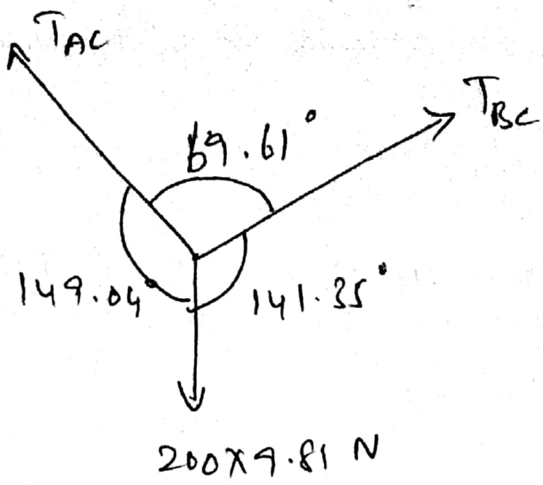
Resultant angle $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

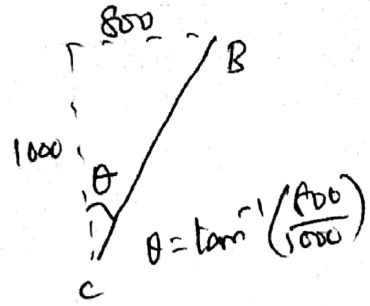
$$= \tan^{-1}\left(\frac{65.52}{146.17}\right)$$

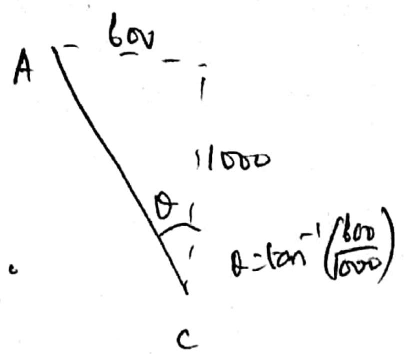
$$\theta = 24.14^\circ$$

(4) Find the tensions in AC and BC as shown in fig.









Apply Lami's theorem,

$$\frac{T_{AC}}{\sin 141.35^\circ} = \frac{T_{BC}}{\sin 149.04^\circ} = \frac{200 \times 9.81}{\sin 69.61^\circ}$$

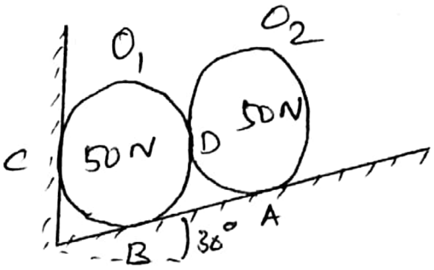
$$\Rightarrow \frac{T_{AC}}{\sin 141.35^\circ} = \frac{200 \times 9.81}{\sin 69.61^\circ}$$

$$T_{AC} = 12977 \text{ N}$$

$$\Rightarrow \frac{T_{BC}}{\sin 149.04^\circ} = \frac{200 \times 9.81}{\sin 69.61^\circ}$$

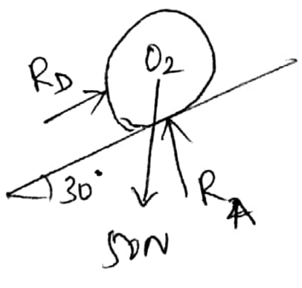
$$T_{BC} = 10675 \text{ N.}$$

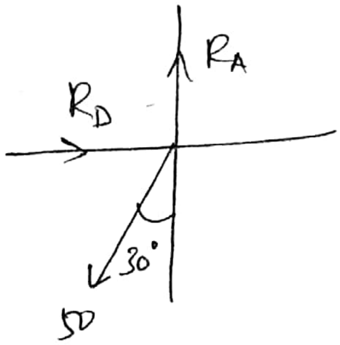
(5) Two identical rollers each of weight 50 N are supported by an inclined plane & vertical wall as shown in fig. Determine the reactions at the point of support A, B & C assuming all the surfaces are smooth. Also find the reaction force between the spheres



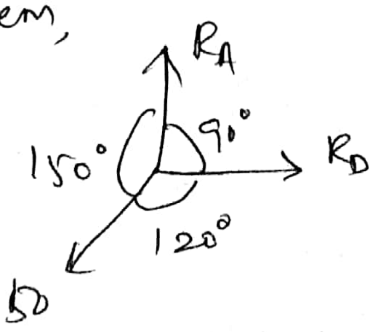
Sol:

At O_2,





By lami's theorem,



$$\frac{R_A}{\sin 120^\circ} = \frac{R_D}{\sin 150^\circ} = \frac{50}{\sin 90^\circ}$$

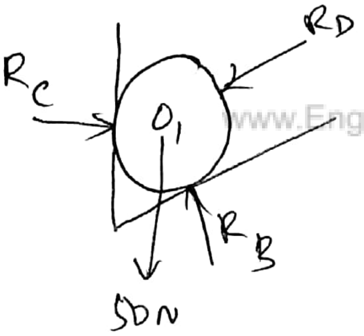
$$\frac{R_A}{\sin 120} = \frac{50}{\sin 90^\circ}$$

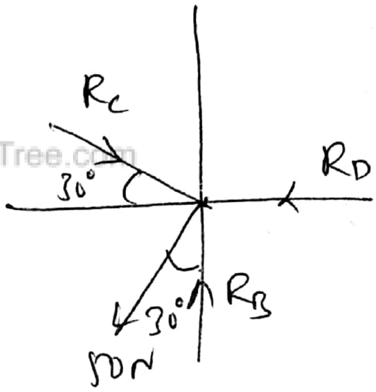
$$R_A = 43.3\,N$$

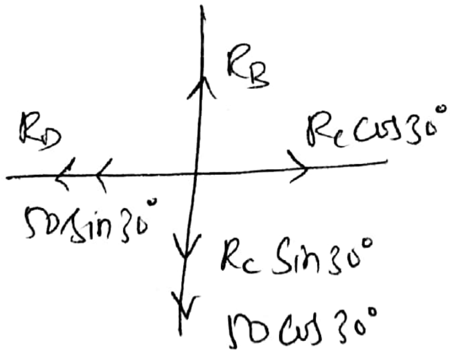
$$\frac{R_D}{\sin 150^\circ} = \frac{50}{\sin 90^\circ}$$

$$R_D = 25\,N.$$

At O_1,







$\Sigma F_x = 0$,

$$R_C \cos 30^\circ - 25 - 50 \sin 30^\circ = 0$$

$$R_C = 57.73\,N.$$

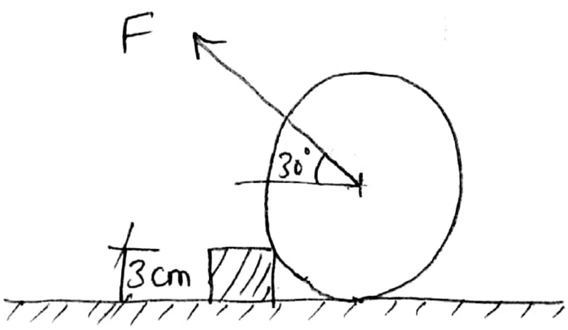
$\Sigma F_y = 0$

$$R_B - 50 \cos 30° - R_C \sin 30° = 0$$

$$R_B - 43.3 - (57.7) \sin 30° = 0$$

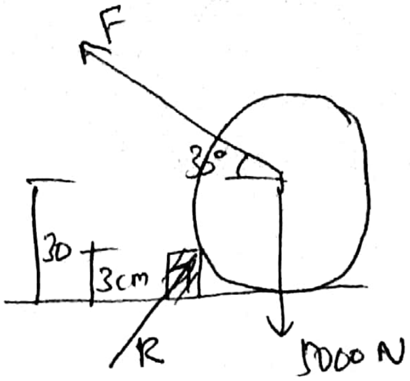
$$R_B = 72.15 \text{ N}.$$

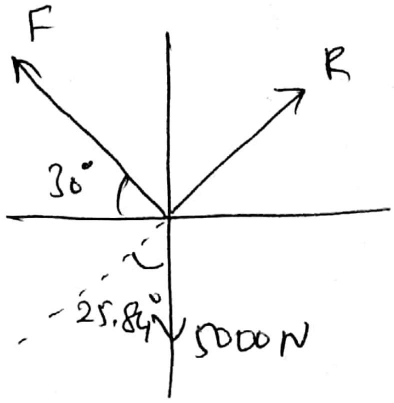
(b) A road roller of weight 5000 N which is of cylindrical shape is pulled by a force (F) acting at an angle of 30° with horizontal as shown in fig. It has to cross an obstacle of height 3cm. Calculate (F) to just cross the obstacle. The radius of the roller is 30cm.

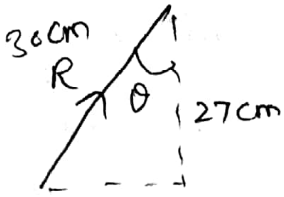


Sol:

For the given system, we are neglecting the cylindrical roller reaction with respect to horizontal plane.



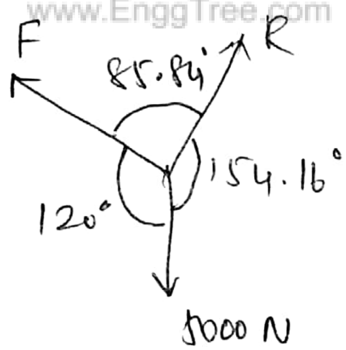




$$\cos\theta = \frac{27}{30}$$

$$\theta = 25.84^\circ$$

Apply Lami's,

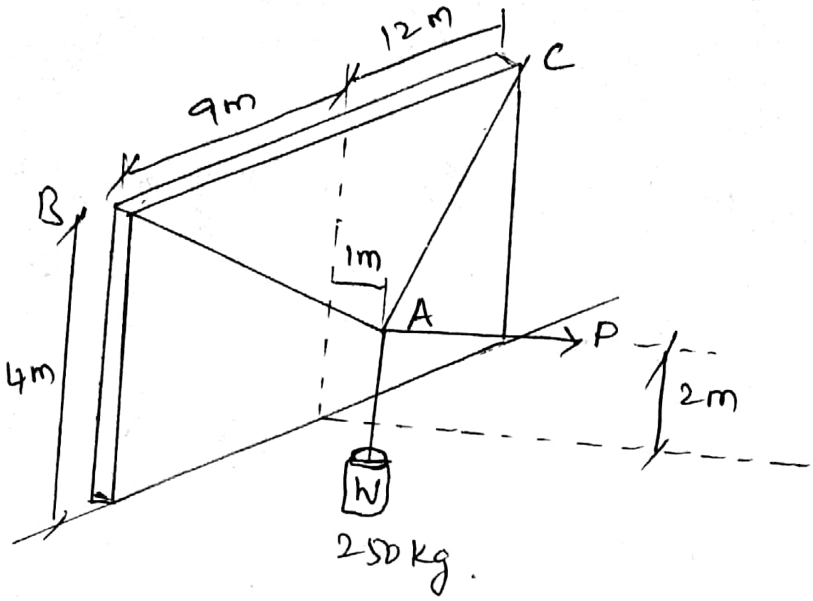


$$\frac{F}{\sin 154.16^\circ} = \frac{R}{\sin 120^\circ} = \frac{5000}{\sin 85.84^\circ}$$

$$\frac{F}{\sin 154.16^\circ} = \frac{5000}{\sin 85.84^\circ}$$

$$\boxed{F = 2185\,N}$$

⑦ A horizontal force P normal to the wall holds the cylinder in the position shown in fig below. Determine the magnitude of P and the tension in each cable.



Sol:

From the diagram, the co-ordinates are,

A (1, 2, 0)   B (0, 14, 9)   C (0, 14, −12)

Tension AB

$$\vec{T_{AB}} = T_{AB} \cdot \lambda_{AB}$$

$$= T_{AB} \left[\frac{-i + 12j + 9k}{\sqrt{(-1)^2 + (12)^2 + 9^2}} \right]$$

$$\vec{T_{AB}} = -0.0665\, T_{AB}\, i + 0.7984\, T_{AB}\, j + 0.598\, T_{AB}\, k \quad \text{—— (1)}$$

Tension in AC,

$$\vec{T_{AC}} = T_{AC} \cdot \lambda_{AC}$$

$$= T_{AC}\left[\frac{-i+12j-12k}{\sqrt{(-1)^2+(12)^2+(-12)^2}}\right]$$

$$\vec{T_{AC}} = -0.058 \cdot T_{AC}\, i + 0.7058\, T_{AC}\, j - 0.7058\, T_{AC}\, k. \quad \text{--- (2)}$$

$$\vec{W} = 250 \times 9.81\,(-j)$$

$$= -2452.5\, j \quad \text{--- (3)}$$

$$\vec{P} = P \cdot i \quad \text{--- (4)}$$

Using equilibrium Conditions,

$\Sigma F_x = 0$,

$$-0.665\, T_{AB} - 0.0588\, T_{AC} + P = 0 \quad \text{--- (5)}$$

$\Sigma F_y = 0$

$$0.7984\, T_{AB} + 0.7058\, T_{AC} - 2452.5 = 0 \quad \text{--- (6)}$$

$\Sigma F_z = 0$

$$0.5988\, T_{AB} - 0.7058\, T_{AC} = 0 \quad \text{--- (7)}.$$

By solving these (5), (6) & (7) eqns,

we can get,

$T_{AB} = 1755.8\, N$ $\quad$ $P = 204.3 N.$

$T_{AC} = 1489.1 N$

Mohan S R

TWO MARKS

① Define Coplaner forces.

If the line of action of all forces lie on the same line, then the forces are said to be Coplaner forces.

② Define Concurrent forces.

If the line of action of all forces meet at common point, then the forces are said to be Concurrent forces.

③ What is resultant force?

If a number of forces acting simultaneously on a particle, then these forces can be replaced by a single force which would produce the same effect as produced by all forces. This single force is called resultant force.

④ State the necessary Conditions for Static equillibrium of a particle in two dimensions.

$\Sigma F_x = 0$

$\Sigma F_y = 0.$

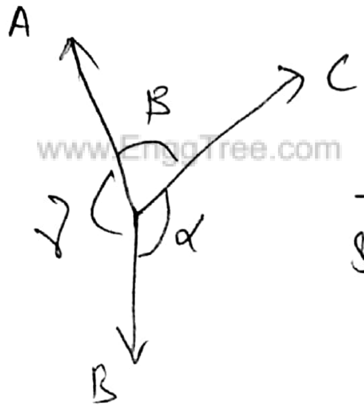
5. What is unit Vector?

A Vector, whose magnitude is unity is called as Unit Vector.

$$\text{Unit Vector, } n = \frac{\vec{AB}}{|AB|}$$

6. State Lami's theorem.

If three Coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of angle between the other two.



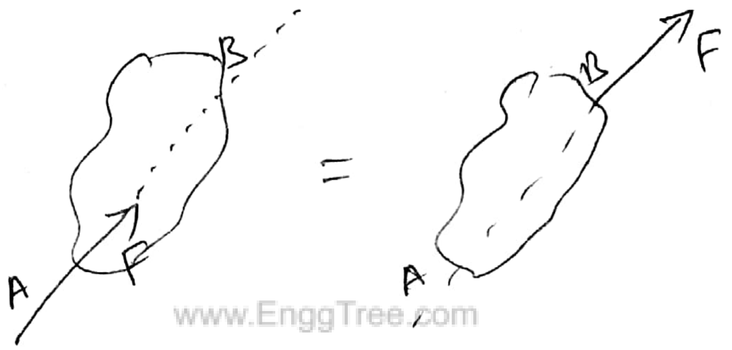
$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}.$$

7. State Varignon's theorem.

If a number of Coplanar forces are acting Simultaneously on a body, the algebraic sum of the moments of all the forces about any point is equal to the moment of resultant force about the same point.

(8) State the principle of transmissibility of forces.

The Conditions of equillibrium of motion of a rigid body remains unchanged, if a force acting at a given point of the rigid body is replaced by a force of same magnitude & direction, but acting at a different point provided that the two forces have the same line of action.



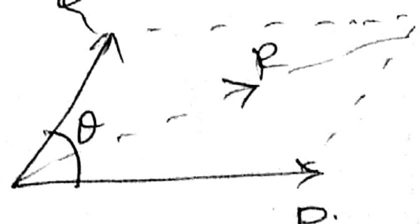
(9) Find the magnitude of the resultant of the two Concurrent forces of magnitude 60 kN & 40 kN with an inclined / included angle of 70° between them.

$P = 60\ kN, \quad Q = 40\ kN, \quad \theta = 70°.$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

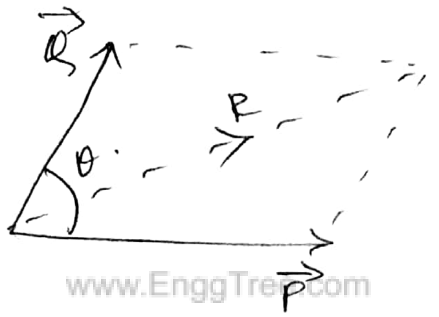
$= \sqrt{60^2 + 40^2 + 2(60)(40)\cos 70°}$

$R = 82.7\ kN$



(10) State the parallelogram law of forces.

It states that "If two forces, acting simultaneously on a particle be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."



(11) Define Scalar Quantities & Vector Quantities.

Scalar quantities are those which are completely defined by their magnitude only.

e.g. 2 kg of mass, 50°C temp.

Vector quantities are those which are defined by their magnitude & direction.

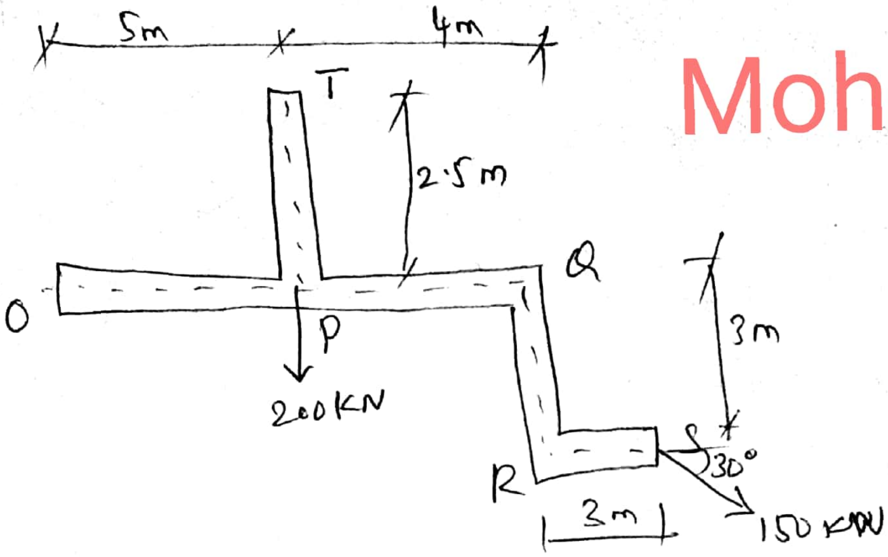
e.g. 20 N force acting vertically downward.

MOHAN S R
MECH/AP

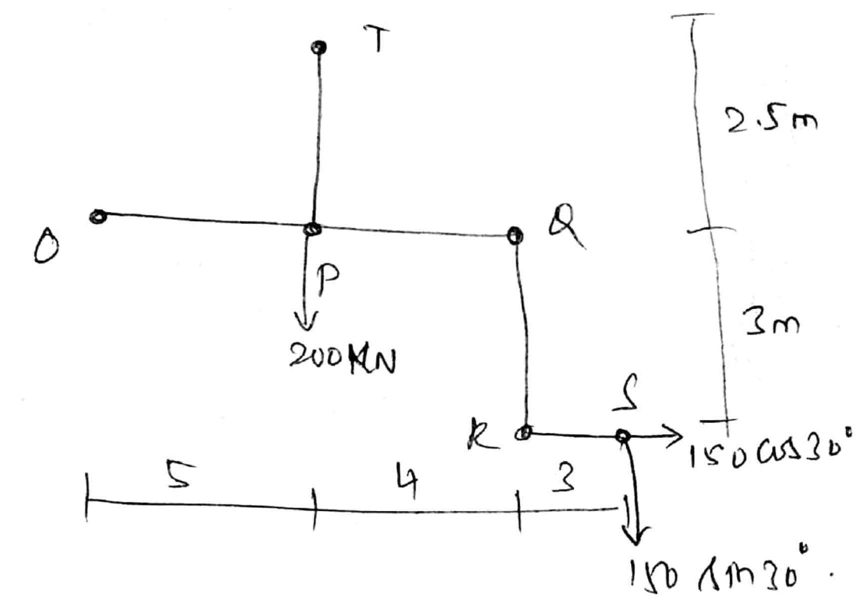
UNIT – II

EQUILIBRIUM OF RIGID BODIES

① Find the moments about the Z axis at point O & T due to the forces as shown in fig.



Sol:



Moment about O,

$$M_0 = (200\times5) + (150\sin30^\circ \times 12) - (150\cos30^\circ \times 3)$$

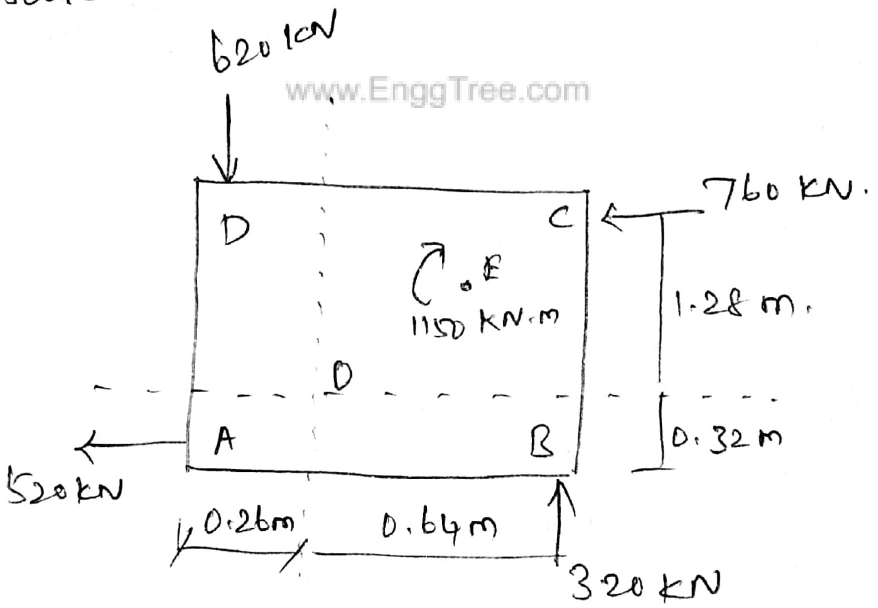
$$M_0 = 1510.2 \text{ kN.m}$$

$$M_T = (150 \sin 30^\circ \times 7) - (150 \cos 30^\circ \times 5.5)$$

$$M_T = -189.47 \text{ kN.m.}$$

$$M_T = 189.47 \text{ kN.m} \circlearrowright$$

(2) Four forces and a Couple are applied to a rectangular plate as shown in fig. Determine the magnitude and direction of the resultant force Couple system also determine the distance x from O along the x axis, where the resultant intersects.



Sol:

$$\Sigma F_x = -520 - 760 = -1280 \text{kN}$$

$$\Sigma F_y = 320 - 620 = -300 \text{kN}.$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$= \sqrt{(-1280)^2 + (-300)^2}$$

$$R = 1314.68 \text{ KN.}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{300}{1280}\right)$$

$$\theta = 13.19^\circ$$

$$\Sigma M_o = \Sigma F_y \cdot x \,.$$

location of resultant.

$$\Sigma M_o = (520 \times 0.32) - (320 \times 0.64) - (760 \times 1.28)$$

$$- (620 \times 0.26) + 1150$$

$$= -22.4 \text{ kN.m}$$

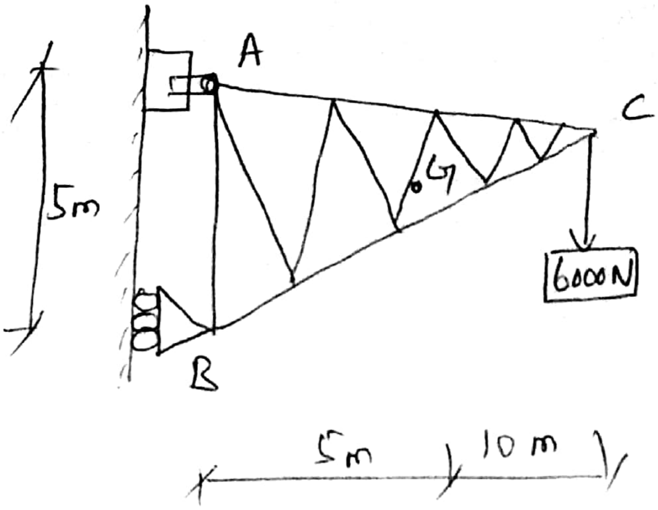
$$\Sigma M_o = 22.4 \text{ kN.m} \circlearrowleft$$

$$\therefore -22.4 = -300 \times x$$

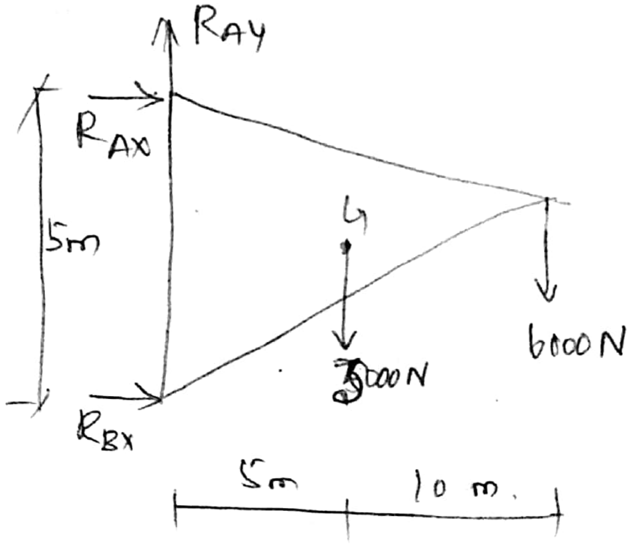
$$x = \frac{22.4}{300}$$

$$x = 0.07 \text{m.}$$

(3) A crane has a weight of 3000 N supports a 6000 N force as shown in fig. Determine the supporting reactions at A (pinned joint) and B (roller support).



Sol:



Apply equillibrium Conditions,

$$\Sigma F_x = 0, \quad \Sigma F_y = 0 \quad \Sigma M_A = 0.$$

$\Sigma F_x = 0$

$$R_{Ax} + R_{Bx} = 0$$

$$R_{Ax} = -R_{Bx} \quad \text{--- (1)}$$

$\Sigma F_y = 0$

$$R_{Ay} - 3000 - 6000 = 0$$

$$R_{Ay} = 9000 \text{ N}$$

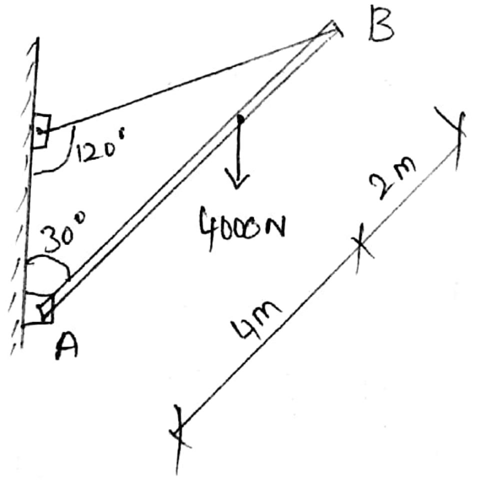
$\Sigma M_A = 0$

$$-(R_{Bx} \times 5) + (3000 \times 5) + (6000 \times 15) = 0$$

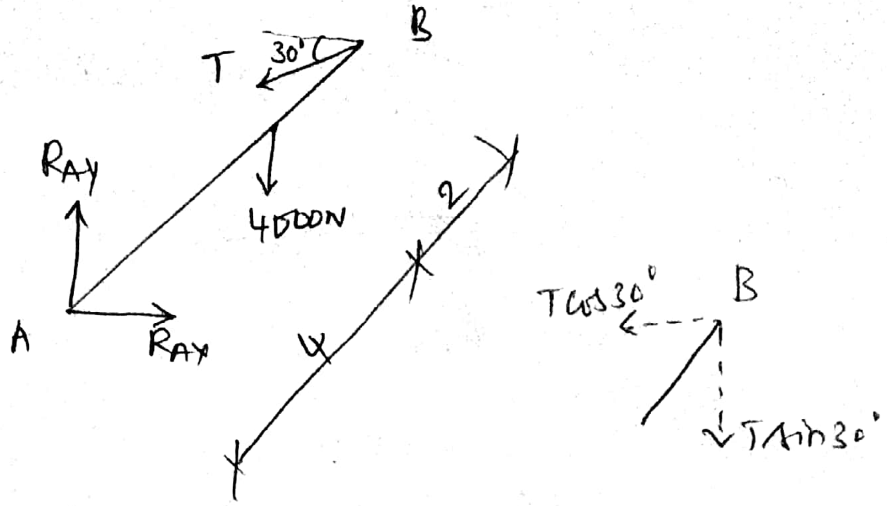
$$R_{Bx} = 21000 \text{ N}$$

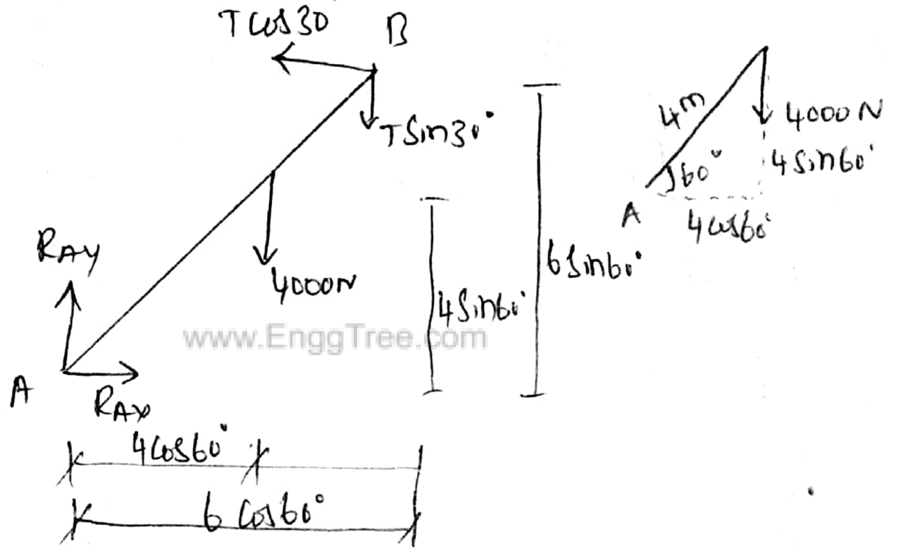
$$\therefore R_{Ax} = -R_{Bx} = -21000 \text{ N.}$$

(4) For the given diagram, determine the tension in Cable BC, neglect the weight of the rod AB.



Sol:





Apply equilibrium Conditions,

$\Sigma F_x = 0$

$$R_{Ax} - T\cos 30° = 0$$

$$R_{Ax} = 0.867 \quad \text{——} \; (1)$$

$\Sigma F_y = 0$

$$R_{Ay} - T\sin 30° - 4000 = 0.$$

$$R_{Ay} - 0.5T = 4000 \quad \text{——} \; (2)$$

$\Sigma M_A = 0$

$$(4000 \times 4\cos 60^\circ) + (T\sin 30^\circ \times 6\cos 60^\circ) - (T\cos 30^\circ \times 6\sin 60^\circ) = 0$$

$$8000 + 1.5T - 4.5T = 0$$

$$8000 - 3T = 0$$

$$\boxed{T = 2666\ N}$$

from eqn ①

$$R_{Ax} = 0.86 \times 2666$$

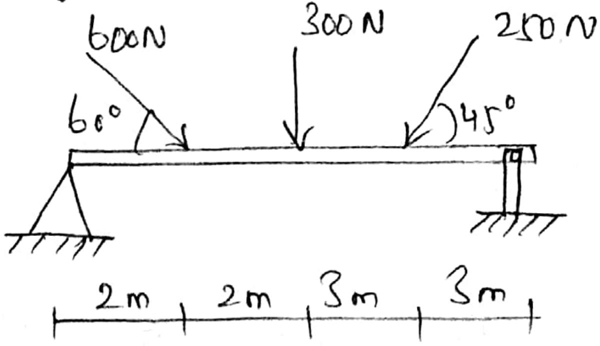
$$R_{Ax} = 2293\ N$$

from eqn ②,

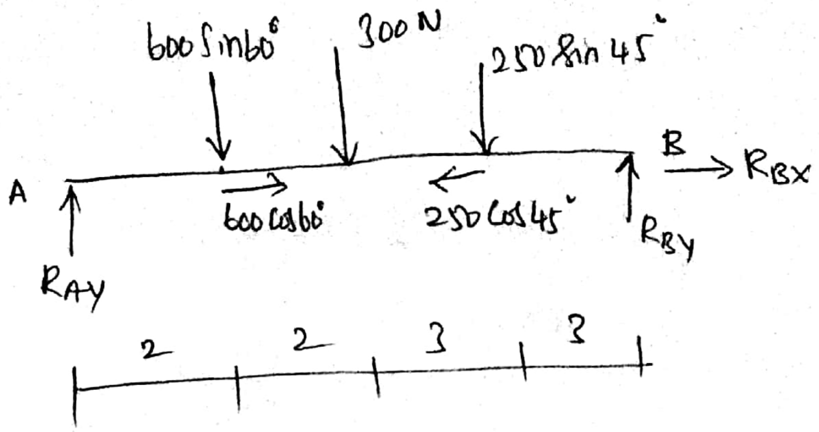
$$R_{Ay} - 0.5(2666) = 4000$$

$$R_{Ay} = 5333\ N.$$

⑤ Find the horizontal and vertical reactions of the support for the beam subjected to loading as shown in fig.



Sol:



Apply equilibrium Conditions,

$\Sigma F_x = 0,$

$R_{Bx} + 600 \cos 60^\circ - 250 \cos 45^\circ = 0$

$R_{Bx} + 123.22 = 0$

$R_{Bx} = -123.22 \text{ N}$

$\Sigma F_y = 0,$

$R_{Ay} + R_{By} - 600 \sin 60^\circ - 300 - 250 \sin 45^\circ = 0$

$R_{Ay} + R_{By} - 996.39 = 0$

$R_{Ay} + R_{By} = 996.39 \text{ N}$ —— (1)

$\Sigma M_B = 0$

$-(250 \sin 45^\circ \times 3) - (300 \times 6) - (600 \sin 60^\circ \times 8)$
$+ (R_{Ay} \times 10) = 0$

$R_{Ay} = 648.32 \text{ N}$

From eqn (1)

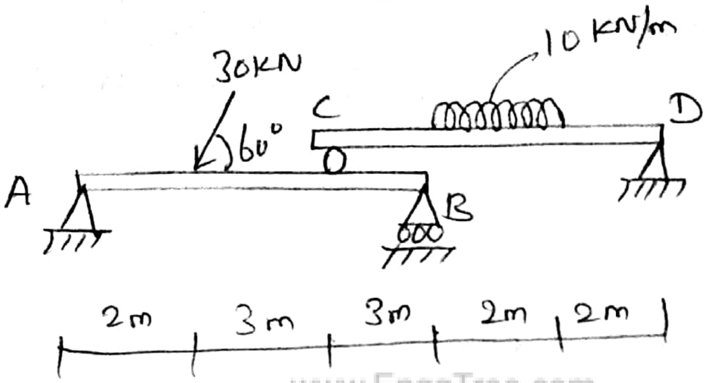
$648.32 + R_{By} = 996.39$

$R_{By} = 348.07 \text{ N}.$

(6) Two beams AB & CD are shown in fig. A & D are hinged supports. B & C are roller supports.

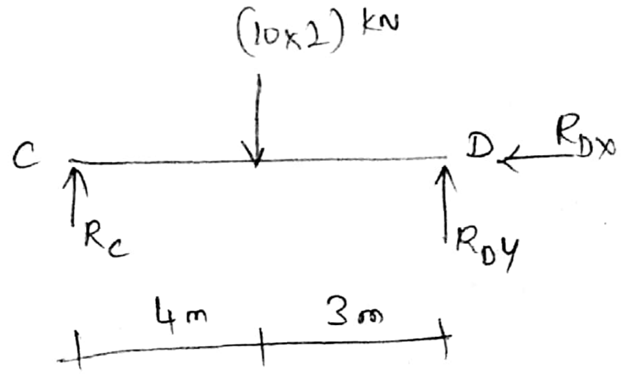
i) Sketch the FBD of the beam AB & determine reactions at the supports A & B.

ii) Sketch the free body diagram of beam CD and determine the reactions at the supports C & D.



Sol:

Beam CD



Apply equilibrium conditions,

$\Sigma F_x = 0$

$R_{Dx} = 0.$

$\Sigma F_y = 0$

$$R_C + R_{DY} - 20 = 0$$

$$R_C + R_{DY} = 20 \quad \text{—— (1)}$$

$\Sigma M_C = 0,$

$$-(20 \times 4) + (R_{DY} \times 7) = 0$$

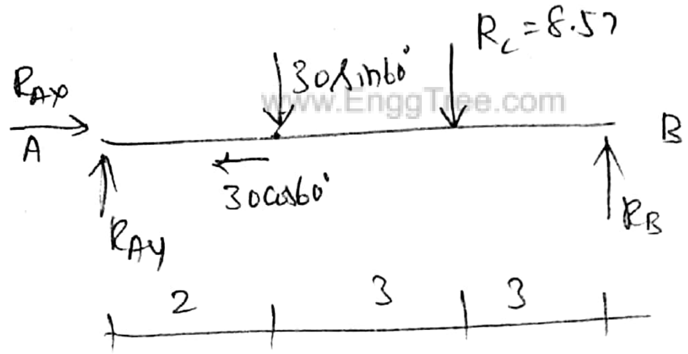
$$R_{DY} = 11.43\ kN$$

Sub in eqn (1)

$$R_C + 11.43 = 20$$

$$R_C = \cancel{31.43\ kN}\ 8.57\ kN.$$

Beam AB



Using equilibrium conditions,

$\Sigma F_x = 0$

$$R_{Ax} - 30\cos 60° = 0$$

$$R_{Ax} = 15\ kN.$$

$\Sigma F_y = 0$

$$R_{Ay} - 30\sin 60° - 8.57 + R_B = 0.$$

$$R_{Ay} + R_B = 34.55 \quad \text{—— (2)}$$

$\Sigma M_A = 0.$

$$-(30\sin 60^\circ \times 2) - (8.57 \times 5) + (R_B \times 8) = 0$$

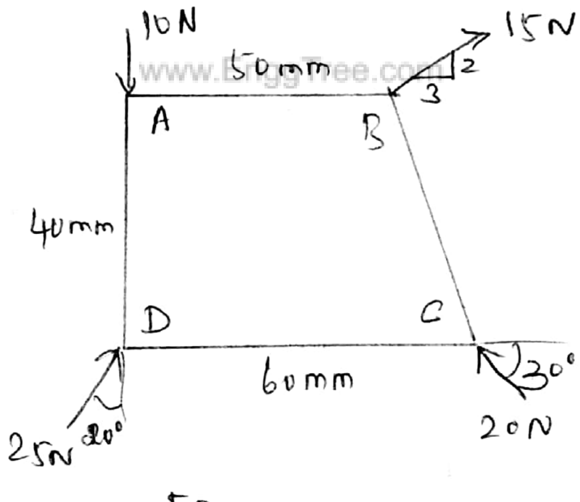
$$R_B = 11.85\ kN.$$

Sub in eqn ②.

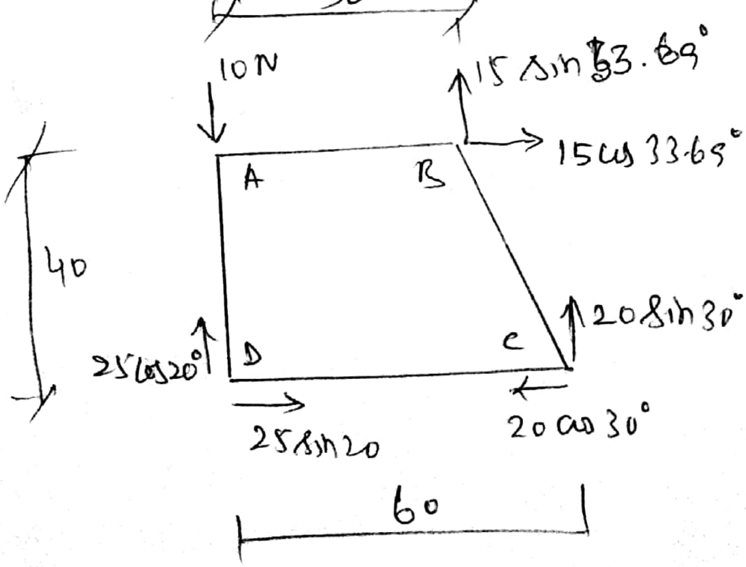
$$R_{Ay} + 11.85 = 34.55$$

$$R_{Ay} = 22.7\ KN.$$

⑦ Replace the given system of forces acting on a plane ABCD shown in fig by a Force - Couple system acting at the point A.



Sol:



$\theta = \tan^{-1}(2/3)$

$\theta = 33.69.$

$\Sigma F_x = 25 \sin 20^\circ - 20 \cos 30^\circ + 15 \cos 33.69$

$= 3.71 N.$

$\Sigma F_y = -10 + 25 \cos 20^\circ + 20 \sin 30^\circ + 15 \sin 33.69^\circ$

$= 31.81 N.$

Resultant, $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

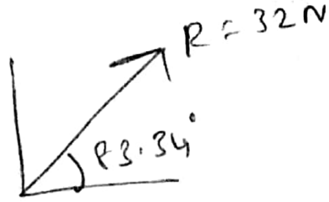
$R = \sqrt{3.71^2 + 31.81^2}$

$R = 32 N$

Resultant angle, $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_y}\right)$

$\theta = \tan^{-1}\left(\frac{31.81}{3.71}\right)$

$\theta = 83.34^\circ$



To find the location of resultant,

$\Sigma M = R \cdot x.$ ↳ location

$\Sigma M_A = (15 \sin 33.69^\circ \times 50) + (25 \sin 20^\circ \times 40) + (20 \sin 30^\circ \times 60) - (20 \cos 30^\circ \times 40)$

$\Sigma M_A = 665.18$ N.mm ↺

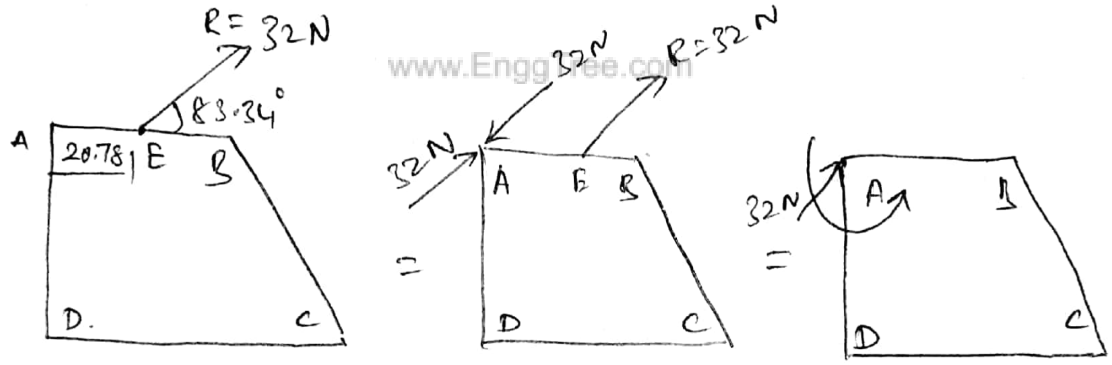
$$665.18 = 32 \cdot x$$

$$x = 20.78 \text{ mm.}$$

To find Force Couple System @ A.

The resultant force acting at point E is shown in fig..

To reduce the resultant force into force-couple system at A, apply two equal & opposite collinear forces at A, parallel to resultant and of same magnitude (32 N) as shown in fig.



Force Couple System.

Mohan S R

TWO MARKS

(1) Define Free Body Diagram.

Free body diagram is a line diagram, representation of force magnitudes & directions.

(2) State the necessary conditions for equilibrium of rigid bodies in two dimensions.

$$\Sigma F_x = 0 \quad , \quad \Sigma F_y = 0 \quad , \quad \Sigma M = 0$$

(3) Distinguish between a Couple and a Moment.

Couple:

* Two equal & parallel forces are acting in opposite direction constitute a couple.
* It doesnot depend on any point or axis.

Moment:

* Moment is the turning effect produced by a force on the body on which it acts.
* It depends on point or axis about which moment is taken.

(4) Why the Couple moment is said to be free vector?

The couple is a pure turning effect which may be moved anywhere in its own plane without change its effect on the body. Hence, couple moment is free vector.

(5) For what Condition Moment of force will be zero?

A force produces zero moment about an axis or reference point which intersects the line of action of the force.

(6) Define Moment of a force.

The turning effect of the force about the point is called Moment of a force (M)

M = Force × perpendical distance.

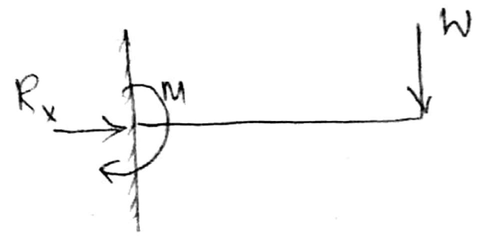
(7) Define equilibriant.

The force which brings the system of forces into equilibrium is called equilibrant.

(8) When is moment of force zero about a line?

* Force is parallel to that line
* Line of action of force intersects that line.

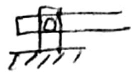
(9) Sketch idealized, graphical and reaction of a Cantilever support at a point.



(10) List the type of loads.

1. point load
2. Uniformely Distributed Load (UDL)
3. Uniformely Varying Load (UVL)

(11) List the types of Support & Sketch.

1. Fixed Support 
2. Roller Support 
3. Hinged Support 

(12) List the types of beams.

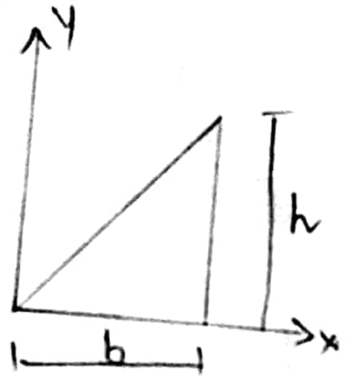
1. Simply Supported beam 
2. Cantilever beam 
3. Fixed beam 

MOHAN S.R
MECH/AP

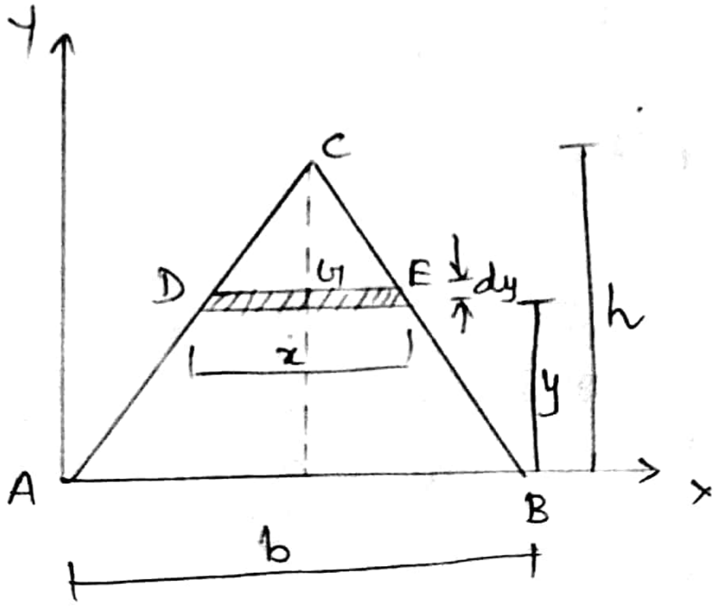
UNIT – III

PROPERTIES OF SURFACES

(1) Derive the expression for the location of the centroid of a triangular are shown in fig by direct integration.



Sol:



Consider a triangle ABC of base width 'b' and height 'h' as shown in fig. Let $\bar{y}$ be the distance of centre of gravity of triangle from its base.

Consider a elemental strip DE, of the triangle of width 'x' and thickness 'dy' at a distance 'y' from AB.

Consider the similar triangle ABC & DCE.

$$\frac{DE}{AB} = \frac{CG}{CF}.$$

$$\frac{x}{b} = \frac{h-y}{h}.$$

$$x = \frac{b(h-y)}{h}.$$

$$x = b\left(1 - y/h\right)$$

Area of elemental strip $dA = x.dy.$

$$dA . b\left(1 - y/h\right).dy.$$

Area of triangle $A = \frac{1}{2} bh.$

We know,

$$\text{Centroid } \bar{y} = \frac{\text{Moment of area}}{\text{Total area}}$$

$$= \frac{\int y.dA}{A}.$$

$$= \frac{\int_0^h y.b\left(1 - y/h\right).dy.}{\frac{bh}{2}}$$

$$= \frac{b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h}{\frac{bh}{2}}$$

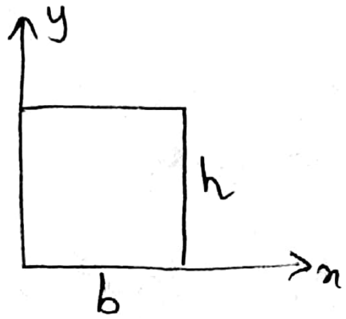
$$= \frac{2\left[\frac{h^2}{2} - \frac{h^3}{3h}\right]}{h}$$

$$= 2\left[\frac{h}{2} - \frac{h^2}{3h}\right]$$

$$= h - \frac{2h}{3}$$

$$\bar{y} = h/3$$

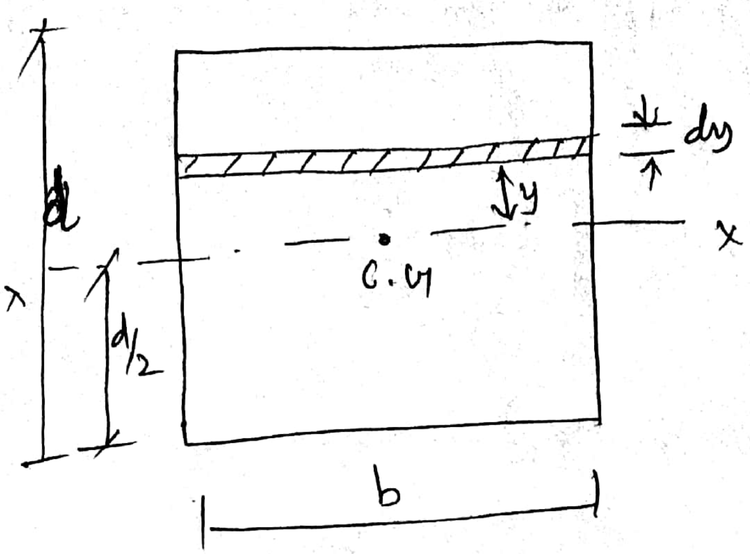
(2) Derive from first principle, the Second Moment of area I_{xx} & I_{yy} for an rectangular area when the axes are as shown below.



Sol:

Consider a rectangle of width 'b' & depth 'd'

Let x-x be the horizontal axis passing through the centroid of the rectangular section.



Consider an elemental strip of thickness 'dy' parallel to x-x axis. and at a distance of 'y' from it.

Area of elemental strip $dA = b.dy$.

Moment of inertia of the elemental strip about the centroidal axis x-x is,

$$dI_{xx} = \text{area} \times y^2$$

$$dI_{xx} = b.dy.y^2.$$

Moment of Inertia of whole area,

$$I_{xx} = \int_{-d/2}^{d/2} dI_{xx}$$

$$= 2\int_{0}^{d/2} by^2.dy.$$

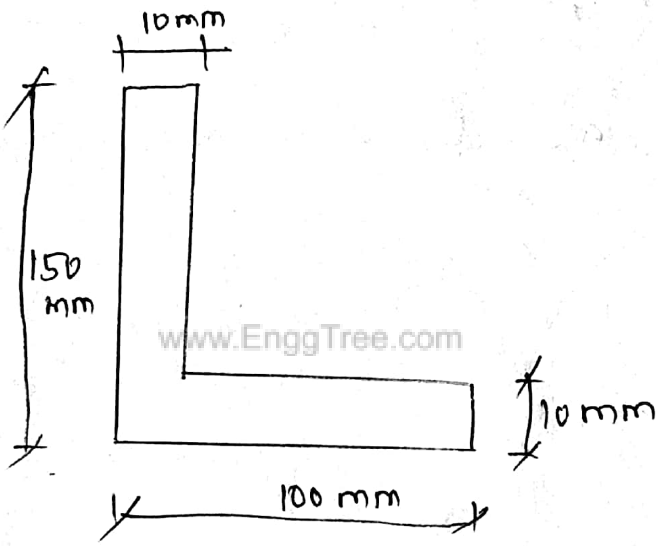
$$= 2b\left[\frac{y^3}{3}\right]_0^{d/2}$$

$$= 2b\left[\frac{d^3}{24}\right]$$

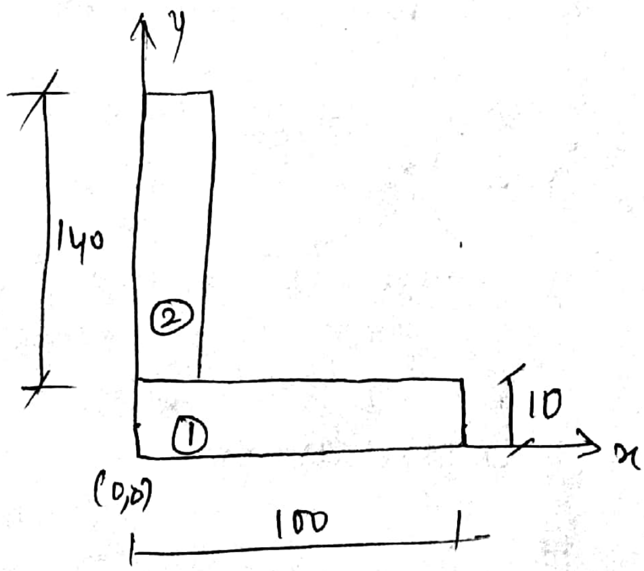
$$I_{xx} = \frac{bd^3}{12}$$

Similarly, we can get $I_{yy} = \frac{db^3}{12}$.

③ Locate the centroid for the following fig.



Sol:



Part ①

$$x_1 = \frac{b}{2} = \frac{100}{2} = 50\text{ mm}$$

$$y_1 = h/2 = 10/2 = 5\text{ mm}$$

$$a_1 = bh = 100 \times 10 = 1000\text{ mm}^2$$

Part ②

$$x_2 = 10/2 = 5\text{ mm}$$

$$y_2 = h/2 + 10 = 140/2 + 10 = 80\text{ mm.}$$

$$a_2 = bh = 10 \times 140 = 1400\text{ mm}^2.$$

Centroid

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{1000(50) + 1400(5)}{1000 + 1400}$$

$$\bar{x} = 23.75\text{ mm}$$

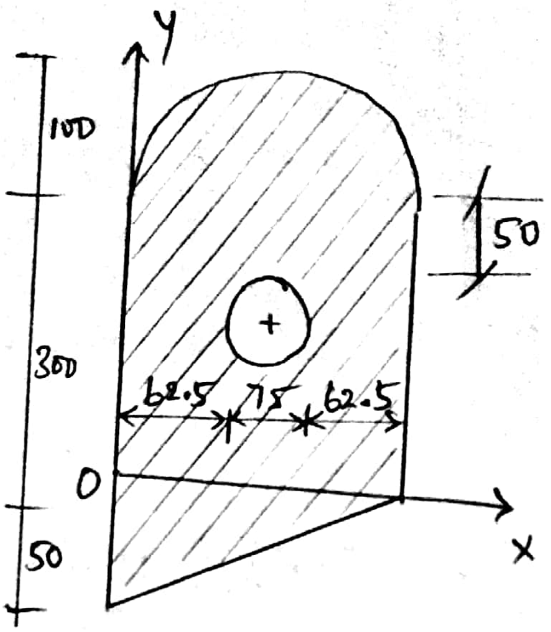
$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{1000(5) + 1400(80)}{1000 + 1400}$$

$$\bar{y} = 48.75\text{ mm.}$$

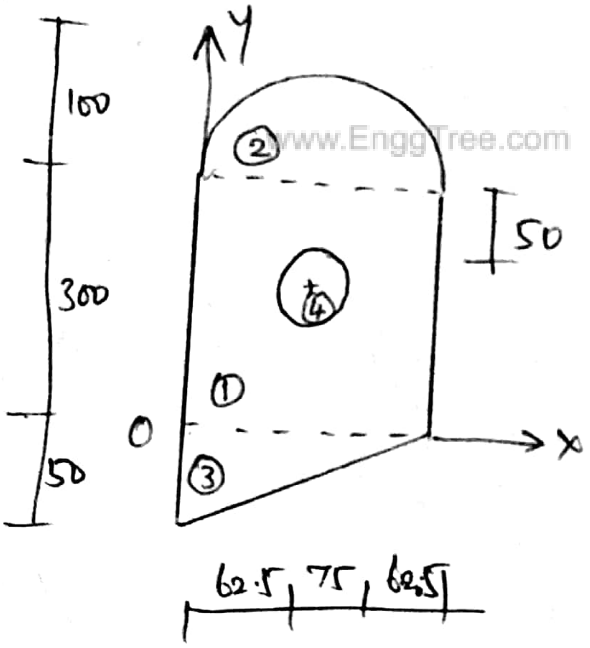
$$\text{Centroid } (\bar{x}, \bar{y}) = (23.75, 48.75)$$

(4) Find the centroid for the following fig.



All dimensions are in mm.

Sol:



part (1)

$x_1 = b/2 = \frac{200}{2} = 100\text{ mm}$

$y_1 = \frac{h}{2} = 300/2 = 150\text{ mm}$

$a_1 = bh = 200 \times 300 = 60000\text{ mm}^2$.

Part ②

$$x_2 = R = 100 \text{ mm}$$

$$y_2 = \frac{4R}{3\pi} + 300 = \frac{4\times100}{3\pi} + 300 = 342.44 \text{mm}$$

$$a_2 = \frac{\pi r^2}{2} = \frac{\pi \times 100^2}{2} = 15700 \text{ mm}^2.$$

Part ③

$$x_3 = \frac{b}{3} = \frac{200}{3} = 66.67 \text{mm}$$

$$y_3 = -\frac{h}{3} = -\frac{50}{3} = -16.67 \text{ mm}$$

$$a_3 = \frac{1}{2} bh = \frac{1}{2} \times 200 \times 50 = 5000 \text{ mm}^2$$

Part ④

$$y_4 = 300 - \left(50 + \frac{75}{2}\right) = 212.5 \text{ mm}$$

$$x_4 = \frac{200}{2} = 100 \text{ mm}.$$

$$a_4 = \pi R^2 = \pi (100)^2 = 31.41 \times 10^3 \text{ mm}^2.$$

Centroid $\bar{x} = \frac{\Sigma ax}{\Sigma a}$.

$$= \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 - a_4 x_4}{a_1 + a_2 + a_3 - a_4}.$$

$$\bar{x} = \frac{6000(100) + 15700(100) + 5000(16.67) - 31410(100)}{6000 + 15700 + 5000 - 31410}$$

$$\bar{x} = 96.57\,mm$$

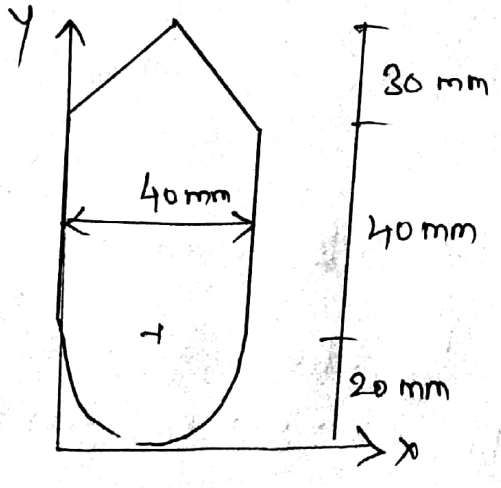
$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{a_1y_1 + a_2y_2 + a_3y_3 - a_4y_4}{a_1 + a_2 + a_3 - a_4}$$

$$= \frac{6000(150) + 15700(342.44) + 5000(-16.67) - 31410(212.5)}{6000 + 15700 + 5000 - 31410.}$$

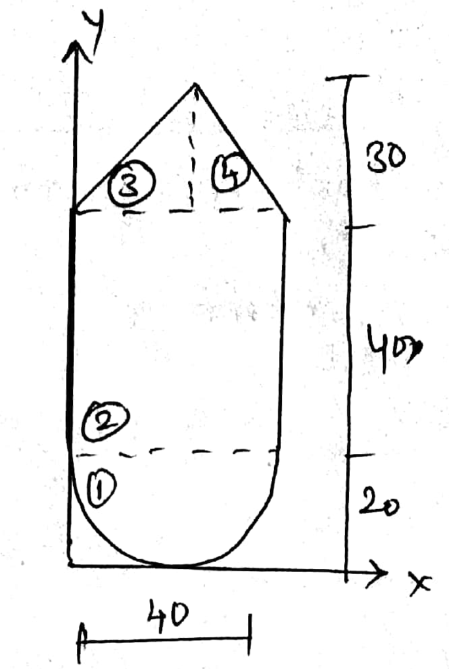
$$\bar{y} = 154.39\ mm,$$

$\therefore$ Centroid $(\bar{x}, \bar{y}) = (96.57, 154.39)$.

(5) Find the Moment of Inertia for the following fig about the centroidal axes.



Sol:



Part ①

$$x_1 = R = 20 \text{ mm}$$

$$y_1 = R - \frac{4R}{3\pi} = 20 - \frac{4(20)}{3\pi} = 11.51 \text{ mm}$$

$$a_1 = \frac{\pi r^2}{2} = \frac{\pi (20)^2}{2} = 628.32 \text{ mm}^2$$

Part ②

$$x_2 = \frac{b}{2} = \frac{40}{2} = 20 \text{ mm}$$

$$y_2 = 20 + h/2 = 20 + 40/2 = 40 \text{ mm}$$

$$a_2 = bh = 40 \times 40 = 1600 \text{ mm}^2$$

Part ③

$$x_3 = \frac{2b}{3} = \frac{2 \times 20}{3} = 13.33 \text{ mm}$$

$$y_3 = h/3 + 60 = \frac{30}{3} + 60 = 70 \text{ mm}$$

$$a_3 = \frac{1}{2} bh = \frac{1}{2} \times 20 \times 30 = 300 \text{ mm}^2$$

Part ④

$$x_4 = 20 + \frac{b}{3} = 26.66 \text{ mm}$$

$$y_4 = h/3 + 60 = 70 \text{ mm}$$

$$a_4 = \frac{1}{2} bh = \frac{1}{2} \times 20 \times 30$$

$$a_4 = 300 \text{ mm}^2$$

Centroid, $\bar{x} = \dfrac{\Sigma a x}{\Sigma a}$.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4}{a_1 + a_2 + a_3 + a_4}$$

$$= \frac{628.32(20) + 1600(20) + 300(13.33) + 300(26.66)}{628.32 + 1600 + 300 + 300}$$

$\bar{x} = 20$ mm.

$$\bar{y} = \frac{\Sigma a y}{\Sigma a}.$$

$$\bar{y} = \frac{77.23 \times 10^3}{2828.32} = 27.3 \text{ mm}.$$

Moment of Inertia:

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3} + I_{xx_4}$$

$$\left[\text{Formula, } I_{xx} = I_{xx \text{ of part}} + a(y - \bar{y})^2\right]$$

$$I_{xx_1} = I_{xx \text{ of part } 1} + a_1(y_1 - \bar{y})^2$$

$$= 0.11 r^4 + 628.32(11.51 - 27.3)^2$$

$$= 0.11(20)^4 + 628.32(11.51 - 27.3)^2$$

$$I_{xx_1} = 174.2 \times 10^3 \text{ mm}^4$$

Similarly,

$I_{xx2} = 471.66 \times 10^3 \ mm^4$

$I_{xx3} = \frac{bh^3}{36} + 300(70 - 27.3)^2$

$= 561.98 \times 10^3$

$I_{xx4} = \frac{bh^3}{36} + 300(70 - 27.3)^2$

$= 561.98 \times 10^3$

$\therefore I_{xx} = 174.2 \times 10^3 + 471.66 \times 10^3 + 561.98 \times 10^3 + 561.98 \times 10^3$

$I_{xx} = 1.76 \times 10^6 \ mm^4.$

Similarly for y-axis,

$I_{yy} = I_{yy1} + I_{yy2} + I_{yy3} + I_{yy4}.$

[Formula $I_{yy} = I_{yy \text{ of part}} + a(x - \bar{x})^2$]

$I_{yy1} = I_{yy \text{ of part (1)}} + a_1(x_1 - \bar{x})^2$

$= \frac{\pi R^4}{8} + a_1(x_1 - \bar{x})^2$

$= \frac{\pi (20)^4}{8} + 628.32(20 - 20)$

$I_{yy1} = 62.83 \times 10^3 \ mm^4.$

$I_{yy2} = 21.3 \times 10^4 \ mm^4.$

$$I_{yy_3} = \frac{hb^3}{36} + a_3(x_3 - \bar{x})^2$$

$$= \frac{30(20)^3}{36} + 300(12.33 - 20)^2$$

$$= 20.01 \times 10^3 \text{ mm}^4.$$

$$I_{yy_4} = \frac{hb^3}{36} + a_4(x_4 - \bar{x})^2$$

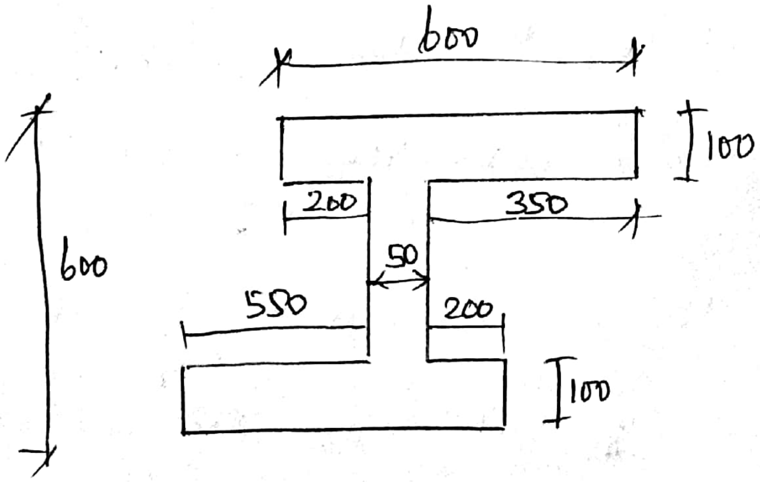
$$= \frac{30(20)^3}{36} + 300(26.66 - 20)^2$$

$$= 19.97 \times 10^3 \text{ mm}^4.$$

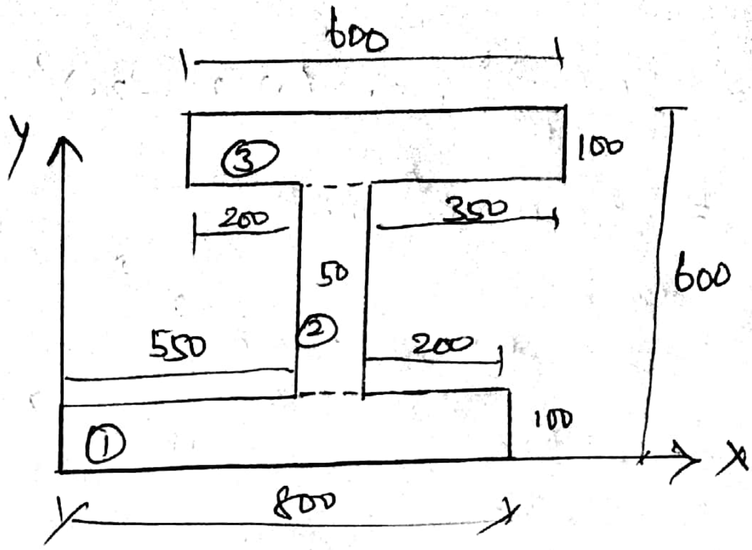
$$I_{yy} = 62.83 \times 10^3 + 21.3 \times 10^4 + 20.01 \times 10^3 + 19.97 \times 10^3$$

$$I_{yy} = 315.83 \times 10^3 \text{ mm}^4.$$

(b) For the Section below find the Moment of Inertia about XX-axis.



Sol:



part ①

$$x_1 = b/2 = 800/2 = 400\text{ mm}$$

$$y_1 = h/2 = 100/2 = 50\text{ mm}$$

$$a_1 = bh = 800 \times 100 = 8\times10^4\text{ mm}^2$$

part ②

$$x_2 = 550 + b/2 = 550 + 50/2 = 575\text{ mm}$$

$$y_2 = 100 + h/2 = 100 + \frac{400}{2} = 300\text{ mm}$$

$$a_2 = bh = 50\times400 = 2\times10^4\text{ mm}^2$$

part ③

$$x_3 = 350 + b/2 = 350 + \frac{600}{2} = 650\text{ mm}$$

$$y_3 = 500 + h/2 = 500 + 100/2 = 550\text{ mm}$$

$$a_3 = bh = 600\times100 = 6\times10^4\text{ mm}^2.$$

Centroid $\bar{x} = \frac{\Sigma ax}{\Sigma a}$.

$$= \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{8\times10^4(400) + 2\times10^4(575) + 6\times10^4(650)}{16\times10^4}$$

$\bar{x} = 515.625$ mm.

$\bar{y} = \frac{\Sigma ay}{\Sigma a}$.

Similarly, $\bar{y} = \frac{8\times10^4(50) + 2\times10^4(300) + 6\times10^4(550)}{16\times10^4}$

$\bar{y} = 268.75$ mm.

Moment of Inertia about xx-axis,

$I_{xx} = I_{xx}$ of part $+ a(y-\bar{y})^2$.

$I_{xx_1} = I_{xx}$ of part ① $+ a_1(y_1-\bar{y})^2$

$$= \frac{bh^3}{12} + a_1(y_1-\bar{y})^2$$

$$= \frac{800\times100^3}{12} + 8\times10^4(50 - 268.75)^2$$

$I_{xx_1} = 3.89\times10^9$ mm^4.

$$I_{xx_2} = \frac{50 \times 400^3}{12} + 2\times10^4 (300 - 268.75)^2$$

$$= 0.286\times10^9 \text{ mm}^4$$

$$I_{xx_3} = \frac{600 \times 100^3}{12} + 6\times10^4 (550 - 268.75)^2$$

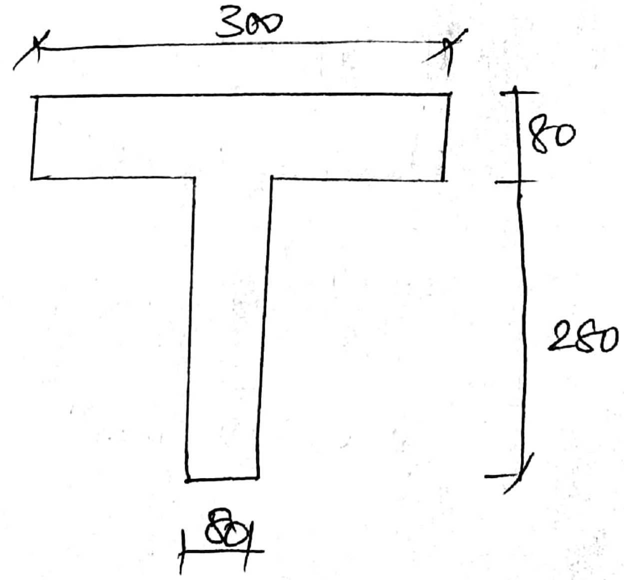
$$= 4.79\times10^9 \text{ mm}^4.$$

$$\therefore I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

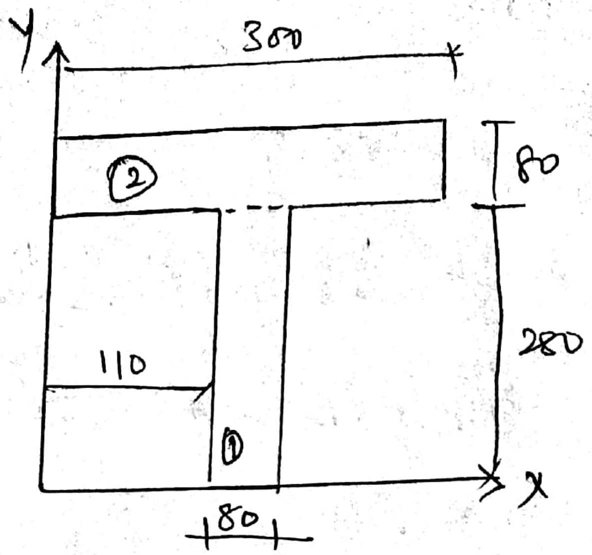
$$= 3.89\times10^9 + 0.286\times10^9 + 4.79\times10^9$$

$$I_{xx} = 8.96\times10^9 \text{ mm}^4.$$

(7) Find the polar moment of inertia of a T section shown in fig. about an axis passing through its centroid. Also find the radius of gyration with respect to the polar axis. (Dimension in mm)



Sol:



part ①

$$a_1 = bh = 80 \times 280 = 22400 \text{ mm}^2$$

$$x_1 = 110 + b/2 = 110 + 80/2 = 150 \text{ mm}$$

$$y_1 = h/2 = \frac{280}{2} = 140 \text{ mm}$$

part ②

$$a_2 = bh = 300 \times 80 = 24000 \text{ mm}^2$$

$$x_2 = b/2 = 300/2 = 150 \text{ mm}$$

$$y_2 = 280 + h/2 = 280 + 80/2 = 320 \text{ mm}.$$

Centroid,

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{22400(150) + 24000(150)}{22400 + 24000}$$

$$\bar{x} = 150 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{22400(140) + 24000(320)}{22400 + 24000}$$

$$\bar{y} = 233.1 \text{ mm}.$$

Moment of Inertia :

$$I_{xx_1} = \frac{bh^3}{12} + a_1(y_1 - \bar{y})^2$$

$$= \frac{80 \times 280^3}{12} + 22400(140 - 233.1)^2$$

$$= 340.5 \times 10^6 \text{ mm}^4.$$

$$I_{xx_2} = \frac{300 \times 80^3}{12} + 24000(320 - 233.1)^2$$

$$= 194.03 \times 10^6 \text{ mm}^4.$$

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= 340.5 \times 10^6 + 194.03 \times 10^6$$

$$= 534.5 \times 10^6 \text{ mm}^4$$

$$I_{yy_1} = \frac{hb^3}{12} + a_1(x_1 - \bar{x})^2$$

$$= \frac{280(80)^3}{12} + 22400(150 - 150)^2$$

$$= 11.94 \times 10^6 \text{ mm}^4$$

$$I_{yy_2} = \frac{80 \times 300^3}{12} + 22400(150 - 150)^2$$

$$= 180 \times 10^6 \text{ mm}^4$$

$$I_{yy} = I_{yy_1} + I_{yy_2}$$

$$= 11.94 \times 10^6 + 180 \times 10^6$$

$$= 191.94 \times 10^6 \text{ mm}^4.$$

Polar Moment of inertia,

$$J = I_{xx} + I_{yy}$$

$$= 534.53 \times 10^6 + 191.94 \times 10^6$$

$$J = 726.47 \times 10^6 \text{ mm}^4$$

Radius of gyration

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{534.53 \times 10^6}{46.4 \times 10^3}}$$

$$K_{xx} = 107.33 \text{ mm.}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{\Sigma a}} = \sqrt{\frac{191.94 \times 10^6}{46.4 \times 10^3}}$$

$$K_{yy} = 64.31 \text{ mm.}$$

Radius of gyration about polar axis is,

$$K_p = \sqrt{K_{xx}^2 + K_{yy}^2}$$

$$= \sqrt{107.33^2 + 64.31^2}$$

$$K_p = 125.12 \text{ mm.}$$

TWO MARKS

① Distinguish between centroid and centre of gravity.

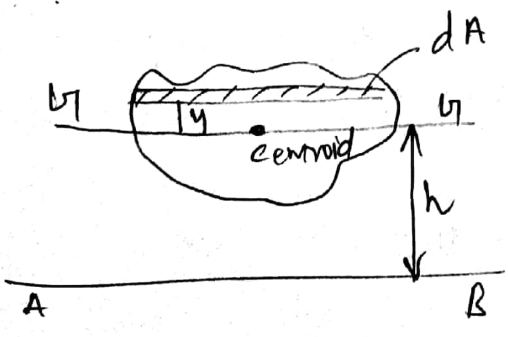
Centroid is defined as the point through which the entire area of the place figure is assumed to be concentrated.

The centre of gravity is defined as a point through which the entire weight of body acts, irrespective of the orientation of body.

② State parallel axis theorem with simple sketch.

The moment of inertia of a plane area about any axis is the sum of the moment of inertia of the area about the axis, passing through the centroid of area parallel to the given axis and the product of area of the plane and the square of the perpendicular distance of its centroid from the axis.

$$I_{AB} = I_{cg} + Ah^2$$



(3) Define the radius of gyration with respect to x-axis of an area.

The radius of gyration is defined as the distance at which the whole area of the body may be assumed to be concentrated with reference to the axis of reference.

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma A}}$$

(4) Define polar Moment of inertia of lamina.

The polar moment of inertia of an about an axis passing through a pole is the sum of moment of inertia about the rectangular x and y axes passing through the pole.

(5) Define first Moment of an area about an axis.

It is defined as the point through which the entire area of the plane figure is assumed to be concentrated.

(6) Write the SI units of the Mass Moment of inertia and of the area moment of inertia of a lamina.

Mass MI $\rightarrow$ $kg.m^2$

Area MI $\rightarrow$ m^4.

(7) Define Principle axes and Principal Moment of Inertia.

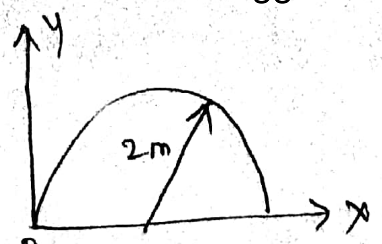
If we rotate the given axes, the sign of product of inertia I_{xy} changes its sign and becomes negative. So it can be concluded that there must be certain direction of the axes for which the product of inertia is zero. The axes taken in these directions are called the principal axes of the area.

The moments of Inertia about the principal axes are called principal moments of Inertia.

(8) When will the product of inertia of a lamina become zero?

The product of Inertia is zero, when either one or both of the x-x and y-y axes, happen to be the axes of symmetry. Because for each element dA of co-ordinates x and y, there is an element with co-ordinates x & y, thus making the $\int xy\, dA$ zero.

(9) Locate the centroid and calculate the moment of inertia about centroidal axes of a semi-circular lamina of radius 2m.



Centroid,

$$\bar{x} = R = 2\,m$$

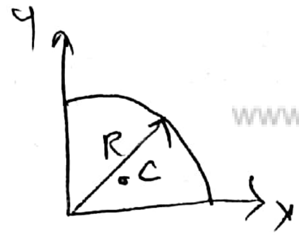
$$\bar{y} = \frac{4R}{3\pi} = \frac{4\times 2}{3\pi} = 0.84\,m$$

Moment of inertia,

$$I_{xx} = 0.11 R^4 = 0.11(2)^4 = 1.76\,m^4.$$

$$I_{yy} = \pi R^4/8 = \frac{\pi(2)^4}{8} = 6.28\,m^4$$

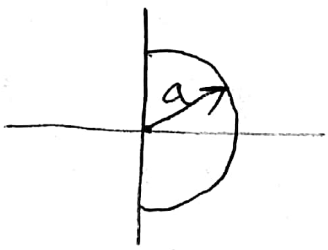
(10) Write the centroidal values of Quarter circle.



$$\bar{x} = \frac{4R}{3\pi}$$

$$\bar{y} = 4R/3\pi.$$

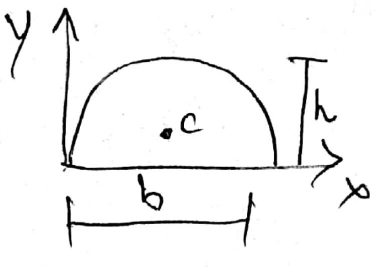
(11) A semicircle of radius 'a' is defined in the first and fourth quadrants. Write down its co-ordinates of centroid



$$\bar{x} = \frac{4R}{3\pi}$$

$$\bar{y} = 0.$$

(12) Write the centroid of the parabola.



$$\bar{x} = \frac{b}{2}$$

$$\bar{y} = \frac{2h}{5}$$

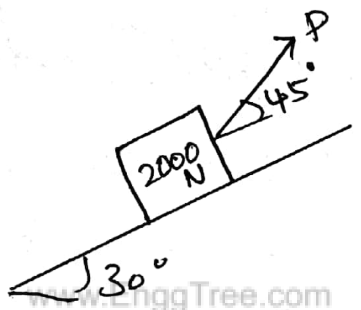
$$\text{area} = \frac{2}{3}bh.$$

MOHAN. S.R
MECH/AP

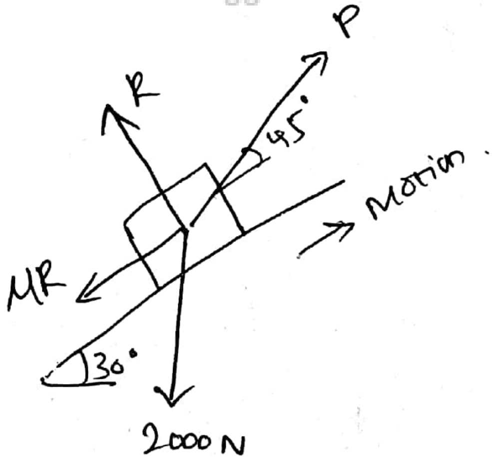
UNIT – V

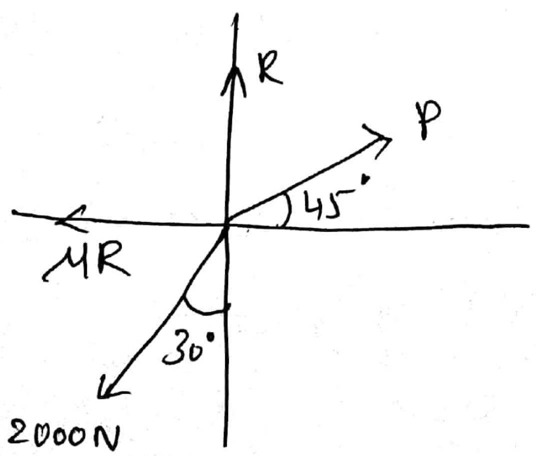
FRICTION

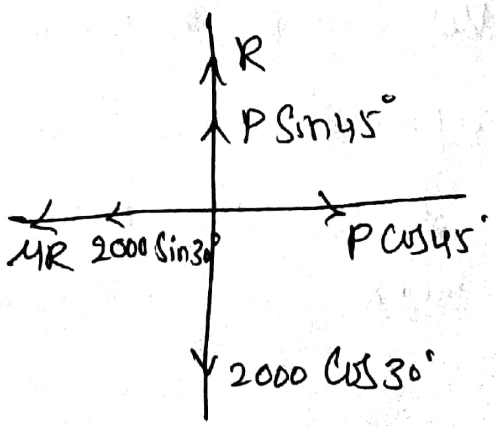
① A body weighing 2000 N is lying on an inclined plane making an angle of 30° with the horizontal as shown in fig. Determine the force applied at 45° to the inclined plane that can just move the body up the plane. The co-efficient of friction between the plane and the body is 0.25.



Sol:







by applying equilibrium,

$\Sigma F_x = 0$

$P \cos 45° - \mu R - 2000 \sin 30° = 0$

$0.707P - 0.25R - 1000 = 0$ —— ①

$\Sigma F_y = 0$

$R + P \sin 45° = 2000 \cos 30°$

$R = 1732 - 0.707P$

Sub eqn in ①,

$0.707P - 0.25(1732 - 0.707P) - 1000 = 0$

$0.707P - 433 + 0.176P - 1000 = 0$

$0.883P - 1433 = 0$

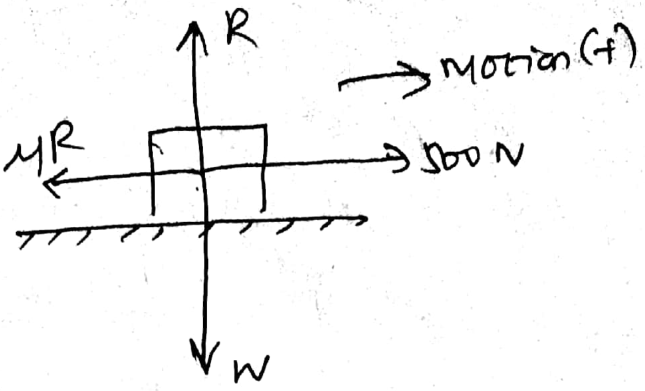
$P = \frac{1433}{0.883}$

$P = 1622.87 \text{ N}$.

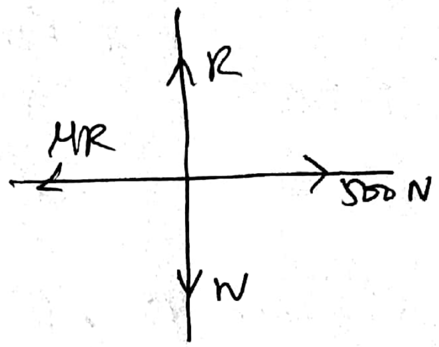
② A body lying on a horizontal plane is able to just start to move when a force of 500N is applied parallel to the horizontal plane. If a force of 400N is replaced such that it is acting 30° to the x-axis and it can also make the body to move, determine the weight and the Co-efficient of friction.

Sol:

Case (i)



Using equilibrium conditions,



$\Sigma F_x = 0,$

$$500 - \mu R = 0$$

$$\mu R = 500 \quad \text{--- (1)}$$

$\Sigma F_y = 0,$

$$R - W = 0$$

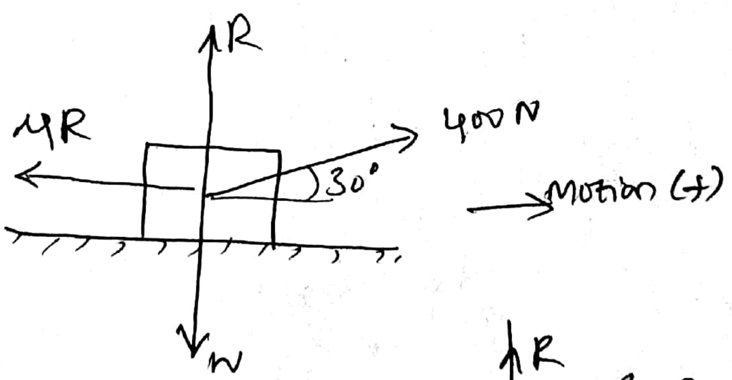
$$R = W$$

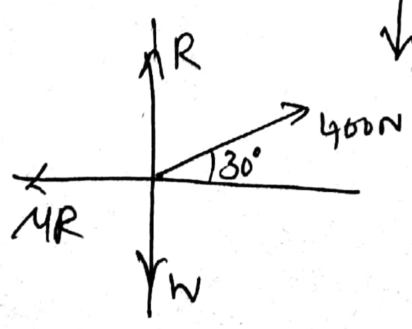
Sub in eqn (1).

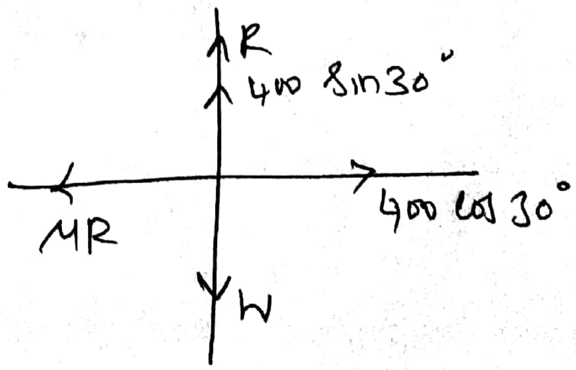
$$\mu W = 500$$

$$W = 500/\mu \quad \text{--- (2)}$$

Case (ii)







Using equilibrium Conditions,

$\Sigma F_x = 0,$

$$400 \cos 30^\circ - \mu R = 0$$

$$\mu R = 400 \cos 30^\circ$$

$$R = \frac{346.4}{\mu} \quad \text{--- (3)}$$

$\Sigma F_y = 0,$

$$R + 400 \sin 30^\circ - W = 0$$

$$\frac{346.4}{\mu} + 200 - W = 0$$

$$W = \frac{346.4}{\mu} + 200 \quad \text{--- (4)}$$

Equate (2) & (4),

$$\frac{500}{\mu} = \frac{346.4}{\mu} + 200$$

$$500 = 346.4 + 200\mu$$

$$200\mu = 153.6$$

$$\mu = 0.76$$

Sub in (2)

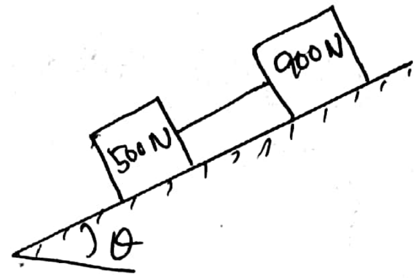
$$W = \frac{500}{\mu}$$

$$W = \frac{500}{0.76}$$

$$W = 651\,N.$$

③ Two blocks of weight 500 N and 900 N connected by a rod are kept on an inclined plane as shown in the fig below. The rod is parallel to the plane. The Co-efficient of friction between 500 N block and the plane is 0.3 and that between 900 N block and the plane is 0.4. Find the inclination of the plane with the horizontal and the tension in the rod when the motion down the plane is just about to start.

Soln:



Sol:

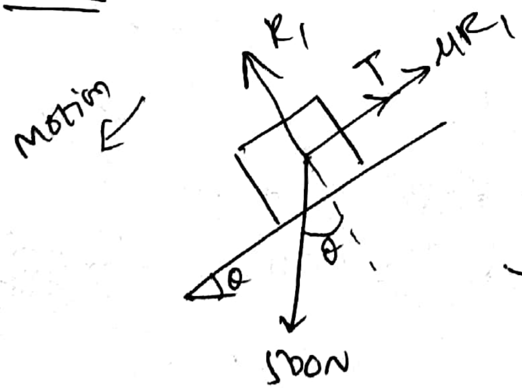
$W_1 = 500\text{ N}$ $\quad W_2 = 900\text{ N}$

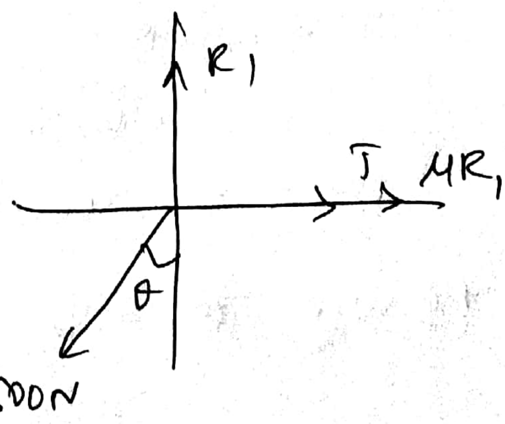
$\mu_1 = 0.3$ $\quad \mu_2 = 0.4$

$\theta = ?$

Tension $= ?$

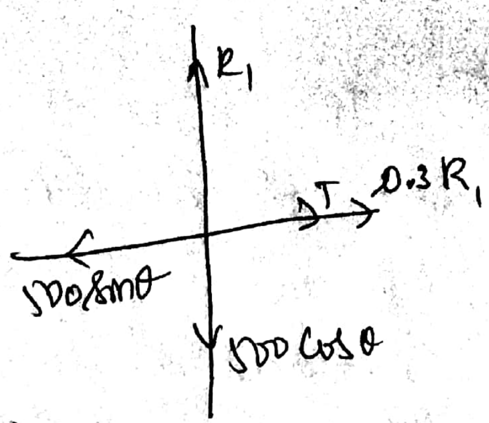
500N block





$\Sigma F_y = 0$

$R_1 = 500 \cos\theta$ —— (1)



$\Sigma F_n = 0,$

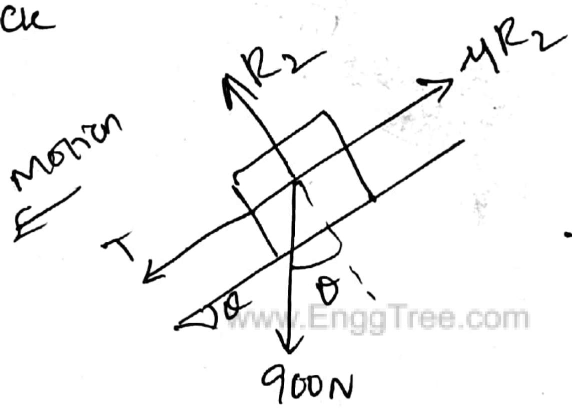
$500 \sin\theta - T - 0.3 R_1 = 0$

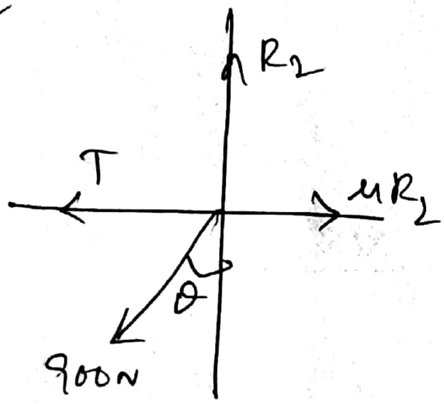
$500 \sin\theta - T - 0.3(500 \cos\theta) = 0$

$500 \sin\theta = T + 150 \cos\theta$ —— (2)

$T = 500 \sin\theta - 150 \cos\theta$

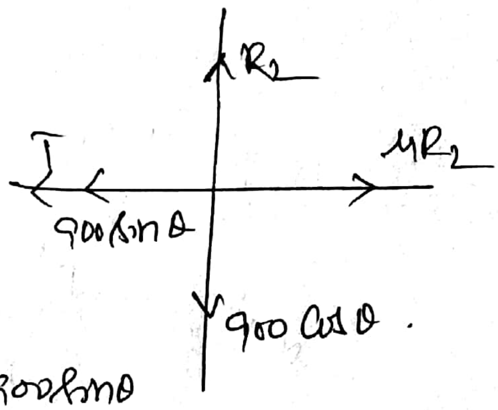
900N block





$\Sigma F_y = 0$

$R_2 = 900 \cos\theta$



$\Sigma F_n = 0,$

$T + 900 \sin\theta = \mu R_2$

$T = 0.4(900 \cos\theta) - 900 \sin\theta$

$T = 360 \cos\theta - 900 \sin\theta$ —— (3)

Equating (2) & (3),

$$500\sin\theta - 150\cos\theta = 360\cos\theta - 900\sin\theta.$$

$$1400\sin\theta = 510\cos\theta.$$

$$\frac{\sin\theta}{\cos\theta} = \frac{510}{1400}$$

$$\tan\theta = \frac{510}{1400}$$

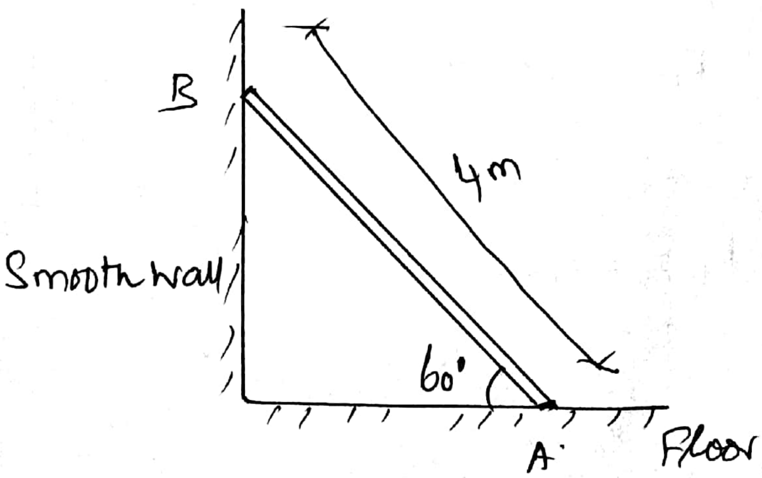
$$\boxed{\theta = 20^\circ}$$

$\therefore$ from (2),

$$T = 500\sin 20^\circ - 150\cos 20^\circ$$

$$\boxed{T = 30\ N}$$

(4) A ladder of weight 1000 N and length 4 m rests as shown in fig. If a 750 N weight is applied at a distance of 3 m from the top of ladder, it is at the point of sliding. Determine the co-efficient of friction between ladder and the floor.

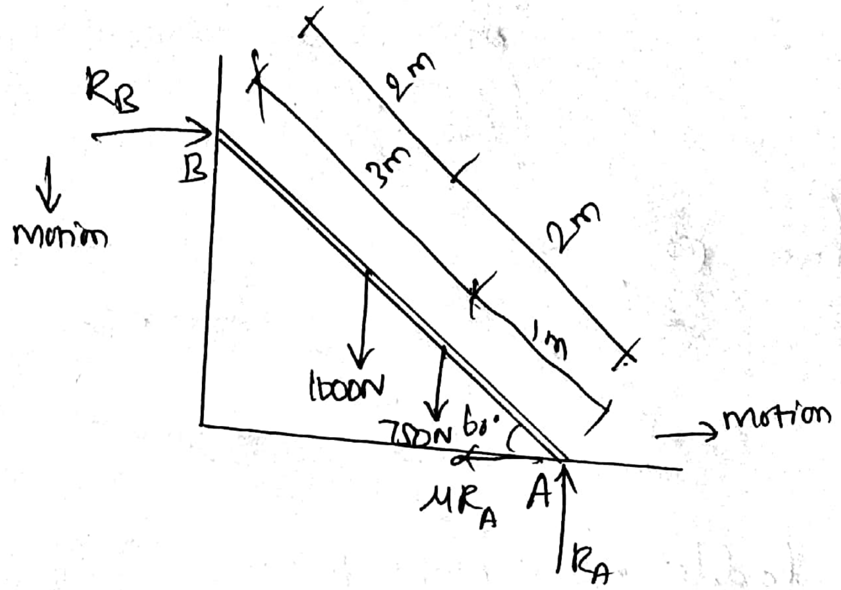


Sol:

Weight of ladder = 1000 N

L = 4m.

Since wall is smooth, so $\mu_{wall} = 0$.



$\Sigma F_x = 0$,

$R_B = \mu R_A$ —— ①

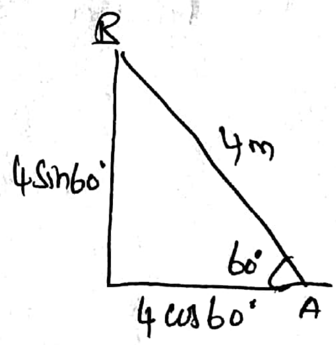
$\Sigma F_y = 0$,

$R_A = 1000 + 750$

$R_A = 1750$ N.

$\therefore R_B = \mu(1750)$

$R_B = 1750\mu$ —— ②



$\Sigma M_A = 0$,

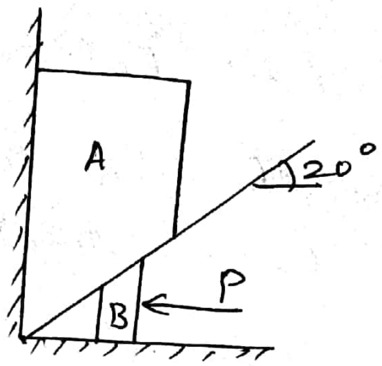
$-(R_B \times 4\sin 60°) + (1000 \times 2\cos 60°) + (750 \times 1\cos 60°) = 0$

$R_B = 369.94$ N

From eqn ②,

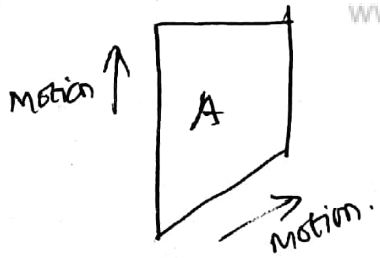
$\mu = \frac{369.94}{1750} = 0.226$

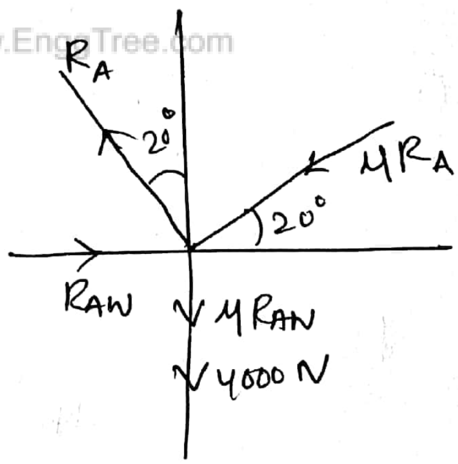
(5) Determine the horizontal force P required for wedge B to raise block A of weight 4000 N as shown in fig. The Co-efficient of friction on all surfaces is equal to 0.3.

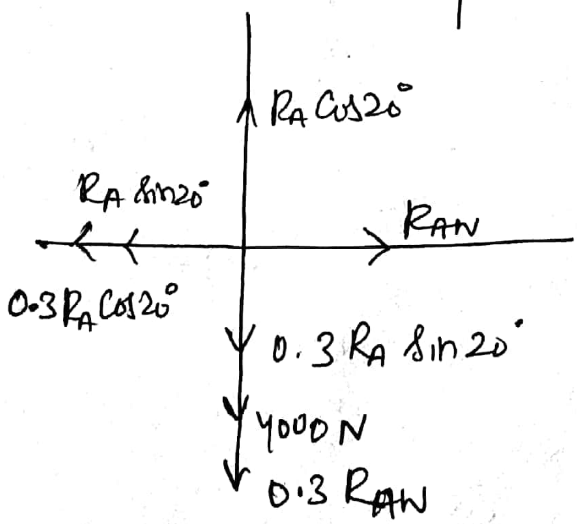


Sol:

Body A FBD







Using equilibrium,

$\Sigma F_y = 0,$

$$R_A \cos 20^\circ - 4000 - 0.3 R_{AW} - 0.3 R_A \sin 20^\circ = 0 \quad \text{—— (1)}$$

$\Sigma F_x = 0,$

$$R_{AW} - R_A \sin 20^\circ - 0.3 R_A \cos 20^\circ = 0$$

$$R_{AW} = 0.34 R_A + 0.281 R_A$$

$$R_{AW} = 0.621 R_A \quad \text{—— (2)}$$

Sub in (1),

$$0.93 R_A - 4000 - 0.3 (0.621 R_A) - 0.102 R_A = 0$$

$$0.641 R_A = 4000$$

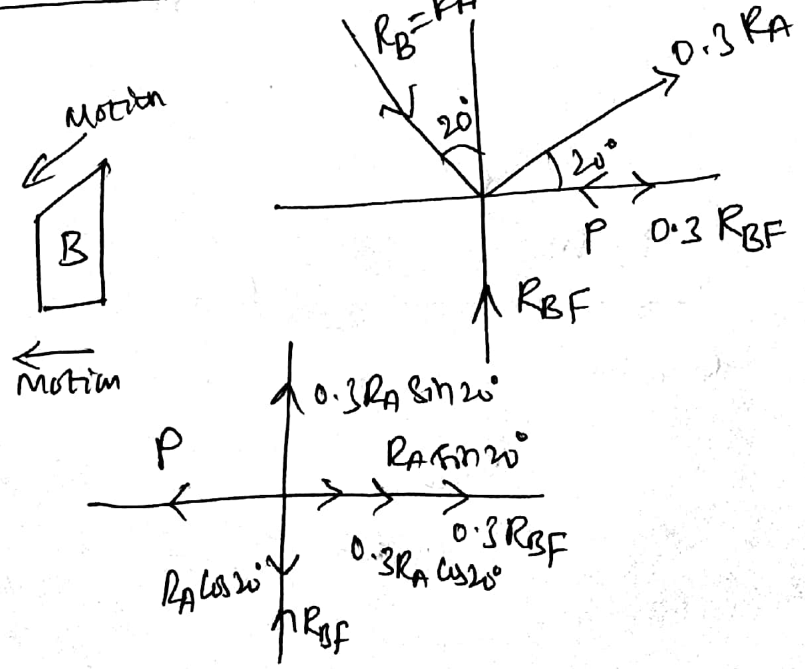
$$R_A = 6233 \text{ N}.$$

from (2),

$$R_{AW} = 0.621 (6233)$$

$$R_{AW} = 3870 \text{ N}$$

Body B FBD



$\Sigma F_x = 0,$

$$R_A \sin 20° + 0.3 R_A \cos 20° + 0.3 R_{BF} = P$$

$$6233 \sin 20° + 0.3\,(6233) \cos 20° + 0.3 R_{BF} = P$$

$$3888 + 0.3 R_{BF} = P. \quad \text{——— (3)}$$

$\Sigma F_y = 0$

$$R_{BF} + 0.3 R_A \sin 20° = R_A \cos 20°$$

$$R_{BF} + 0.3\,(6233) \sin 20° = 6233 \cos 20°$$

$$R_{BF} = 5217 \text{ N}$$

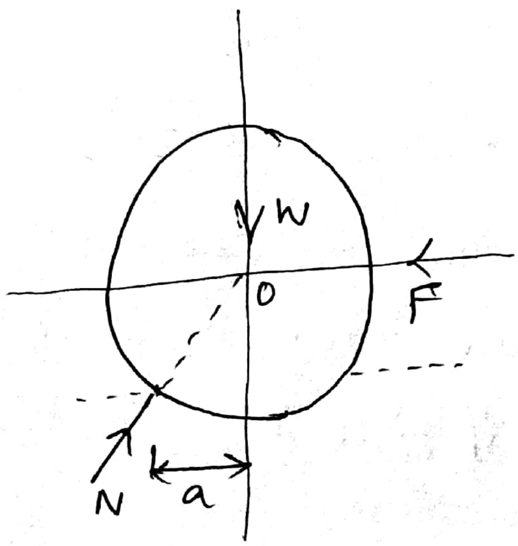
Sub in (3),

$$3888 + 0.3\,(5217) = P$$

$$P = 5453 \text{ N}.$$

(6) A cylinder of weight 1500 N and radius 400 mm is required to move on a horizontal surface. Find the force required to roll the cylinder without slipping if the co-efficient of rolling resistance is equal to 16 mm.

Sol:



$W = 1500\,N \quad r = 0.4\,m \quad a = 0.016\,m$

At the start of rolling and at any instant of time during rolling, the cylinder must satisfy the moment equilibrium condition about Z-axis at point P.

$$\Sigma M_P = 0. \qquad (\curvearrowleft + \; -)$$

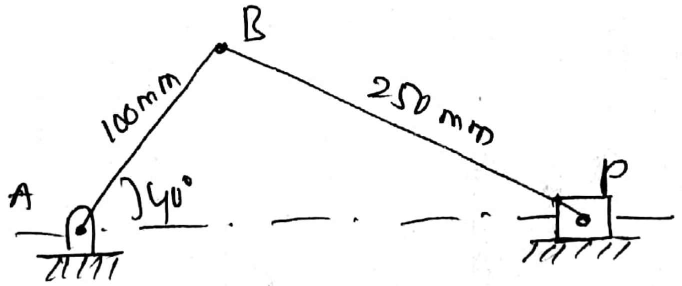
$$Fr - W(a) = 0$$

$$F = \frac{Wa}{r}$$

$$F = \frac{1500\,(0.016)}{0.4}$$

$$F = 60\,N.$$

⑦ In the engine system shown in fig, the crank AB has a constant clockwise angular speed of 3000 rpm.



For the crank position indicated, find

i) the angular velocity of the connecting rod BP.

ii) Velocity of piston P.

Sol:

MOHAN. S.R
MECH - AP

$r = 100\,mm$ $\quad l = 250\,mm = 0.25\,m$

$r = 0.1\,m$

$N = 3000\,rpm.$

Considering motion of crank,

angular velocity of crank $\omega = \frac{2\pi N}{60}$

$$\omega = \frac{2\pi \times 3000}{60} = 314.15\ rad/sec.$$

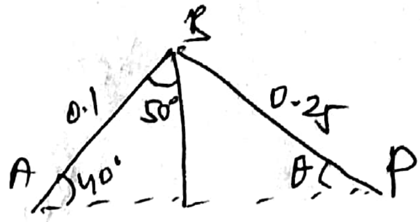
Tangential velocity of end B,

$$V_B = r\omega = 0.1 \times 314.15$$

$$V_B = 31.4\ m/s.$$

Considering motion of connecting rod BP,

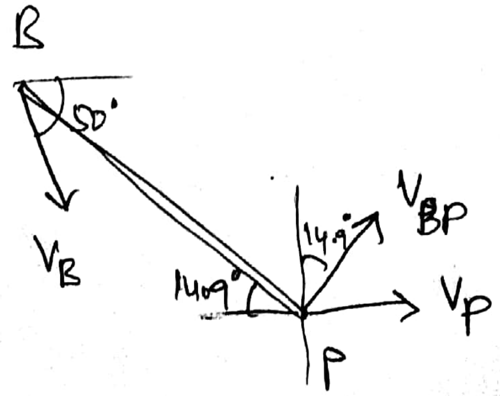
From triangle ABP,



$$0.1 \sin 40° = 0.2 \sin\theta$$

$$\sin\theta = \frac{0.1 \sin 40°}{0.2}$$

$$\theta = 14.9°.$$



Tangential velocity of point P with respect to B.

$$V_{BP} = l\,\omega_{BP} = 0.25\,\omega_{BP}.$$

The resultant velocity at P is horizontal.

Considering vertical components of velocities,

$$0 = V_B \sin 50^\circ + V_{BP} \cos 14.9^\circ$$

$$0 = 31.4 \sin 50^\circ + 0.25\,\omega_{BP} \cos 14.9^\circ.$$

$$\omega_{BP} = 99.56 \text{ rad/sec}.$$

Considering horizontal components,

$$V_{BP} = V_B \cos 50^\circ + V_{BP} \sin 14.9^\circ$$

$$V_{BP} = 31.4 \cos 50^\circ + 0.25\,(99.56) \sin 14.9^\circ$$

$$V_{BP} = 26.53 \text{ m/s}.$$

TWO MARKS

MOHAN. S.R
MECH / AP.

① State laws of dry friction.

* The frictional force always acts in a direction opposite to that in which the body tends to move.

* The force of friction depends upon the nature of surfaces in contact.

* The frictional force does not depend on the area and shape of surfaces in contact.

② Define Co-efficient of kinetic friction.

When the bodies are in relative motion, the co-efficient of friction is called kinetic. The kinetic co-efficient of friction is always less than static co-efficient of friction.

③ What is Coloumb friction?

The friction that exists between two unlubricated surfaces is called Coloumb friction.

④ Define Co-efficient of static friction.

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{R}.$$

(5) List the different types of friction.

* Dry friction
* Fluid friction
* Sliding friction
* Roller friction.

(6) When do we say that the motion of a body is impending?

When the applied force over a body is just sufficient to overcome the friction, then the motion of a body is at impending stage.

(7) What is general plane motion?

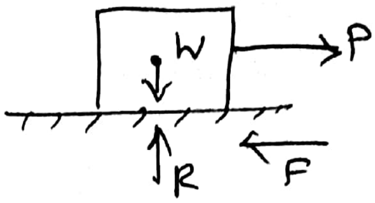
The motion of a rigid body is said to have general plane motion when the body undergoes a combination of translational and rotational.

(8) A rigid body rotates about a fixed axis. Write the expression for angular velocity when the rotation is uniformly accelerated.

$$\omega = \frac{2\pi N}{60}.$$

(9) What is limiting friction?

The maximum value of frictional force, which comes into play, when the body just begins to slide over the surface of the other body is known as limiting friction.

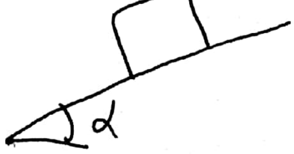


(10) State the factors influencing friction.

* Types of materials
* Roughness of contact surface.
* Weight of the body moving over the surface.
* Nature of motion body.

(11) Define angle of repose.

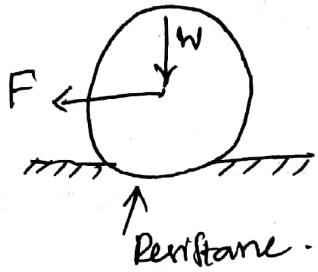
The angle that the plane of contact between two bodies makes with the horizontal when the upper body is just on the point of sliding.



α - angle of repose

(12) Define Rolling Resistance.

Rolling resistance is the force resisting the motion when a body rolls on a surface.



MOHAN. S.R
MECH/AP

UNIT - IV

DYNAMICS OF PARTICLES

(1) Two trains A & B leave the same Station on parallel lines. A Starts with a uniform acceleration of $0.15\ m/s^2$ and attains the speed of 24 km/hr after which its speed remains constant. B leaves 40 seconds later with uniform accelaration of $0.3\ m/s^2$ to attain a maximum of 48 km/hr, its speed also becomes constant thereafter. When will B overtake A?

Sol:

Train A

initial velocity $u = 0$

final velocity $V = 24\ km/hr = \frac{24 \times 1000}{3600}$

$v = 6.67\ m/s^2$

$a = 0.15\ m/s^2$

WKT,

$V = u + a t_A$

$6.67 = 0 + 0.15\, t_A$

$t_A = 44.67\ s.$

Distance travelled by 44.67 sec,

$S_1 = u t_A + \frac{1}{2} a t_A^2$

$S_1 = 0 + \frac{1}{2} \times 0.15 (44.67)^2$

$S_1 = 150m.$

Since train B leaves 40 seconds later,
So that the train A travelled T+40 sec.

$$\therefore \quad S_A = S_1 + V[(T+40) - t_A]$$

$$S_A = 150 + 6.67[(T+40) - 44.67] \quad \text{--- (1)}$$

Train B

$$\because u = 0$$

$$V = 48 \text{ km/hr} = \frac{48 \times 1000}{3600} = 13.34 \text{ m/s}.$$

$$a = 0.3 \text{ m/s}^2$$

WKT,

$$V = u + a t_B$$

$$13.34 = 0 + 0.3 t_B$$

$$t_B = 44.47 \text{ sec}.$$

Distance travelled by 44.47 sec,

$$S_2 = u t_B + \frac{1}{2} a t_B^2$$

$$= 0 + \frac{1}{2} \times 0.3 \times 44.47^2$$

$$S_2 = 296.63 \text{ m}$$

Distance travelled by T sec,

$$S_B = S_2 + V(T - t_B)$$

$$S_B = 296.63 + 13.34(T - 44.47) \quad \text{--- (2)}$$

$\therefore S_A = S_B$

$$150 + 6.67\left((T+t_0) - 44.67\right) = 296.63 + 13.34\,(T - 44.57)$$

$$150 + 6.67T + 266.8 - 297.94 = 296.63 + 13.34T - 595.8$$

$$6.67T = 418.03$$

$$T = 62.67 \text{ sec.}$$

(2) Car A accelerates uniformly from rest on a Straight level road. Car B starting from the same point 6 seconds later with zero initial velocity accelerates at $6\,m/s^2$. It overtakes the car A at 400m from the starting point. What is the acceleration of the Car A?

<u>Sol:</u>

initial velocity of Car A $u_A = 0$

initial velocity of Car B $u_B = 0$

$$a_B = 6\,m/s^2$$

$$a_A = ?$$

$$S_A = S_B = 400\,m.$$

Let t_A be the time taken by Car A.

$$t_B = t_A - 6.$$

Consider Car A,

$$S_A = u_A t_A + \tfrac{1}{2} a_A t_A^2.$$

$$400 = 0 + \frac{1}{2} a_A t_A^2$$

$$a_A t_A^2 = 800 \quad \text{——— (1)}$$

Consider Car B,

$$S_B = u_B t_B + \frac{1}{2} a_B t_B^2$$

$$400 = 0 + \frac{1}{2} 6 (t_A - 6)^2$$

$$\frac{400}{3} = t_A^2 - 12 t_A + 36$$

$$t_A^2 - 12 t_A - 97.33 = 0.$$

by solving, $t_A = 17.54$ sec.

Sub in eqn (1)

$$a_A (17.54)^2 = 800$$

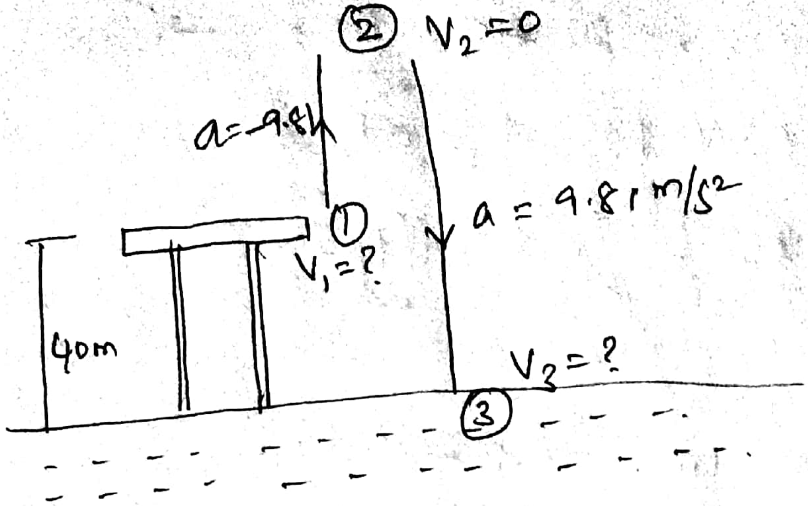
$$a_A = 2.6 \ m/s^2$$

(3) A stone is thrown vertically upwards at a point on a bridge located 40 m above the water. If it strikes the water after 4 sec determine

i) the speed at which the stone was thrown up

ii) The speed at which the stone strikes the water.

Sol:



$$t_{1-2-3} = 4$$

$$t_{1-2} + t_{2-3} = 4$$

$$S_{2-3} = S_{1-2} + 40.$$

① - ②

$$V = u + at$$

$$V_2 = V_1 + a t_{1-2}$$

$$0 = V_1 + (-9.81) t_{1-2}$$

$$V_1 = 9.81 \, t_{1-2} \quad \text{— ①}$$

$$S = ut + \tfrac{1}{2} a t^2$$

$$S_{1-2} = V_1 t_{1-2} + \tfrac{1}{2} a t_{1-2}^2$$

$$S_{1-2} = 9.81 t_{1-2}^2 + \tfrac{1}{2}(-9.81) t_{1-2}^2$$

$$S_{1-2} = 4.9 \, t_{1-2}^2 \quad \text{— ②}$$

② - ③

$$V_3 = V_2 + a t_{2-3}$$

$$V_3 = 0 + 9.81 \, t_{2-3}$$

$$V_3 = 9.81 (4 - t_{1-2})$$

$$V_3 = 39.24 - 9.81\, t_{1-2} \quad \text{--- (3)}$$

$$S_{2-3} = V_2 t_{2-3} + \tfrac{1}{2} a t_{2-3}^2$$

$$S_{1-2} + 40 = \tfrac{1}{2}\, 9.81\, (4 - t_{1-2})^2$$

$$4.9\, t_{1-2}^2 + 40 = 4.9\, (16 + t_{1-2}^2 - 8t_{1-2})$$

$$40 = 78.4 - 39.2\, t_{1-2}$$

$$39.2\, t_{1-2} = 38.4$$

$$t_{1-2} = 0.97\, s$$

Sub in (1), $V_1 = 9.81\,(0.97)$

$$V_1 = 9.5 \text{ m/s}$$

Sub in (3)

$$V_3 = 39.24 - 9.81\,(0.97)$$

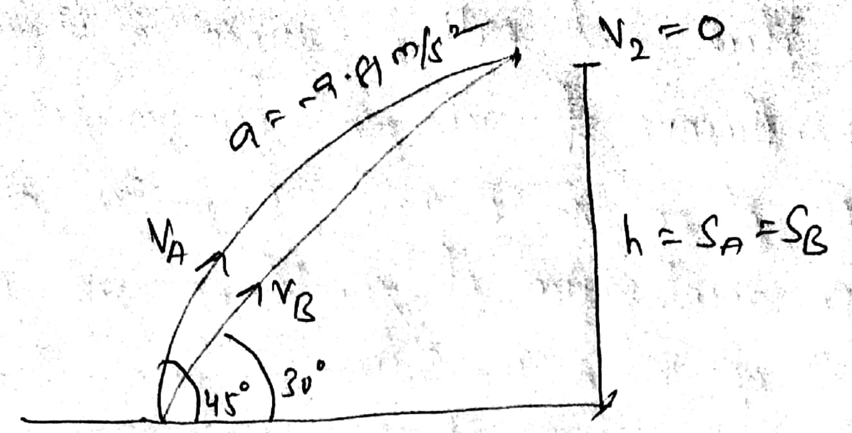
$$V_3 = 29.7 \text{ m/s}.$$

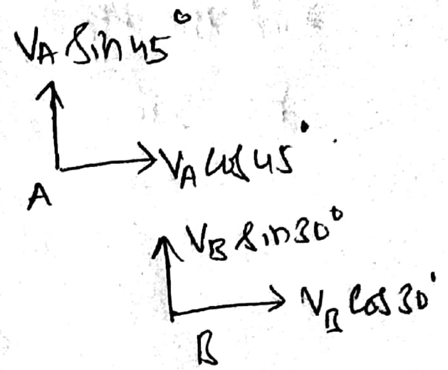
(4) Two Stones A & B are projected from the same point at 45° and 30° respectively, inclined to the horizontal. Find the ratio of the velocity of projection of A and B of the maximum height reached by both is the same.

Sol:

Stone A

$$v^2 - u^2 = 2as$$





$$V_2^2 - V_1^2 = 2aS_A$$

$$0 - (V_A \sin 45°)^2 = 2 \times (-9.81)h$$

$$-0.5 V_A^2 = -19.62h$$

$$V_A^2 = 39.29h$$

$$V_A = 6.26\sqrt{h} \quad \text{—} (1)$$

<u>Stone B</u>

$$V^2 - u^2 = 2as$$

$$V_2^2 - V_1^2 = 2aS_B$$

$$0 - (V_B \sin 30°)^2 = 2(-9.81)h$$

$$-0.25 V_B^2 = -19.62h$$

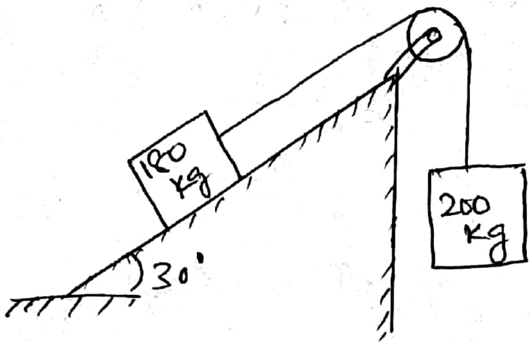
$$V_B = 8.85\sqrt{h} \quad \text{—} (2)$$

ratio of velocities,

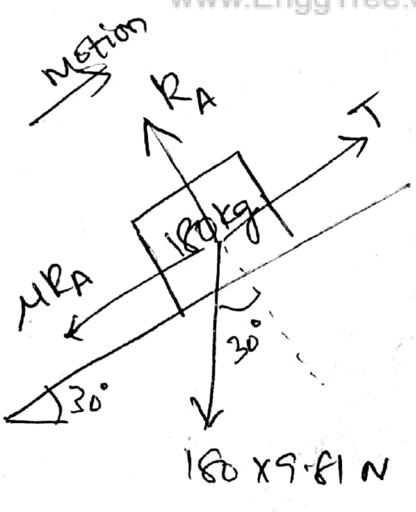
$$\frac{V_A}{V_B} = \frac{6.26\sqrt{h}}{8.85\sqrt{h}}$$

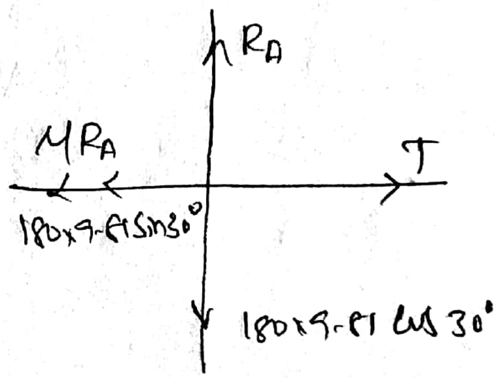
$$\frac{V_A}{V_B} = 0.707$$

(5) A block and pulley system is shown in fig below. The co-efficient of kinetic friction between the block and the plane is 0.25. The pulley is friction less. Find the acceleration of the blocks and the tension in the string when the system is just released. Also find the time required for 200 kg block to come down by 2m.



Sol:





Apply equilibrium methodology,

$\Sigma F_y = 0$

$$R_A = 180 \times 9.81 \cos 30°$$

$$R_A = 1529 \text{ N}.$$

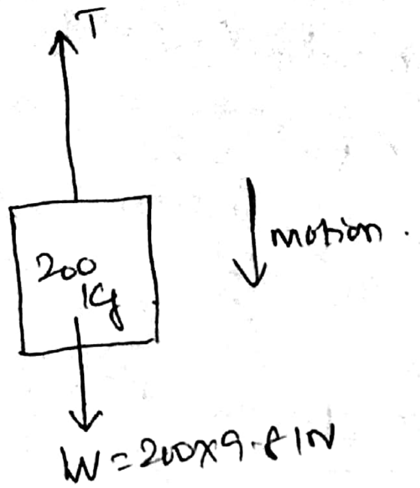
$$\Sigma F_{motion} = ma$$

$$(T - \mu R_A) - 180 \times 9.81 \sin 30^\circ = 180\,a.$$

$$T - 0.25(1529) - 882.9 = 180\,a$$

$$T - 1265.1 = 180\,a.$$

$$T = 180\,a + 1265.1 \quad \text{—} (1)$$



Using equilibrium,

$$\Sigma F_x = 0$$

$$\Sigma F_{motion} = ma.$$

$$(200 \times 9.81) - T = 200\,a$$

$$1962 - T = 200\,a.$$

$$T = 1962 - 200\,a. \quad \text{—} (2)$$

Using (1) & (2)

$$180\,a + 1265.1 = 1962 - 200\,a.$$

$$380\,a = 696.9$$

$$a = 1.83\ m/s^2.$$

from eqn ①,

$$T = 180(1.83) + 1265.1$$

$$T = 1594.5 \text{ N}.$$

time required 200 kg block moves 2m,

$$S = ut + \frac{1}{2}at^2$$

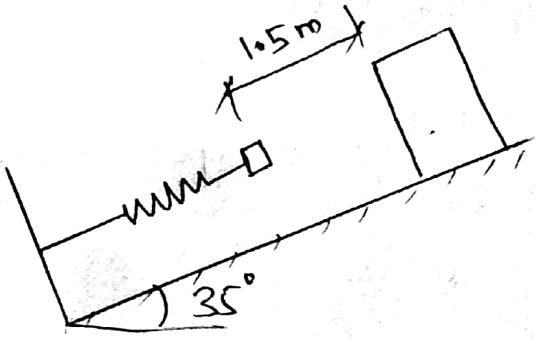
$$2 = 0 + \frac{1}{2}(1.83)t^2$$

$$0.915t^2 = 2$$

$$t^2 = 2.185$$

$$t = 1.47 \text{ sec}.$$

⑥ A block of mass 50 kg slides down a 35° inclined and strikes a spring 1.5m away from it as shown in fig. below. The maximum compression of the spring is 300 mm when the block comes to rest. If the spring constant is 1 kN/m. find the co-efficient of kinetic friction between the block and the plane.



Sol:

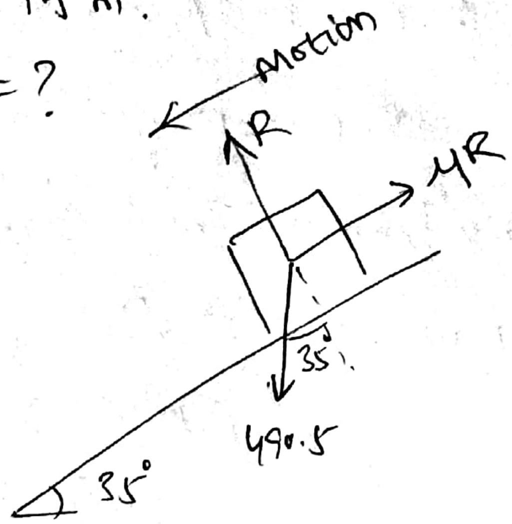
$m = 50\ kg = 50 \times 9.81 = 490.5\ N$

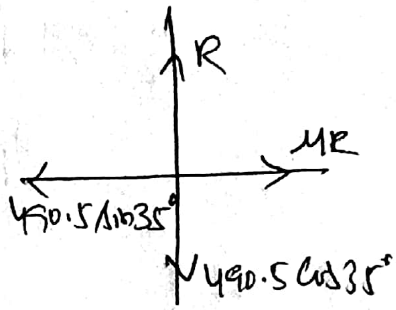
$K = 1\ kN/m = 1000\ N/m.$

$x = 300\ mm = 0.3\ m$

$d = 1.5\ m.$

$\mu = ?$





Using equilibrium conditions,

$\Sigma F_y = 0$

$R = 490.5 \cos 35°$

$R = 401.79\ N$

Total distance moved by block,

$S = 1.5\ m + 0.3\ m$

$S = 1.8\ m$

Workdone by block, $= \Sigma F_{along\ motion} \times$ distance.

$= (490.5 \sin 35° - \mu(401.79)) \times 1.8$

$= 506.412 - 723.22\mu$

Workdone by Spring $= -\frac{1}{2} K x^2$

$= -\frac{1}{2} \times 1000 \times 0.3^2$

$= -45$ N·m.

Change in K.E $= \frac{1}{2} m (v^2 - u^2)$

K.E $= 0$ $\quad (\because u = 0, v = 0)$

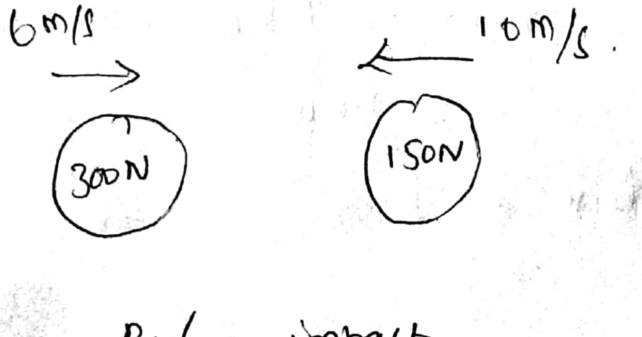
$\therefore$ Total workdone = Total Change in K.E

$461.412 - 723.22\,\mu = 0$

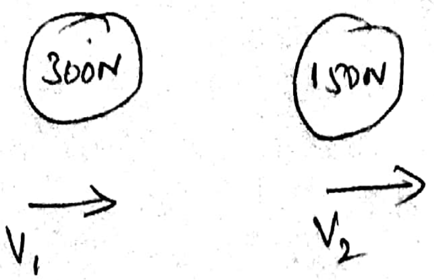
$\mu = 0.637$

(7) Direct central impact occurs between a 300N body moving to the right with a velocity of 6 m/s and 150 N body moving to the left with a velocity of 10 m/s. Find the velocity of each body after impact if the Co-efficient of restitution e is 0.8.

Sol:



Before impact



After impact.

From the principle of Conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$u_1, u_2 \rightarrow$ Before impact Velocity

$v_1, v_2 \rightarrow$ After "

$$300(6) + 150(-10) = 300 v_1 + 150 v_2$$

$$2 v_1 + v_2 = 2 \quad \text{——— (1)}$$

From Co-efficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$0.8 = \frac{v_2 - v_1}{6 - (-10)}$$

$$v_2 - v_1 = 12.8 \quad \text{——— (2)}$$

by solving (1) & (2)

$$v_1 = -3.6 \text{ m/s}$$

$$v_2 = 9.2 \text{ m/s.}$$

Two Marks

① Differentiate between kinematics & kinetics.

Kinematics is the branch of mechanics deals with the motion without reference to force or mass.

Kinetics is the branch of mechanics which deals with the forces that cause motion of the bodies.

② State the principle of work and energy.

Principle of work and energy states that "The change in kinetic energy is equal to the total work done by the particle".

③ What is D'Alembert's principle?

The body will be in equilibrium under the action of external force (F) and the inertia force (−ma).

④ What do you mean by impact of elastic bodies?

The phenomenon of collision of two bodies, occurs in a very small interval of time and during which the two bodies exert a very large force and each other is called an impact.

(5) State Newton's law concerning equilibrium of particle.

Everybody continuous in its state of rest or uniform motion in a straight line, unless it is compelled by some external force to change that state.

(6) A stone is dropped from the top of a tower. It strikes the ground after four seconds. Find the height of the tower.

$t = 4\ s.$

$u = 0$

$a = 9.81\ m/s^2$

$S = ut + \frac{1}{2} at^2$

$S = 0 + \frac{1}{2}\ 9.81\ (4)^2$

$S = 78.48\ m.$

(7) What is impulse force?

The impulsive force is defined as the force which acts for a very short time and yet produces a great change of momentum on the bodies on which it acts.

e.g. blow of hammer.

collision of two bodies.

(8) State the law of Conservation of Momentum.

The law of Conservation of Momentum states that "Total momentum of any group of objects always remains constant, provided if no external forces area acting on them."

(9) Define Co-efficient of restitutions.

$$e = \frac{\text{Relative Velocity of Separation}}{\text{Relative Velocity of approach}}.$$

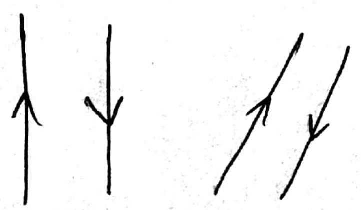
(10) What is projectile?

When the object is thrown into space, the traces of the object makes parabolic curve. This is called projectile.

(11) What is Rectilinear Motion?

When the object moves along the same line of action, that motion is called Rectilinear motion.

e.g.



(12) Write the equations for impulse Momentum method.

$$K.E_{1} + W.D = K.E_{2}$$

$$m.V_1 + \Sigma F_{\text{along motion}} . \Delta t = m.V_2$$