UNIT I -CLASSIFICATION OF SIGNALS AND SYSTEMS
PART A

1. What are the major classifications ofsignals?
Signals are classified as Continuous Time (CT) and Discrete Time(DT) signals.
Both CT and DT signals are further classified as
Deterministic and Random signals, Even and Odd signals, Energy and Power signals,
Periodic and Aperiodic signals

2. With suitable examples distinguish a deterministic signal from a random signal. Define a random signal.(Nov2019)
Deterministic signal: A signal which can be modeled (represented) by a mathematical equation.
Example: cosine signal
Random signal: A signal which cannot be modeled by a mathematical equation is called random signal. **Example:** Speech Signal

3. Define energy signal and power signal.(May2015)
A signal x(t) is said to be energy signal if , Energyisfinite   i.e. $0 < E <\infty$ andaverage power is zero i.e. P=0 Where E =energy and   P = Averagepower
A signal x(t) is said to be power signal if powerisfinite   i.e $0 < P <\infty$ and energy is infinitei.e.
E = $\infty$where E =energy   and   P = Averagepower

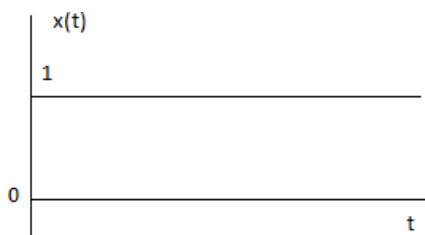**4. Givethemathematicaland graphicalrepresentationoframpsequence.(Dec2018)**
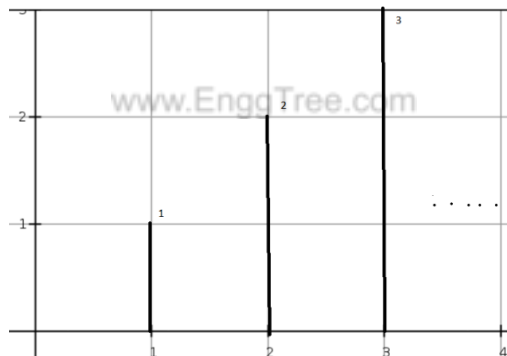
$r[n] = n$ for $n \geq 0$

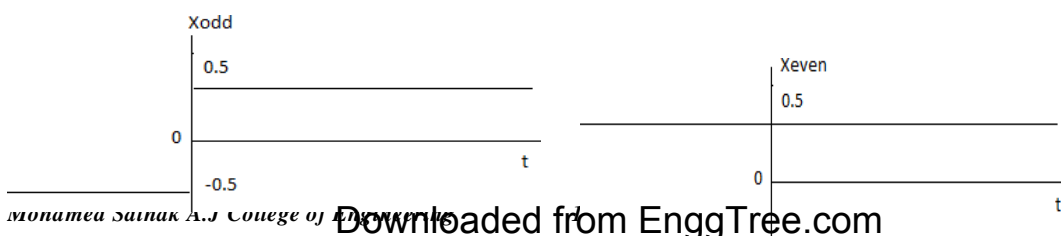**5. Evaluate the integral $\int_{-1}^{1}(2t^2 + 3)\delta(t)dt$. (Dec2018)**
Weknowthat, $\int_{-\infty}^{\infty} \emptyset(t)\delta(t - t_0)dt = \emptyset(t_0)$
Therefore t_0=0 and Answer=$2(0)^2+3=3$

6. Find the even and odd part of the signal?(Apr2019)

Odd part of the signal x(t)=x(t)-x(-t)/2 Even part of the signal x(t)=x(t)+x(-t)/2

7. State two properties of unit impulse function. (Dec2014)

Shifting property:

signal with shifted impulse simply shifts the signal.

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_o)\,dt = x(t_o)$$

$\delta(t)$ is the limit of graphs of area 1, the area under its graph is $1.\delta(t)$peak response at origin.

8. Define symmetric and anti-symmetric signal. Symmetric signal: It is a even signal, A signal $x(-t)=x(t)$.

$x(t)$ is said to be symmetric signal if

Example: $x(t) = A\cos\omega t$

Anti symmetric signal:

A signal $x(t)$ is said to be anti-symmetric signal if

$x(-t) = -x(t)$ .Example: $x(t) = A\sin\omega t$

8. Verify whether $x(t) = Ae^{-at}u(t)$, $a > 0$ **is an energy signal or not.**

$x(t) = Ae^{-at}u(t)$, $a > 0$:

$$Energy = \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2\,dt = \lim_{T\to\infty}\int_{0}^{T}|Ae^{-at}|^2\,dt = \lim_{T\to\infty}\left|A^2\frac{e^{-2at}}{-2a}\right|_0^T = \frac{A^2}{2a}\ Joules$$

$$power = \lim_{T\to\infty}\frac{1}{2T}\int_{=T}^{T}|x(t)|^2\,dt = \lim_{T\to\infty}\frac{1}{2T}\left|A^2\frac{e^{-2at}}{-2a}\right|_0^T = 0\ Watts$$

Energy is finite, Power is zero. The signal is energy signal

9. Determine the power and RMS value of the signal $x(t) = e^{jat}\cos\omega t$ ·$_o$

$$power = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|e^{jat}\cos\omega_0 t|^2\,dt = \lim_{T\to\infty}\frac{1}{4T}(\int_{-T}^{T}1 + \cos2\omega_0 t\,dt) \qquad since\ |e^{jat}| = 1$$

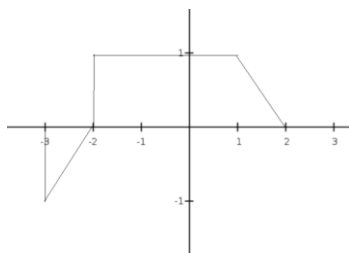$$= \lim_{T\to\infty}\frac{1}{4T}(2T) = \frac{1}{2}watt \qquad ;RMS value = \sqrt{\frac{1}{2}}$$

10. Find the average power of the signal $u(n) - u(n-N)$.
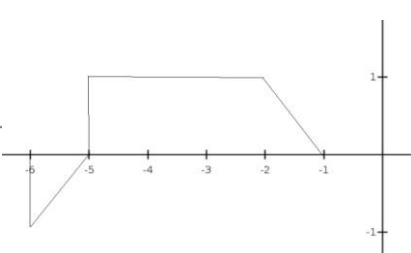
Average power of a DT signal x(n) is

$$Power = \lim_{N\to\infty}\frac{1}{(2N+1)}\sum_{n=-N}|x(n)|^2 = \lim_{N\to\infty}\frac{1}{(2N+1)}\sum_{n=0}(1)^2 = 0\ watt$$

12. Plot x(3-5t) (May 2018)

i)x(t)   **ii)x(t+3)**

13. Distinguish static system from dynamic system.

Static system: Static system is a system with no memory or energy storage element. Output of a static system at any specific time depends on the input at that particular time.

Dynamic system:

Dynamic systems have memory or energy storage elements. Output of a dynamic system at any specific time depends on the inputs at that specific time and at other times.

14. Define a time invariant system.

A system is said to be time invariant if its input-output characteristics do not change with time.

Let $y(t) = F[x(t)]$; *F denotes some transformation (operation) on x(t)*; x(t)-input, y(t)- output

Let $y(t, t_o)$ denote the output due to delayed input $x(t-t_0)$ i.e., $y(t, t_0) = F[x(t - t_0)]$

let $y(t - t_0)$ be the output delayed by t_0 if $y(t - t_0) = y(t, t_0)$ then the system is time invariant

15. Define a continuous time LTI system. Give the conditions for a system to be LTI system. (Dec2013)

A continuous time system which posses two properties i) linearity (Obeys superposition principle) ii) Time invariance(Input –output characteristics do not vary with time) is a CT LTI system.

16. Determine whether the system described by the following input-output relationship is linear and causal y(t) =x(-t)

$y(t) = x(-t) \rightarrow input - output relationship$ $y(t) = output$ & $x(t) = input$

Checking for linearity:

For an input $x_1(t)$, the output $y_1(t)$ is, $y_1(t) = x_1(-t)$

For an input $x_2(t)$, the output $y_2(t)$ is, $y_2(t) = x_2(-t)$

For an input $a x_1(t) + b x_2(t)$, the output $y_3(t)$ is, $y_3(t) = a x_1(-t) + b x_2(-t)$

$y_3(t) = a y_1(t) + b y_2(t)$ The system obeys superposition principle. Therefore the system is linear

Checking for causality:

For $t = -1$, $y(-1) = x(1)$ For negative values of time 't', the output depends on the future input. Therefore the system is non-causal.

17. Compute the average energy and power of the signal x(t)=r(t)-(r-2)(May2018)

$$\text{Energy} = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_0^2 t^2 \, dt + \int_2^{+\infty} 2 dt = \infty$$

$$\text{Power} = \lim_{T \to \infty} \left(\frac{1}{2T}\right) \int_{-T}^{+T} |x(t)|^2 dt = \lim_{T \to \infty} \left(\frac{1}{2T}\right) \left\{ \int_0^2 |t|^2 dt + \int_2^T 4 dt \right\} = 2W$$

Energy is infinite, Power is 2W. The signal is power signal

18. Check whether the following system is static (or) dynamic and causal (or) non-causal: $y(n) = x(2n)$ (Dec 2012)

For a given 'n' the output depends on the future input. Therefore the system is non-causal. The system is a dynamic system.

19. Verify whether the system described by the equation is linear and time invariant. $y(t)=x(t^2)$

Linearity:

$y(t)=x(t^2)$; $y(t)=F[x(t)]=x(t^2)$

For an input $x_1(t)$, $y_1(t)=F[x_1(t)]=x_1(t^2)$

For an input $x_2(t)$, $y_2(t)=F[x_2(t)]=x_2(t^2)$

Weighted sum of outputs is given by $ay_1(t)+by_2(t)=ax_1(t^2)+bx_2(t^2)$

Output due to weighted sum of inputs is $y_3(t)=F[ax_1(t)+bx_2(t)]=[ax_1(t^2)+bx_2(t^2)]$
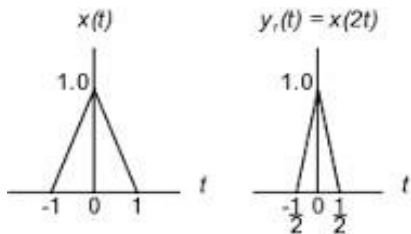
Therefore, the system is linear.

Time invariance:

$y(t)=x(t^2)$, $y(t)=F[x(t)]=x(t^2)$

If the input is delayed by k units of time then the output is , $y(t,k)=F[x(t-k)]=x((t-k)^2)$

Output delayed by k units of time is, $y(t-k)=x(t^2-k)$, $y(t,k) \neq y(t-k)$

    Therefore, the system is time –variant.

20. Sketch the following signal $y(t)=x(2t)$**(May2014)**

21. State the Difference between Causal and non-causal system (Nov2016)

Causal System: A system is said to be causal if the present output depends on present input and past input.

Non-Causal System: A system is said to be Non-causal if the present output depends on future input.

22. Check whether the following system is Time Invariant/ Time variant and also causal/ non-causal: Y(t)=X(t/3). (Nov2017)

A system is said to be time invariant if its input-output characteristics do not change with time.

Let Y(t)=X(t/3); *F denotes some transformation* (*operation*) *on x(t)* ;x(t)-input, y(t)- output

Let $y(t,t_o)$ denote the output due to delayed input X(t/3-t_0)   i.e, $y(t,t_0)=F[x(t-t_0)]$

let $y(t-t_0)$ be the output delayed by t_0 if $y(t-t_0)=y(t,t_0)$ then the system is time invariant

23. Determine if the signal X[n]=sin($\frac{6\pi n}{7}+1$) given below is periodic. If yes, give its fundamental period. If not, state why it is aperiodic. (Nov2017)

For given problem $w=\frac{6\pi}{7}=2\pi f$

$f=\frac{3}{7}=\frac{k}{N}$, *periodic signal with fundamental period* $N=7$

24. Define a Linear systems? (Apr2017)

A system is said to be linear, it should satisfy superposition principle i.e additivity and Homogeneity property.
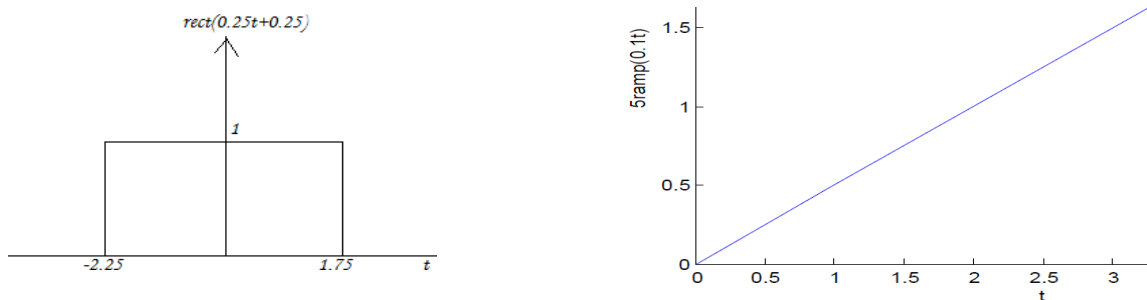
$$F(a_1x_1(t)+a_2x_2(t))=a_1y_1(t)+a_2y_2(t)$$

$x_1(t), x_2(t)$ *are input signals* $y_1(t), y_2(t)$ *are output signals*

25. Give the mathematical and graphical representation of a continuous time and discrete time unit impulse functions.(Nov2016)

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \qquad \delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$
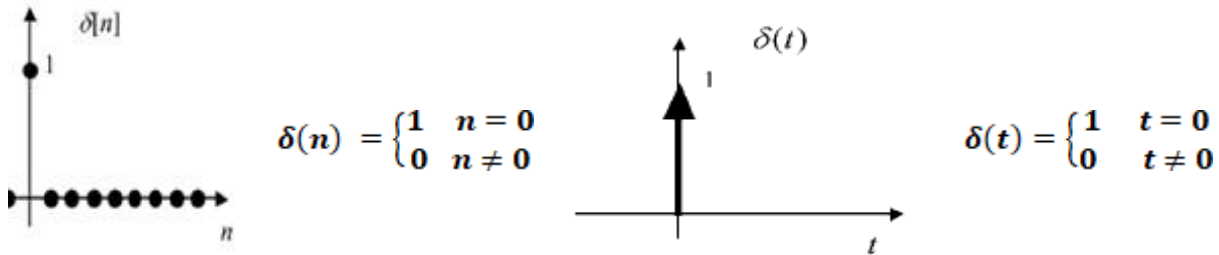
26. Sketch thefollowingsignals: $\text{rect}\left(\frac{t+1}{4}\right); 5\text{ramp}(0.1t)$ (May2016)

27. Fine the summationof $x(n) = \sum_{n=-\infty}^{+\infty} \delta(n-1)\sin 2n$ (May2017)

$$\delta(n-1) = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases}$$

x(n) = Sin2=0.0348

28. Determine whether the given discrete time sequence is periodic or not. If the sequence is periodic, find the fundamental period. $x(n)=\cos(n/8)+\cos(n\pi/8)$.**(Apr 2019)**
N1=2π/(1/8)=16π:N2=2π/(π/8)=16. Since N1 and N2 cannot be represented as ratio of integers the given signal isaperiodic.

29. Determine whetherthesignal $x(t) = \sin\sqrt{2}t$ is periodic or not.(Nov2019)
To find the fundamentalperiodT=2π/ω=2π/ $\sqrt{2}$=$\sqrt{2}$π
Since this cannot be expressed as ratio of integers. Therefore, the given signal is not periodic.

30. DetermineaveragepowerP∞forthesignalx(t)=2cos(t).(Nov2020)
The general form is Acos2πft.
2πft = t ; f=1/2π ; 1/T=1/2π; T=2π
Power:

$$P = \lim_{T \to \infty} \frac{1}{T} \int_0^T |x(t)|^2 dt$$

$$P = \lim_{T \to \infty} \frac{1}{2\pi} \int_0^{2\pi} |2cost|^2 dt$$

$$P = \lim_{T \to \infty} \frac{1}{2\pi} \int_0^{2\pi} 4cos^2 t \, dt$$

$$P = \lim_{T \to \infty} \frac{2}{\pi} \int_0^{2\pi} cos^2 t \, dt$$

$$P = \lim_{T \to \infty} \frac{2}{\pi} \int_0^{2\pi} \left(\frac{1 + cos2t}{2}\right) dt$$

$$P = \lim_{T \to \infty} \frac{1}{\pi} \int_0^{2\pi} (1 + cos2t)\, dt$$

$$P = \lim_{T \to \infty} \frac{1}{\pi} \left\{ \int_0^{2\pi} t\, dt + \int_0^{2\pi} cos2t\, dt \right\}$$

$$P = \lim_{T \to \infty} \frac{1}{\pi} |t|_0^{2\pi}$$

In the above equation cosine waves are integrated over full cycles. Therefore the integration will be zero.

$$P = \lim_{T \to \infty} \frac{1}{\pi} (2\pi)$$

P=2W

31. Express discrete time unit impulse signal in terms of discrete time unit step signal and express discrete time unit step signal in terms of discrete time unit impulse signal. (Nov2020)
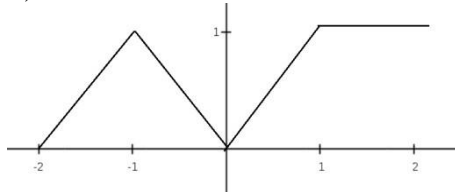
$\delta[n]=u[n]-u[n-1]$

$$u[n] = \sum_{k=0}^{\infty} \delta[n - k]$$

PART B(C204.1)

1.i) Draw the waveform for the signal x(t)=u(t)+r(t)-2r(t-1)+r(t-2)-u(t-2).

ii)Determine and sketch the odd and even part of the following signal

iii)A continuous system is given by $y(t) = \int_{-\infty}^{2t} x(t)\,dt$ is Linear/Time invariant/static/causal.

(May 2018)

2.(i)A continuous system is given by $y(t) = \begin{cases} 0 & \text{for } x \geq 0 \\ x(t) + x(t - 2) & \text{for } x(t) < 0 \end{cases}$ check whether the system is Linear/Time invariant/static/causal.

(ii)Draw the waveform for the signal x(t)=r(t)-2r(t-1)+r(t-2)

(iii)Find whether the signal is periodic or not.

$x(t) = e^{j(2\pi/3)^n} + e^{j\left(\frac{2\pi}{4}\right)n}$ **(May2018)**

3.i) Define an energy and power signal

ii)Determine whether the following signals are energy or power and calculate their energy or power.(**May 2013**)

(i) $x(n)= \left\lceil \frac{1}{2} \right\rceil^n u(n)$ (ii) x(t) =rect$\left\lceil \dfrac{t}{T_o} \right\rceil$ (iii)$x(t) = cos^2(\Omega_o t)$

4. Determine whether the discrete time system   y(n)=x(n)cos(ωn) is(i)memoryless

(ii) Stable (iii) causal(iv) linear (v)time invariant.(**Dec2013**)

5.i)Determine whether the signal x(t)=sin20πt+sin5πt is periodic and if it is periodic find the fundamental period(**Dec2013**)

ii)Discuss various forms of real &complex exponential signals with graphical representations(**Dec2013**)

iii) State the precedence rule for combined time scaling and time shifting operation.

6. Check whether the system is linear,causal,time invariant and or stable(**Apr/Dec2019**)

i)y(n)= nx(n) ii) y(n)=x(n)-x(n-1) iii) $y(t)= \dfrac{d}{dt}x(t)$

7. (i) Given $x(t) = \dfrac{1}{6}(t+2)$ $-2 \le t \le 4$ *otherwise* 0   Sketch (1)x(t) (2) x(t+1) (3) x(2t) (4)x(t/2).

(ii) Determine whether the discrete time sequence

$x[n] = \sin\left(\dfrac{3\pi}{7}n + \dfrac{\pi}{4}\right) + \cos\left(\dfrac{\pi}{3}\right)$ is periodic or not.**(May2014)**

8. Check the following systems are linear, stable

(i) $y(t) = e^{x(t)}$ (ii) $y(n) = x(n-1)$. **(May2014)**

11. Given $x(n) = [1,4,3,-1,2]$. Plot the following signals **(Dec2015)**

i)$x(n-1)$ ii)$x\left(\dfrac{-n}{2}\right)$ iii)$x(-2n+1)$ iv)$x\left(\dfrac{-n}{2}\right)+2$

9. Given the input-output relationship of a continuous time system y(t)=tx(-t). Determine whether the system is linear, causal ,time invariant and stable **(Dec2015)**

11. i)Draw the waveforms represented by the following step functions.

$f_1(t)=2u(t-1)$, $f_2(t)=-2u(t-2)$,$f(t)= f_1(t)+ f_2(t)$,$f(t)= f_1(t)- f_2(t)$.

ii)Determine the energy and power of the given signal x(t)=tu(t).

iii)Check whether the system is linear or not $y(t)=x^2(t)$.**(Apr 2019)**

10.i)Check if x(t)=4cos(3πt+π/4)+2cos(4πt) is periodic.**(May2015)**

ii) For the system y(n) = log[x(n)], Check for linearity, causality, time invariance and stability.

11. (i) Find whether the following signals are periodic or aperiodic. If periodic find the fundamental period and fundamental frequency

$x_1(t) = \sin 2\pi t + \cos \pi t$,$x_2(n) = \sin\left(\dfrac{n\pi}{3}\right)\cos\left(\dfrac{n\pi}{5}\right)$**(May2016)**

(ii) Find whether the following signals are power or energy signals. Determine power and energy of the signals.

$x(t) = 5\cos\left(17\pi t + \dfrac{\pi}{4}\right) + 2\sin\left(19\pi t + \dfrac{\pi}{3}\right)$,$x(n) = (0.5)^n u(n)$**(May 2016)**

12. Find whether the following systems are time invariant, linear, stable, Memoryless and causal y(t)=t x(t-1). **(Dec2018)**
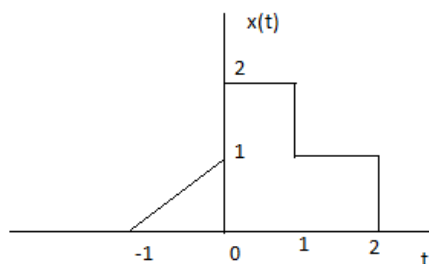
13. Determine whether the system is linear, Time Invariant, Causal and memoryless

$y(t) = \dfrac{1}{2}\int_{-\infty}^{t} x(z)\,dz$ **(Dec2016)**

14. A discrete time signal is given as x[n]={1,2,1,2,1,2,1}Plot the following signals i)x[n -1] ii)x[n/2] iii)x[n/2-1] iv)x[-n/2-1] v)x[2n] **(Dec 2018/2019)**

15.i) A continuous time signal x(t) is shown in figure below, sketch and label each of the following signals. x(t-2),x(2t+3) and x(-t+1)

ii) Determine the energy and power of the given signal x[n]=cos(πn/4) **(Apr 2019)**

16. (i)Find out whether the following signals are periodic or not. If periodic, find the period Determine whether

$x(t) = 2\cos(10t+1) - 1\sin(4t-1)$, $x(n) = \cos(0.1\pi n)$ **(May 2017)**

(ii) Find out whether the following signals are energy or power signal or neither power nor energy. Determine power or energy as the case may be for the signal

$x(t) = u(t) + 5u(t-1) - 2u(t-2)$

17. Determine whether the following given systems are Linear, Causal, Time invariant and dynamic **(May2017)**

(i) $\dfrac{d^2 y(t)}{dt^2} + 3t\dfrac{dy(t)}{dt} + y(t) = x(t)$

(ii) $y_1(n) = x(n^2) + x(n)$

(iii) $y_2(n) = log_{10} x(n)$

18. i)Considerthesystemdescribedbytheinputoutputrelation,y(t)=[cos(3t)]x(t).Here x(t)standsforinputandy(t)foroutput.Statewithjustificationwhetherthesystemislinear   and/or timeinvariant.

ii) Consider the system described by the input output relation, y(t) = [cos (3t)]x(t). Here x(t) standsforinputandy(t)foroutput.Statewithjustificationwhetherthesystemislinearand/or timeinvariant.

iii) State whether the LTI system described by impulse response h[n]= $(1/4)^n$u[-n] is causal and stable with justification.**(Nov2020)**

19. i)Forthesignalx(t)showninFigure,sketch

$$x\left(2 - \frac{t}{2}\right)$$

ii) Sketchtheevenandoddpartofthesignalx(t)showninFigure.
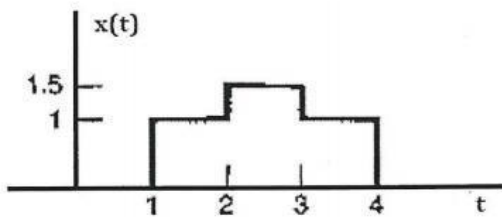
iii) Let x[n]=u[n+4];h[n]=δ[n]-δ[n-2]. Sketch the convolution of x[n] and h[n]. **(Nov**

2020)UNIT II-ANALYSIS OF CONTINUOUSTIME SIGNALS (C204.2)
PART-A
1. State the conditions for convergence of Fourier series.(May2017)

The Fourier Series exists only when the function x(t) satisfies the following three conditions:

a) Thefunctionx(t)haveonlyafinitenumberof maximaandminima.

b) Thefunctionx(t)haveafinitenumberofdiscontinuities.

c) $x(t)$ is absolutelyintegrable.

i.e, $\int\limits_{0}^{T} |x(t)| \, dt < \infty$

2. State Dirichlet's conditions for Fourier Transform.(May 2018/Dec2018)

The Fourier transform does not exist for all aperiodic functions. The conditions for x(t) to have Fourier transform are:

a) x(t)isabsolutelyintegrableover$(-\infty,\infty)$

ie., $\int\limits_{-\infty}^{\infty} |x(t)| \, dt < \infty$

b) x(t)hasfinitenumberofdiscontinuitiesandafinitenumberofmaximaandminimain every finite timeinterval.

3. GivetheFouriertransformandInverseFouriertransformpairequation.

$X(j\Omega) = F[x(t)]$ and $x(t) = F^{-1}[X(j\Omega)]$

$$X(j\Omega) = \int\limits_{-\infty}^{\infty} x(t)e^{-j\Omega t} \, dt \qquad for\ all\ \Omega$$

$$x(t) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t} \, d\Omega \qquad for\ all\ t$$

4. If X(jΩ) is the Fourier transform of the signal x(t), what is the Fourier transformof the signal x(3t) in terms X(jΩ)? (Dec2018)

According to the property of Fourier of transform

$x(at) \xrightarrow{FT} \frac{1}{|a|} X\left(\frac{j\Omega}{a}\right)$ Therefore $x(3t) \xrightarrow{FT} \frac{1}{3} X\left(\frac{j\Omega}{3}\right)$

5. State convolution (time) property of Fourier transform.

$$F[x(t)] = X(j\Omega) \; ; \; F[h(t)] = H(j\Omega)$$

$$F[x(t) * h(t)] = X(j\Omega)H(j\Omega)$$

6. What is the Fourier transform of $x(t) = e^{-at}u(t)$? (Dec 2017)

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt = \int_0^\infty e^{-at}e^{-j\Omega t}dt = \left[-\frac{e^{-(a+j\Omega)t}}{(a+j\Omega)} \right]_0^\infty = \left(\frac{1}{j\Omega + a} \right)$$

7. What is the Inverse Fourier transform of $X(j\Omega) = \dfrac{1}{(a+j\Omega)^2}$?

$$x(t) = t \, e^{-at} u(t)$$

8. Find the Fourier transform of the impulse signal.

$$x(t) = \delta(t) \quad ; X(j\Omega) = \int_{-\infty}^{\infty} \delta(t) \, e^{-j\Omega t} \, dt = 1$$

9. Determine Laplace transform of $x(t) = e^{-at}\sin(\Omega t)u(t)$.

$$L[\sin(\Omega t)u(t)] = \frac{\Omega}{s^2 + \Omega^2}$$

$$X(S) = L[e^{-at}\sin(\Omega t)u(t)] = \frac{\Omega}{(s+a)^2 + \Omega^2}$$

10. Find the ROC of the Laplace transform of $x(t) = u(t)$. (Nov 2014)

$$X(s) = \frac{1}{(s+1)} \; ; \; \text{Re}\{s\} > -1$$

The ROC of u(t) is Re{s}>-1.

11. What is the inverse Laplace transform of $\dfrac{1}{(s+2)}$; Re$\{s\} < -2$

Inverse Laplace transform of $\dfrac{1}{(s+2)}$; Re$\{s\} < -2$ is, $-e^{-2t} u(-t)$

12. What is the inverse Laplace transform of $\dfrac{1}{(s+1)}$; Re$\{s\} > -1$

Inverse Laplace transform of $\dfrac{1}{(s+1)}$; Re$\{s\} > -1$ is, $e^{-t}u(t)$

13. Define ROC of Laplace transform. (Dec 2012)

Laplace transform of x(t) is given by the following formula

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

The range of values of 's' for which the integral in the equation converges is referred to as the region of convergence (ROC).

14. What is the Laplace transform of (i) u(t) (ii) t u(t) ? Also specify the ROC.

(i) Laplace transform of u(t) $= \int_0^\infty e^{-st}dt = \left[-\frac{e^{-st}}{s} \right]_0^\infty = \frac{1}{s}$ $ROC: \; \text{Re}\{s\} > 0$

(ii) Laplace transform of $tu(t) = \int\limits_{0}^{\infty} te^{-st}dt = \left[-\left(\dfrac{te^{-st}}{s} - \dfrac{e^{-st}}{s^2} \right) \right]_{0}^{\infty}$

$= \dfrac{1}{s^2}$ ROC: $Re\{s\} > 0$

15. Determine the Laplace transform and ROC for the signal $y(t) = -e^{at}u(-t)$.

$Y(s) = \int\limits_{-\infty}^{\infty} -e^{-at}u(-t)\, e^{-st}\, dt = -\int\limits_{-\infty}^{0} e^{-(s-a)t}\, dt = \dfrac{1}{(s-a)}$ ROC : $Re\{s\} < a$

16. Determine the Laplace transform and ROC of x(t)=u(t-5).(May2012)

$x(t) \longleftarrow LT \longrightarrow X(s)$ $; x(t-a) \longleftarrow LT \longrightarrow e^{-as}X(s)$

Laplace Transform of $u(t-5) = e^{-5s}L[u(t)] = e^{-5s} \times \dfrac{1}{s}$

$L[u(t-5)] = \dfrac{e^{-5s}}{s}$ ROC : $Re\{s\} > 0$

17. What is the unilateral Laplace transform of $\dfrac{d}{dt}x(t)$?

$\dfrac{dx(t)}{dt} \longleftarrow\!\!\overset{unilateral}{LT} \longrightarrow s\, X(s) - x(0^-)$

18. State the convolution (in time) property of Laplace transform.

$x(t) \longleftarrow LT \longrightarrow X(s); h(t) \longleftarrow LT \longrightarrow H(s)$

$x(t) * h(t) \longleftarrow LT \longrightarrow X(s)H(s)$

19. State the time scaling property of Laplace transform.(May2013)

$x(t) \longleftarrow LT \longrightarrow X(s)$ $ROC : R$

$x(at) \longleftarrow LT \longrightarrow \dfrac{1}{|a|}X\!\left(\dfrac{s}{a}\right)$ $ROC : \dfrac{R}{|a|}$
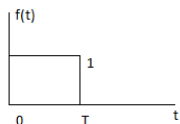
20. State initial value theorem and final value theorem of Laplace transform.

$L[x(t)] = X(s)$

Initial Value theorem: $\underset{t\to 0}{lt}\; x(t) = \underset{s\to\infty}{lt}\; s\, X(s)$

Final Value theorem: $\underset{t\to\infty}{lt}\; x(t) = \underset{s\to 0}{lt}\; s\, X(s)$

21. Find the Laplace transform of the following signal. (Apr2019)

$X(s) = \int\limits_{-\infty}^{\infty} x(t)e^{-st}dt = \int\limits_{0}^{T} e^{-st}dt = \dfrac{1-e^{-sT}}{s}$

22. Give synthesis and analysis equations of CT Fourier transform.(Dec2012)

$X(j\Omega) = F[x(t)]$ and $x(t) = F^{-1}[X(j\Omega)]$

$X(j\Omega) = \int\limits_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}dt$ for all $\Omega \to$ analysis equation

$x(t) = \dfrac{1}{2\pi}\int\limits_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t}d\Omega$ for all $t \to$ synthesis equation

23. **Consider a periodic signal x(t) with fundamental frequency 2π and $a_0=1, a_{-1}=1/4, a_2=a_{-2}=1/2, a_3, a_{-3}=1/3$. Express x(t) in general Fourier series formula. (May2018)**

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$$

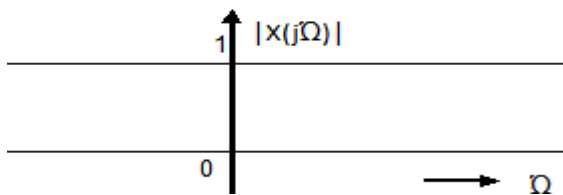*from the given problem* $\Omega_o = 2\pi$

From the given equation, $x(t) = \frac{1}{3}e^{-j6\pi t} + \frac{1}{2}e^{-j4\pi t} + 1 + \frac{1}{2}e^{j4\pi t} + \frac{1}{3}e^{j6\pi t}$

24. What is the Fourier transform of a DC signal of amplitude 1? (May2013)

$x(t) = 1$

$$X(j\Omega) = \int_{-\infty}^{\infty} 1 \cdot e^{-j\Omega t} dt = 1$$

$|X(j\Omega)| = 1$ *for all* Ω $angle(X(j\Omega))=0$ *for all* Ω

25. Give the Laplace Transform of $x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$ with ROC. (May 2016)

$$L(x(t)) = X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \quad, \quad L(e^{-at}u(t)) = \frac{1}{S+a}$$

$$L(x(t)) = X(S) = \frac{3}{S+2} - \frac{2}{S+1} = \frac{S-1}{(S^2+2S+2)} \quad Re\{S\} > -1$$

26. What is the Laplace transform and Fourier Transform of $\delta(t)$? (Apr2015)

$$L(x(t)) = X(S) = \int_{-\infty}^{\infty} \delta(t) e^{-St} dt = 1$$

$$F(x(t)) = X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt = 1$$

27. State Gibb's Phenonmenon. (Nov2019)

The Gibbs phenomenon is an overshoot (or "ringing") of Fourier series and other Eigenfunction series occurring at simple discontinuities. Gibbs phenomenon occurs due to the non- uniform convergence of the Fourier series at a discontinuity. Thus, the frequency response so obtained contains ripples in the frequency domain.

28. State any two properties of ROC of Laplace transform X(S) of a signal x(t). (May2014)

i) If x(t) is absolutely integrable and of finite duration, then the ROC is the entire s-plane (the Laplace transform integral is finite, i.e., X(S) exists, for any s.

ii) The ROC of X(S) consists of strips parallel to the $j\omega$-axis in the s-plane.

iii) If x(t) is right sided and Re(s)=σ is in the ROC, then any s to the right of σ (i.e., Re(s)>σ) is also in the ROC, i.e., ROC is a right sided half plane.

iv) If x(t) is left sided and Re(S)= σ is in the ROC, then any s to the left of σ (i.e., Re(S)<σ) is also in the ROC, i.e., ROC is a left sided half plane.

29. Draw the Spectrum of CT Rectangular Pulse?(Apr2015)
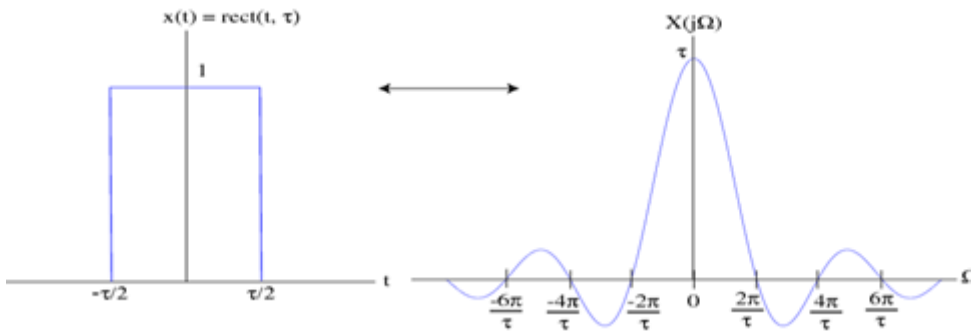
30. What is the inverse Fourier transform of (i) $e^{-j2\pi f t_o}$ **(ii)** $\delta(f - f_o)$ **(May 2016)**

(i)$\delta(t - t_0)$ (ii)$\frac{1}{2\pi} e^{jf_0 t}$

31. Find the Fourier coefficients of the signal.(Apr/Dec2019)

$$x(t) = 1 + \sin\left(\frac{\pi}{2}\right)t$$

Linear combination of harmonically related complex exponential of the form

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_o t}$

$= 1 + \frac{1}{2j} e^{j\left(\frac{\pi}{2}\right)t} - \frac{1}{2j} e^{-j\left(\frac{\pi}{2}\right)t}$

On comparing the above equations $a_0 = 1, a_1 = 1/2j, a_{-1} = -1/2j$

32. Find the Laplace transform of $x(t) = e^{-at} u(t$ **(Dec2016)**

$$L(x(t)) = X(S) = \int_{-\infty}^{\infty} x(t) e^{-St} dt ,$$

$$X(S) = \int_{0}^{\infty} e^{-at} e^{-st} dt = \int_{0}^{\infty} e^{-(S+a)t} dt = \left[\frac{e^{-(S+a)t}}{-(S+a)}\right]_{0}^{\infty} = \left[\frac{e^{-\infty}}{-(S+a)} + \frac{1}{S+a}\right]$$

$$= \frac{1}{S+a} \quad Re\{s\} > -a$$

33. Find the Laplace transform of following impulse response h(t),h(t)= te⁻ᵗ u(t).
(Dec2017)

$$L(h(t)) = H(S) = \int_{-\infty}^{\infty} h(t) e^{-St} dt$$

The Laplace transform of the given signal is H(s)=1/(s+1)²

34. StateParseval'stheoremforcontinuoustimeaperiodicsignal(Dec2020)

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\Omega)|^2 d\Omega$$

35. DetermineFouriertransformforunitstepsignal.(Nov2020)

sgn(t)=2u(t)-1

$$u(t) = \frac{1}{2}\{1 + sgn(t)\}$$

Taking Fourier transform on both sides,

$$FT\{u(t)\} = \frac{1}{2}\{FT\{1\} + FT\{sgn(t)\}\}$$

$$FT\{u(t)\} = \frac{1}{2}\left[2\pi\delta(\omega) + \frac{2}{j\omega}\right]$$

$$FT\{u(t)\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

**36. DeterminetheLaplacetransformforthesignalx(t)=e^{-4t}u(t)(Nov2020)**

$$L(x(t)) = X(S) = \int_{-\infty}^{\infty} x(t)e^{-St}\,dt \ ,$$

$$X(S) = \int_{0}^{\infty} e^{-4t}e^{-st}\,dt = \int_{0}^{\infty} e^{-t(s+4)}\,dt = \frac{e^{-t(s+4)}}{-(s+4)}$$

$$= \frac{1}{S+4}$$

ROC:Re$\{s\}>$-4

PART B (C204.2)

1. i) Find the Fourier transform of thesigal $x(t) = \begin{cases} 1, & |t| < T_1 \\ 0. & |t| > T. \end{cases}$

ii) Find the Laplace transform of the signal x(t)=e^{-2}tu(t)+e^{-t}cos(3t)u(t)**(May 2018)**

2. i)Using the properties of Fourier transform find X(jω) and sketch itsmagnitudespectrum

$x(t) = e^{-a|t|}u(t): a > 0$, δ(t-5) **(Dec 2018/Apr2019)**

3. Find the inverse Laplace transform of $X(s) = \frac{3s^2+8s+6}{(s+2)(s^2+2s+1)}$**(May2018)**

4. i)DeterminetheFouriertransformrepresentationofthehalfwaverectifieroutput.

ii) Write the properties of ROC of Laplace transform. **(May 2013)**

5. i)FindtheexponentialFourierseriesof thewaveform.**(Dec2013)**
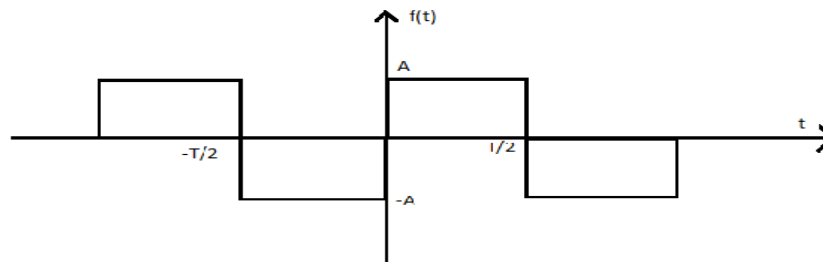
6. i)Find the Laplace transform ofthesignal $x(t) = e^{-a|t|}$ and it ROC and also indicatewhether the Fourier transform exists.**(Dec2020)**

7. Find the Fourier series coefficients of thesignal shown below.**(May 2014)**
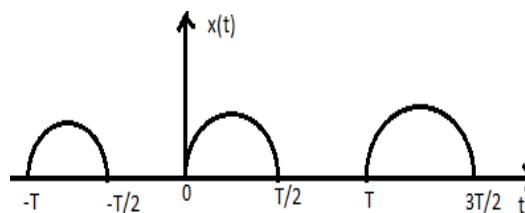

8. Find the inverse Laplace transform of $X(S) = \frac{1}{(S+3)(S+5)}$for the ROC's(i)-5<Re$\{s\}<$3

(ii) Re$\{s\}>$3

9. FindtheFourierseriescoefficientsofthefollowingsignal.**(Nov2020)**
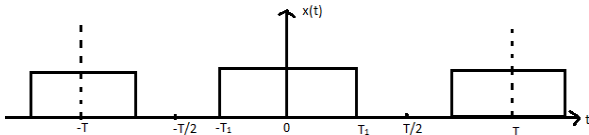
10. Find thespectrumof $x(t) = e^{-2|t|}$.Plotthespectrumofthesignal.**(Dec2014)**

11. State and prove any four properties of Fourier Transform**(Dec2015)**

12. FindtheLaplacetransformanditsassociatedROCforthesignal $x(t) = te^{-2|t|}$**(Dec2015)**

13. i)DeterminetheFourierseriesexpansionforaperiodicrampsignalwithunitamplitudeand a period T(10)

ii) Find the Fourier transform of $x(t) = te^{-at}u(t)$**(May2015)**

14.   i)   If $x(t) \leftrightarrow X(j\Omega)$,   then   using   time   shifting   property   show   that $x(t + T) + x(t - T) \leftrightarrow 2cos\Omega T\ X(j\Omega)$

ii) Find the inverse Laplace transform of $X(S) = \dfrac{8S + 10}{(S + 1)(S + 2)^3}$ **(May 2015)**

15. ObtaintheFourierseriescoefficients&plotthespectrumforgivenwaveform**(May2016)**
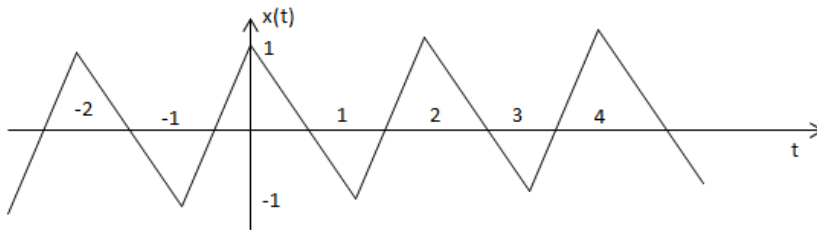
16. (i)Frombasicformul,determinetheFouriertransformofthegiven signals.Obtain magnitudeandphasespectraofthegivensignals.**(May2016)**

a) $x(t) = te^{-at}\ u(t)\ \ a > 0$

b) $x(t) = e^{-a|t|}\ \ a > 0$

(ii) State and prove Rayleigh's Energy theorem. **(May 2016)**

17. Find theFouriertransformofthe Signal$x(t) = cos\ \Omega_o t\ u(t)$**(Dec2016)**

18. Stateandprovethemultiplication&convolutionpropertyofFourierTransform.**(Dec2016)**

19. SpecifyallpossibleROC'sforthefunctionX(s)givenbelow.Andalsofindx(t)ineachcase **(Nov 2019)**

$$X(s) = \frac{4s}{(s + 2)(s + 4)}$$

20. i) Determine the Fourier transform for double exponential pulse whose function is given by x(t)= e^{-a|t|}, a>0. Also draw its amplitude and phase spectra. **(Dec 2017/Dec2019)**
   ii) Obtain the inverse Laplace transform of the function.

$$x(S)=\frac{1}{s^2+3s+2}$$, ROC: -2<Re{s}<-1. **(Dec 2017)**

21. Obtain the trigonometric Fourier co-efficient and write the quadrature form of full wave rectified sine wave. **(May 2017/Apr2019)**

22. DeterminetheinverseLaplacetransformofthefollowing**(May2017)**

$$X(s)= \frac{1 - 2s^2 - 14s}{s(s + 3)(s + 4)}$$

$$X(s)= \frac{2s^2 + 10s + 7}{(s + 1)(s^2 + 3s + 2)}$$

23. i)Considertheperiodicsignal.Derivetheexpression fortheFourierseriescoefficient$a_m$of the complex exponential$e^{jm\omega_0 t}$.
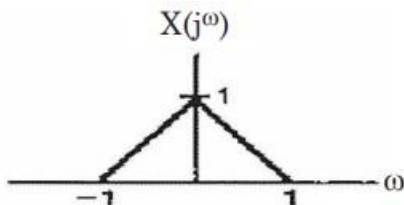
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

ii) Let

$$x(t) = \delta(t) - \frac{2}{3}e^{-2t}u(t) + \frac{1}{3}e^{-4t}u(t)$$

Determine Laplace transform for the signal x(t). Plot pole zero and mark region of convergence
(Nov 2020)

24. Consider a signal x(t) with X(jω) shown in Figure Let p(t) = sin(t).sin(2t). Determine the Fouriertransformforthesignaly(t)generatedbytheproductofx(t)andp(t)givenbyy(t)= x(t).p(t).SketchthespectrumY(jω)**(PartC-Nov2020)**

25. Given x[n] has Fourier transform $X(e^{j\omega n})$ Express Fourier transform for the following signals**(Part C- Nov2020)**
i) $x_1[n]=x[2-n]+x[-2-n]$
ii)) $x_2[n]=(n-1)^2x[n]$

UNIT III-LINEAR TIME INVARIANT CONTINUOUS TIME SYSTEMS (C204.3)
PARTA

1. What is the overall impulse response h(t), when two systems with impulse responses $h_1(t) = e^{-2t}u(t)$ and $h_2(t)=\delta(t-1)$ are in series? (Dec2018)
　　　Over all impulse response h(t) of two LTI systems with impulse responses $h_1(t)$ and $h_2(t)$ connected in cascade(series) is the convolution of the individual impulse responses.
Overall impulse response is 　$h(t) = h_1(t) * h_2(t)$
Given h(t)= $e^{-2t}u(t)*\delta(t-1)$
According to the property ofDiracdelta　$x(t)*\delta(t-t_0)=x(t-t_0)$
Therefore h(t) = $e^{-2(t-1)}u(t-1)$

2. What is the overall impulse response h(t) when two systems with impulse responses $h_1(t)$ and $h_2(t)$ are in parallel? (Nov2019)
Overall impulse response h(t) = sum of the individual impulse responses.
$h(t) = h_1(t) + h_2(t)$

3. Whatarethedrawbacksofrepresentingasystemusingitstransferfunction?
(a) Thetransferfunctiondescribesonlythezerostateresponseofasystem.
(b) It describes only the relationship between the input and output of a system, but doesnot provideanyinformationregardingtheinternalstateofasystem.
(c) It is limited to single input and single outputsystems.
(d) It is applicable only for LTIsystems.

4. Checkwhethergivensystemiscausalandstable.h(t)=e^{-4t}u(t+10).(Apr2019)
If the system is absolutely summable then the system isstable.

$$h(t) = \int_{-10}^{\infty} e^{-4t}\ dt = \frac{e^{40}}{4} = \text{Summable}$$

Sincetheh(t)isnotequalto0fort<0thesystemisnoncausal.

5. If the system $H(s) = 4 - \frac{3}{s+2}$ $Re(s) > -2$, find the impulse response h(t).(Dec2018)
h(t)=$4\delta(t)-3e^{-2t}u(t)$

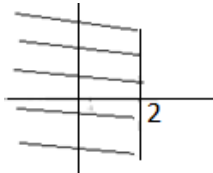**6. Determine the frequency response of the system having impulse response**
$h(t) = \delta(t) - 2e^{-2t}u(t)$ **.**

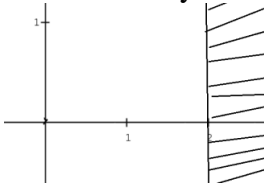Frequency response = Fourier transform of impulse response.

$$F[h(t)] = H(j\Omega) = 1 - \frac{2}{(j\Omega + 2)} = \frac{j\Omega}{(j\Omega + 2)}$$

7. The pole and zero of a function is given by Poles (-3+4j),(2+j),(2-j),(-3-4j) zeros(-5+2j),(-5-2j),(-2),(+3) Plot the ROC when the system is causal and stable.(May 2018)

A stable system must include jω axis in its ROC

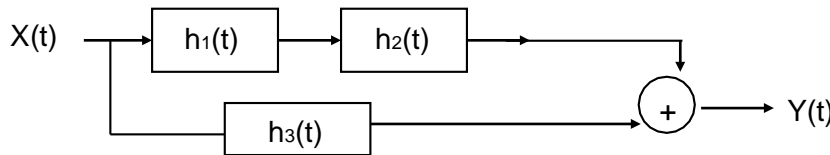For a causal system the function should be right hand sided

8. The impulse response h(n) is given below. Check the system is stable / causal
h(n)=(1/3)^{n}u(n)(May2018)

This system is absolutely summable. Therefore the system is stable.
Since the h(n)=0 for n<0 the system is causal

9. Find the overall impulse response h(t) of the system shown.

Overall impulse eresponse $= h(t) = \big[h_1(t) * h_2(t)\big] + h_3(t)$

10. What is the overall impulse response h(t) when two systems with impulse responses
h$_1$(t)= $\delta(t)$ and h$_2$(t) $=e^{-t}u(t)$ are in series?

Over all impulse response h(t) of two LTI systems with impulse responses h$_1$(t) and h$_2$(t) connected in cascade(series) is the convolution of the individual impulse responses.
Overall impulse response is $\quad h(t) = h_1(t) * h_2(t)$
in 's'domain $\quad H(s) = H_1(s)H_2(s)$

$$H_1(s) = L[\delta(t)] = 1 \qquad \& H_2(s) = \frac{1}{(s+1)}: \ H(s) = 1 \times \frac{1}{(s+1)} = \frac{1}{(s+1)}: h(t) = L^{-1}[H(s)] = e^{-t}u(t)$$

11. Check the stability of the CT system whose impulse response is $h(t) = e^{-3t}u(t)$ **.**

$$h(t) = e^{-3t}u(t); H(s) = L[h(t)] = \frac{1}{(s+3)} = transfer\ function$$

Pole is at s= - 3 which is on the left half of S-plane. Therefore the system is stable.

12. Compare the hardware requirements of Direct form I and Direct form II realization.

Direct form II structures require lesser number of integrators.

13. Define the convolution integral.(May2017)

$$y(t)=h(\tau)*x(\tau); y(t)= \int_{-\infty}^{\infty} h(\tau)\, x\,(t-\tau)\, d\tau,\ where,\ h(t) = impulse\ response,$$

$x(t) = input$

14. What is the condition for a LTI system tobe stable?(May2013)

The poles of the LTI system should be on the left half of S-plane.

15. What are the three elementary operations in block diagram representation of CT system.(Dec2013)

(i) Summing, (ii) Scalar multiplication,(iii)Integration.

16. Check whether the systemis stable $H(S) = \dfrac{1}{S-2}$ **transfer function(Dec2013)**

Pole is at s= 2 which is on the right half of S-plane. Therefore the system is unstable.

17. State the necessary and sufficient condition for an LTI continuous time system tobe causal. (May2014)

An LTI continuous time system is causal if and only if its impulse response is zero for negative values of t.

18. Find the differential equation relating the input and output of a CT system represented by(May2014).

$$H(j\Omega)= \frac{Y(j\Omega)}{X(j\Omega)} = \frac{4}{(j\Omega)^2 + 8\, j\Omega + 4}$$

On cross multiplying, By taking inverse Fourier transform corresponding differential

equation $\dfrac{d^2 y(t)}{dt^2} + 8\dfrac{dy(t)}{dt} + 4\, y(t) = 4.x(t)$

19. The input output relationship is given by the following equation system. Find H(s) for the following system. (Nov2019)

$$\frac{d^2 y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}$$

Take Laplace transform on both sides,

$S^2 Y(s)+sY(s)+2Y(s)=sX(s)$

$H(s)=Y(s)/X(s)=s/(s^2+s+2)$

20. Find y(n)=x(n-1)*δ(n+2).(Dec2014)

From $x(n)*\delta(n-n_0)=x(n_0)$

y(n)=x(n-1+2)=x(n+1)

21. Given the differential equation representation ofa

system $\dfrac{d^2 y(t)}{dt^2} + 2\dfrac{dy(t)}{dt} - 3y(t) = 2.x(t)$ **Find the frequency response H(jΩ) (Dec2015).**

$$H(j\Omega)= \frac{Y(j\Omega)}{X(j\Omega)} = \frac{2}{((j\Omega)^2 + 2\, j\Omega - 3)}$$

22. Find whether the following system whose impulse response is given is causaland stable $h(t) = e^{-2t} u(t-1)$. **(May 2016)**

$$H(S) = \int_{1}^{\infty} e^{-2t} e^{-St} dt = \left(\frac{e^{-t(S+2)}}{-(S+2)}\right)_{1}^{\infty} = \frac{e^{-(S+2)}}{(S+2)}\quad Re\{S\} > -2$$

The given system is causal and stable because poles are located in the left half of S plane i.e poles are having negative real parts.

For causal system ROC is right half of right most pole for stability ROC must includes the $j\Omega$ axis.

23. Realize the block diagram representing the system $H(S) = \frac{S}{S+1}$.(May2016)
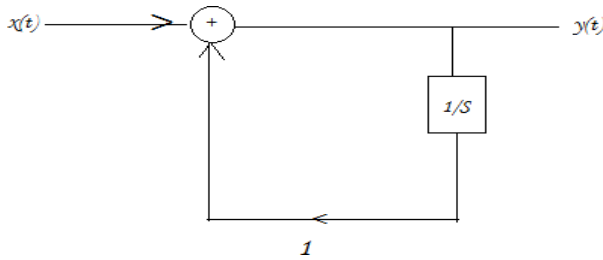
24. Convolve the signals $u(t-1)$ and $\delta(t-1)$ (Dec2016)

The convolution of a signal with shifted impulse simply shifts the signal.

$u(t-1) * \delta(t-1) = u(t-2)$

Proof: $L(x(t) * h(t)) = X(S)H(S) = Y(S)$

$$L\big(u(t-1)\big) = \frac{e^{-S}}{S}, L\big(\delta(t-1)\big) = e^{-S}, i.e \; L\big(u(t-1) * \delta(t-1)\big) = \frac{e^{-2S}}{S}, L^{-1}\left(\frac{e^{-2S}}{S}\right)$$

$$= u(t-2)$$

25. Given $H(S) = \frac{S}{S^2+2S+1}$ Find the differential equation of the system.(Dec2016)

$$\frac{Y(S)}{X(S)} = \frac{S}{S^2 + 2S + 1} = Y(S)(S^2 + 2S + 1) = SX(S)$$

By taking inverse Laplace transform the corresponding differential equation

$$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt}$$

26. will there be two different signals having same laplace transform? Give anexample. How do you differentiate these two signals? (Dec2017)

For example Laplace transform of given signals are same but ROC will differ

$$x(t) = -e^{-at}u(-t), X(s) = \frac{1}{s+a} \; \text{Re}(s) < -a, \quad x(t) = e^{-at}u(t), X(s) = \frac{1}{s+a} \; \text{Re}(s) > -a$$

27. consider an LTI system with transfer function H(s) is given by H(s)= $\frac{1}{(s+1)(s+3)}$

Re(s)>3; determine h(t). (Dec 2017)

By applying partial fraction,

$$\frac{1}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3} = \frac{A(s+3) + B(s+1)}{(s+1)(s+3)}$$

.

$1 = A(s+3) + B(s+1), \; s = -1 \text{ in the equation } A = \frac{1}{2}, s = -3; B = \frac{-1}{2}$

$H(s) = \frac{\frac{1}{2}}{s+1} + \frac{\frac{-1}{2}}{s+3}$ by taking inverse laplace $L^{-1}(H(s)) = h(t) = \frac{1}{2}(e^{-t} - e^{-3t})u(t)$
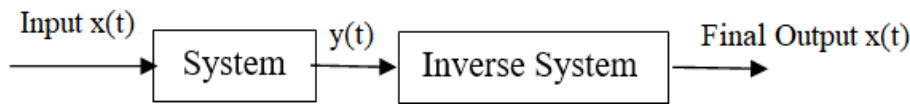
28. Given h(t),what is the step response of a CT LTI systems (May2017)

Step response in terms of impulse response is nothing but integral of Impulse response with respect to time.

$$y(t)=h(\tau)*x(\tau); y(t)= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau; where, h(t)=impulse\ response,$$ $x(t)=input=u(t)$
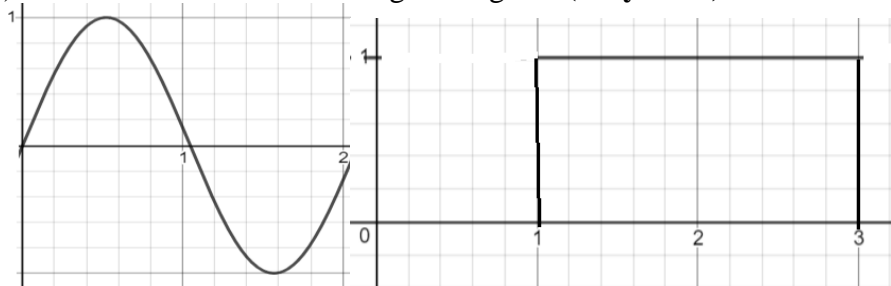
29. Define an invertible continuous time system. (Nov2020)

A system is said to be invertible if there is unique output for every unique input.

PART B (C204.3,C204.4)

1.i) Find the convolution for the given signals.**(May 2018)**

2. Find the output y(t) of the system H(s)=1/(s+2);Re{s}>-2 for the input x(t)=e⁻³ᵗu(t)
(Dec 2018)

3. Determine the impulse response h(t) of the system given by the differential equation
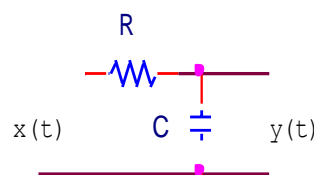**(May 2018)**

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4\ y(t) = \frac{dx(t)}{d(t)}$$

4. A causal LTI system satisfies the linear differential equation
$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 12y(t) = \frac{dx(t)}{dt} + 2x(t)$ Find the frequency response and output y(t) for the
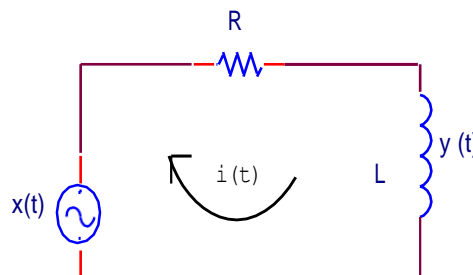input x(t)=e⁻²ᵗu(t).**(Dec 2018)**

5. Determine the output response of RC Lowpass network shown in figure due to input
(Dec2014)

$$x(t) = te^{-\frac{t}{RC}}$$ by convolution.

6. Using convolution integral, determine the response of a CTLTI system y(t) given input
x(t)=e⁻ᵅᵗu(t) and impulse response h(t)= $e^{-\beta t}$ u(t),|α|<1,|β|<1.**(May2014)**

7. Find the frequency response of the system shown below:**(May2014)**

**8.** Convolve the following signals: x(t) = e⁻²ᵗu(t-2) , h(t) = e⁻³ᵗu(t)**(Dec2015)**

9. The input-output of a causal LTI system are related by the differential equation
$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$ .Use Fourier transform

i)   Find the impulse response h(t)

ii)   Find the response y(t) of this system if x(t)=u(t) **(Dec2015)**

10. i. Solve the differential equation $(D^2+ 3D+2)y(t) = Dx(t)$ using the input $x(t) = 10\ e^{-3t}$ and with initial condition $y(0^+) =2$ and $y\cdot(0^+)=3$ **(May2015)**

   ii). Draw the block diagram representation for $H(s) = (\ 4s+28\ ) / (s^2+6s +5)$

11. i) For a LTI system with $H(s)=(s+5)/(s^2+4s+3)$ find the differential equation. Find the system output y(t) to the output $x(t)=e^{-2t}u(t)$

ii) Using graphical method convolve $x(t) = x(t) = e^{-2t}u(t)$ with $h(t) =u(t+2)$ **(May 2015)**

12. (i) Using graphical convolution, Find the response of the system whose impulse response is

$$h(t) = e^{-2t}u(t)\ \text{for an input}\ x(t) = \begin{cases} A, & for\ 0 \le t \le 2 \\ 0, & otherwise \end{cases}$$ **(May2016)**

   (ii) Realize the following in Direct Form –II

$$\frac{d^3 y(t)}{dt^3} + 4\ \frac{d^2 y(t)}{dt^2} + 7\ \frac{dy(t)}{dt} + 8y(t) = 5\frac{d^2 x(t)}{dt^2} + 4\ \frac{dx(t)}{dt} + 7x(t)$$ **(May2016)**

13. (i) An LTI system is defined by differential equation $\frac{d^2 y(t)}{dt^2} - 4\ \frac{dy(t)}{dt} + 5y(t) = 5x(t)$

Find the response of the system y(t) for an input $x(t) = u(t)$, if the initial conditions are

$$y(0) = 1; \frac{dy(t)}{dt}\Big|_{t=0} = 2$$ **(May2016)**

(ii) Determine Frequency response and impulse response for the system described by the following difference equation. $\frac{dy(t)}{dt} + 3y(t) = x(t)$ **(May2016)**

14. Convolve the following signals $x(t) = e^{-3t}u(t)\ \ and\ h(t) = u(t + 3)$. **(Dec2016)**

15. A system is described by the differential equation $\frac{d^2}{dt^2} y(t) + 6\frac{d}{dt} y(t) + 8y(t) = \frac{d}{dt}x(t) + x(t).$ Find the transfer function and the output signal $y(t)\ for\ x(t) = \delta(t)$. **(Dec2016)**

16. Convolve the following signals x(t)=u(t) and h(t)=u(t)-u(t-2). **(Nov2019)**

17. Find the condition for which Fourier transform exists for x(t). Find the Laplace transform of x(t) and its ROC. $x(t) = e^{-at}u(-t)$. **(Dec 2017)**

18. Using Laplace transform determine the response of the system described by the equation $\frac{d^2}{dt^2} y(t) + 5\frac{d}{dt} y(t) + 4y(t) = \frac{d}{dt}x(t)$ with initial condition $y(0) = 0; \frac{dy(t)}{dt}\Big|_{t=0} = 1$ for the input $x(t) = e^{-2t}u(t)$ **(May2017)**

19. A causal LTI system having frequency response is $H(j\Omega)= \dfrac{1}{j\Omega+3}$ producing an output $y(t) =e^{-3t}\ u(t) -e^{-4t}\ u(t)$ for a particular input x(t). Determine x(t). **(May 2017)**

20. Realize the given systems in parallel form **(May2017)**

$$H(s)= \frac{s(s + 2)}{(s^3+8s^2+19s+12)}$$

21. i) Consider the cascade interconnection of three signal stage causal LTI system with impulse response h1[n],h2[n],h3[n] shown below. The impulse response h2[n]=u[n]-u[n-2].The overall response h[n] is also given below. Find the impulse response h1[n] and the response y[n] given x[n]=δ[n]-δ[n-1].**(Apr2019)**
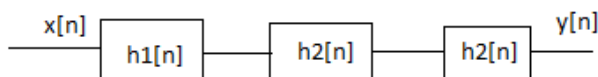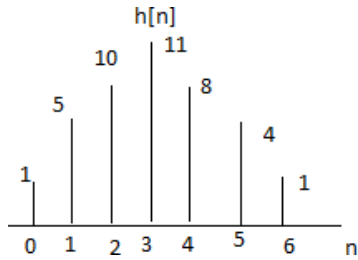
ii) Let h(t) be a triangular pulse and let x(t) be the impulse train. Determine and sketch y(t) for the following values of T (i) T=4 (ii) T=2 (iii) T=1 (iv) T=3/2
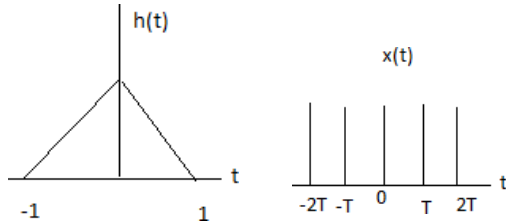
22. i) Find the convolution between x[n] and h[n] , where $x[n] = \alpha^n u[n]; 0 < \alpha < 1$, h[n]=u[n]

ii)Find the convolution of x(t) and h(t) **(Apr 2019)**

$$x(t) = \begin{cases} 2 & for -2 \leq t \leq 2 \\ 0 & otherwise \end{cases} \quad h(t) = \begin{cases} 4 & for\ 0 \leq t \leq 2 \\ 0 & otherwise \end{cases}$$

23. Consider an LTI system described by differential equation.**(Nov2020)**

$$\frac{d^2(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt} + 2x(t)$$

Here x(t) and y(t) are the input and output of the system respectively.

   i) Determine the transfer function H(s) of the system, if the system is causal and stable.

   ii) Considering the system to be causal and stable, if the input is defined as x(t)=e⁻³ᵗu(t),

   Determine the response y(t).

24.(i)Suppose that the signal eʲωᵗ is applied to the excitation to a linear, time invariant system that has an impulse response h(t). By using the convolution integral Show that the resulting output is H(ω) eʲωᵗ where

$$H(\omega) = \int_{-\infty}^{\infty} h(\tau)e^{j\omega\tau}d\tau$$

(ii) Assume the first order differential equation $\frac{dy(t)}{dt} + ay(t) = x(t)$ if x(t)= eʲωᵗ then y(t)= H(ω)eʲωᵗ.By substituting the differential equation determine H(ω).**(Apr2019)**

25. The feedback interconnection of two causal subsystems with system function   F(s) and G(s) is shown below. Find the overall system function H(s) for this feedback system.**(Nov 2019)**
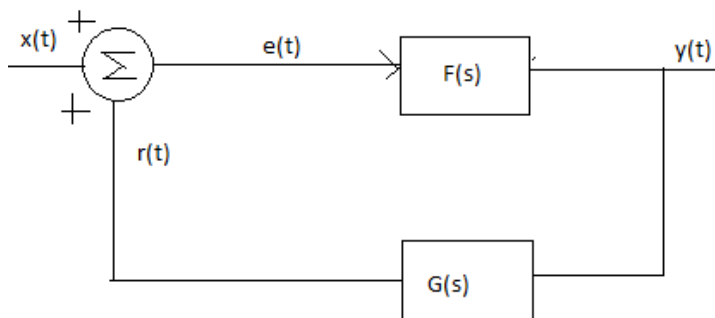
26. Let x(t)=u(t-2)-u(t-5) and h(t)=e⁻⁵ᵗ. Compute the convolution y(t)=x(t)*h(t) and sketch the signal y(t). **(Nov2020)**

UNIT IV-ANALYSIS OF DISCRETE TIME SIGNALS (C204.4)

PARTA

1. State sampling theorem.(Apr2019)

A continuous time (CT) can be completely represented in its samples and recovered back if the sampling frequency $F_s \geq 2F_m$.

$$F_s = sampling \ \ frequency$$

$$F_m = highest \ \ frequency component present in the signal$$

2. What is an antialiasingfilter?(May2014)

A filter that is used to reject high frequency signals before it is sampled to reduce the aliasing is called an anti aliasing filter.

3. Write the relationship between DTFT and Z transform. (Apr2019)

DTFT isdefinedas $X[e^{j\omega}] = \sum_{n=-\infty}^{n=+\infty} x[n]e^{-j\omega n}$

DTFT interms of Ztransform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \Big| where \ z = e^{j\omega}$$
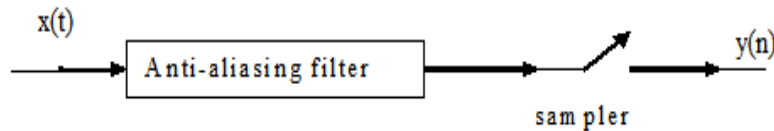
4. Define Nyquistrate.

A continuous time (CT) can be completely represented in its samples and recovered back if the sampling

$F_s \geq 2F_m$. F_s sampling frequency, F_m Maximum Frequency. The limiting sampling rate

$F_s = 2F_m$ is called as Nyquist sampling rate.

5. Find the DTFT of the signal x[n]=(1/3)ⁿu(n)(May2018)

$$DTFT\ (a^n u(n)) = \frac{1}{1 - ae^{-j\omega}}$$

Therefore $DTFT\ \left(\frac{1}{3}u(n)\right) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$

6. For the analog signal(t)=3cos(50πt)+10sin(300πt)-cos(100πt), What is the minimum sampling rate required to avoid aliasing?(Nov 2020)

$$\cos(50\pi t) \Rightarrow \qquad \Omega_1 = 50\pi \qquad F_1 = \frac{50\pi}{2\pi} = 25\ Hz$$

$$\sin(300\ \pi t) \Rightarrow \Omega_2 = 300\pi \qquad F_2 = \frac{300\pi}{2\pi} = 150\ Hz$$

$$\cos(100\pi t) \Rightarrow \Omega_3 = 100\pi \qquad F_3 = \frac{100\pi}{2\pi} = 50Hz$$

Highest frequency = 150 Hz = Fm
Minimum sampling rate = Nyquist rate = 2Fm = 300 Hz

7. State the linearity and periodicity properties of Discrete-Time FourierTransform.
Linearity:

$$x(n) \xleftarrow{DTFT} X_1(e^{j\omega}); x(n) \xleftarrow{DTFT} X_2(e^{j\omega})$$

$$ax_1(n) + bx_2(n) \xleftarrow{DTFT} aX_1\left(e^{j\omega}\right) + bX_2\left(e^{j\omega}\right) \text{where a, b are constants.}$$

Periodicity:

DTFT is periodic with period 2π

$$x(n) \longleftrightarrow \text{DTFT} \longrightarrow X(e^{j\omega})$$

$$X\left(e^{j(\omega+2k\pi)}\right) = X\left(e^{j\omega}\right) \text{ where 'k' is an integer.}$$

8. A continuous time signal has the following real Fourier transform $X(j\Omega)=\{1,|\Omega|\leq 10\pi$ Find the Nyquist rate.(Dec2018)

Given : $2\pi f = 10\pi$; fmax=5Hz; fsampling=2fmax=10Hz

9. Compute the discrete time Fourier transform of thesignal $x(n) = u(n-2) - u(n-6)$.

$$u(n-2) \xleftrightarrow{\text{DTFT}} \frac{e^{-j2\omega}}{1-e^{-j\omega}}; u(n-6) \xleftrightarrow{\text{DTFT}} \frac{e^{-j\omega 6}}{1-e^{-j\omega}}; Xe^{(j\omega)} = \frac{e^{-j2\omega}}{1-e^{-j\omega}} - \frac{e^{-j6\omega}}{1-e^{-j\omega}}$$

10. The DTFT of a discrete time signal x(n) is given as $X(e^{j\omega})=2e^{2j\omega}+3+4e^{-4j\omega}-2e^{-2j\omega}$ (Dec2018)

DTFT is definedas $X[e^{j\omega}] = \sum_{n=-\infty}^{n=+\infty} x[n]e^{-j\omega n}$

Therefore by comparing the coefficients x(-2)=2,x(-1)=0,x(0)=3, x(1)=0,x(2)=-2,x(3)=0,x(4)=4

11. DefineunilateralandbilateralZ-transforms.(Dec2013)

$$X(Z) = \sum_{n=0}^{\infty} x(n)z^{-n} \text{-Unilateral ZT}; X(Z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \text{- Bilateral ZT}$$

12. State the convolution property ofthe z-transform.

$x(n) \xleftrightarrow{z} X_1(z)$; $x_2(n) \xleftrightarrow{z} X_2(z)$; $x_1(n) * x_2(n) \xleftrightarrow{z} X_1(z)X_2(z)$

13. What is the z transform of$\delta(n+k)$?(May2013)

Using time shifting property of z-transform,

$$X(z) = z^k[z-transformof\ \delta(n)] = z^k$$

14. What isaliasing?(May2013)

When sampling rate is less than the Nyquist rate, high frequency fold in and appear as low frequency. The superimposition of the high frequency behavior on to the low f requency is known as aliasing or frequency folding.

15. Statethefinalvaluetheoremofz-transform.

For causal signal, $x(n)$; $x(n) \xleftrightarrow{z^+} X^+(z)$

If poles of $X^+(z)$ are within the unit circle in z-plane, then $x(\infty) = \lim_{z\to1}(z-1)X^+(z)$

16. State and prove the time folding property of Z-transform. (Dec2014)

Statement:x(-n)=X(z^{-1})

Proof: $z\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n)(z^{-n}) = \sum_{n=-\infty}^{\infty} x(-n)(z^{-1})^{-n} = X(z^{-1})$

17. State the multiplication propertyofDTFT. (May2014)

Multiplicationproperty:

$y(n) = x_1(n)x_2(n)$then

$$Y(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$$

18. For the analog signal(t)=sin(200πt)+3sin²(120πt), What is the minimum sampling rate required to avoid aliasing? (Apr15)

$$\sin(200\ \pi t) \Rightarrow \Omega_1 = 200\pi \qquad F_1 = \frac{200\pi}{2\pi} = 100\ Hz$$

$$\sin^2(120\pi t) \Rightarrow \left(\frac{1-\cos(240\pi t)}{2}\right) \qquad \Omega_2 = 240\pi \qquad F_2 = \frac{240\pi}{2\pi} = 120Hz$$

Highest frequency = 120 Hz = F_m

Minimum sampling rate = Nyquist rate = $2F_m$ = 240 Hz

19. Find the Z-Transform of the signalx[n]=cos(nωT)u(n)(May2018)

$$X(z) = \frac{z^2 - z\cos(\omega T)}{z^2 - 2z\cos(\omega T) + 1}, |z| > 1$$

20. Writetheconditionsforexistenceof DTFT.(May2016)

1. Thus, the absolute summability of $x[n]$ is a sufficient condition for the existence of the DTFT$X(e^{jw})$

2. TheDTFT $X(e^{jw})$ of$x[n]$isacontinuousfunctionofω

3. Itisalsoaperiodicfunctionofωwithaperiod2π

21. Find the final value of the given $X(Z) = \frac{1}{1+2z^{-1}-3z^{-2}}$(May 2016)

$$X(Z) = \frac{Z^2}{Z^2 + 2Z - 3} = \frac{Z^2}{(Z+3)(Z-1)}$$

From final value theorem $\quad \lim_{Z\to 1}(Z-1)X(Z) = x(\infty) = \lim_{Z\to 1}(Z-1)\frac{Z^2}{(Z+3)(Z-1)} = \frac{1}{4}$

22. Find the Nyquist rate ofthe signal $x(t) = sin200\pi t - cos100\pi t$(Dec 2016)

$\Omega_1 = 200\pi = 2\pi F_1, F_1 = 100\ Hz\ \&\ \Omega_2 = 100\pi = 2\pi F_2, F_2 = 50\ Hz$

$Nyquist\ Rate = F_N = 2F_{max} = 2(100) = 200Hz$

23. ListtheRocPropertiesofZ-transform.(Dec2017)

If x(n) is a causal finite sequence then the ROC is the entire z-plane except at z = 0.

If x(n) is a anti causal sequence of finite duration then the ROC is the entire z-plane except at z = ∞.

ROC cannot contain any poles.

If x(n) is a finite duration two sided sequence the ROC is entire z-plane except at z = 0 and z = ∞.

24. Find the Z-transform of the signal & its associated ROC $x[n] = \left\{ \begin{matrix} 2, -1, 3, 0, 2 \\ \uparrow \end{matrix} \right\}$(Dec 2016)

$$X(Z) = \sum_{n=-2}^{2} x(n)Z^{-n} = x(-2)Z^2 + x(-1)Z^1 + x(0)Z^0 + x(1)Z^{-1} + x(2)Z^{-2}$$

$$X(Z) = 2Z^2 - Z^1 + 3 + 2Z^{-2}\ ROC\ is\ entire\ Z\ plane\ Except\ Z = 0\ and\ Z = \infty$$

25. Find the Z-transform of a sequence x[n]=cos(nωT) u[n]. (Dec2017)

$\cos(n\omega T)\ u[n] = \cos(n\omega T)\ u(n) = \left(\frac{e^{jnwT} + e^{-jnwt}}{2}\right)u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=-\infty}^{n=\infty} \cos(n\omega T)u(n)z^{-n} = \sum_{n=-\infty}^{\infty}\left(\frac{e^{jnwT} + e^{-jnwt}}{2}\right)u(n)z^{-n}$$

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$$= \sum_{n=0}^{\infty} \left(\frac{e^{jnwT} + e^{-jnwt}}{2}\right) z^{-n} = \frac{1}{2}\left(\frac{1}{1 - e^{j\omega T}z^{-1}} + \frac{1}{1 - e^{-j\omega T}z^{-1}}\right)$$

$$X(z) = \frac{1}{2}\left(\frac{1 - e^{-j\omega T}z^{-1} + 1 - e^{j\omega T}z^{-1}}{(1 - e^{j\omega T}z^{-1})(1 - e^{-j\omega T}z^{-1})}\right) = \frac{1}{2}\left(\frac{2(1 - \cos(\omega T))}{1 - 2\cos(\omega T)z^{-1} + z^{-2}}\right)$$

$$X(z) = \left(\frac{1 - \cos(\omega T)}{1 - 2\cos(\omega T)z^{-1} + z^{-2}}\right) |z| >$$

26. What is the Z transform of a unit step sequence (May2017)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=-\infty}^{\infty} u(n)z^{-n} = \sum_{n=0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + ---$$

$$X(z) = \frac{1}{1 - z^{-1}} |z| > 1$$

27. Find x(∞) ofthe given signal for with the Z transform is given by
$$X(Z) = \frac{Z+1}{3(Z-1)(Z+0.9)} \text{ (May2017)}$$

$$X(Z) = \frac{Z + 1}{3(Z - 1)(Z + 0.9)}$$

From   final   value
theorem

$$\lim_{Z \to 1}(Z - 1)X(Z) = x(\infty) = \lim_{Z \to 1}(Z - 1)\frac{Z+1}{3(Z-1)(Z+0.9)} = \frac{2}{5.7} = 0.3508$$

28. Find the Z-transform and its associated ROC for the signal.x[n]=δ[n+1]+2δ[n]-3δ[n-2] (Nov2019)
X(z)=z+2-3z^{-2}

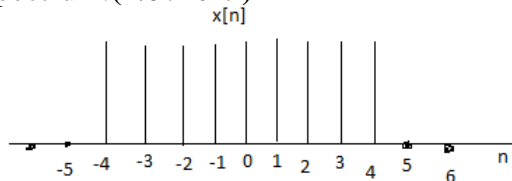**29. Find the Fourier transform for the discrete time signal x[n]= δ[n]+ δ[n-1]+ δ[n+1] and draw its spectrum.(Nov2020)**
DTFT{δ[n]}=1
Applying the Shifting property
$$DTFT\{x[n - n_0]\} = e^{-j\Omega n_0}X(\Omega)$$
DTFT{x[n]}=1+e$^{-j\Omega}$+e$^{j\Omega}$

PART B (C204.4)

1. Find the DTFT of the rectangular pulse sequence shown below and also plot the spectrum.**(Nov2019)**

2. i) Determine the sequence x(n) from the following function using Partial fraction expansion.**(May2018)**

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}} \quad ROC \text{ is } | z | > 1$$

ii) Find the DTFT of the signal $x(n) = u(n - 2)$

3. i)DeterminetheZtransformof x(n)=sin(ω_0n)u(n)**(Dec2018)**
ii) Determine the inverse Z transform of

$$X(Z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}} \quad |Z| > 1$$

4. State and prove the properties of DFT.**(May2018)**

5. i)DeterminetheDiscretetimeFouriertransformofx(n)=a$^{|n|}$,|a|<1.**(Dec2013)**
ii) Find the z-transform and ROC of the sequence x(n)= r^{n}cos(nθ)u(n).

6. (i) Find the inverse Laplace transform of s+4/(2s^2+5s+3) ;ROC:Re{s}>-1

(ii)Consider an LTI system with impulse response h[n]=α^nu[n], |α< 1| and x[n]= β^nu[n], |β<1|. Find the response of the LTI system. **(Apr 2019)**

7. State and prove sampling theorem for a bandlimited signal.**(Dec 2014)**

8. Find inverse Z-transform of

$$X(Z)= \frac{Z^{-1}}{(1-0.25Z^{-1}-0.375Z^{-2})} Z \quad |\ |>0.75 \quad |Z|<0.5 \quad \textbf{(Dec2014)}$$

9. Using convolution property of DTFT of $X(e^{jw})= \frac{1}{(1-\alpha e^{-jw})^2}$ |α|<1**(May2014)**

10. Find the inverse Z-transform of

$$X(Z) = \frac{Z^2}{(Z-0.5)(Z-1)^2} \quad |Z|>1 \quad \textbf{(May2014)}$$

11. Let X($e^{j\omega}$) be the Fourier transform of the sequence x[n].Determine x[n] for the following sequences using DTFT properties

(i) X($e^{j(\omega-\omega 0)}$) (ii)X*($e^{j(\omega)}$) (iii) jd/dωX($e^{j\omega}$) (iv) 1/2πX₁($e^{j\omega}$)⊗X₂($e^{j\omega}$)**(Dec 2018)**

12. State and prove any two properties of DTFT and any two properties of Z-Transform. **(Dec2015)**

13. i)A continuous time sinusoid cos(2πft+θ) is sampled at a rate f$_s$=1000Hz.Determine the resulting signal samples if the input signal frequency f is 400 Hz and 1000 Hz respectively **(May2015)**

ii) Prove the following DTFT Properties a) nx(n) <-> j d X(Ω)/dΩ b) x(n) $e^{j\Omega c\,n}$ <-> X(Ω-Ω$_c$)

14.i)Find the DTFT of x(n) = (1/2)$^{n-1}$u(n-1)

ii) Using suitable z transform properties find X(z) if x(n) =(n-2)(1/3)$^{n-2}$u(n-2)

iii) Find the z transform of x(n) = α $^{|n|}$ 0<α <1.**(May 2015)**

15.(i) State and prove sampling theorem.

(ii) What is aliasing? Explain the steps to be taken to avoid aliasing **(May 2016)**

16.(i)Consider a discrete time LTI system with impulse response h(n)=(1/2)^{n}u[n]. Use Fourier Transform to find the response of the system to the input x[n]=(3/4)^{n}u[n]

(ii)A difference equation of the system is given as y(n)-y(n-1)+(1/4)y(n-2)=x(n)+1 /4x(n-1)-1/8x(n-2)**.** Determine the transfer function of the inverse system. Check whether the inverse system is causal and stable. **(Apr 2019)**

17.(i) Discuss the effects of under sampling a signal using necessary diagrams. **(Dec 2016)**

(ii)Find the Z transform of $x[n] = a^n u[n] - b^n u[-n-1]$ and specify it's ROC. **(Dec 2016)**

18.(i) Give the relationship between DTFT and Z transform.**(Dec 2016)**

(ii)State & prove the time shifting property & time reversal property of Z-transform. **(Dec 2016)**

19. i)Find the z-transform and sketch the ROC of the following sequence x[n]=2^{n}u[n]+3^n u[n-1].

ii) consider an analog signal x(t)= 5cos 200πt.

a) Determine the minimum sampling rate to avoid aliasing.

b) If sampling rate F$_s$=400Hz.What is the DT signal after sampling?**(Dec2017)**

20. i)Determine unit step response of the LTI system defined by d^{2}y/dt^2+5dy/dt+6y(t)=dx/dt +x(t).

ii) Find the Inverse z-transform using partial fraction method.

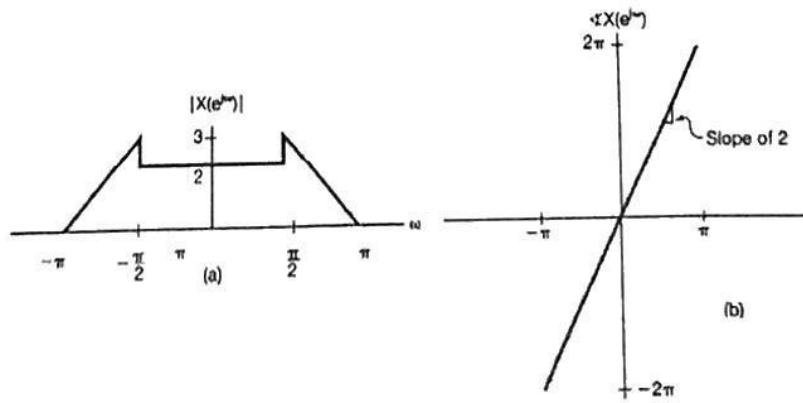$$X(z)= \frac{3-(5/6)z^{-1}}{(1-(1/4)z^{-1})(1-(1/3)z^{-1})} \quad ;|z|>1/3 \textbf{(Dec 2017)}$$

21. State and prove the following properties of DTFT**(May2017)**

(i)Differentiation in frequency

(ii)Convolution in frequency domain

22. Consider the sequence $x(n)$ whose Fourier transform $X(e^{j\omega})$ is depicted for

$-\pi \leq \omega \leq \pi$ in the figure below. Determine whether or not , in the time domain, $x(n)$ is periodic,real,even,and/or of finite energy.

(i) What is the transfer function and the impulse response of low pass RC circuit?

(ii) Find the necessary and sufficient condition on the impulse response $h(n)$

Such that for any input $x(n)$ $max\{|x(n)|\} \geq max\{|y(n)|\}$ where $y(n) = x(n) * h(n)$

23. Given the z Transform of the sequence x[n] as X(z)=z/z-1 Find the Z transform of the following signals in terms of x(z) using properties of z transform i) x[n-1] ii) x[-n] iii) α^nx[n] iv)nx[n] **(Nov2019)**

24. Consider the signal y[n]
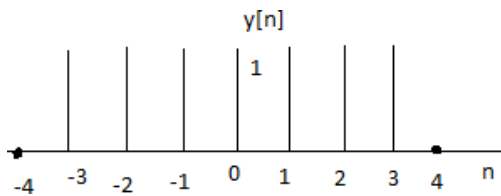
(i)Find a signal x[n] such that even$\{x[n]\}$= y[n] for n≥0 and odd$\{x[n]\}$=y[n] for n<0.

(ii)Suppose that even $\{w[n]\}$=y[n] for all n. Also assume that w[n]=0 for n<0. Find w[n].**(Part C Apr 2019)**

25. i)Determine DTFT of the signal x[n]

$$x[n] = \begin{cases} 1, & |n| < N_1 \\ 0, & |n| > N_1 \end{cases}$$ Sketch it spectrum for N_1=4.

Consider a signal x[n]= 5(1/3)^{n}u[n]-3(1/4)^{n}u[n] , determine its z-transform X[z] and mark its ROC. **(Nov 2020)**

26.i)Determine the z-transform for the signal x[n]=(1/2)^{n}sin(π/8n)u[n] and mark its ROC.

ii) Determine the Fourier transform for the signal x[n] = u[n – 3] – u[n – 7]. **(Nov 2020)**

UNIT V-LINEAR TIME INVARIANT DISCRETE TIME SYSTEMS
(C204.3,C204.4)PART A

1. Find the system function for the given difference equation y(n)=0.5y(n-1)+x(n).

Taking Z transform of the equation,

$$Y(z) - 0.5z^{-1}Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.5z^{-1}}$$

2. What is a F I R system?

FIR-Finite Impulse Response system. Impulse response of the system is of finite duration. General form of difference equation describing the system is

$$y(n) = \sum_{k=0}^{M} b_k x(n-k)$$

3.What is an IIR system?

IIR-infinite impulse response system. Impulse response of the system is of infinite duration. General form of difference equation describing the system is:

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k-0}^{M} b_k x(n-k)$$

4. The input x(n)={1,2,3,4} and output y(n)={0,1,2,3,4}. Find the impulse response h(n) of the LTI system. (Dec2018)

According to the property of z-transform

$$x(n)\circledcirc h(n) = y(n) \xrightarrow{Z-Tranform} X(z)H(z) = Y(z)$$

$H(z)=Y(z)/X(z)$; $Y(z)=z^{-1}+2z^{-2}+3z^{-3}+4z^{-4}$ and $X(z)=1+2z^{-1}+3z^{-2}+4z^{-3}$

Dividing $H(z)= (z^{-1}+2z^{-2}+3z^{-3}+4z^{-4})/(1+2z^{-1}+3z^{-2}+4z^{-3})= z^{-1}$

Oncomparingwithstandardz-transformequationh(n)={0,1}

5. Given the system function $H(z)=z^{-1}/(z^{-2}+2z^{-1}+4)$. Find the difference equation of the system. (Dec2018)

We know that $H(z)=Y(z)/X(z)= z^{-1}/(z^{-2}+2z^{-1}+4)$

$z^{-2}Y(z)+2z^{-1}Y(z)+4Y(z)=z^{-1}X(z)$

Taking Inverse z-transform $y(n-2)+2y(n-1)+4y(n)=x(n-1)$

6. Is the discrete time system described by the difference equation y(n) = x(-n) causal? (May2013)

When n= -1 y(n)=x(-(-1))=x(1) - Future value **;** Therefore the system is non-causal.

7. X(ω) is the DTFT of x(n), what is the DTFT of $x^*(-n)$? (May2013)

From complex conjugation and time reversal property $x^*(-n) = X(-\omega)$

8. A Causal LTI system hasimpulse response $h(n)$, for which the z-transformis

$$H(z)= \frac{1+z^{-1}}{(1-0.5z^{-1})(1+0.25z^{-1})}.$$ **Is the system stable? Explain. (Dec 2012)**

Poles are at z = 0.5 and at z = -0.25. The poles lie within the unit circle in z-plane. Therefore the system is stable.

9. Determine the Z-transform of the following signals. Note that the two have samealgebraic equation and only differ in ROC: $x_1[n]=(1/2)^n u(n)$ and $x_2[n]=-(1/2)^n u[-n-1]$. (Apr2019)

Z transform for both the functions are z/z-2. The ROC for $x_1[n]=Re\{z\}>1/2$, for $x_2[n]=Re\{z\}<1/2$

10. In terms of ROC, state the condition for an LTI system discrete time system to be causal and stable. (Nov2020)

A discrete LTI system with rational system function H(z) is causal if and only if the ROC is the exterior of the circle of the outer most pole and stable if and only if all of the poles of H(z) lies inside the unit circle.

11. Find x(∞) if X(z)=z+1/3(z-1)(z+0.9) (May 2018)

x(∞)=lim [z-1]X(z)
      z→1
x(∞)=0.35

12. Find the transfer function of the system described by the equation y(n-2)-3y(n-1)+2y(n)=x(n-1)

Taking z-transform of the equation,

$$z^{-2}Y(z)-3z^{-1}Y(z)+2Y(z)=z^{-1}X(z); H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{z^{-2}-3z^{-1}+2}$$

13. Whatarethedrawbacksoftransferfunctionrepresentationofthesystem?

(i)The transfer function describes only the zero state response of a system.(ii) It describes only the relationship between the input and output of a system, but does not provide any information regarding the internal state of a system. (iii) It is limited to single-input single-output systems.(iv) It is applicable only for linear time-invariant systems.

14. What is the necessary and sufficient condition for a DT LTI system to be stable, what is the necessary and sufficient condition on impulse response for stability of a causal LTIsystem?

Necessaryandsufficientcondition:ROCofsystemfunctionmustincludeunitcircle,ALTI causalsystemissaidtobestableif andonlyifPolesofsystemfunctionH(z)mustliewithin the unit circle in z -plane.

15. Check whether the system is causaland stable.(Dec2013)

$$H(Z)= \frac{1}{1-\frac{1}{2}Z^{-1}} + \frac{1}{1-2Z^{-1}} = \frac{(1-2Z^{-1})+(1-\frac{1}{2}Z^{-1})}{(1-1/2Z^{-1})(1-2Z^{-1})}$$

Poles at Z=1/2, Z=2 .The system is causal-Output does not depend on future I/P

The system is unstable since the poles lie outside the unit circle.

16. Given the impulse response of a linear time invariant system as h(n)=sinπn, Check whether the system is stable or not.(Apr2019)

Sinπn=0 for n=…-2,-1,0,1,2…. Hence h(n) is absolutely summable and the system is stable.

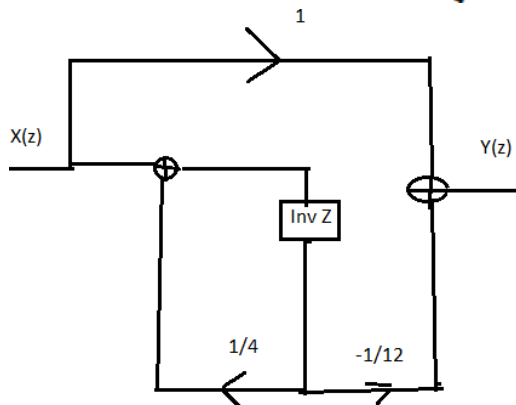**17. UsingZ-Transformcheckwhetherthefollowingsystemisstable.(May2014)**

$$H(Z)= \frac{Z}{Z-\frac{1}{2}} , \frac{1}{<3} <Z$$

$$+\frac{Z}{Z-3}$$

Here the ROC of system function includes the unit circle.So the given system is stable.

18. Implementthesysteminparallelform. $\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$ **(May2018)**

Simplifying we get $X(z) = 1 - \frac{1}{12\left(z-\frac{1}{4}\right)}$

19. DistinguishBetweenRecursiveandNonRecursiveSystem?(May2017)

A recursive system is a system in which present output depends on previous output and input, Non recursive system is a system in which present output depends on previous input.

Recursive: $y(n)=- \sum^{N} \quad a_k y(n- \quad k)+ \quad k=1$

$$\sum_{k=0}^{M} b_k \qquad x(n-k)$$

Non Recursive: $y(n) = \sum\limits_{k=0}^{M} b_k \; x(n-k)$

IIR filter is example for Recursive system, FIR filter is example for Non Recursive system

20. From discrete convolution sum, find the step response in terms of h(n).(May2016)

$$y(n) = \sum\limits_{k=-\infty}^{\infty} x(k)h(n-k)$$

21. Determinewhetherthefollowingsystemisrecursivesystemandjustifyyouranswer.
y[n]=2x[n]+3x[n-1]-2x[n-2] (Nov2019)

The given system is a non recursive system since the outputdepends on previous input only

22. Convolve the following sequences $x[n] = [1,2,3] \; h[n] = [1,1,2]$(Nov2019)

	1	2	1
1	1	2	1
2	2	4	2
3	3	6	3

linear convolution y(n) = (1,4,8,8,3)

circular convolution between x[n]and h[n]is $\begin{bmatrix}1 & 3 & 2\\2 & 1 & 3\\3 & 2 & 1\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix} = \begin{bmatrix}1+3+4\\2+1+6\\3+2+2\end{bmatrix} = \begin{bmatrix}8\\9\\7\end{bmatrix}$
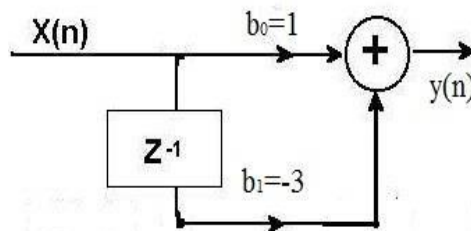
23. Given the system function $H(Z) = 2 + 3Z^{-1} + 4Z^{-3} - 5Z^{-4}$. Determine the impulse response h(n).(Dec2016)

$h(n) = 2\delta(n) + 3\delta(n-1) + 4\delta(n-3) - 5\delta(n-4), h(n) = (2,3,0,4,-5)$

24. Write the condition for stability of a DT-LTI system respect to the position of poles.(Dec2017)

DT-LTI is said to be stable if the ROC of system function includes the unit circle.

25. Realizethedifferenceequationy[n]=x[n]-3x[n-1]indirectformI.(Dec2017)

26. Determine Z-transform of unit impulse signal δ[n] and sketch itsROC.

$$X(z) = \sum\limits_{n=0}^{\infty} x[n]z^{-n}$$

$$X(z) = \sum\limits_{n=0}^{\infty} \delta[n]z^{-n}$$

$$\delta[n] = \begin{cases}1 \; for \; n = 0\\0 \; for \; n \ne 0\end{cases}$$

Therefore ZT{δ[n]}=1 ROC: Entire Z-Plane.

PART B (C204.5)

1. Consider thesystem $H(z) = \dfrac{0.2z}{(z+0.4)(z-0.2)}$; ROC $|z| > 0.4$

i) Findtheimpulseresponseofthesystem.

ii) Is DTFT exists for the system. If sohow?

iii) Find the DTFT. **(May2018)**

2. Drawthecascadeformofthefollowingsystemfunction

       y(n)-1/4y(n-1)-1/8y(n-2)=x(n)+3x(n-1)+2x(n-2)**(May 2018)**

3.Givenh(n)={-2(1/3)ⁿ+3(1/2)ⁿ}u(n).FindH(z)andstepresponseofthesystem.**(Dec2018)**

4. A causal DT LTI system is given by y[n-2]-7/10y[n-1]+1/10y[n]=x[n]. Find whether systemisstableornotusingpolezeroplot.**(Dec2018)**

5. Compute convolution sum of the following sequences. x[n]=αⁿu[n];h[n]=u[n-1] **(Nov 2019)**

6. i)Determinethetransferfunctionand impulseresponseforthecausalLTIsystem describedbythedifferenceequationusingz-transform

$$y(n) - \frac{1}{4}y(n-1) - \frac{3}{8}y(n-2) = -x(n) + 2x(n-1).$$

7. Given theimpulseresponseofadiscretetimeLTIsystemh[n]=-2(1/3)ⁿu(n)+3(1/2)ⁿu(n) (i)FindH(z)(ii)Findthedifferenceequationofthesystem(iii)Findthestepresponse ofthe system**(May2018)**

8. The I/O relation is given by y[n]-1/4y[n-1]=x[n].Find y[n] if $X\left(e^{j\omega}\right) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$**(Dec2018)**

9. ConsideranLTIsystemwithimpulseresponseh[n]=αⁿu[n]andtheinputtothissystem is x(n) =βⁿ u(n) with │α│ & │β│<1. Determine the response y[n]. i) When α = β and ii) When α ≠ β using DTFT. **(Apr2019)**

10. Let y(n)=x(n)*h(n) where x(n)=(1/3)nu(n) and h(n)=(1/5)nu(n) Find Y(z) using property andalsofindy(n)usingpartialfractionmethod.**(Dec2018)**

11. Let y[n]=x[n]*h[n] where x[n]=(1/3)ⁿu[n] and h[n]=(1/5)ⁿu[n].Find y(z) by using the convolutionpropertyofztransformandalsofindy(n)usingpartialfractionmethod.

(Dec 2018)

12. A causal system has input $x(n)$ and output $y(n)$.Find the

     (i)System function $H(Z)$ www.EnggTree.com

     (ii) Impulse response $h(n)$

     (iii)Frequency response $H(e^{jw})$

$x(n) = \delta(n) + \frac{1}{6}\delta(n-1) - \frac{1}{6}\delta(n-2), h(n) = \delta(n) - \frac{2}{3}\delta(n-1).$**(Dec 2016)**

13. i)Obtaintheparallelrealizationofthesystemgivenbyy(n)-3y(n-1)+2y(n-2)=x(n).

     ii) Determine the direct form II structure for the system given by difference equation

       y(n)= (-3/8)y(n-1)+(3/32)y(n-2)+(1/64)y(n-3)+x(n)+3x(n-1)+2x(n-2) **(Apr 2019)**

14. Usingthepropertiesof inverseZ-transformsolve:**(Dec2017)**

i) X(z)= log (1+az⁻¹); |z|>|a| and X(z)= $\dfrac{az^{-1}}{(1-az^{-1})^2}$; |z|>|a|

     ii) Checkwhetherthesystemfunctioniscausalornot

H(zs)= $\dfrac{1}{1-(1/2)z^{-1}} + \dfrac{1}{1-2z^{-1}}$;|z|>2

     iii) ConsiderasystemwithimpulseresponseH(s)=eˢ/s+1;Re{s}>-1.Checkwhetherthe systemfunctioniscausalornot.

15. Perform Convolution to find the response of the systems $h_1[n]$and $h_2[n]$f or the given inputsequence $x_1[n]$ and $x_2[n]$

 (1) $x_1[n] = \{1, -1, 2, 3\}$,$h_1[n] = \{1, -2, 3, -1\}$

 (2) $x_2[n] = \{1, 2, 3, 2\}$,$h_2[n] = \{1, 2, 2\}$**(May2017)**

16. For a causal LTI system the input x(n) and output y(n) are related throughdifference equation $y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$ Determine the frequency response H(eʲʷ) and the impulse response h(n) of the system. **(May 2017)**

17. Determinethesteadystateresponseforthesystemwithimpulseresponse$h[n] = [j\, 0.5]^n$foran input $x[n] = \cos(\pi n)\, u(n)$(**May2017)**

18. ConsideraDTLTIsystemisgivenbyH(z)=z/z-0.5,|z|>0.5Findthestepresponseofthe system. **(Nov2019)**

19. Consider the discrete LTI system shown below. Find the frequency response and the impulseresponseofthesystem.Andalsosketchthemagnituderesponse.**(PartCNov2019)**
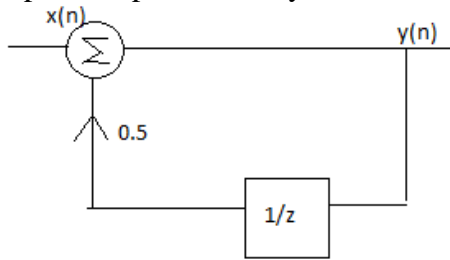
20. i)ADiscretetimeLTIsystemprovidesresponsey[n]=0.4^nu[n]forinputx[n]=0.2^nu[n]. Determinefrequencyresponse$H(e^{j\omega})$ofthesystem.

ii) Consider second order LTI system described by

$$H(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)}$$

Determine the impulse response if the system is causal. **(Nov 2020)**

21. Determineandplottheconvolutionofx[n]andh[n]definedby**(Nov2020)**

$$x[n] = \left(\frac{1}{3}\right)^{n-1} u[n-1]$$

and h[n]=u[n+3]