UNIT I
ANALYSIS OF TRUSSES

INTRODUCTION

A structure made up of several bars (or members) riveted or welded together is known as frame.

TYPES OF FRAMES

The different types of frame are :
(i). Perfect frame and (ii) Imperfect frame.
Imperfect frame may be a deficient frame or redundant frame.

PERFECT FRAME.

The frame which is composed of such members, which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external load, is known as perfect frame.

The simplest perfect frame is a triangle as shown in Fig.5.1

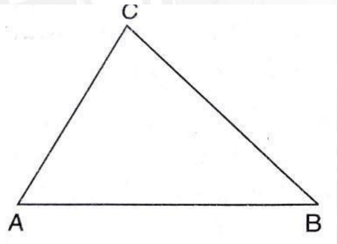

Fig.1.1

It consists of three members AB, BC and AC whereas the three joints are A,B and C. This frame can be easily analysed by the condition of equilibrium given below.

$n = 2j - 3$

Where n = Number of members and j = Number of joints.

IMPERFECT FRAME

A frame in which the number of members and number of joints are not given by $n = 2j - 3$ is known as imperfect frame.

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DEFICIENT FRAME AND REDUNDANT FRAME

If the number of members in an imperfect frame are less than 2j – 3, then the frame is known as deficient frame and if the number of members in an imperfect frame are more than 2j – 3, then the frame is known as redundant frame

ASSUMPTIONS MADE IN FINDING OUT THE FORCES IN A FRAME

The assumptions made in finding out the forces in a frame are:
(i) The frame is a perfect frame (ii)The frame carries load at the joints (iii)All the members are pin jointed.

ANALYSIS OF A FRAME

A frame is analysed by the following methods:
(i) Method of joints,
(ii) Method of sections,
(iii) Tension Coefficient method and (iv) Graphical method.

1.2 METHOD OF JOINTS

In this method, after determining the reactions at the supports, the equilibrium of every joint is considered. This means the sum of all the vertical forces as well as the horizontal forces acting on a joint is equated to zero. The joint should be selected in such a way that at any time there are only two members, in which the forces are unknown. The force in the member will be compressive if the member pushes the joint to which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.

Example 1.2.1 A truss of 8m span consisting of seven members each of 4m length supported at its ends and loaded as shown in Fig.5.2. Determine the forces in the members by method of joints

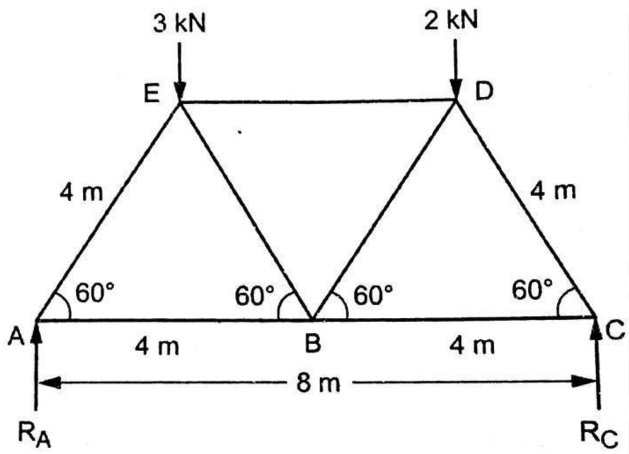


Fig.1.2.a

Solution:

Determine the reactions at A and C

Taking moment about A

$$R_C \times 8 = R_D \times 6 \times R_E \times 2$$

Or $R_C \times 8 = 2 \times 6 \times 3 \times 2$

Or $\mathbf{R_C = 2.25\ kN}$

We know that ,

Upward vertical reaction = Download vertical reaction

$$RA + RC \quad = 3 + 2$$

Or $R_A + 2.25 = 5$

Or $\mathbf{R_A = 2.75\ kN}$

Consider the joint A.

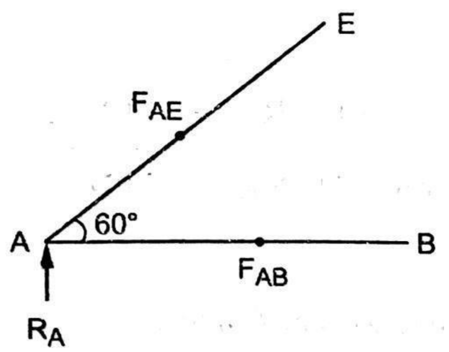


Fig.1.2.b

Assume the forces (F_{AE} and F_{AB}) acting on joint A are tensile forces(acting away from joint A). If we get negative value the force in that member is compressive.

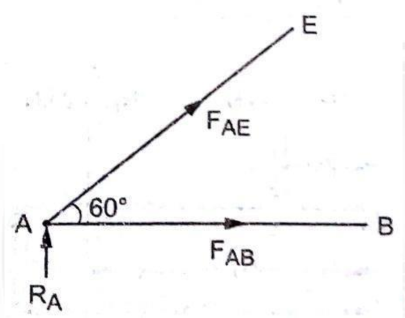


Fig.1.2.c

At joint A:

Resolving the force (F_{AE}) vertically, we know that the sum of vertical forces =0.

$R_A + F_{AE} \sin 60^o = 0$

Or $2.75 = -F_{AE} \sin 60^o$

Or $\mathbf{F_{AE} = -3.17KN}$

(Compression)

Resolving the force (F_{AE}) horizontally,

Sum of horizontal forces = 0

$F_{AB} + F_{AE} \cos 60^o = 0$

Or $F_{AB} = -F_{AE} \cos 60^o$

Or $\mathbf{F_{AB} = -1.58kN}$ (Tension)

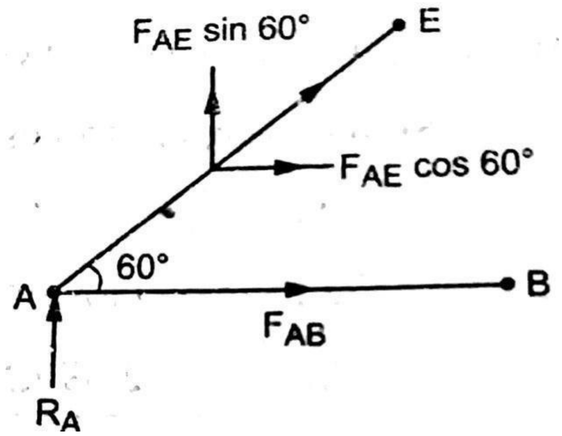


Fig.1.2.d

Consider the joint C.

Assume the forces (F_{DC} and F_{BC}) acting on joint A are tensile forces (acting away from joint A). If we get negative value the force in that member is compressive.

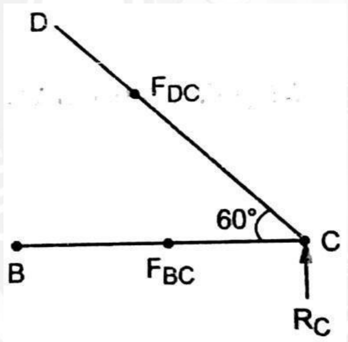


Fig.5.2.e

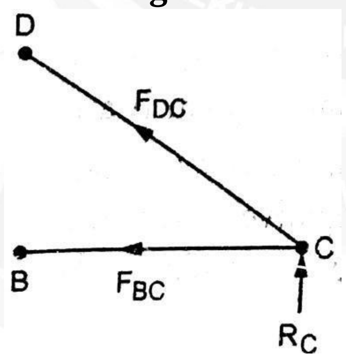


Fig.1.2.f

At joint C:

Resolving the force (F_{DC}) vertically,

Sum of horizontal forces = 0

Or $R_C + F_{DC} \sin 60^o = 0$

Or $2.25 = -F_{DC} \sin 60^o$

Or $\mathbf{F_{DC} = -2.59kN}$

(Compression)

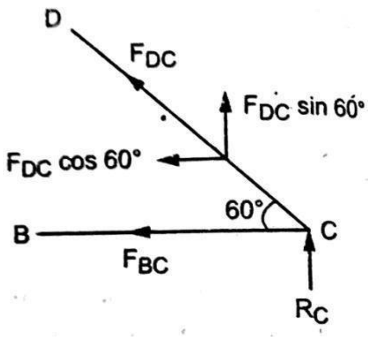


Fig.1.2.h

Resolving the force horizontally,

Sum of horizontal forces = 0

$- F_{BC} - F_{DC} \cos 60^o = 0$

(Force acting towards left side is –ve)

Or $F_{BC} = -F_{DC} \cos 60^o$

Or $F_{BC} = - 2.59 \times \cos 60^o$

Or **F_{BC} = 1.295kN** (Tension)

Consider the joint B.

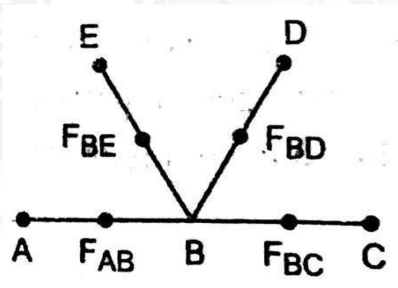


Fig.1.2.i

Assume the forces (F_{AB}, F_{BC}, F_{BD} and F_{BC}) acting on joint B are tensile forces (acting away from joint B). If we get negative value the force in that member is compressive.

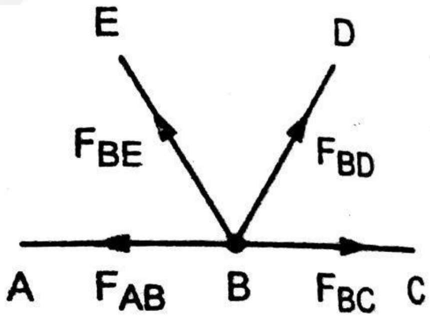


Fig.1.2.j

At joint B:

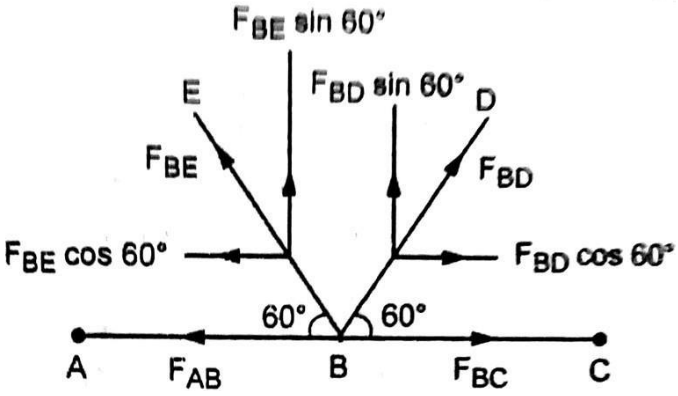


Fig.1.2.k

Resolving the force (F_{BE} & F_{BD}) vertically,

Sum of horizontal forces = 0

$F_{BE} \sin 60^o + F_{BD} \sin 60^o = 0$

Or $F_{BE} \sin 60^o = -F_{BD} \sin 60^o$

Or $\mathbf{F_{BE} = - F_{BD}}$

Resolving the force (F_{BE} & F_{BD}) horizontally,

Sum of horizontal forces = 0

$\Sigma H = 0$

$- F_{AB} - F_{BE} \cos 60^o + F_{BC} + F_{BD} \cos 60^o = 0$

(Force acting towards right side is –ve, force towards left side is –ve)

Or $- 1.58 - F_{BE} \cos 60^o + 1.29 + F_{BD} \cos 60^o = 0$

Or $- 0.29 + 2F_{BD} \cos 60^o = 0$

(Since, $F_{BE} = - F_{BD}$)

Or **FBD = 0.29 kN** (Tension)

Or $\mathbf{F_{BE} = -0.29\ kN}$ (Compression)

Consider the joint D.

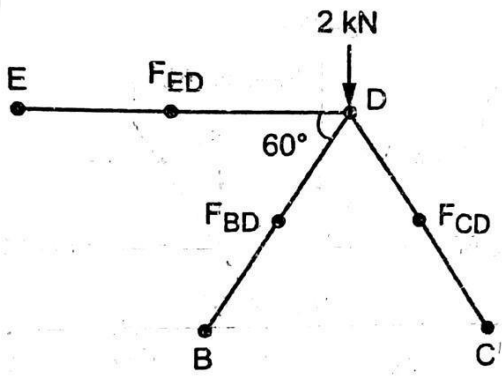


Fig.1.2.l

Assume the forces (F_{ED}, F_{BD}, and F_{CD}) acting on joint D are tensile forces (acting away from joint D). If we get negative value, the force on that member is compressive.

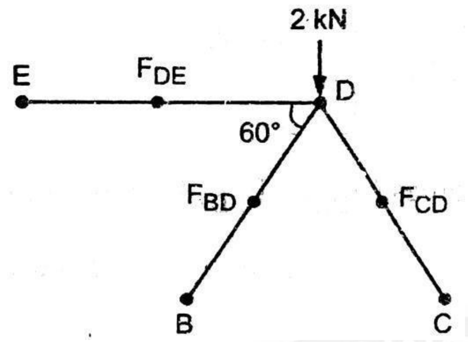


Fig.1.2.m

At joint D:

Resolving the force (F_{BD} & F_{CD}) horizontally, sum of horizontal forces = 0

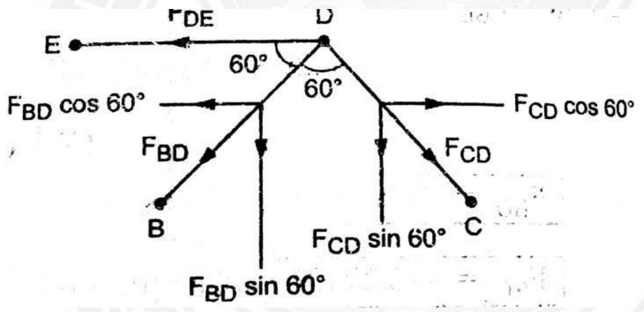


Fig.1.2.n

$$- F_{DE} - F_{BD} \cos 60^o + F_{CD} + F_{BD} \cos 60^o = 0$$

(Force towards right side +ve, force towards left side –ve)

Or $- F_{DE} - 0.29 \times \cos 60^o - 2.59 \times \cos 60^o = 0$

Or **F_{DE} = -1.44 kN** (Compression)

Result:

Sl.No.	Member	Force (kN)	Nature of force.
1	**AE**	**-3.17**	**compression**
2	**AB**	**1.58**	**Tension**
3	**CD**	**-2.59**	**compression**
4	**BC**	**1.29**	**Tension**
5	**BD**	**0.29**	**Tension**
6	**BE**	**-0.29**	**compression**

7	DE	-1.44	compression

Example 1.2.2 Determine the forces in the truss shown in Fig 5.3. It carries a horizontal load of 16 kN and vertical load of 24 kN.

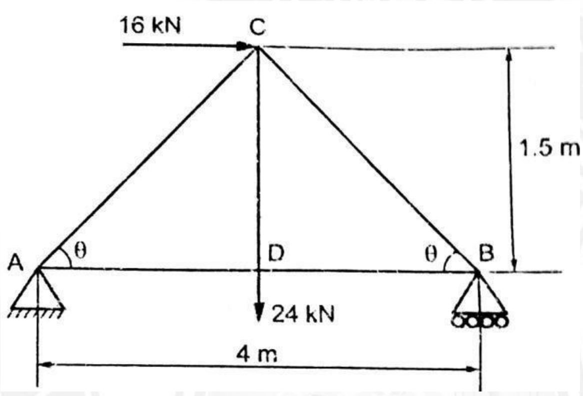


Fig.1.3.a

Solution: The truss is supported on rollers at B and hence the reaction at B should be vertical 9(R_B).

The truss in hinged at A and hence end A consists of a horizontal reaction (H_A) and vertical reaction (R_A).

Determine the reaction at A and B (R_A and R_B).

Taking moment about A.

$$R_B \times 4 = 24 \times 2 + 16 \times 1.5$$

$\mathbf{R_B = 18\ kN}$

We know that,

Upward vertical load = Download vertical load

$$R_A + R_B = 24$$

$$R_A + 18 = 24$$

$$\mathbf{R_A = 6\ kN}$$

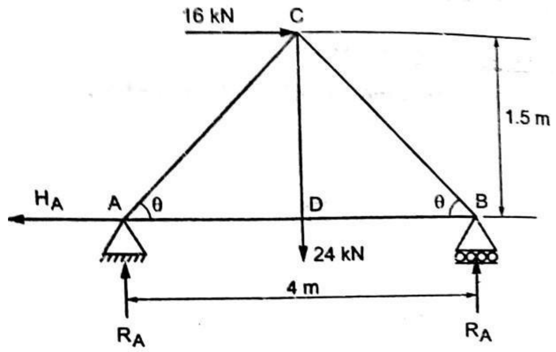


Fig.1.3.b

Right side horizontal load = Left side horizontal load

$$16 = H_A$$

HA = 16kN

In the triangle BCD

$$BC^2 = CD^2 + BD^2$$
$$BC^2 = (1.5)^2 + 2^2$$

BC = 2.5 m

$$\text{Sin } \theta = \frac{DC}{BC} = \frac{1.5}{2.5}$$
$$\text{Sin } \theta = 0.6$$
$$\boldsymbol{\theta = 36.8^0}$$

Consider the joint A.

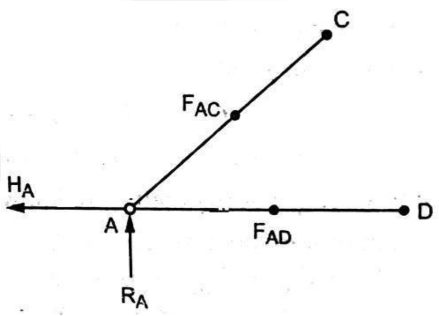


Fig.1.3.c

Assume the forces F_{AC} and F_{AD} acting on joint A are tensile forces (acting away from joint A). If we get negative value force on that member is compressive.

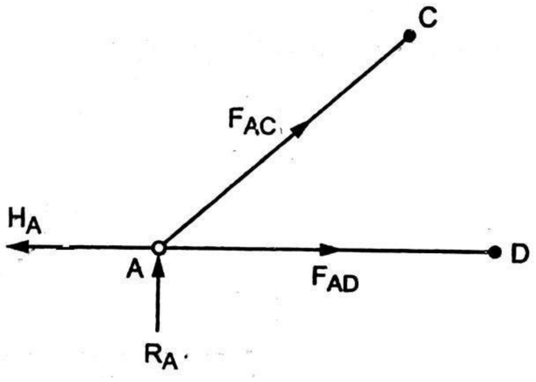


Fig.1.3.d

At joint A:

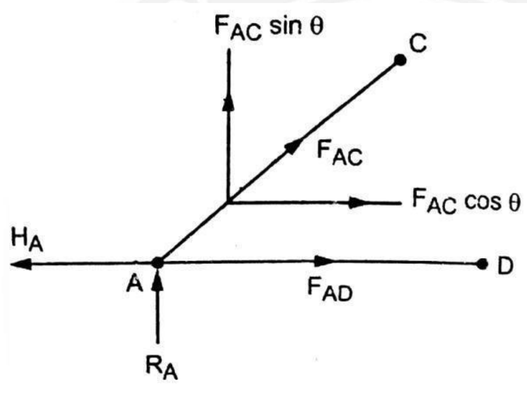


Fig.1.3.e

Resolving the force (F_{AC}) vertically, we know that

Sum of vertical forces = 0

$R_A + F_{AC} \sin \theta = 0$

$6 + F_{AC} \sin 36.8^0 = 0$

$6 = - F_{AC} \sin 36.8^0$

F_{AC} = - 10KN (Compressive)

Resolving the force (F_{AC}) horizontally, we know that,

Sum of horizontal forces = 0

$-H_A + F_{AD} + F_{AC} \cos \theta = 0$

$-16 + F_{AD} + -10 \times \cos 36.8^0 = 0$

F_{AD} = 24kN (Tension)

Consider the joint B.

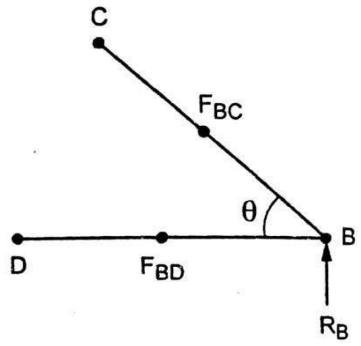


Fig.1.3.f

Assume the forces F_{BC} and F_{BD} acting on joint B are tensile forces (acting away from B). If we get negative value, force in that member is compressive.

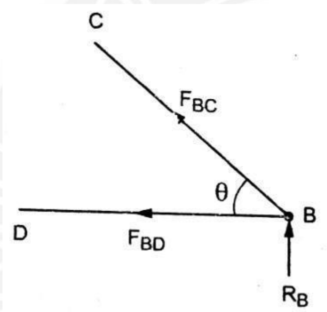


Fig.1.3.g

At joint B:

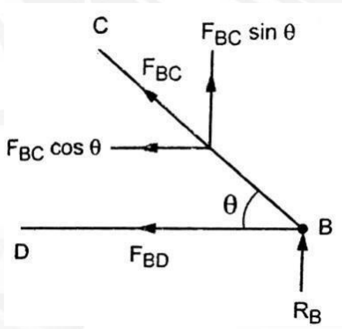


Fig.1.3.g

Resolving the force (F_{BC}) vertically, we know that,

Sum of vertical forces = 0

$R_B + F_{BC} \text{ Sin } \theta = 0$

$18 + F_{BC} \text{ Sin } \theta = 0$

$F_{BC} = -\frac{18}{\sin 36.8}$

$\mathbf{F_{BC} = -30kN}$ (Compressive)

Resolving the force (F_{BC}) horizontally,
we know that,

Sum of horizontal forces = 0

$- F_{BC} \cos\theta - F_{BD} = 0$

$- F_{BC} \cos\theta = F_{BD}$

$$30 \cos\theta = F_{BD}$$

$$30 \cos 36.8^0 = F_{BD}$$

F_{BD} = 24kN (Tension)

Result:

Sl.No.	Member	Force (kN)	Nature of force
1	AC	-10	Compression
2	AD	24	Tension
3	BC	-30	Compression
4	BD	24	Tension

Example 1.2.3 A truss is loaded as in Fig 5.4. Determine the forces in all the members of that truss.

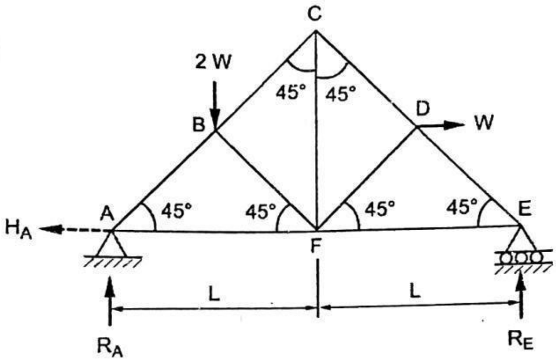


Fig 1.4.a

Solution: To solve the above problem, consider W = 1kN

To find the reactions at the support:

$$R_A + R_E = 2W = 2$$

$H_A = W = 1$

Taking moment about point A,

$$i.e., \quad \Sigma MA = 0$$

$$-(RE \times 2L) \times \left(2 \times \frac{L}{2}\right) + \left(1 \times \frac{L}{2}\right) = 0$$

$$\left(L \times \frac{L}{2}\right) = R_E \times 2L$$

$$\frac{3}{2} L = R_E \times 2L$$

$$\mathbf{R_E = 0.75}$$

$$R_A + R_E = 2$$

$$R_A = 2 - R_E = 2 - 0.75$$

$$R_A = 1.25$$

Solving the above problem using method of joints:

At joint A:

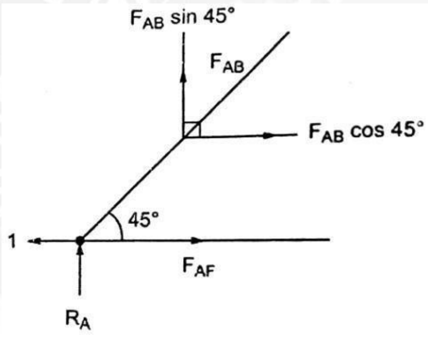


Fig 1.4.b

Sum of horizontal forces = Σ H = 0

Sum of vertical forces = Σ V = 0

Σ V = 0, $\quad R_A + F_{AB}.\sin 45^0 = 0$

$$F_{AB}.\sin 45^0 = -R_A$$

$$F_{AB} = \frac{-R_A}{\sin 45^\circ} = \frac{-1.25}{\sin 45^\circ}$$

F_{AB} = -1.77 (Compression)

Σ H = 0, $\quad -1 + F_{AB} \cos 45^0 + F_{AF} = 0$

$$-1 + (-1.77 \times \cos 45^0) + F_{AF} = 0$$

F_{AF} = 2.25 (Tension)

At joint E:

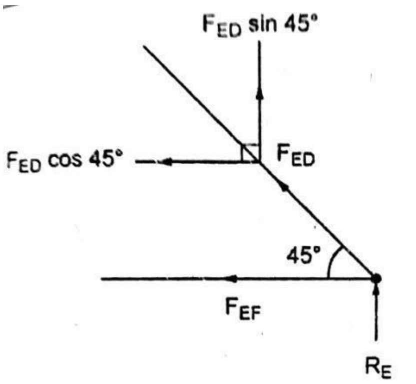


Fig 1.4.c

$\Sigma V = 0$

$F_{ED}. \sin 45 + R_E = 0$

$F_{ED} = \frac{-R_E}{\sin 45°} = \frac{-0.75}{\sin 45°}$

F_{ED} = -1.06 (Compressive)

$\Sigma H = 0$

$-F_{EF} - F_{ED} \cos 45^0 = 0$

$-F_{EF} - (-1.06 \times \cos 45^0) = 0$

$-F_{EF} + 0.75 = 0$

F_{EF} = 0.75 (Tensile)

At joint B:

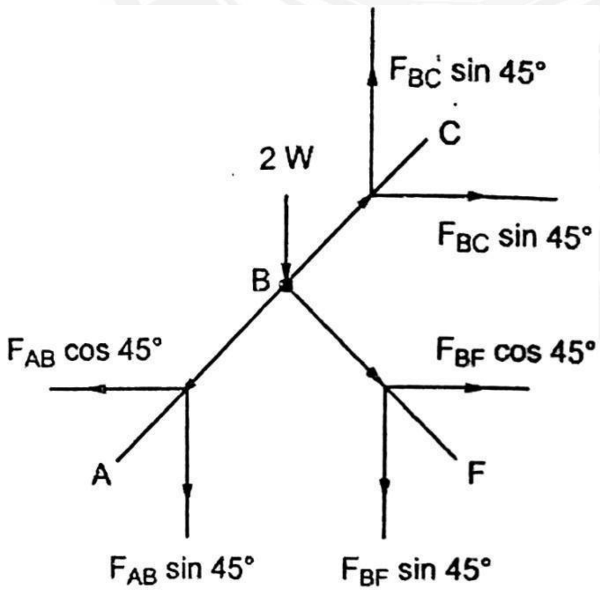


Fig 1.4.d

$\Sigma H = 0$

$- F_{AB} \cos 45^0 + F_{BC} \cos 45^0 + F_{BF} \cos 45^0 = 0$

$\cos 450 [- F_{AB} + F_{BC} + F_{BF}] = 0$

$F_{BC} + F_{BF} = + F_{AB} = -1.77$

$$F_{BC} + F_{BF} = -1.77 \qquad \ldots\ldots (1)$$

$$\Sigma V = 0$$

$$F_{BC} \sin 45^0 - F_{AB} \sin 45^0 - F_{BF} \sin 45^0 - 2 = 0$$

$$F_{BC} \sin 45^0 - F_{BF} \sin 45^0 = 2 + F_{BA} . \sin 45^0$$

$$F_{BC} \sin 45^0 - F_{BF} \sin 45^0 = 2 + (-1.77 \sin 45^0)$$

$$(F_{BC} - F_{BF}) \sin 45^0 = 0.75$$

$$F_{BC} - F_{BF} = \frac{0.75}{\sin 45°} = 1.06$$

$$F_{BC} - F_{BF} = 1.06 \qquad \ldots\ldots (2)$$

Solving equations (1) and (2), we get

F_{BC} = - 0.36 (Compression)

F_{BF} = - 1.42 (Compression)

At joint D:

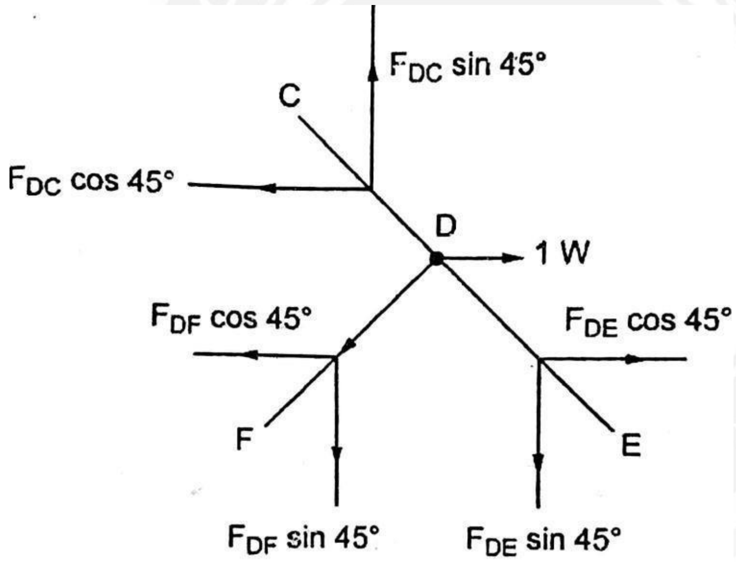


Fig 1.4.d

$$\Sigma H = 0$$

$$- F_{DC} \cos 45^0 - F_{DF} \cos 45^0 + F_{DE} \cos 45^0 + 1 = 0$$

$$- F_{DC} \cos 45^0 - F_{DF} \cos 45^0 - (1.06 \times \cos 45^0) + 1 = 0$$

$$- F_{DC} \cos 45^0 - F_{DF} \cos 45^0 = - 0.25$$

$$- (F_{DC} + F_{DF}) \cos 45^0 = - 0.25$$

$$F_{DC} + F_{DF} = +\frac{0.25}{\cos 45°}$$

$$F_{DC} + F_{DF} = 0.35 \qquad \ldots\ldots (3)$$

$$\Sigma V = 0$$

$$+ F_{DC} \sin 45^0 - F_{DF} \sin 45^0 - F_{DE} \sin 45^0 = 0$$

$$F_{DC} - F_{DF} - F_{DE} = O$$

$$F_{DC} - F_{DF} = + F_{DE}$$

$F_{DC} - F_{DF} = -1.06$ ….. (4)

Solving (3) and (4), we get

$\mathbf{F_{DC} = -0.35}$

(Compression)

$\mathbf{F_{DF} = 0.71}$

(Tension)

Member	Force (kN)	Nature of force
AB	-1.77W	Compression
BC	-0.36W	Compression
CD	-0.35W	Compression
DE	-1.06W	Compression
EF	0.75W	Tension
FA	2.25W	Tension
FD	0.71W	Tension
CF	0.5W	Tension
BF	-1.42W	Compression

Example 1.2.4 Analyse the truss shown in Fig.5.5. using methods of joints.

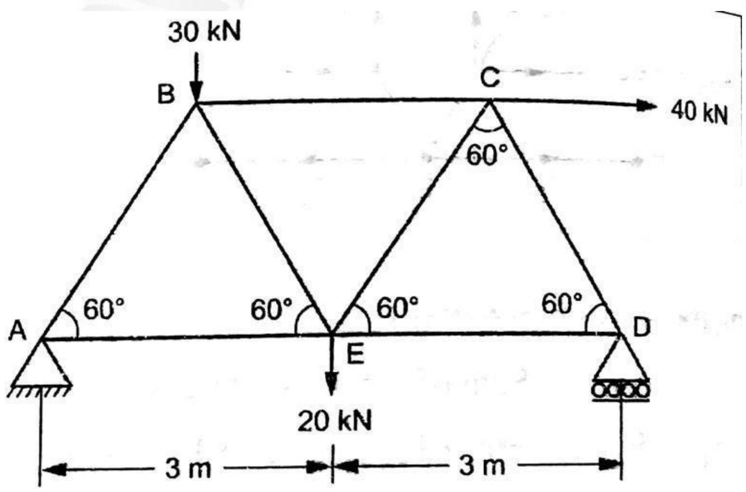


Fig 1.5.a

Solution:

The truss is supported on rollers at D and hence the reaction at D should be vertical (R_D).

The truss is hinged at A and hence end A consists of horizontal reaction (H_A) and vertical reaction (R_A).

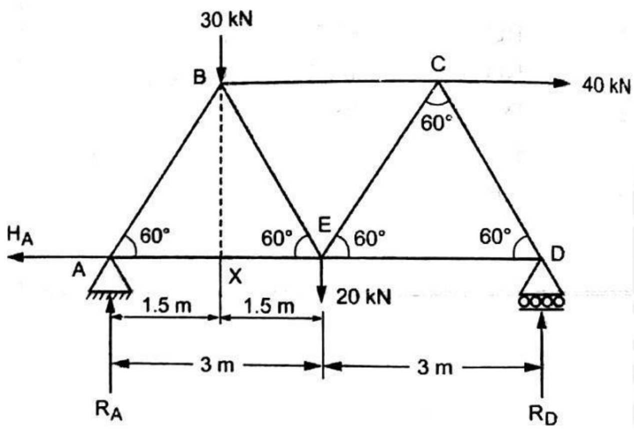


Fig 1.5.b

From Δ^{le} BEX,

$\tan 60° = \frac{BX}{XE} = \frac{BX}{1.5}$

$BX = \tan 60° \times 1.5$

$\mathbf{BX = 2.6\ m}$

Determine the reactions. Taking moment about A

$R_D \times 6 = 40 \times BX + 20 \times 3 + 30 \times 1.5$

$= 40 \times 2.6 + 20 \times 3 + 30 \times 1.5$

$\mathbf{R_D = 34.8}$

kN

We know that,

Upward vertical forces = Downward vertical forces

$R_A + R_D = 30 + 20$

$R_A = 50 - R_D$

$RA = 50 – 34.8$

$\mathbf{R_A = 15.2\ kN}$

We know that,

Horizontal forces towards right side = Horizontal forces towards left side

$40kN = H_{A,}$ $\therefore \mathbf{H_A = 40kN}$

Consider the joint A.

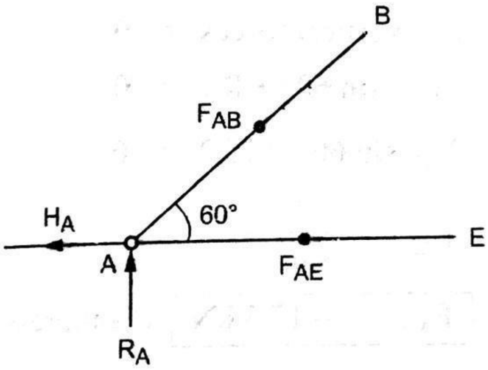


Fig 1.5.c

Assume the forces F_{AB} and F_{AE} acting on joint A are tensile forces (acting away from A). If we get negative value, the force in that member is compressive.

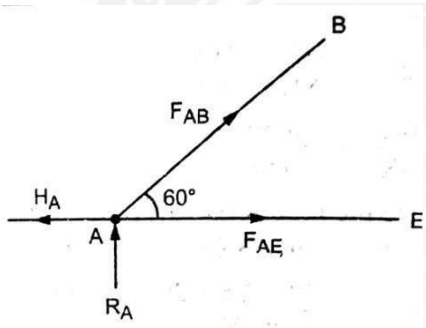


Fig 1.5.d

At joint A:

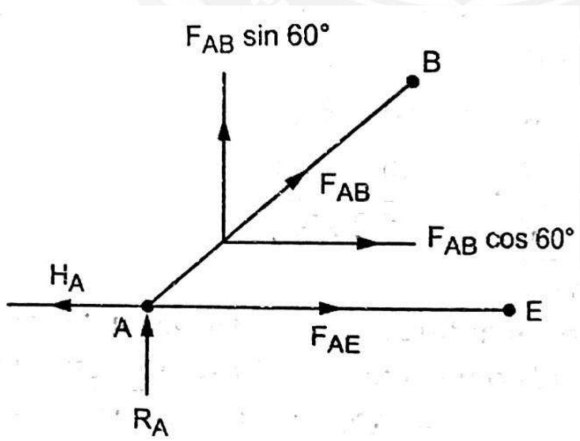


Fig 1.5.e

Resolving the force vertically,

Sum of vertical forces = 0

F_{AB} Sin 60° + R_A = 0

F_{AB} Sin 60° + 15.2 = 0 15 2

$F_{AB} = \frac{- \quad .}{\sin 60°}$

F_{AB} = - 17.5kN (Compressive)

Resolving the force horizontally,

Sum of horizontal forces = 0

$F_{AB} \cos 60^0 - H_A + F_{AE} = 0$

$-17.5 \cos 60^0 - 40 + F_{AE} = 0$

$-17.5(0.5) - 40 + F_{AE} = 0$

F_{AE} = 48.75kN (Tension)

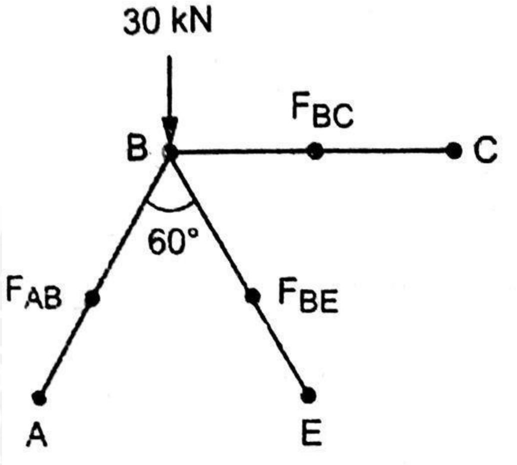


Fig 1.5.f

Consider the joint B.

Assume the forces acting on joint B are tensile forces . If we get negative value, force in that member is compressive.

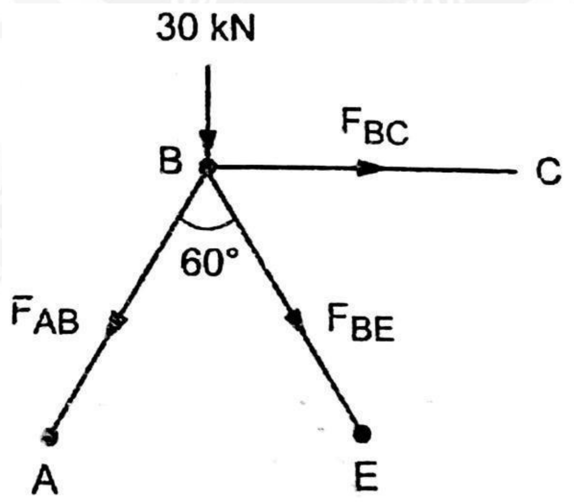


Fig 1.5.f

At joint B

Resolving the force vertically, Sum of vertical forces = 0

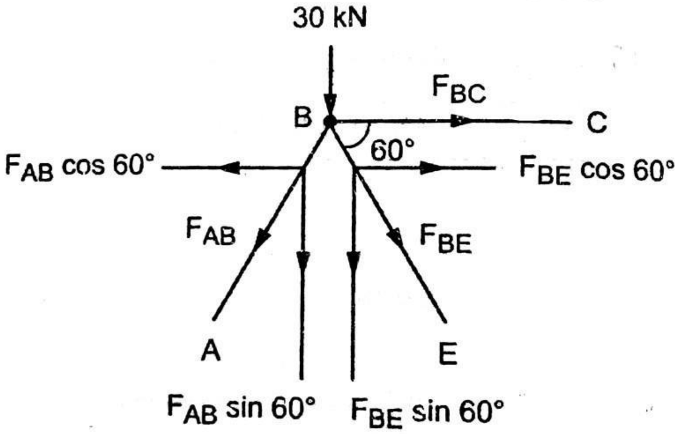


Fig 1.5.g

$30kN - F_{AB} \sin 60^0 - F_{BE} \sin 60^0 = 0$

$-30 + 17.5 \times \sin 60^0 = F_{BE} \sin 60^0$

$\mathbf{F_{BE} = -17.1kN}$ (Compression)

Resolving the force horizontally,

Sum of horizontal forces = 0

$F_{BC} + F_{BE} \cos 60^0 - F_{AB} \cos 60^0 = 0$

$F_{BC} - 17.1(0.5) + 17.1(0.5) = 0$

$\mathbf{F_{BC} = -0.2kN}$ (Compression)

Consider the joint C:

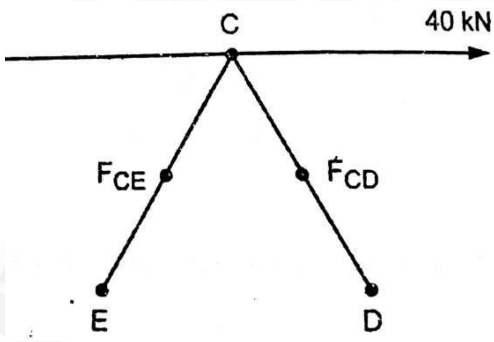


Fig 1.5.h

Assume the forces acting on C are tensile forces . If we get negative value, the force on that member is compressive.

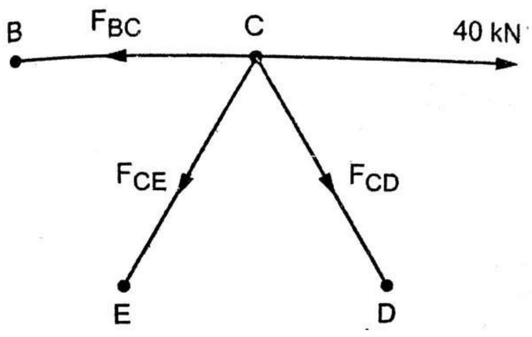


Fig 1.5.i

At joint B:

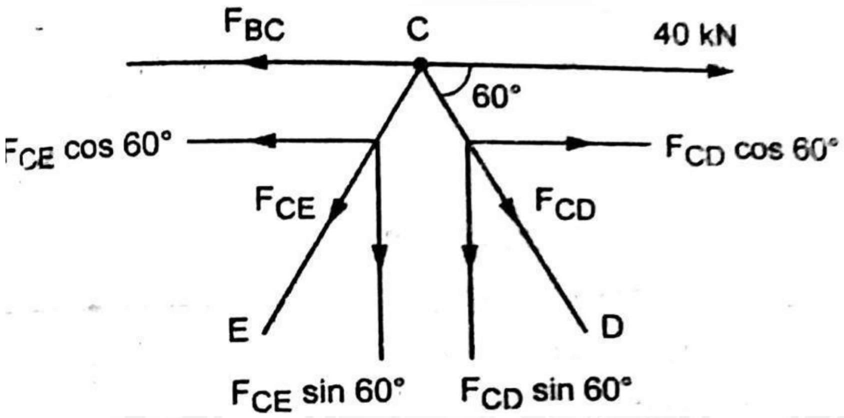


Fig 1.5.j

Resolving the force vertically,

Sum of vertical forces = 0

$-F_{CE} \sin 60^o - F_{CD} \sin 60^o = 0$

$\mathbf{F_{CE} = - F_{CD}}$ ….. (A)

Resolving the force horizontally,

Sum of horizontal forces = 0

$- F_{BC} + 40 - F_{CE} \cos 60^o + F_{CD} \cos 60^o = 0$

$0.2 + 40 + 2 F_{CD} \cos 60^o = 0$

(Since, $F_{CE} = - F_{CD}$)

$\mathbf{F_{CD} = -40.2\ kN}$ (Compression)

Apply in (A),

$\mathbf{F_{CE} = 40.2\ kN}$ (Tension)

Consider the joint D:

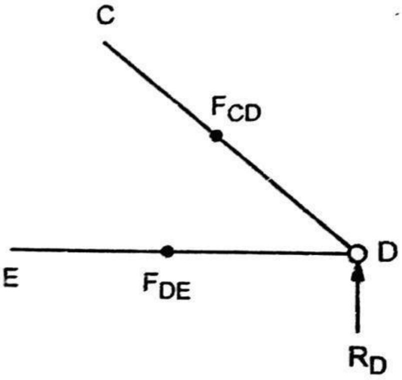


Fig 1.5.k

Assume the forces acting on joint D are tensile forces if we get negative value, the force in that member is compressive.

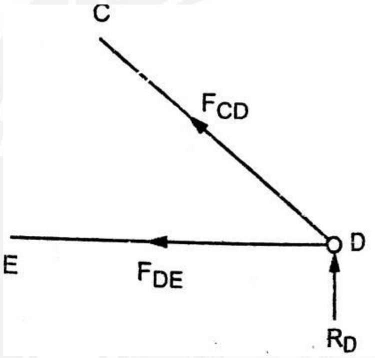


Fig 1.5.l

Resolving the force horizontally,

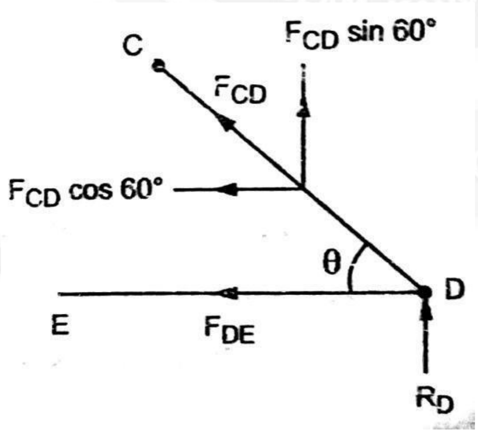


Fig 1.5.m

Sum of horizontal forces = 0

$- F_{DE} - F_{CD} \cos 60^0 = 0$

$- F_{DE} + 40.2 \cos 60^0 = 0$

$\mathbf{F_{DE} = 20.1kN}$ (Tension)

Sl.No.	Member	Force (kN)	Nature of force

1	**AB**	**-17.5**	**Compression**
2	**AE**	**48.75**	**Tension**
3	**BE**	**-17.1**	**Compression**
4	**BC**	**-0.2**	**Compression**
5	**CD**	**-40.2**	**Compression**
6	**CE**	**40.2**	**Tension**
7	**DE**	**20.1**	**Tension**

1.3 METHOD OF SECTIONS

When the forces in a few members of a truss are to be determined, then the method of section is mostly used. This method is very quick as it does not involve the solution of other joints of the truss.

In this method, a section line is passed through the members, in which the forces are to be determined. The section line should be drawn in such a way that it does not cut more than three members in which the forces are unknown. The part of the truss on any one side of the section line is treated as free body in equilibrium under the action of external forces on that part and forces in the members cut by the section line. The unknown forces in the members are then determined by using equations of equilibrium as

Σ Fx = 0, Σ Fy = 0 and Σ M = 0,

Example 1.3.1 A truss of span 6 m is loaded as shown in Fig.5.6. Find the reactions and forces in the members of the truss by method of section.

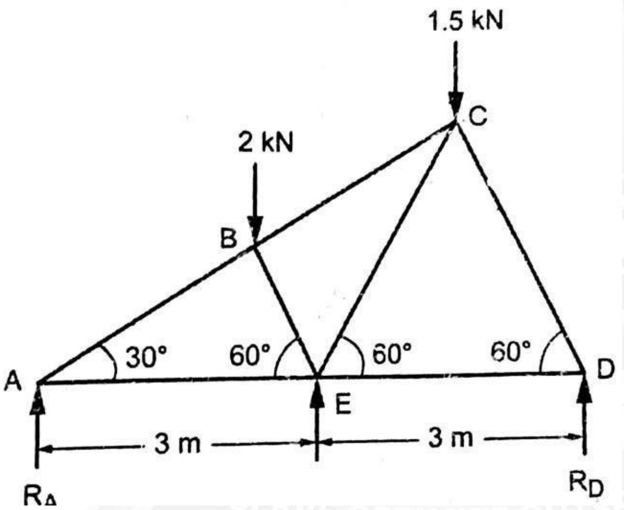


Fig 1.6.a

Solution:

Reaction at R_C = 1.5 kN

Reaction at R_B = 2 kN

Determine the reactions at A and D (R_A and R_D)

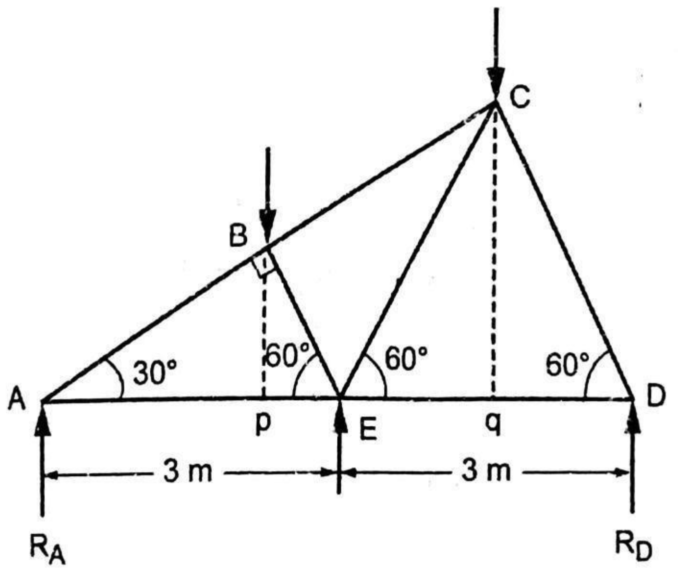


Fig 1.6.b

From ΔCED,

$$Eq = qD = 1.5 \text{ m}$$

From ΔABE,

$$\sin 60° = \frac{AB}{AE}$$

$$AB = AE \times \sin 60° = 3 \times 0.866$$

AB = 2.59 m

From ΔABp,

$$\cos 30° = \frac{Ap}{AB}$$

$$Ap = AB \times \cos 30° = 2.59 \times 0.866$$

Ap = 2.25 m

Taking moment about A:

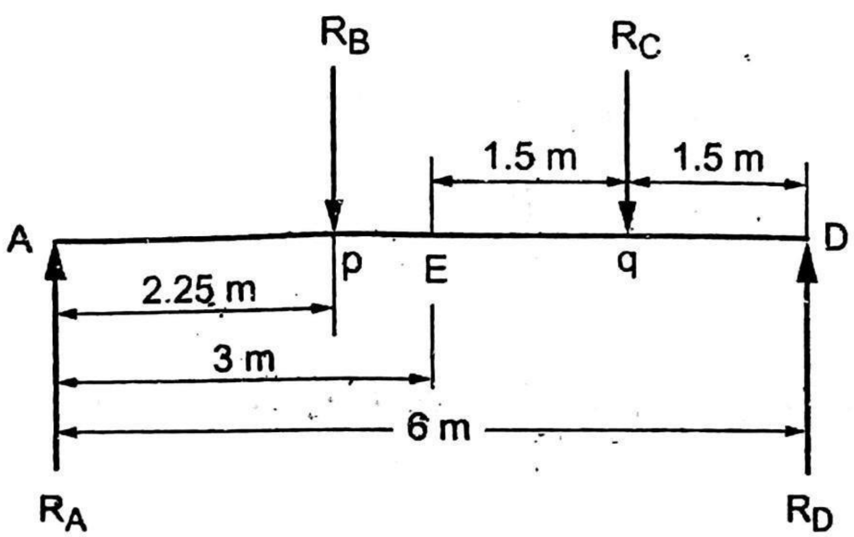


Fig 1.6.c

We know that,

Clockwise moment = Anticlockwise moment

$R_B \times Ap + R_C \times Aq = R_D \times AD$

$R_B \times 2.25 + R_C \times 4.5 = R_D \times 6$

$2 \times 2.25 + 1.5 \times 4.5 = R_D \times 6$

$\mathbf{R_D = 1.875\ KN}$

We know that,

Upward reaction = Downward reaction

$R_A + R_D = R_B + R_C$

RA + 1.875 = 2 + 1.5

$R_A = 1.625$ KN

Draw the section line (1, 1) cutting the members AB and AE

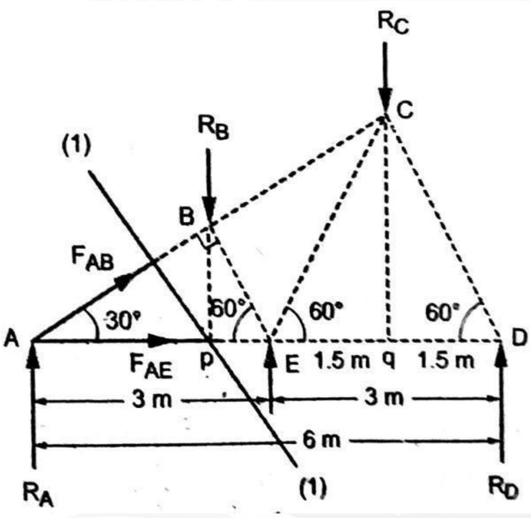


Fig 1.6.d

Now consider the equilibrium of the left part of the truss (since, it is similar than right part).

Taking moments of all forces acting from the left of the section (R_A, F_{AE} and F_{AB}) about point E.

Since force F_{AE} passing through the point E, moment about point E is zero. So, we have to consider R_A and F_{AB} forces only.

We know that,

Sum of Clockwise moment = Sum of Anticlockwise moment

Force (R_A) × perpendicular distance between the force (R_A) and a point E + Force (R_{AB}) × perpendicular distance between the force (R_{AB}) and a point E = 0

(since, there is no anticlockwise moment)

$R_A \times 3 + F_{AB} \times BE = 0$

$$1.625 \times 3 + F_{AB} \times BE = 0 \qquad \ldots(1)$$

From Δ AEB,

$$\sin 30° = \frac{BE}{AE}$$

$$BE = AE \times \sin 30°$$

$$= 3 \times 0.5$$

$$BE = 1.5 \text{ m}$$

$$1.625 \times 3 + F_{AB} \times 1.5 = 0$$

$$F_{AB} = -3.25\text{kN (Compression)}$$

(since, we are assuming all the forces are tensile forces. If we get negative value, the force in that member is compressive.)

Now taking moment of all forces acting to the left of section(R_A,F_{AB} and F_{AE}) about point C.

Since force F_{AB} passing through the point C, the moment about point C is zero.

So, we have to consider R_A and F_{AE} force only.

We know that, sum of Clockwise moment = sum of Anticlockwise moment

Or Force (R_A X perpendicular distance between the force R_A and a point C + Force (F_{AE}) X perpendicular distance between the force F_{AE} and a point C =0

Or $$R_A \text{ X } A_q = F_{AE} \text{ X } C_q$$

Or $$1.625 \text{ X } 4.5 = FAE \text{ X } Cq \qquad \ldots(2)$$

From Δ CqD

$$\text{Cos } 60^o = \frac{qD}{CD} = \frac{1.5}{CD}$$

∴ $$CD = 3\text{m}$$

$$\text{in } 60^o = \frac{Cq}{CD}$$

Or $$Cq = CD \text{ X } \sin 60 = 3 \text{ X } 0.866$$

= 2.59m

From equation (2)

$$1.625 \text{ X } 4.5 = F_{AE} \text{ X } 2.59$$

$$F_{AE} = 2.8 \text{ kN (Tension)}$$

Draw a section line (2,2) cutting the members BC, BE and AE.

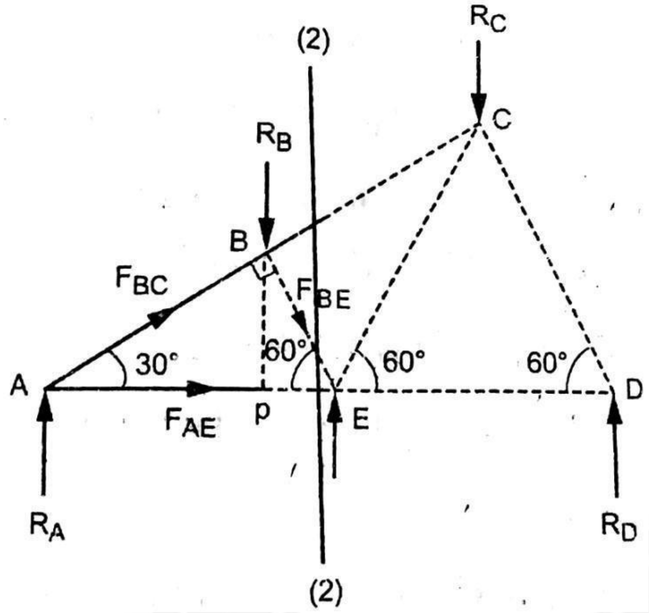


Fig 1.6.e

(only two unknown forces are permitted while considering a section. Here, BC,BE – unknown forces AE – known Force)

Now taking moment of all forces acting to the left of section (R_A, F_{AE}, F_{BC}, R_B and F_{BE}) about point A.

Consider R_B and F_{BE} forces only.

(Since R_A, F_{AE} and F_{BC} passing through a point A)

We know that,

Sum of Clockwise moment = Sum of Anticlockwise moment

Or Force (F_{BE}) X perpendicular distance between the force (F_{BE}) and a point A + Force R_B X perpendicular distance between the force R_B and a point A = 0

Or F_{BE} X AB + R_B X A_p = 0

Or F_{BE} X 2.59 +2 X 2.25 = 0

F_{BE} = -1.73kN Compression

(Since F_{BE} and F_{AE} passing through a point E.)

We know that,

Sum of Clockwise moment = Sum of Anticlockwise moment

Force R_A X Perpendicular distance between the force R_A and a point E + Force F_{BC} X Perpendicular distance between the force F_{BC} and a point E = Force(R_B) X Perpendicular distance between R_B and a point E

Or R_A X AE + F_{BC} X BE = R_B X pE

Or 1.625 X 3 + F_{BC} X 1.5 = R_B X pE

Or 1.625 X 3 + F_{BC} X 1.5 = 2 X 0.75

(AP = 2.25, AE = 3m and pE = 3-2.25 = 0.75m)

F_{BC} = -2.25 kN Compression

Draw a section line (3,3) cutting the members BC, CE and ED. (Only two unknown forces are permitted while considering a section. Here, BC is known force, CE and ED are unknown forces).

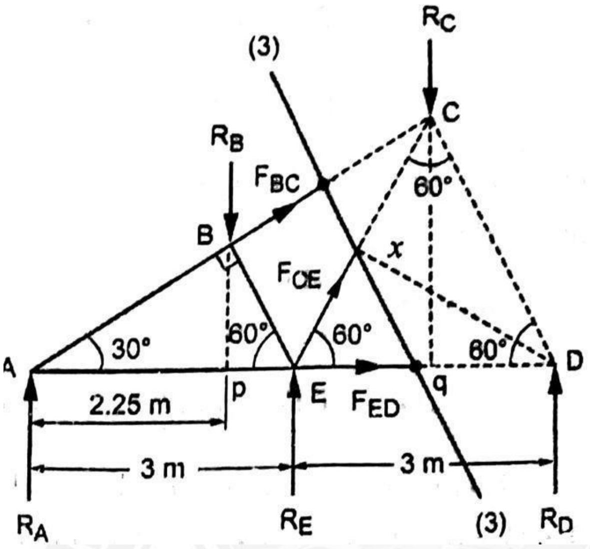


Fig 1.6.f

Taking moment of all forces acting from the left of section (R_A,R_B,F_{BC}, F_{CE} and F_{ED}) about point D.

Consider R_A, R_B , F_{BC}, F_{CE} forces only.

(Since F_{ED} passing through a point D)

We know that, Sum of Clockwise moment = Sum of Anticlockwise moment

Force R_A X perpendicular distance between the force R_A and a point D + Force F_{BC} X perpendicular distance between the force F_{BC} and a point D + Force F_{CE} X perpendicular distance between F_{CE} and a point D = Force R_B X Perpendicular distance between R_B and a point D

Or R_A X AD + F_{BC} X CD + F_{CE} X xD = R_B X pD or

1.625 X 6 + -2.25 X 3 + F_{CE} X xD = 2 X 3.75

(ED = CE = CD = 3m)

ED =3m, Ap = 2.25m,pD = 3+0.75 = 3.75m

From Δ CxD,

$$\text{Sin } 60^o = \frac{xD}{CE}$$

xD = CD X Sin 60º = 3 X Sin 60º or 1.625 X 6 - 2.25 X 3 + F_{CE} X xD = 2 X 3.75 or 1.625 X 6 -2.25 X 3 + F_{CE} X 3 X Sin 60º = 2 X 3.75

∴ F_{CE} = 0.96 kN (Tension)

Now taking moment of all the forces acting from the left of the section (R_A, R_B, F_{BC}, F_{CE} and F_{ED}) about point C.

Consider R_A, R_B, F_{ED} forces only.

(since F_{BC}, F_{CE} passing through a point C)

Force R_A X Perpendicular distance between R_A and a point C = Force R_B X Perpendicular distance between R_B and a point C + Force F_{ED} X perpendicular distance between F_{ED} and a point C

Or $R_A \times A_q = R_B \times Pq + F_{ED} \times Cq$

Or $1.625 \times 4.5 = 2 \times 2.25 + F_{ED} \times 3 \times \sin 60^o$

∴ $F_{ED} = 1.08$ kN (Tension)

We know that From fig (iii),

Aq = 4.5m

Ap = 2.25

(PE = 3-2.25 = 0.75m

Pq = PE + Eq = 0.75+1.5 = 2.25m

Cq = xD = 3 X sin60°)

Consider a section line (4,4) cutting the members CD and ED.

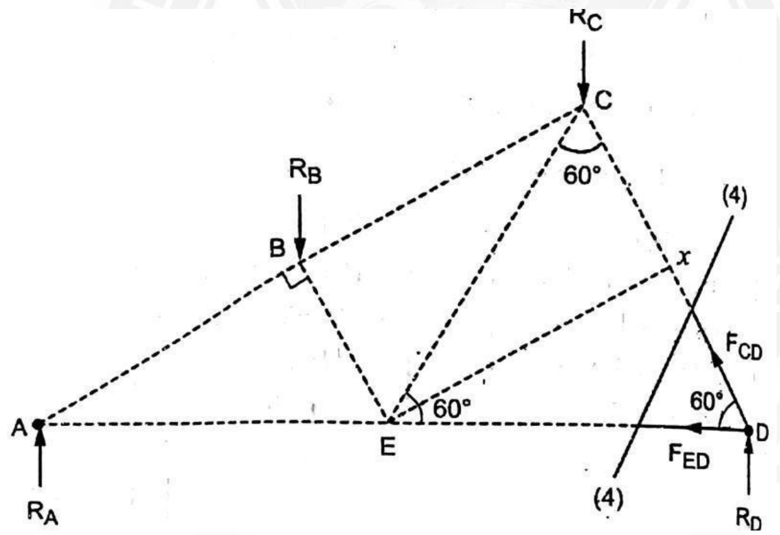


Fig 1.6.g

Consider the equilibrium of the right part of truss (since It is smaller than left part). Now taking moment of all forces acting from the right of the section (R_D,F_{CD} and F_{CE}) about point E.

We know that, Sum of Clockwise moment = Sum of Anticlockwise moment

Or Force R_D X perpendicular distance between the force R_D and a point E + Force F_{CD} X perpendicular distance beteen the force F_{CD} and a point E. = 0

Or $R_D \times DE + F_{CD} \times xE = 0$

Or $1.875 \times 3 + F_{CD} \times 3 \times \sin 60^o = 0$

From Δ CEx, $\sin 60^o = \frac{xE}{CE}$

Or $xE = CE \times \sin 60^o = 3 \times \sin 60^o$

∴ F_{CD} = -2.16 kN Compression **Result:**

Sl.No.	Member	Force (kN)	Nature of force
1	AB	-3.25	Compression
2	AE	2.8	Tension
3	BE	-1.73	Compression
4	BC	-2.25	Compression
5	CE	0.96	Tension
6	ED	1.08	Tension
7	CD	-2.16	Compression

1.4 TENSION COEFFICIENT METHOD

Tension Coefficient is defined as the ratio between pull and length of member

$$T = P/L$$

Where, T = Tension Coefficient

P = pull

L = Length

Example 1.4.1 A truss of 8m span consisting of seven members each 4m length supported at its ends and loaded as shown in Fig.5.7. Determine the forces on the members by tension coefficient method.

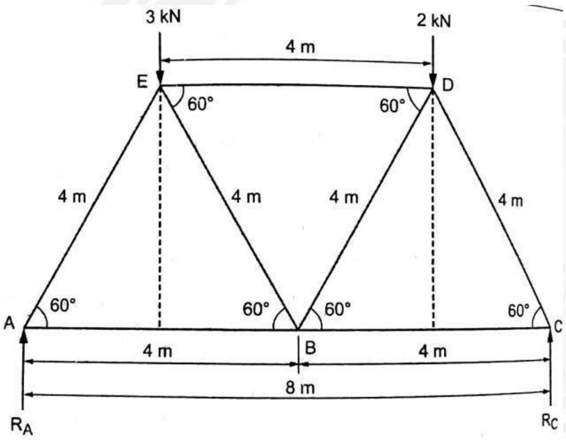


Fig.1.7.a

Solution:

Taking moment about A,

R_E X 2 +R_D X 6 = R_C X 8

Or 3 X 2 + 2 X 6 = R_C X 8

∴ R_C = 2.25 kN

R_A + RC = 3+2

R_A +2.25 = 5

R_A =5-2.25 =2.75 kN Consider a joint A.

In X direction (Take 'A' as a reference point)

At joint A, F_{AB} and F_{AE} forces are acting, so tension coefficient equation is

$$T_{AB}(X_B – x_A) + T_{AE}(X_E – X_A) =0 \qquad …(1)$$

Where,

T_{AB} = Tension coefficient at joint A and B

T_{AE} = Tension Coefficient at joint A and E

X_B = Horizontal distance between point A and B = 4m

X_A = Horizontal distance between point A and A = 0m

X_E = Horizontal distance between point A and E = Ap

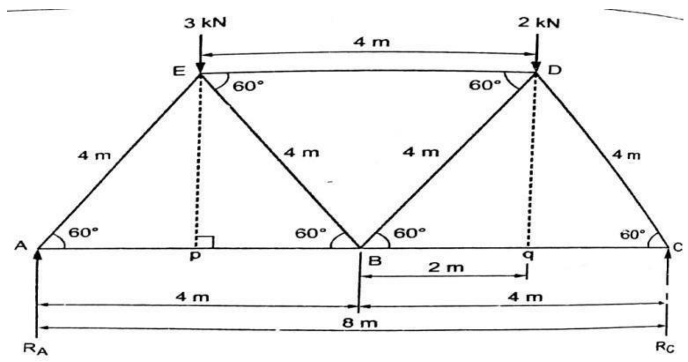


Fig.1.7.b

(1) → T_{AB} (4-0) + T_{AE} (Ap – 0) = 0

Or T_{AB} X 4 + T_{AE} X 2 = 0 …(2)

Similarly,

In Y direction (Take A as a reference point)

T_{AB} (Y_B – Y_A) + T_{AE} (Y_E – Y_A) + R_A = 0 …(3)

Where,

Y_B = Vertical distance between points A and B = 0

Y_A = Vertical distance between points A and A = 0

Y_E = Vertical distance between points A and E = Ep

(3) → T_{AB} (0-0) + T_{AE} (Ep – 0) + 2.75 = 0

Or 0+ T_{AE} (Ep – 0) = -2.75

From Δ APE,

$$\text{Sin}60^\circ = \frac{Ep}{AE} = \frac{Ep}{4}$$

∴ Ep = 3.46m T_{AE} X 3.56 = -2.75

∴ T_{AE} = -0.79

Substituting T_{AE} value in equation (2),

T_{AB} X 4 - 0.79 X 2 = 0

4 T_{AB} = 1.58

∴ T_{AB} =0.39

Consider joint C.

In x direction (Take A as a reference point)

T_{CB}(X_B – X_C) + T_{CD}(X_D – X_C) = 0

Where,

X_B = Horizontal distance between points A and B = 4m

X_C = Horizontal distance between points A and C = 8m

X_D = Horizontal distance between points A and D = Aq

T_{CB}(4 – 8) + T_{CD}(6 – 8) = 0

Or T_{CB} X -4 + T_{CD} X -2 = 0 …(4)

Similarly,

In Y direction (Take 'A' as a reference point)

$$T_{CB}(Y_B - Y_C) + T_{CD}(Y_D - Y_C) + R_C = 0$$

Where,

Y_B = Vertical distance between A and B = 0

Y_C = Vertical distance between A and C = 0

Y_D = Vertical distance between A and D = Dq = Eq

Then, $T_{CB}(0 - 0) + T_{CD}(Ep - 0) + R_C = 0$

T_{CD} X 3.46 + 2.25 = 0

$\therefore T_{CD} = -0.65$

Substituting, T_{CD} value in equation (4).

T_{CB} X (-4) + (-0.65)X (-2) = 0

$-4 T_{CB} = -1.3$

$\therefore T_{CB} = 0.325$

Consider the joint D.

In X direction (Take 'A' as a reference point)

$T_{DC} (X_C - X_D) + T_{DE}(X_E - X_D) + T_{DB}(X_B - X_D) = 0$

Where,

X_C = Horizontal distance between points A and C = 8m

X_D = Horizontal distance between points A and D = 6m

X_E = Horizontal distance between points A and E = 2m

X_B = Horizontal distance between points A and B = 4m

$T_{DC} (8 - 6) + T_{DE}(2 - 6) + T_{DB}(4 - 6) = 0$

or $2T_{DC} - 4 T_{DE} - 2T_{DB} = 0$ …..(5)

Similarly,

In Y direction (Take 'A' as a reference point)

$T_{DC} (Y_C - Y_D) + T_{DE}(Y_E - Y_D) + T_{DB}(Y_B - Y_D) - 2kN = 0$

Where,

Y_C = Vertical distance between points A and C = 0m

Y_D = Vertical distance between points A and D = Dq = Ep = 3.46m

Y_E = Vertical distance between points A and E = Ep = 3.46m

Y_B = Vertical distance between points A and E = 0m

$T_{DC} (0 - 3.46) + T_{DE}(3.46 - 3.46) + T_{DB}(0 - 3.46) - 2 = 0$

$- 3.46 T_{DC} + 0 - 3.46 T_{DB} = 2$

$- 3.46 (-0.65) - 3.46 T_{DB} = 2$

(we know that, $T_{DC} = -0.65$)

$\mathbf{T_{DB} = 0.07}$

Substituting, T_{DC} value in equation (5),

$2T_{DC} – 4\ T_{DE} – 2(0.07)$

= 0 $2(-0.65) – 4$

$T_{DE} – 2(0.07) = 0$

T_{DE} = -0.36

Consider joint E.

In X direction (Take 'A' as a reference point)

$$T_{ED}\ (X_D – X_E) + T_{EA}(X_A – X_E) + T_{EB}(X_B – X_E) = 0$$

Where,

X_D = Horizontal distance between points A and D = 6m

X_E = Horizontal distance between points A and E = 2m

X_A = Horizontal distance between points A and A = 0

X_B = Horizontal distance between points A and B = 4m

$$T_{ED}\ (6 – 2) + T_{EA}(0 –2) + T_{EB}(4 – 2) = 0$$

$4\ T_{ED} – 2\ T_{EA} + 2T_{EB}) = 0$

$4(-0.36) – 2(-0.79) + 2T_{EB} = 0$

T_{EB} = -0.07

Result

Sl.No.	Member	Tension coefficient	Length (m)	Force(kN) = Tension Coefficient X Length	Nature
1	**AE**	**-0.79**	**4**	**-3.16**	**Compression**
2	**AB**	**0.39**	**4**	**1.57**	**Tension**
3	**CD**	**-0.65**	**4**	**-2.6**	**Compression**
4	**CB**	**0.325**	**4**	**1.3**	**Tension**
5	**DB**	**0.07**	**4**	**0.28**	**Tension**
6	**DE**	**-0.36**	**4**	**-1.44**	**Compression**
7	**EB**	**-0.07**	**4**	**-0.28**	**Compression**

TWO MARK QUESTIONS AND ANSWERS

1. What is meant by frame?

A structure made up of several bars (or members) riveted or welded together is known as frame.

2. What are the different types of frames? The different types of frame are :

(i). Perfect frame and (ii). Impefect frame.

Imperfect frame may be a deficient frame or redundant frame.

3.what is meant by Perfect frame?

The frame which is composed of such members, which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external load, is known as perfect frame.

The simplest perfect frame is a triangle as shown in Fig.5.1

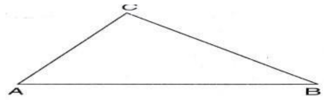


Fig.1.1

It consists of three members AB, BC and AC whereas the three joints are A,B and C. This frame can be easily analysed by the condition of equilibrium given below.

$n = 2j - 3$

Where n = Number of members and j = Number of joints.

4. What is meant by Imperfect frame?

A frame in which the number of members and number of joints are not given by $n = 2j - 3$ is known as imperfect frame.

5.Define Deficient frame and Redundant Frame

If the number of members in an imperfect frame are less than $2j – 3$, then the frame is known as deficient frame. and If the number of members in an imperfect frame are more than $2j – 3$, then the frame is known as redundant frame

6.What are the assumptions made in finding the forces in a truss? (Apr/May 05)

(a) All the members are pin – jointed
(b) The frame is loaded only at the joints
(c) The frame is a perfect frame
(d) The self – weight of the members is neglected

7.What are the methods to analyse the forces in the members of the frame?

a) Analytical method and
b) Graphical method

8.What is meant by method of Joints?

In this method, after determining the reactions at the supports, the equilibrium of every joint is considered. This means the sum of all the vertical forces as well as the horizontal forces acting on a joint is equated to zero. The joint should be selected in such a way that at any time there are only two members, in which the forces are unknown. The force in the member will be compressive if the member pushes the joint to which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.

9. What is maeant by Method of Sections?

When the forces in a few members of a truss are to be determined, then the method of section is mostly used. This method is very quick as it does not involve the solution of other joints of the truss.

In this method, a section line is passed through the members, in which the forces are to be determined. The section line should be drawn in such a way that it does not cut more than three members in which the forces are unknown. The part of the truss on any one side of the section line is treated as free body in equilibrium under the action of external forces on that part and forces in the members cut by the section line. The unknown forces in the members are then determined by using equations of equilibrium as $\Sigma Fx = 0$, $\Sigma Fy = 0$ and $\Sigma M = 0$,

10. What is meant by Tension Coefficient method?

Tension Coefficient is defined as the ratio between pull and length of member

$$T = P/L$$

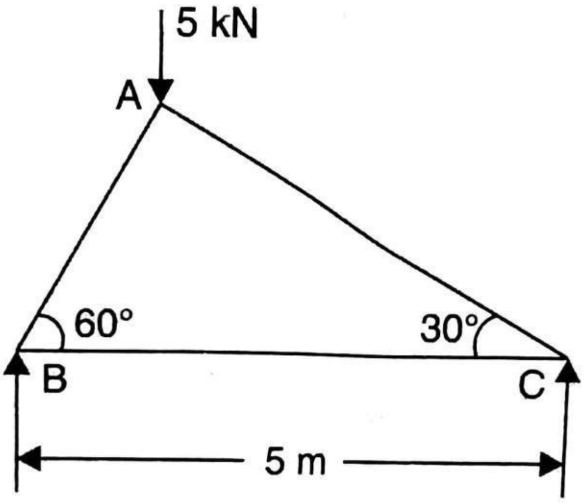
Where, T = Tension Coefficient
P = pull
L = Length

REVIEW QUESTIONS (PART –A)

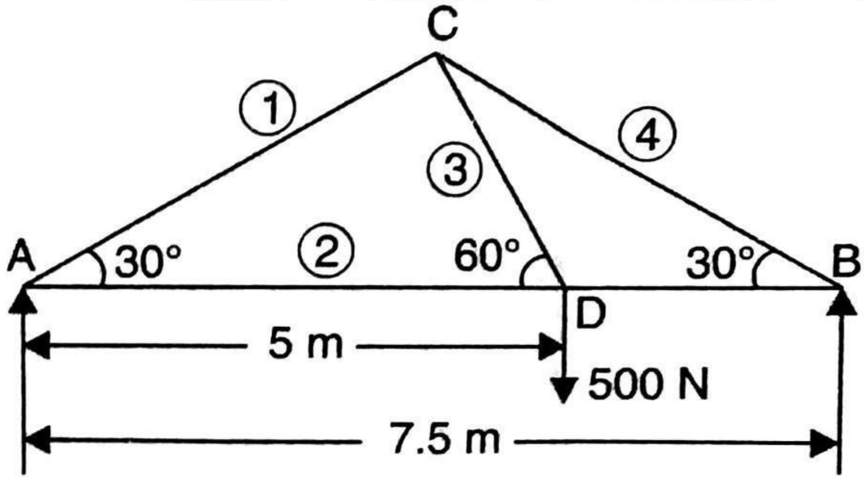
1. What is mean by perfect frame?
2. What are the different types of frames?
3. What is mean by Imperfect frame?
4. What is mean by deficient frame?
5. What is mean by redundant frame?
6. What are the assumptions made in finding out the forces in a frame?
7. What are the reactions of supports of a frame?
8. How will you Analysis of a frame?
10. What are the methods for Analysis the frame?
11. How method of joints applied to Trusses carrying Horizontal loads.
12. How method of joints applied to Trusses carrying inclined loads.
13. How will you determine the forces in a member by method of joints?

REVIEW QUESTIONS (PART – B)

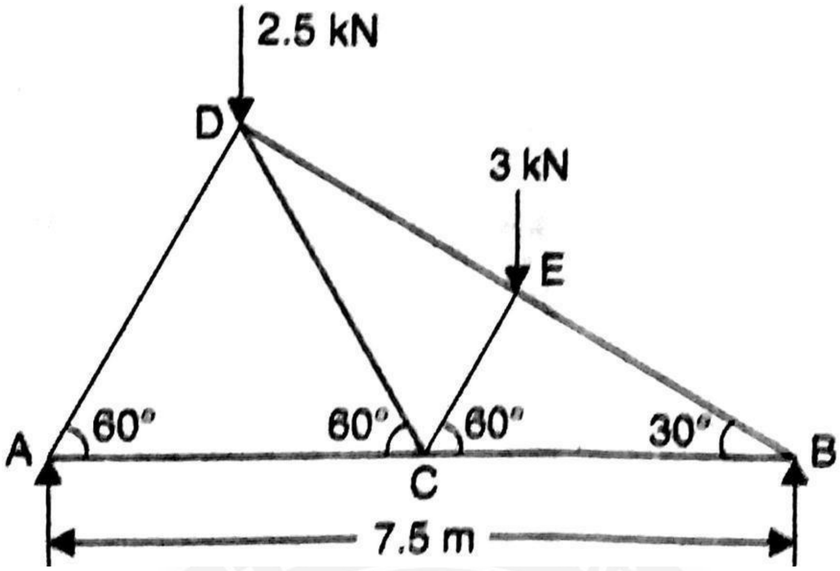
1. Determine the forces in the following figure by method of joints.



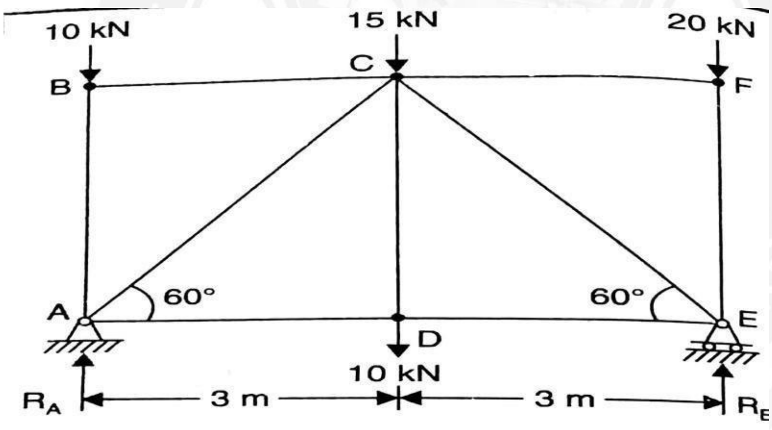
2. Determine the forces in the members by method of joints as given below.



3. Determine the forces in the members by method of joints as given below.



4. Determine the forces in the members by method of joints as given below.

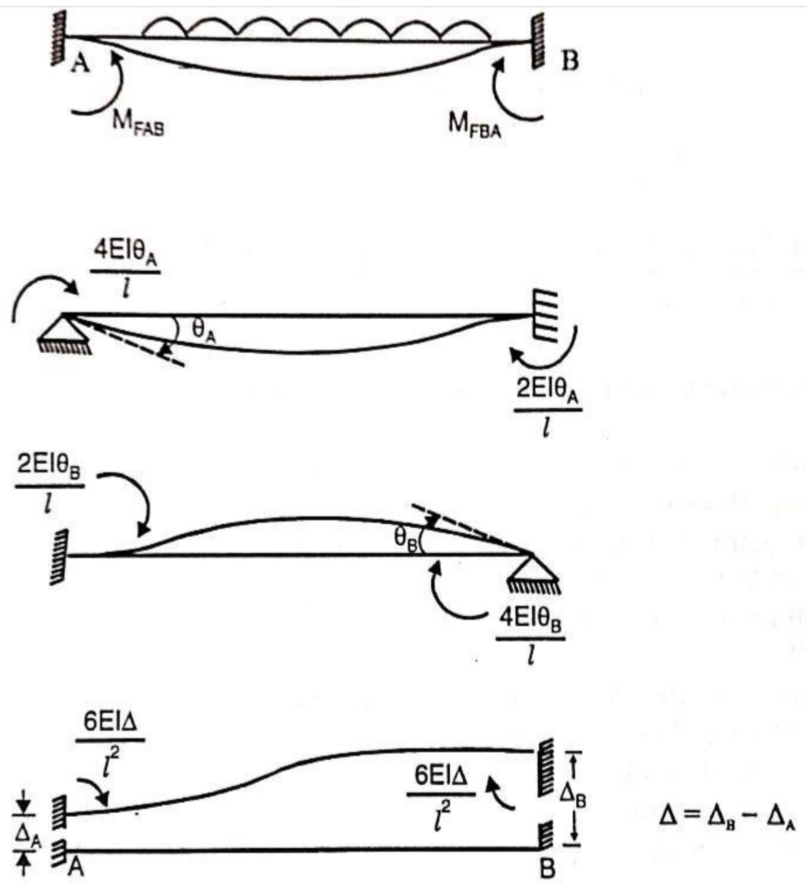


2.1 SLOPE DEFLECTION EQUATION

Slope-Deflection Method of Analysis of Indeterminate Structures

2.1.1 INTRODUCTION

In 1915, George A. Maney introduced the slope-deflection method as one of the classical methods of analysis of indeterminate beams and frames. The method accounts for flexural deformations, but ignores axial and shear deformations. Thus, the unknowns in the slope-deflection method of analysis are the rotations and the relative joint displacements. For the determination of the end moments of members at the joint, this method requires the solution of simultaneous equations consisting of rotations, joint displacements, stiffness, and lengths of members.



positive if its tangent turns in a clockwise direction. The rotation of the chord connecting the ends of a member (Δ/L) the displacement of one end of a member relative to the other, is positive if the member turns in a clockwise direction.

2.1.3 SLOPE DEFLECTION EQUATIONS

The slope-deflection equations, consider a beam of length L and of constant flexural rigidity EI loaded. The member experiences the end moments M_FAB & M_FBA at A and B,respectively.And undergoes the deformed shape with the assumption that the right end B of the member settles by an amount Δ. The end moments are the summation of the moments caused by the rotations of the joints at the ends A and B (θ_A and θ_B) of the beam,and it fixed at both ends referred to as fixed end moments (M_FAB & M_FBA)

The slope equation is;

For member AB;

$$MAB = MFAB + 2EI/L(2\theta A+ \theta B+3\delta/L)$$

$$MBA = MFBA + 2EI/L(2\theta B+ \theta A+3\delta/L)$$

Where,

MAB & MBA – Final Moments at members AB and BA.

MFAB & MFBA – Fixed end moments.

E & I – young´s modulus and moment of inertia.

θA & θB – slope angles.

δ – Deflection

L - Length

2.1.4 EQUILIBRIUM CONDITIONS

- Joint equilibrium conditions
- Shear equilibrium conditions

JOINT EQUILIBRIUM

Joint equilibrium conditions imply that each joint with a degree of freedom should have no unbalanced moments i.e. be in equilibrium.Therefore,

Sum of (end moments + fixed end moments) = Sum of external moments directly applied at the joint.

$$MBA + MBC = 0;$$

SHEAR EQUILIBRIUM

When there are chord rotations in a frame, additional equilibrium conditions, namely the shear equilibrium conditions need to be taken into account.

2.1.5 ANALYSIS OF INDETERMINATE BEAMS

The procedure for the analysis of indeterminate beams by the slope-deflection method is summarized below.

Procedure for Analysis of Indeterminate Beams and Non-Sway Frames by the Slope-Deflection Method

• Determine the fixed-end moments for the members of the beam.

• Determine the rotations of the chord if there is any support settlement.

• Write the slope-deflection equation for the members' end moments in terms of unknown rotations.

• Write the equilibrium equations at each joint that is free to rotate in terms of the end moments of members connected at that joint.

• Solve the system of equations obtained simultaneously to determine the unknown joint rotations.

• Substitute the computed joint rotations into the equations obtained in step 3 to determine the members' end moments.

• Draw a free-body diagram of the indeterminate beams indicating the end moments at the joint.

• Draw the shearing force diagrams of the beam by considering the freebody diagram of each span of the beam in the case of a multi-span structure.

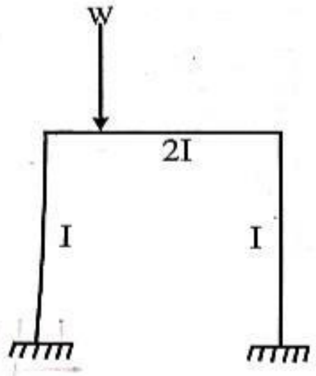
2.1.6 ANALYSIS OF INDETERMINATE FRAMES

Indeterminate frames are categorized as frames with or without side-sway. A frame with side-sway is one that permits a lateral moment or a swaying to one side due to the asymmetrical nature of its structure or loading. The analysis of frames without side-sway is similar to the analysis of beams considered in the preceding section, while the analysis of frames with side-sway requires taking into consideration the effect of the lateral movement of the structure.

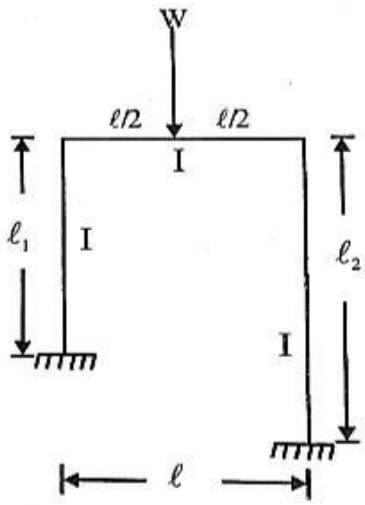
2.1.7. RIGID FRAMES WITH SWAY IN SLOPE DEFLECTION METHOD.

Portal frames may sway due to one of the following reasons:

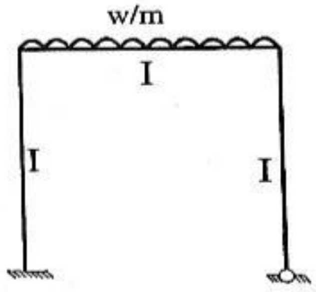
- Eccentric or unsymmetrical loading on the portal frames.
- Unsymmetrical shape of the frames.
- Different end conditions of the columns of the portal frames.
- Non uniform section of the members of the frame.
- Horizontal loading on the columns of the frame.
- Settlement of the supports of the frame.
- A combination of the above.



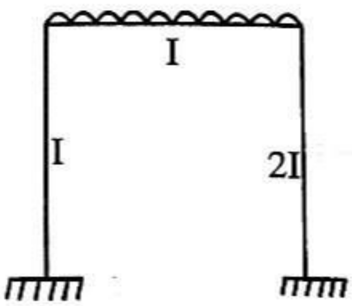
(1)



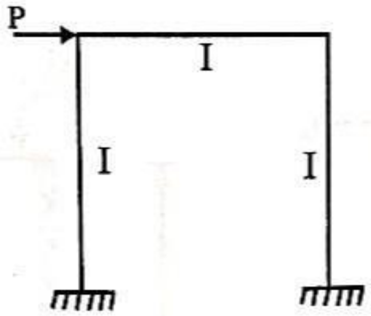
(2)



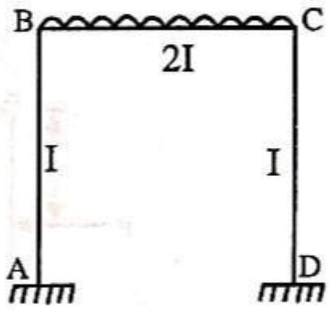
(3)



(4)

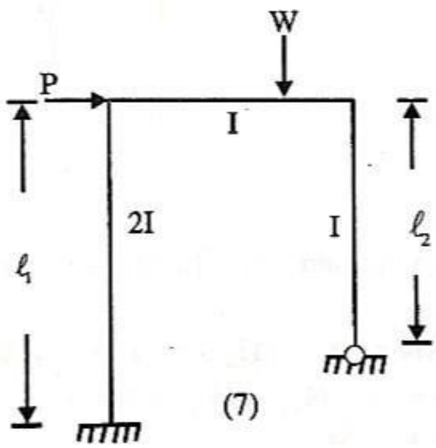


(5)



(6)

D Settles down / Sinks by δ



(7)

2.2 ANALYSIS OF CONTINUOUS BEAMS IN SLOPE DEFLECTION METHOD.

2.2.1 NUMERICAL EXAMPLES ON(CONTINUOUS BEAMS):

PROBLEM NO:01

Analysis the continuous beam shown in fig.2.8,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.

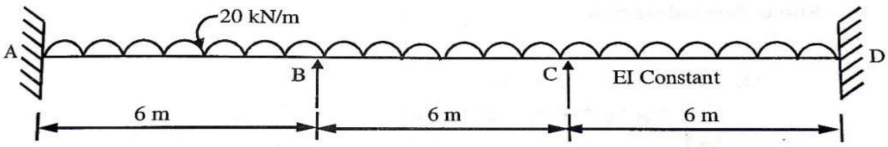


Fig. 2.8

Solutions:

- **Fixed End Moments:**
  MFAB = MFBC = MFCD = $-Wl^2/12$ = - $20x6^2/12$ = - 60 kNm
  MFBA = MFCB = MFDC = $Wl^2/12$ = $20x6^2/12$ = 60 kNm
- **Slope Deflection Equations:**

The structure is symmetrical.So is the loadind.There is no sinking of supports.Hence the following conditions prevail.

- $\theta A = \theta D = 0$
- $\delta = 0$ for all spans
- $\theta B = \theta C$
  Hence there is only one unknown displacement,namely θB.For span AB,the general slope deflection equation is

  MAB = MFAB + 2EI/6(2θA + θB + 3δ/l)

  MAB = - 60 + 2EI/6(θB)--------(2.1)

Since θA = 0 and δ = 0

MAB = 60 + 2EI/6(θB) ------- (2.2)

No other slope deflection equation is needed.

Since θB is the only unknown.

For span BC,

$$MBC = MFBC + 2EI/6(2\theta B + \theta C + 3\delta/l)$$

$$MBC = -60 + 2EI/6(3\theta B) \text{ -------(2.3)}$$

- **Joint Equilibrium Equations:**

MAB + MBC = 0

60 + 2EIθB/3 – 60 + EIθB = 0

Hence, θB = 0

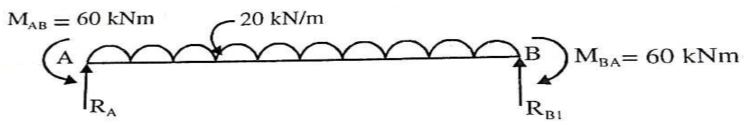
- **Final Moments:**

  MAB = MBC = MCD = - 60 kNm

  MBA = MCB = MDC = 60 kNm

- **Shear Force Diagram:**

  Span AB:



Taking moments about A,on the free body diagram of span AB,

$$- RB1 \times 6 – MAB + MBA + wl^2/2 = 0$$

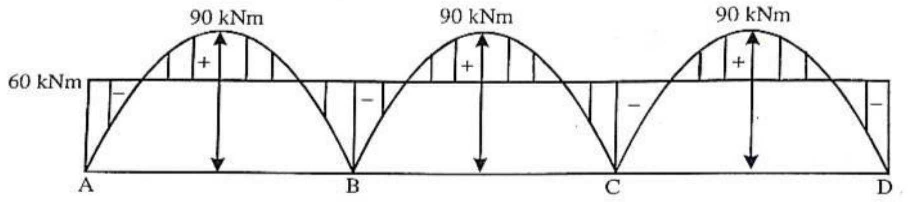
$$- RB1 \times 6 – 60 + 60 + 20 \times 6^2/2 = 0$$

RB1 = 60KN ; RA = 60KN

Similarly in span BC, RB2 = RC1 = 60

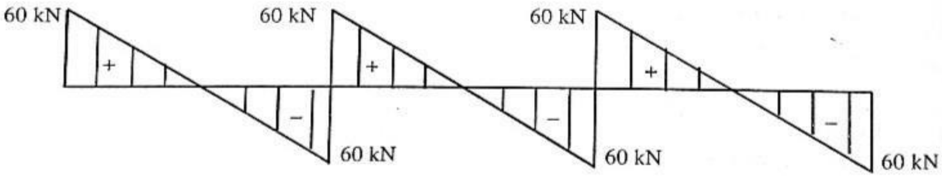
RB = RB1 + RB2 = 120 KN

- **BMD and SFD:**



Bending moment diagram

Simply supported span bending moment = $\frac{wl^2}{8} = \frac{20 \times 6^2}{8} = 90$ kNm



Shear force diagram

PROBLEM NO:02

Analysis the continuous beam shown in fig.2.10,Calculate the support moments using slope deflection method.Support B sinks by 10mm.Take E = 2 x 10^5 N/mm^2,I = 16 x 10^7 mm^4.Sketch the SF and BM diagrams.

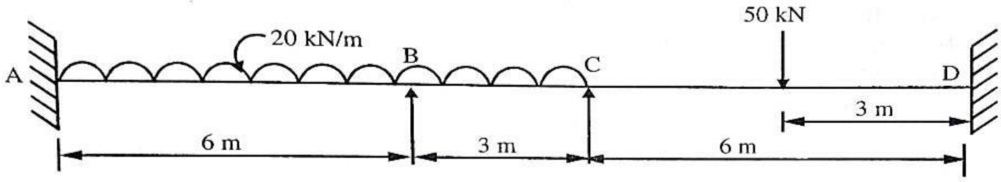


Fig. 2.10

Solution:

- **Fixed End Moments:**

  MFAB = -Wl^2/12 = - 20x6^2/12 = - 60 kNm;

  MFBA = Wl^2/12 = 20x6^2/12 = 60 kNm;

  MFBC = -Wl^2/12 = - 20x3^2/12 = - 15 kNm;

  MFCB = Wl^2/12 = 20x3^2/12 = 15 kNm;

  MFCD = -Wl/8 = - 50x6/8 = - 37.5 kNm;

  MFDC = Wl/8 = 50x6/8 = 37.5 kNm;

- **Slope Deflection Equations:**

  MAB = MFAB + 2EI/6(2θA + θB + 3δ/l)

  = - 60 + EI/3(0 + θB – 1/200) --- (1)

  MBA = MFBA + 2EI/6(2θB + θA + 3δ/l)

$= 60 + EI/3(2\theta B – 3 \times 10/6000)$ --- (2)

$MBC = MFBC + 2EI/3(2\theta B + \theta C + 3\delta/l)$

$= - 15 + 2EI/3(2\theta B + \theta C + 1/100)$ --- (3)

$MCB = MFCB + 2EI/3(2\theta C + \theta B + 3\delta/l)$

$= 15 + 2EI/3(2\theta C + \theta B + 1/100)$ --- (4)

$MCD = MFCD + 2EI/6(2\theta C + \theta D + 3\delta/l)$

$= - 37.5 + EI/3(2\theta C)$ --- (5)

$MDC = MFDC + 2EI/6(2\theta D + \theta C + 3\delta/l)$

$= 37.5 + EI/3(\theta C)$ --- (6)

- **Joint Equilibrium Equations:**

Joint B:

$MBA + MBC = 0$

$EI/3(6\theta B+2\theta C+3/200) = - 135$ --- (7)

Joint C:

$MCB + MCD = 0$

$EI(\theta B+3\theta C+1/100) = 33.75$ --- (8)

Equvating (7 & 8);we get

$\theta C = - 1/464$; $\theta B = - 1/402$

- **Final Moments:**

MAB = - 139.843 kNm;

MBA = - 46.354 kNm;

MBC = 46.3 kNm;

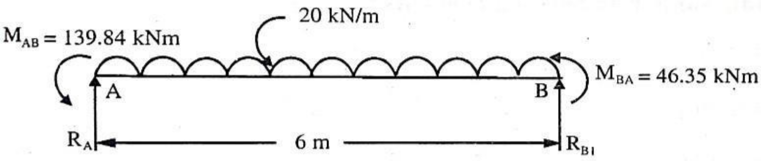
MCB = 83.35 kNm;

MCD = - 83.477 kNm;

MDC = 14.51 kNm;

- **To Draw S.F.D:**

Span AB:

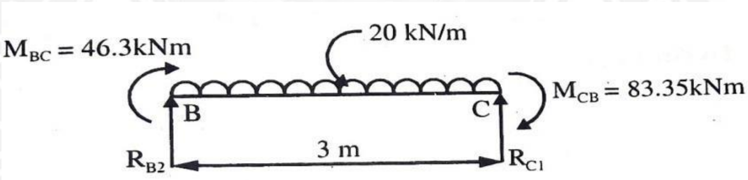


Taking moments about A.

$20 \times 6^2/2 - 46.35 - 139.84 - RB1(6) = 0$; RB1 = 28.97 KN

RA = 20 x 6 – 28.97; RA = 91.03 KN

Span BC:

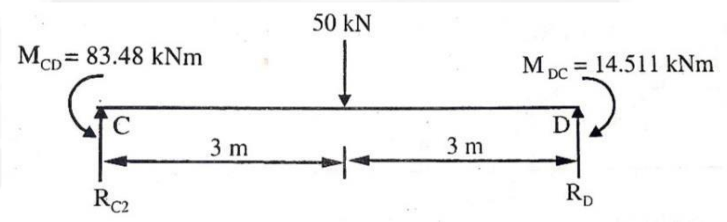


Taking moments about B.

$20 \times 3^2/2 + 83.35 + 46.3 - RC1(3) = 0$; RC1 = 73.22 KN

RB2 = 20 x 3 – 73.22; RB2 = - 13.21 KN

Span CD:



Taking moments about C.

14.511 + 50(3) – 83.48 – RD(6) = 0;

RD = 13.5 KN; RC2 = 36.5 KN

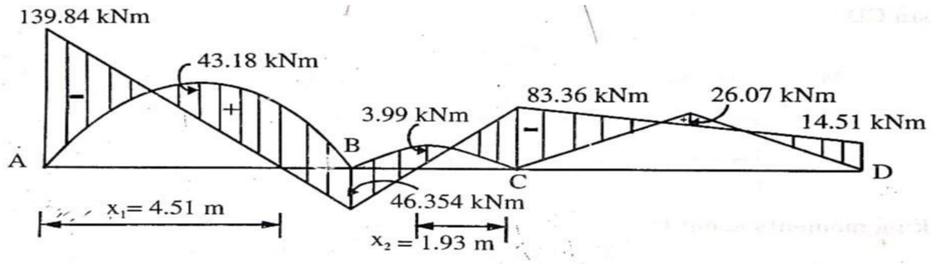
- **Free BMD:**

  MAB = $Wl^2/8$ = 20 x $6^2/8$ = 90 kNm

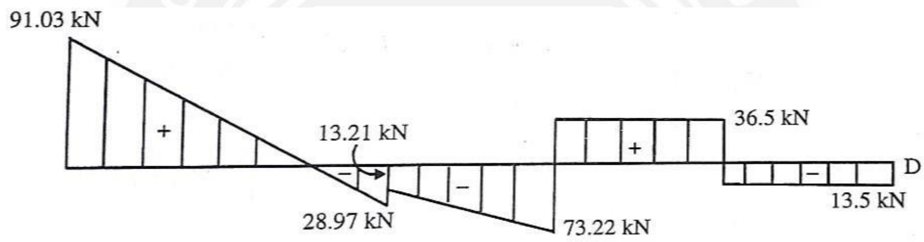
  MBC = $Wl^2/8$ = 20 x $3^2/8$ = 22.5 kNm

  MCD = Wl/4 = 50 x 6/4 = 75 kNm

- **BMD and SFD:**



Bending moment diagram



Shear force diagram

PROBLEM NO:03

Analysis the continuous beam shown in fig.2.3,Calculate the support moments using slope deflection method. Sketch the BM diagrams.

2IAB = IBC = 2ICD = 2I

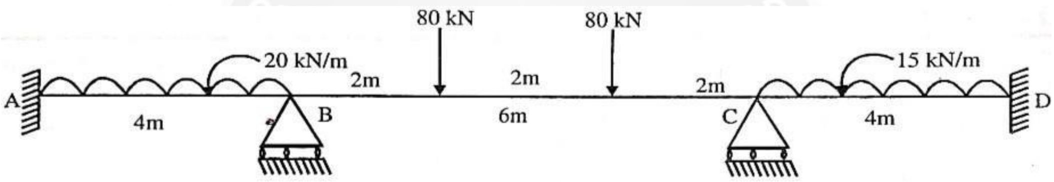


Fig. 2.3

IAB = ICD = I, IBC = 2I, θA = θD = 0 (A and D are fixed)

Solution:

- **Fixed End Moments:**

  MFAB = $-Wl^2/12$ = - 20x$4^2/12$ = - 26.67 kNm;

  MFBA = $Wl^2/12$ = 20x$4^2/12$ = 26.67 kNm;

MFBC = -Wa(a + c)/6 = - 80(4 + 2)/6 = - 106.67 kNm;

MFCB = Wa(a + c)/6 = 80(4 + 2)/6 = 106.67 kNm;

MFCD = $-Wl^2/12$ = - $15x4^2/12$ = - 20 kNm;

MFDC = $Wl^2/12$ = $15x4^2/12$ = 20 kNm;

- **Slope Deflection Equations:**

MAB = MFAB + 2EI/6(2θA + θB + 3δ/l)

= - 26.67 + 0.5EIθB --- (1)

MBA = MFBA + 2EI/6(2θB + θA + 3δ/l)

= 26.67 + 0.5EIθB --- (2)

MBC = MFBC + 2EI/3(2θB + θC + 3δ/l)

= - 106.67 + EI (1.332θB + 0.666θC) --- (3)

MCB = MFCB + 2EI/3(2θC + θB + 3δ/l)

= 106.67 + EI (1.332θC + 0.666θB) --- (4)

MCD = MFCD + 2EI/6(2θC + θD + 3δ/l)

= - 20 + EIθC --- (5)

MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

= 20 + EI (0.5θC) --- (6)

- **Joint Equilibrium Equations:**

Joint B:

MBA + MBC = 0

2.333θB + 0.666θC = 80/EI --- (7)

Joint C:

MCB + MCD = 0

0.666θB + 2.333θC = 86.67/EI --- (8)

Equvating (7 & 8);we get

θC = - 51.11/EI; θB = 48.88/EI;

- **Final Moments:**

MAB = - 2.23 kNm;

MBA = 75.55 kNm;

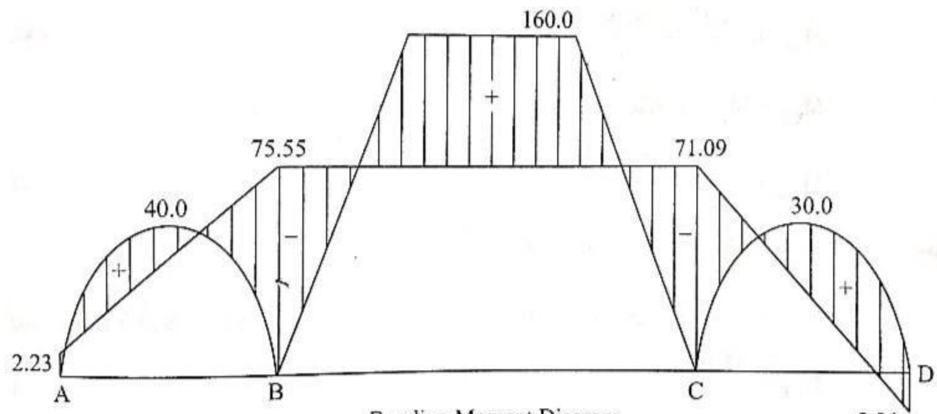
MBC = - 75.55 kNm;

MCB = 71.09 kNm;

MCD = - 71.09 kNm;

MDC = - 5.56 kNm;

- **BMD:**



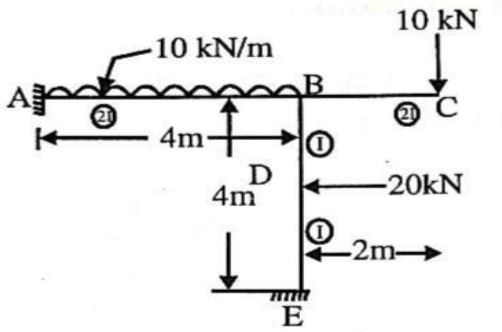
Bending Moment Diagram

2.3 ANALYSIS OF RIGID FRAMES IN SLOPE DEFLECTION METHOD.

2.3.1 NUMERICAL EXAMPLES ON (RIGID FRAMES):

PROBLEM NO:01

Analysis the rigid frame shown in fig.,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.



Solution:

- **Fixed End Moments:**

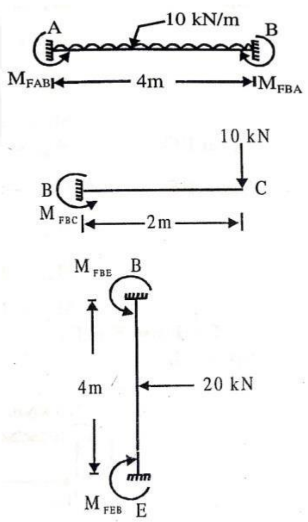
  MFAB = $-Wl^2/12 = -10x4^2/12 = -13.33$ kNm;

  MFBA = $Wl^2/12 = 10x4^2/12 = 13.33$ kNm;

  MFBC = -10 x 2 = - 20 kNm;

  MFBE = -Wl/8 = - 20x4/8 = - 10 kNm;

  MFEB = Wl/8 = 20x4/8 = 10 kNm;



- **Slope Deflection Equations:**

MAB = MFAB + 2E(2I)/4(2θA + θB + 3δ/l)

= - 13.33 + EIθB --- (1)

MBA = MFBA + 2E(2I)/4(2θB + θA + 3δ/l)

= 13.33 + EIθB --- (2)

MBE = MFBE + 2EI/3(2θB + θE + 3δ/l)

= - 10 + EIθB --- (3)

MEB = MFEB + 2EI/3(2θE + θB + 3δ/l)

= 10 + 0.5EIθB --- (4)

- **Joint Equilibrium Equations:**

Joint B, ΣM = 0;

MBA + MBC + MBE = 0

13.33 + 2EIθB – 10 + EIθB – 20 = 0

3EIθB – 16.67 = 0

θB = 5.557/EI;

- **Final Moments:**

MAB = - 7.773 kNm;

MBA = 24.44 kNm;

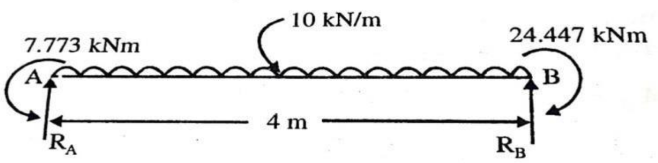
MBC = - 20 kNm;

MCD = - 4.33 kNm;

MDC = 12.78 kNm;

- **To Draw S.F.D:**

Span AB:

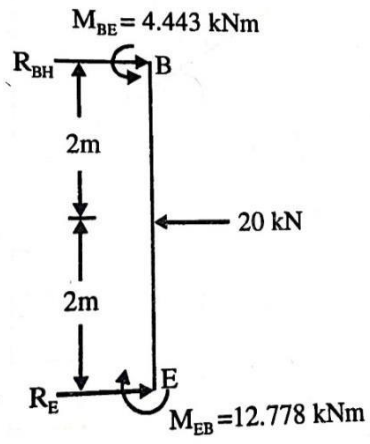


Taking moments about A.

RA x 4 – 10 x 4 x 4/2 – 7.773 + 24.447 = 0; RA = 15.83 KN

RBH = 10 x 4 – RA; RBH = 24.168 KN

Span BE:

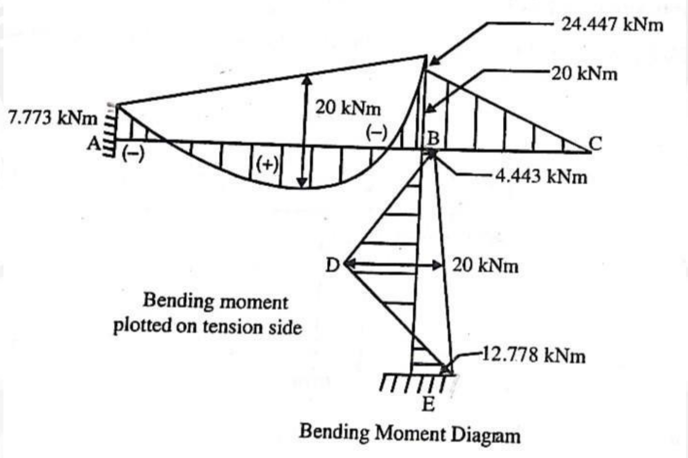


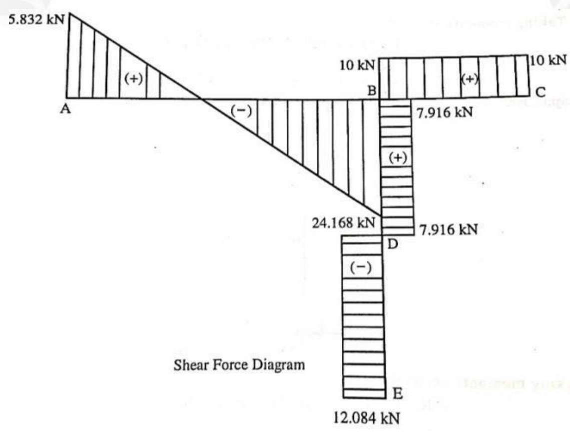
Taking moments about B.

-RE x 4 – 4.443 + 12.778 + 20 x 2 = 0; RE = 12.083 KN

RBH = Total load – RE = 7.916 KN

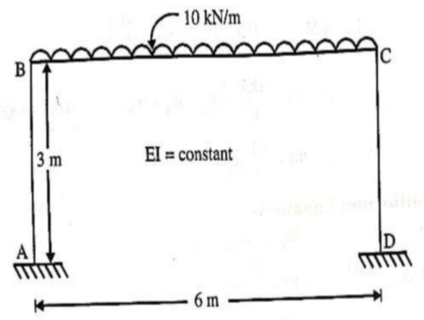
- **BMD and SFD:**





PROBLEM NO:02

Analysis the rigid frame shown in fig.,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.



Solution:

- **Fixed End Moments:**

  MFAB = MFBA = 0

  MFBC = $-Wl^2/12 = -10\text{x}6^2/12 = -30$ kNm

  MFCB = $Wl^2/12 = 10\text{x}6^2/12 = 30$ kNm

  MFCD = MFDC = 0

- **Slope Deflection Equations:**

  MAB = MFAB + 2EI/3(2θA + θB + 3δ/l)

  = 2/3EIθB --- (1)

  MBA = MFBA + 2EI/3(2θB + θA + 3δ/l)

  = 4/3EIθB --- (2)

  MBC = MFBC + 2EI/6(2θB + θC + 3δ/l)

  = - 30 + 1/3EIθB --- (3)

  MCB = MFCB + 2EI/3(2θC + θB + 3δ/l)

  = 30 + 1/3EIθB --- (4)

  MCD = MFCD + 2EI/6(2θC + θD + 3δ/l)

  = 4/3EIθB --- (5)

  MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

  = 2/3EIθB --- (6)

- **Joint Equilibrium Equations:**

  Joint B:

  MBA + MBC = 0

  4/3θB – 30 + 1/3EIθB = 0 --- (7)

  Joint C:

  MCB + MCD = 0

  4/3θC – 30 + 1/3EIθC = 0 --- (8)

  Equvating (7 & 8);we get

  θC = - 18/EI; θB = 18/EI;

- **Final Moments:**

  MAB = 12 kNm;

  MBA = 24 kNm;

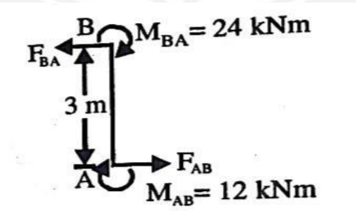
  MBC = - 24 kNm;

  MCB = 24 kNm;

  MCD = - 24 kNm;

  MDC = - 12 kNm;

- **To Draw S.F.D:**



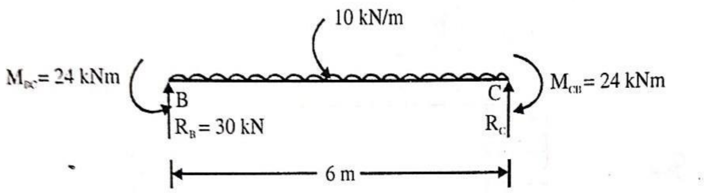
Span AB:

Taking moments about A.

-FBA x 3 + MBA + MAB = 0;

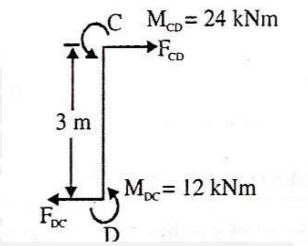
24 + 12 = FBA x 3; FBA = FAB = 12 KN

Span BC:



RB = RC = Total load/2 = 10 x 6/2 = 30 KN

Span CD:

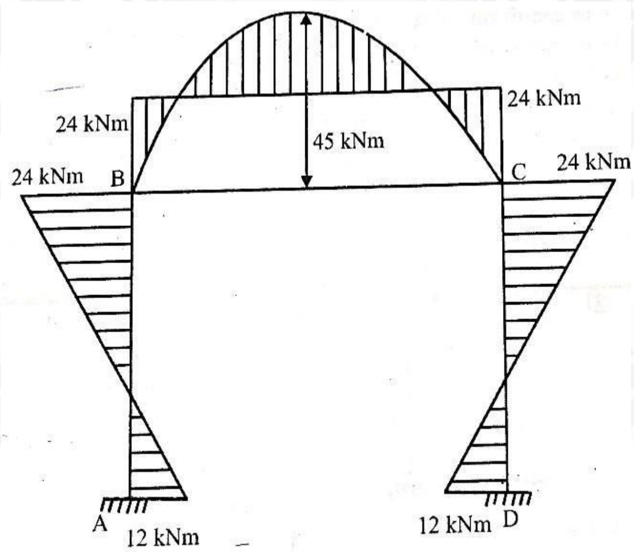


FCD = FDC = 12KN (by symmetry)

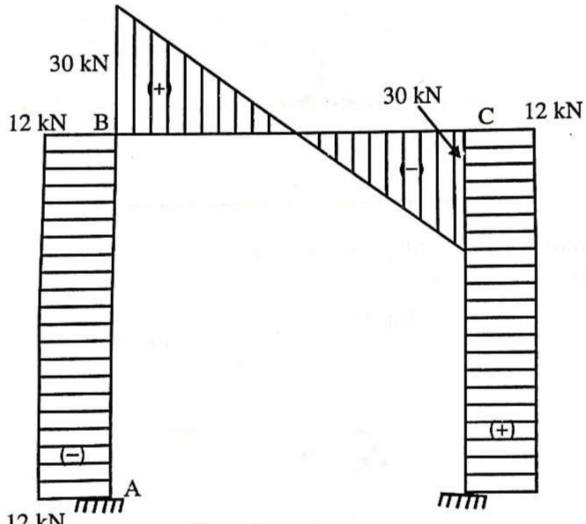
- **Free BMD:**

  MBC = $Wl^2/8 = 10 \times 6^2/8 = 45$ kNm

- **BMD and SFD:**



Bending Moment Diagram

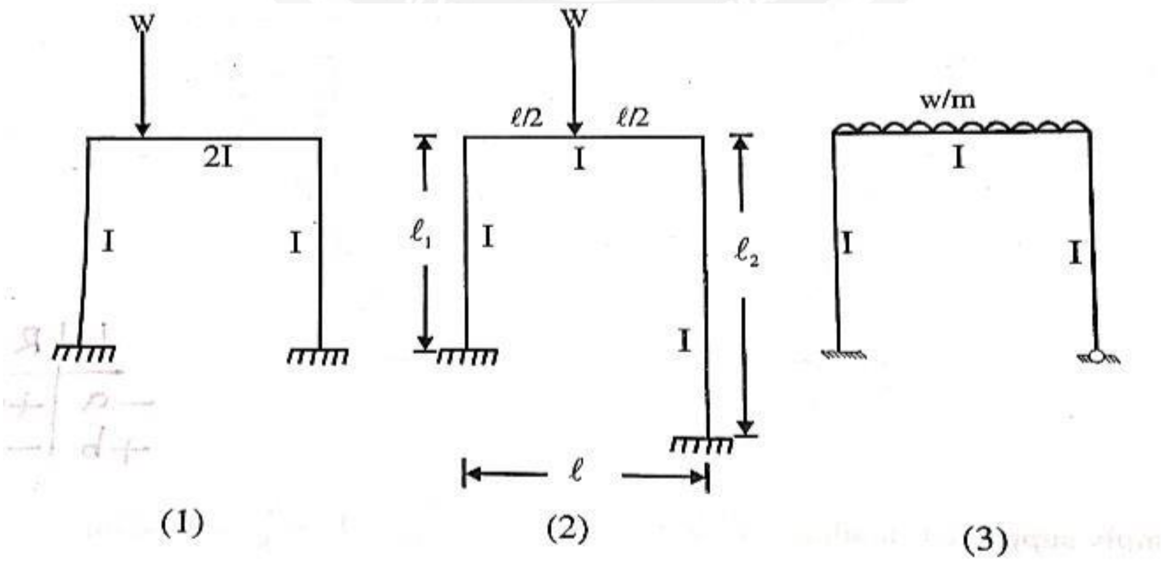


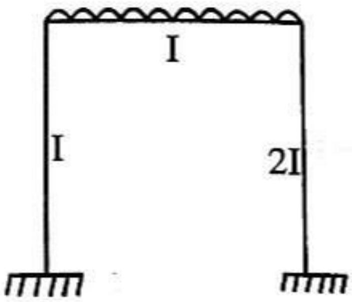
Shear force diagram

2.3.2. RIGID FRAMES WITH SWAY IN SLOPE DEFLECTION METHOD.

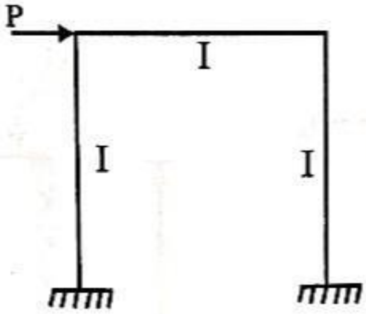
Portal frames may sway due to one of the following reasons:

- Eccentric or unsymmetrical loading on the portal frames.
- Unsymmetrical shape of the frames.
- Different end conditions of the columns of the portal frames.
- Non uniform section of the members of the frame.
- Horizontal loading on the columns of the frame.
- Settlement of the supports of the frame.
- A combination of the above.

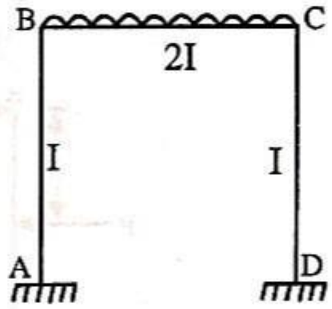




(4)

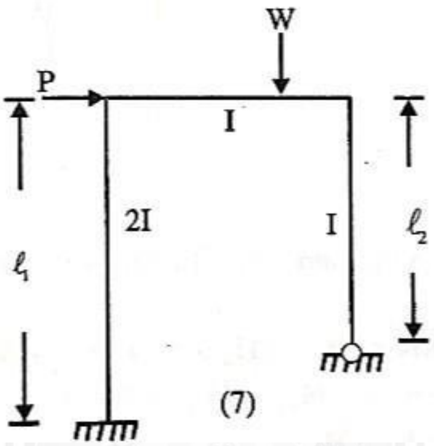


(5)



(6)

D Settles down / Sinks by δ



(7)

2.3.3. NUMERICAL EXAMPLES ON (RIGID FRAMES WITH SWAY):

PROBLEM NO:03

Analysis the rigid frame shown in fig.,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.

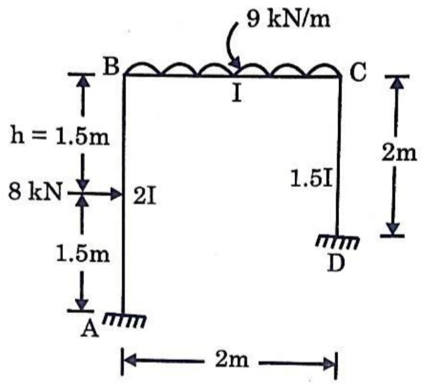


Fig. 2.16

Solution:

- **Fixed End Moments:**

  MFAB = -Wl/8 = - 8x3/8 = - 3 kNm;

  MFBA = Wl/8 = 8x3/8 = 3 kNm;

  MFAB = $-Wl^2/12 = -20x4^2/12 = -26.67$ kNm;

  MFBA = $Wl^2/12 = 20x4^2/12 = 26.67$ kNm;

  MFCD = 0;

  MFDC = 0;

- **Slope Deflection Equations:**

  MAB = MFAB + 2E(2I)/3(2θA + θB + 3δ/l)

  = - 3 + 4/3EI(θB – δ) --- (1)

  MBA = MFBA + 2E(2I)/3(2θB + θA + 3δ/l)

  = 3 + 4/3EI(2θB – δ) --- (2)

  MBC = MFBC + 2EI/2(2θB + θC + 3δ/l)

  = - 3 + EI(2θB + θC) --- (3)

  MCB = MFCB + 2EI/2(2θC + θB + 3δ/l)

  = 3 + EI(2θC + θB) --- (4)

  MCD = MFCD + 2E(1.5I)/2(2θC + θD + 3δ/l)

  = 1.5EI(2θC - 3δ/2) --- (5)

  MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

  = 1.5EI(2θC - 3δ/2) --- (6)

- **Equilibrium and Shear Equations:**

  MBA + MBC = 0

  14 θB - 4δ + 3θC = 0 ---(7)

  MCB + MCD = 0

  θB - 2.25δ + 5θC = 0 ---(8)

  Using Shear Equations,we get;

  MAB + MBA – Ph/l +MCD + MDC/l + P = 0

  (θC = - 0.044/EI; θB = 0.414/EI).

- **Final Moments:**

  MAB = - 4.34 kNm;

  MBA = 2.24 kNm;

  MBC = - 2.24 kNm;

  MCB = 3.33 kNm;

  MCD = - 3.33 kNm;

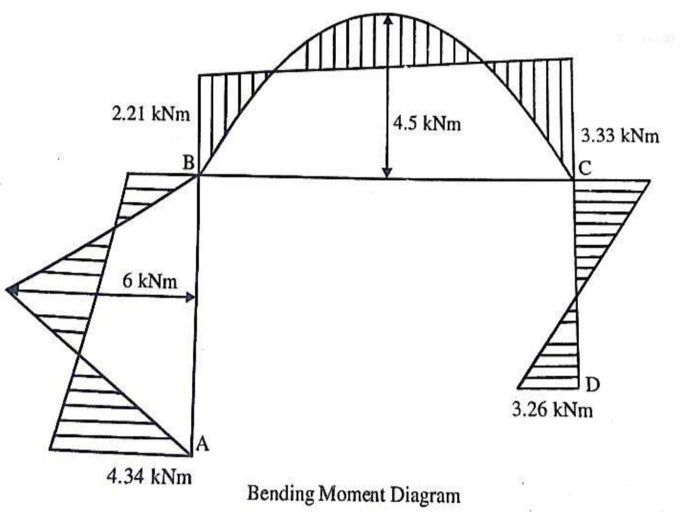
  MDC = - 3.26 kNm;

- **Free Bending Moments:**

  AB = Wl/4 = 8 x 3/4 = 6 kNm

  BC = $Wl^2/8 = 9 \times 2^2/8$ = 4.5 kNm

- **BMD:**



Bending Moment Diagram

2.4. SUPPORT SETTLEMENTS IN SLOPE DEFLECTION METHOD.

2.4.1 SUPPORT SETTLEMENT IN STRUCTURAL ANALYSIS:

Support settlements may be caused by **soil erosion**, dynamic soil effects during earthquakes, or by partial failure or settlement of supporting structural elements. Supports could also potentially heave due to frost effects (this could be considered a negative settlement).

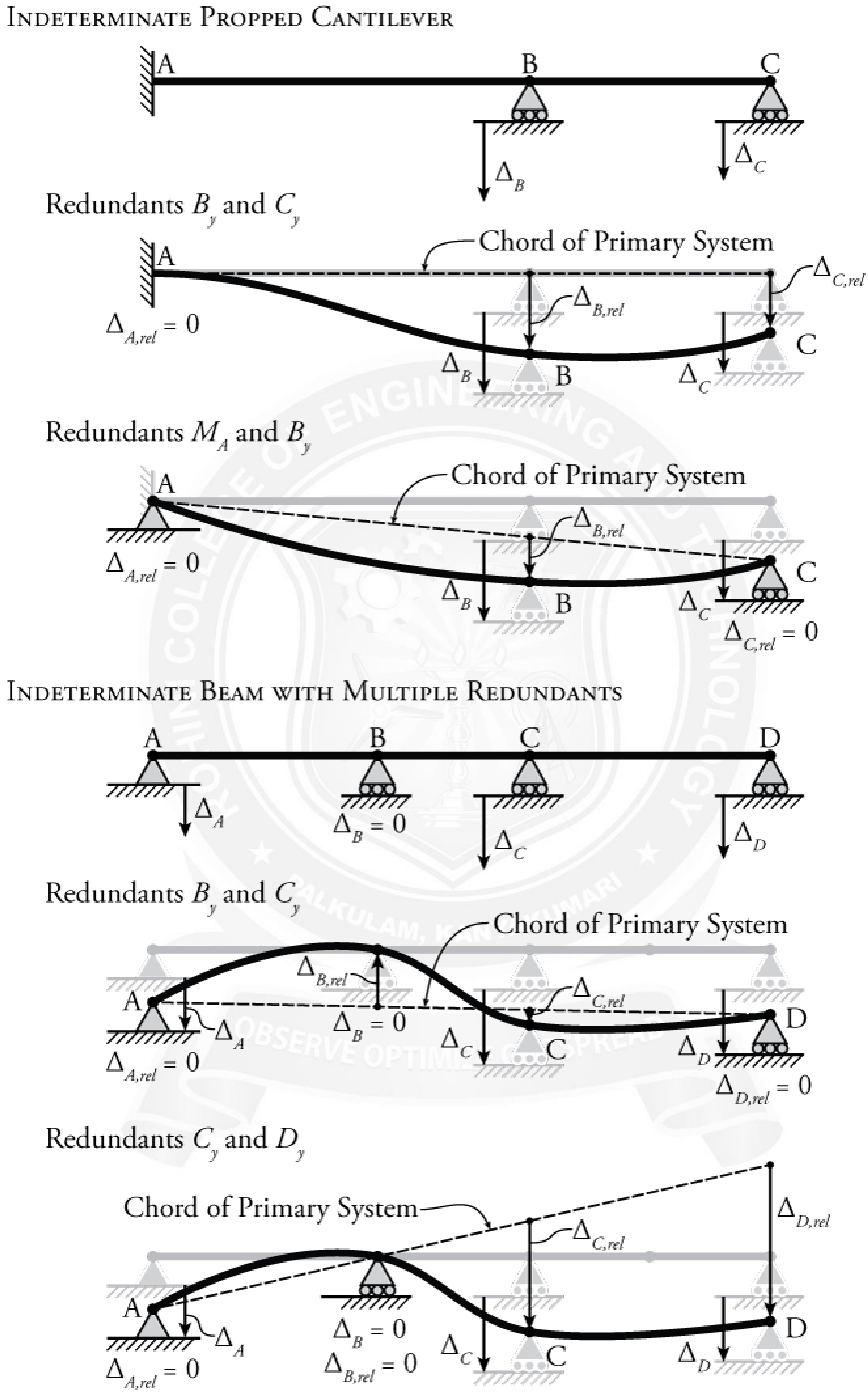
2.4.2. INTRODUCTION:

In the last lesson, the force method of analysis of statically indeterminate beams subjected to external loads was discussed. It is however, assumed in the analysis that the supports are unyielding and the temperature remains constant. In the design of indeterminate structure, it is required to make necessary provision for future unequal vertical settlement of supports or probable rotation of supports. It may be observed here that, in case of determinate structures no stresses are developed due to settlement of supports. The whole structure displaces as a rigid body. Hence, construction of determinate structures is easier than indeterminate structures.

The statically determinate structure changes their shape due to support settlement and this would in turn induce reactions and stresses in the system. Since, there is no external force system acting on the structures, these forces form a balanced force system by themselves and the structure would be in equilibrium. The effect of temperature changes, support settlement can also be easily included in the force method of analysis. In this lesson few problems, concerning the effect of support settlement are solved to illustrate the procedure.

2.4.3. SUPPORT DISPLACEMENTS:

The whole structure displaces as a rigid body. Hence, construction of determinate structures is easier than indeterminate structures. **The statically determinate structure changes their shape due to support settlement** and this would in turn induce reactions and stresses in the system.



Support settlements in continuous beams

2.4.4. NUMERICAL EXAMPLES ON(CONTINUOUS BEAMS):

PROBLEM NO:01

Analysis the continuous beam shown in fig.2.10,Calculate the support moments using slope deflection method.Support B settlememts by 10mm.Take E = 2 x 10^5 N/mm^2,I = 16 x 10^7 mm^4.Sketch the SF and BM diagrams.

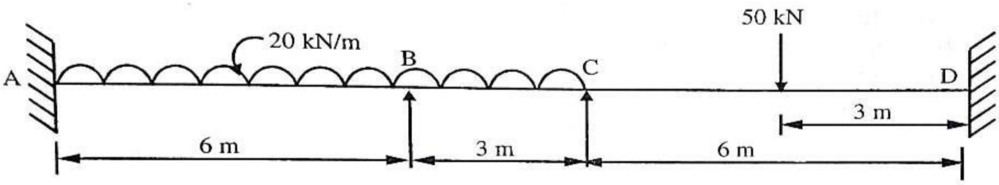


Fig. 2.10

Solution:

- **Fixed End Moments:**

  MFAB = $-Wl^2/12$ = - $20x6^2/12$ = - 60 kNm;

  MFBA = $Wl^2/12$ = $20x6^2/12$ = 60 kNm;

  MFBC = $-Wl^2/12$ = - $20x3^2/12$ = - 15 kNm;

  MFCB = $Wl^2/12$ = $20x3^2/12$ = 15 kNm;

  MFCD = -Wl/8 = - 50x6/8 = - 37.5 kNm;

  MFDC = Wl/8 = 50x6/8 = 37.5 kNm;

- **Slope Deflection Equations:**

  MAB = MFAB + 2EI/6(2θA + θB + 3δ/l)

  = - 60 + EI/3(0 + θB – 1/200) --- (1)

  MBA = MFBA + 2EI/6(2θB + θA + 3δ/l)

  = 60 = EI/3(2θB – 3 x 10/6000) --- (2)

  MBC = MFBC + 2EI/3(2θB + θC + 3δ/l)

  = - 15 + 2EI/3(2θB + θC + 1/100) --- (3)

  MCB = MFCB + 2EI/3(2θC + θB + 3δ/l)

  = 15 + 2EI/3(2θC + θB + 1/100) --- (4)

  MCD = MFCD + 2EI/6(2θC + θD + 3δ/l)

  = - 37.5 + EI/3(2θC) --- (5)

MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

= 37.5 + EI/3(θC) --- (6)

- **Joint Equilibrium Equations:**

Joint B:

MBA + MBC = 0

EI/3(6θB+2θC+3/200) = - 135 --- (7)

Joint C:

MCB + MCD = 0

EI(θB+3θC+1/100) = 33.75 --- (8)

Equvating (7 & 8);we get

θC = - 1/464; θB = - 1/402

- **Final Moments:**

MAB = - 139.843 kNm;

MBA = - 46.354 kNm;

MBC = 46.3 kNm;

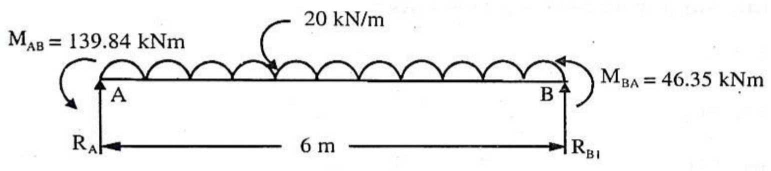
MCB = 83.35 kNm;

MCD = - 83.477 kNm;

MDC = 14.51 kNm;

- **To Draw S.F.D:**

Span AB:

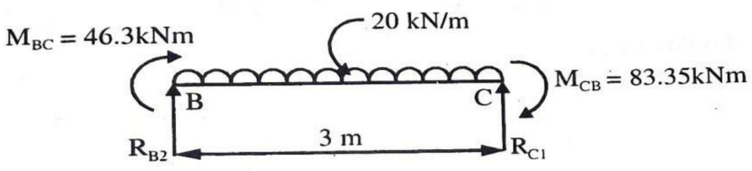


Taking moments about A.

$20 \times 6^2/2 - 46.35 - 139.84 - RB1(6) = 0; RB1 = 28.97$ KN

$RA = 20 \times 6 - 28.97; RA = 91.03$ KN

Span BC:

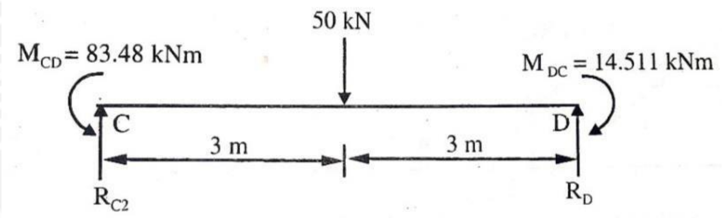


Taking moments about B.

$20 \times 3^2/2 + 83.35 + 46.3 - RC1(3) = 0; RC1 = 73.22$ KN

$RB2 = 20 \times 3 - 73.22; RB2 = -13.21$ KN

Span CD:



Taking moments about C.

$14.511 + 50(3) - 83.48 - RD(6) = 0;$

$RD = 13.5$ KN; $RC2 = 36.5$ KN

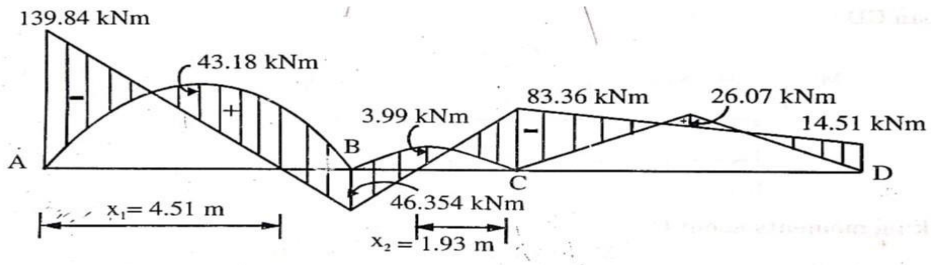
- **Free BMD:**

  $MAB = Wl^2/8 = 20 \times 6^2/8 = 90$ kNm

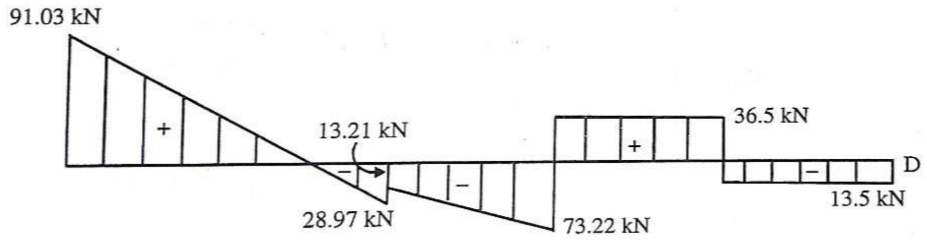
  $MBC = Wl^2/8 = 20 \times 3^2/8 = 22.5$ kNm

  $MCD = Wl/4 = 50 \times 6/4 = 75$ kNm

- **BMD and SFD:**



Bending moment diagram

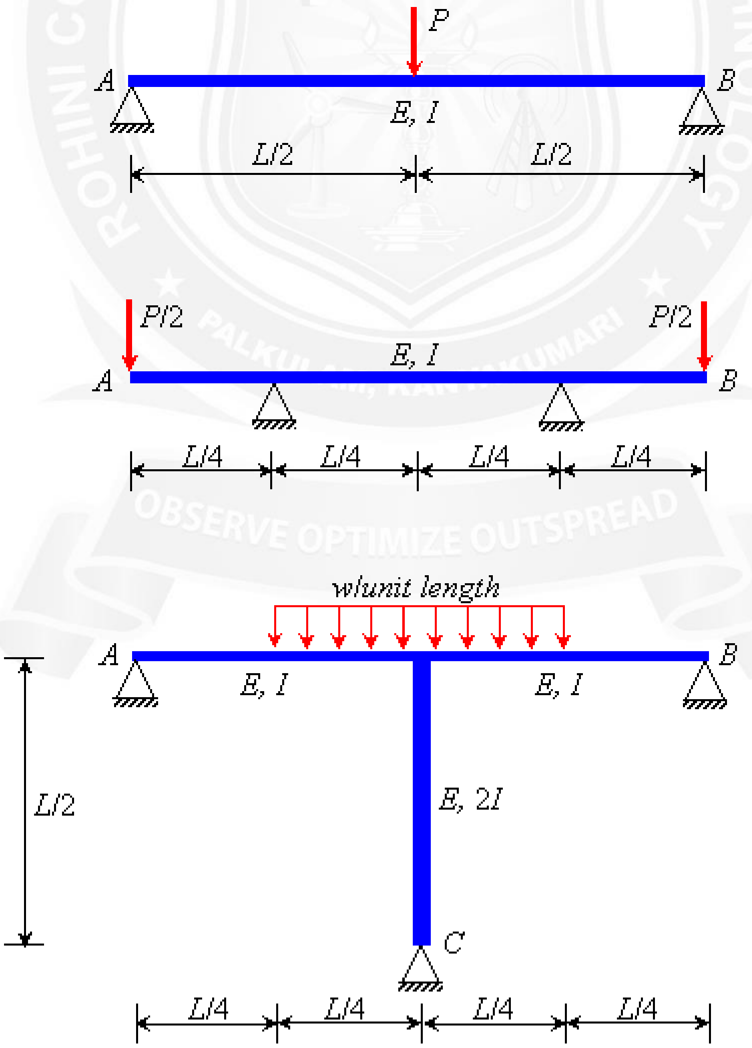


Shear force diagram

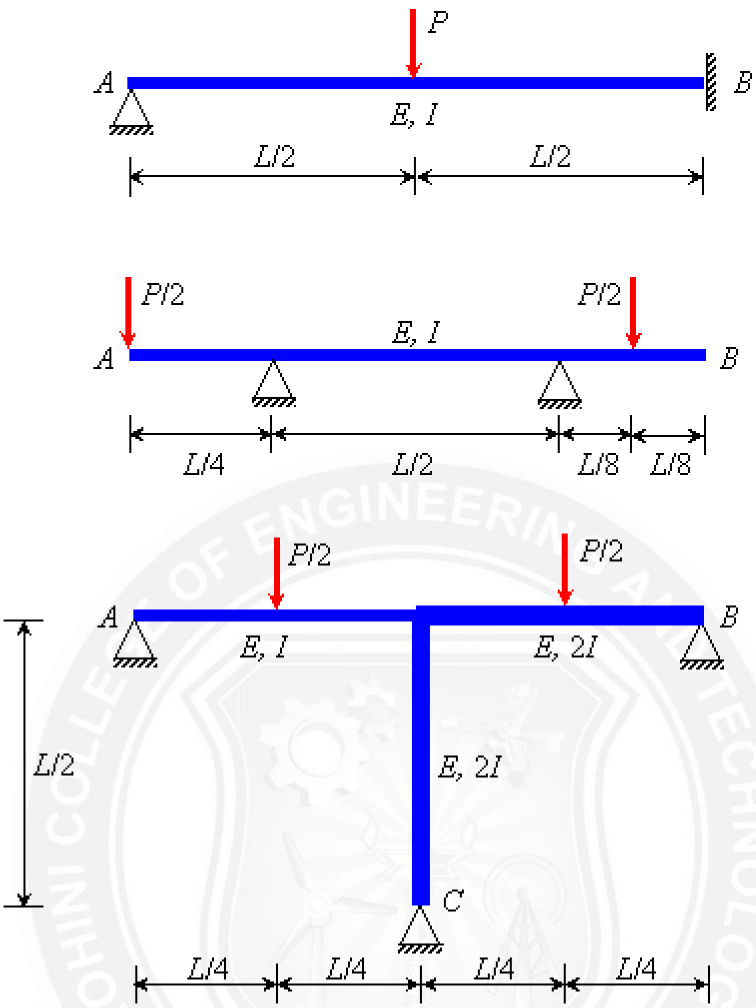
2.5 SYMMETRIC FRAMES WITH SYMMETRIC AND SKEW-SYMMETRIC LOADINGS

2.5.1 SYMMETRY AND ANTISYMMETRY

Symmetry or antisymmetry in a structural system can be effectively exploited for the purpose of analyzing structural systems. Symmetry and anti-symmetry can be found in many real-life structural systems (or, in the idealized model of a real-life structural system). It is very important to remember that when we say symmetry in a structural system, it implies the existence of symmetry both in the structure itself including the support conditions and also in the loading on that structure. The systems shown in Fig. are symmetric because, for each individual case, the structure is symmetric and the loading is symmetric as well. However, the systems shown in Fig. are not symmetric because either the structure or the loading is not symmetric.



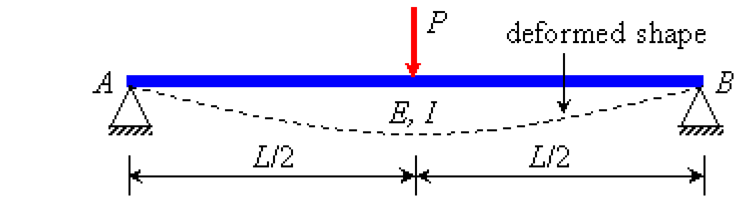
1.1. Symmetric structural systems

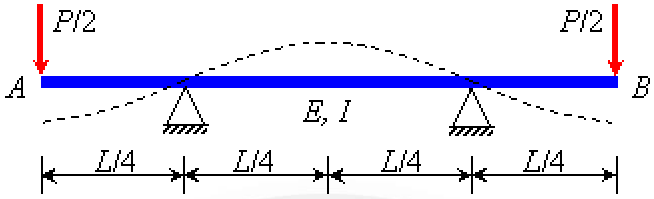


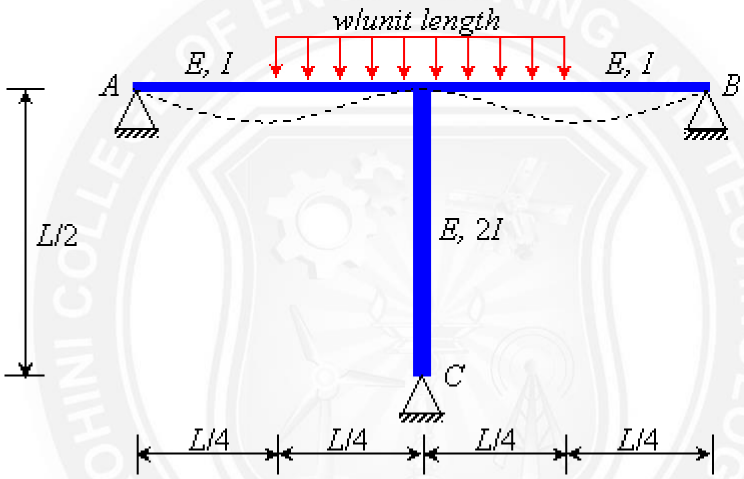
1.2. Non-symmetric (asymmetric) structural systems

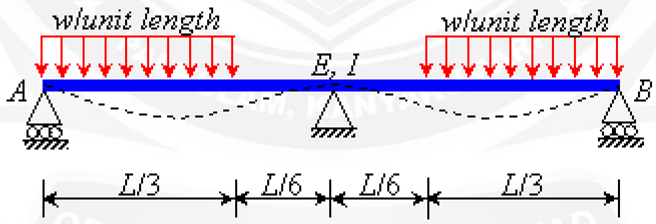
For an antisymmetric system the structure (including support conditions) remains symmetric,however, the loading is antisymmetric.The fig.1.2 ,shows the example of antisymmetric structural systems.

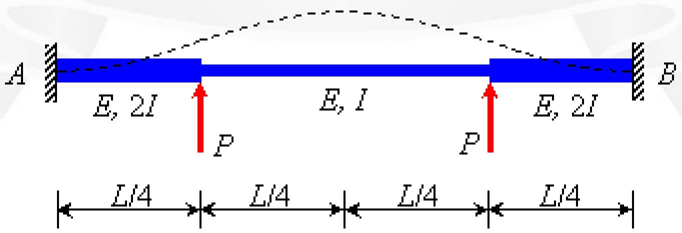
It is not difficult to see that the deformation for a symmetric structure will be symmetric about the same line of symmetry. This fact is illustrated in Fig. 1.3, where we can see that every symmetric structure undergoes symmetric deformation. It can be proved using the rules of structural mechanics (namely, equilibrium conditions, compatibility conditions and constitutive relations), that deformation for a symmetric system is always symmetric. Similarly, we always get antisymmetric deformation for antisymmetric structural systems



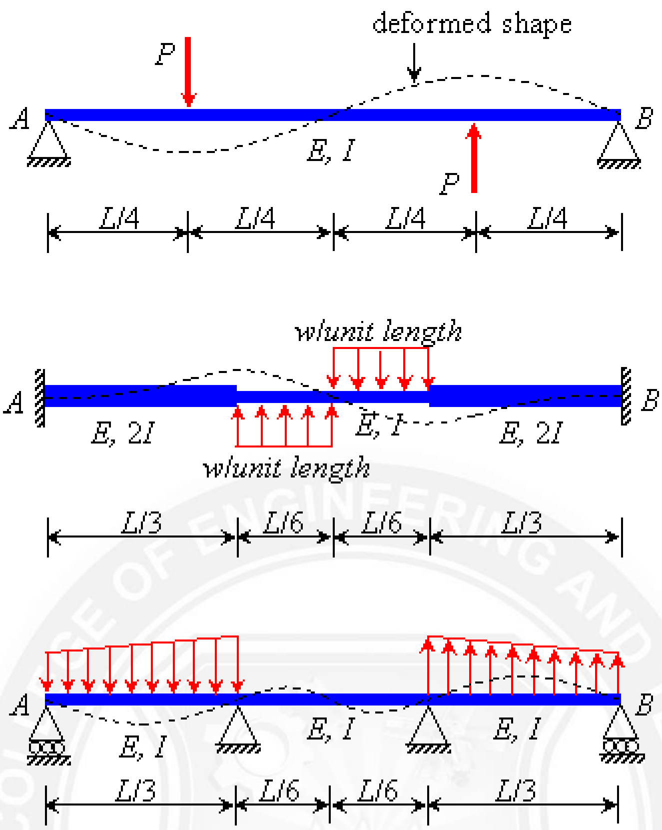






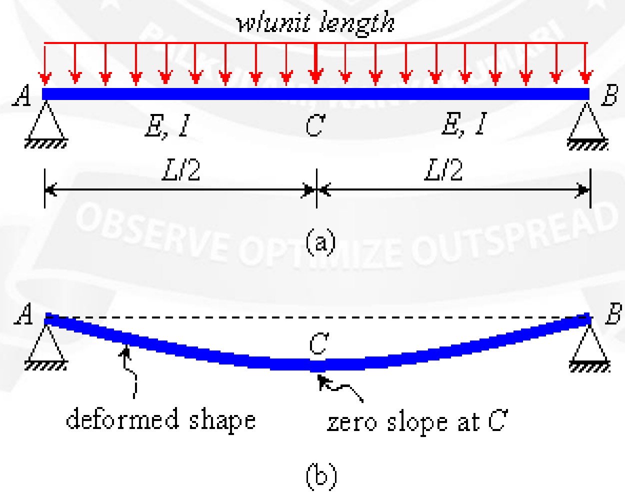


1.3. Deformation in symmetric systems



1.4. Deformation in antisymmetric systems

Let us look at beam AB in Fig. 1.20(a), which is symmetric about point C. The deformed shape of the structure will be symmetric as well (Fig. 1.20(b)). So, if we

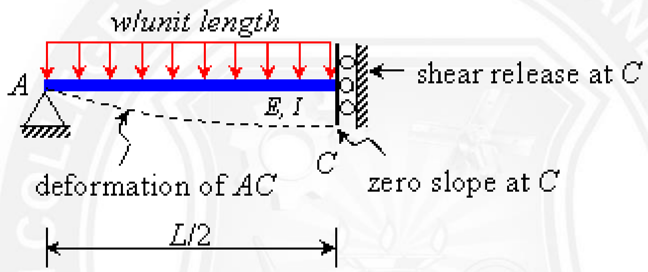


solve for the forces and deformations in part AC of the beam, we do not need to solve for part CB separately. The symmetry (or antisymmetry) in deformation gives us additional information prior to analyzing the structure and these information can be used to reduce the size of the structure that needs to be considered for analysis.

To elaborate on this fact, we need to look at the deformation condition at the point/line of symmetry (or antisymmetry) in a system. The following general rules about deformation can be deduced looking at the examples in Fig. 1.3 and Fig. 1.4:

- For a symmetric structure: slope at the point/line of symmetry is zero.
- For an antisymmetric structure: deflection at the point/line of symmetry is zero.

These information have to be incorporated when we reduce a symmetric (or antisymmetric) structure to a smaller one. If we want to reduce the symmetric beam in Fig. 1.20 to its one symmetric half AC , we have to integrate the fact the slope at point



1.6.Reduced system AC is adopted for analysis for beam AB

C for the reduced system AC will have to be zero. This will be a necessary boundary condition for the reduced system AC . We can achieve this by providing a support at C, which restricts any rotation, but allows vertical displacement, as shown in Fig. 1.6 (Note: this specific type of support is known as a "shear-release" or "shear-hinge"). Everything else (loading, other support conditions) remains unchanged in the reduced system. We can use this system AC for our analysis instead of the whole beam AB .

3.5.2 INTERNAL FORCE DIAGRAMS FOR A) A SYMMETRIC SYSTEM, AND B) AN ANTISYMMETRIC SYSTEM

Having a priory knowledge about symmetry/antisymmetry in the structural system and in its deformed shape helps us know about symmetry/antisymmetry in internal forces in that system. (Symmetry in the system implies symmetry in equilibrium and constitutive relations, while symmetry in deformed shape implies symmetry in geometric compatibility.) Internal forces in a symmetric system are also symmetric about the same axis and similarly antisymmetric systems have antisymmetric internal forces. Detailed discussion on different types of internal forces in various structural systems and on internal force diagrams are provided in the next module (Module 2: Analysis of Statically Determinate Structures). Once we know about these diagrams we can easily see the following:

- A symmetric beam-column system has a symmetric bending moment diagram.
- A symmetric beam-column system has an antisymmetric shear force diagram.
- An antisymmetric beam-column system has an antisymmetric bending moment diagram.
- An antisymmetric beam-column system has a symmetric shear force diagram.

3.1 MOMENT DISTRIBUTION METHOD

Objectives:

Definition of stiffness, carry over factor, distribution factor. Analysis of continuous beams without support yielding – Analysis of continuous beams with support yielding – Analysis of portal frames – Naylor's method of cantilever moment distribution – Analysis of inclined frames – Analysis of Gable frames.

3.1.1 INTRODUCTION

The end moments of a redundant framed structure are determined by using the classical methods, viz. Clapeyron's theorem of three moments, strain energy method and slope deflection method. These methods of analysis require a solution of set of simultaneous equations. Solving equations is a laborious task if the unknown quantities are more than three in number. In such situations, the moment distribution method developed by Professor Hardy Cross is useful. This method is essentially balancing the moments at a joint or junction. It can be described as a method which gives solution by successive approximations of slope deflection equations.

In the moment distribution method, initially the structure is rigidly fixed at every joint or support. The fixed end moments are calculated for any loading under consideration. Subsequently, one joint at a time is then released. When the moment is released at the joint, the joint moment becomes unbalanced. The equilibrium at this joint is maintained by distributing the unbalanced moment. This joint is temporarily fixed again until all other joints have been released and restrained in the new position. This procedure of fixing the moment and releasing them is repeated several times until the desired accuracy is obtained. The experience of designers points that about five cycles of moment distribution lead to satisfactory converging results.

Basically, in the slope deflection method, the end moments are computed using the slopes and deflection at the ends. Contrarily in the moment distribution method, as a first step — the slopes at the ends are made zero. This is done by fixing the joints. Then with successive release and balancing the joint moments, the state of equilibrium is obtained. The release-balance cycles can be carried out using the following theorems.

In Conclusion,when a positive moment M is applied to the hinged end of a beam and a positive moment of M will be transformed to the fixed end.

3.1.2 IMPORTANT FACTORS

- Carry over moment carry over factor
- Relative stiffness or stiffness factor
- Distribution moment and distribution factor.

3.1.3 BASIC DEFINITIONS OF TERMS IN THE MOMENT DISTRIBUTION METHOD

(a) Stiffness

Rotational stiffness can be defined as the moment required to rotate through a unit angle (radian) without translation of either end.

(b) Stiffness Factor

(i) It is the moment that must be applied at one end of a constant section member (which is unyielding supports at both ends) to produce a unit rotation of that end when the other end is fixed, i.e. $k = 4EI/l$.

(ii) It is the moment required to rotate the near end of a prismatic member through a unit angle without translation, the far end being hinged is $k = 3EI/l$.

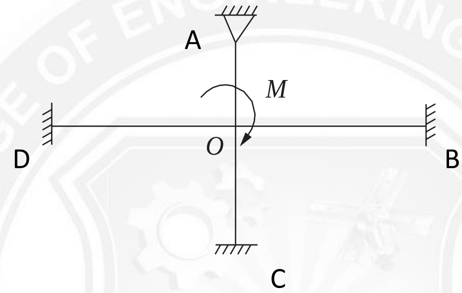
(c) Carry over factor

It is the ratio of induced moment to the applied moment (Theorem 1). The carry over factor is always (1/2) for members of constant moment of inertia (prismatic section). If the end is hinged/pin connected, the carry over factor is zero. It should be mentioned here that carry over factors values differ for non-prismatic members. For non-prismatic beams (beams with variable moment of inertia); the carry over factor is not half and is different for both ends.

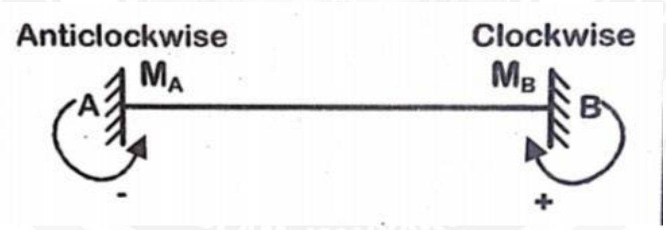
Carry over factor = Carry over moment / Applied moment

(d) Distribution Factors

Consider a frame with members *OA*, *OB*, *OC* and *OD* rigidly connected at *O* as shown in Fig. 2.6. Let *M* be the applied moment at joint *O* in the clockwise direction. Let the joint rotate through an angle θ. The members *OA*,*OB*,*OC* and *OD* also rotate by the same angle θ.



3.1.4 SIGN CONVENTION



Clockwise moments are considered positive and anticlockwise moments negative.

3.1.5 BASIC PROCEDURES IN THE MOMENT DISTRIBUTION METHOD

- Assuming all the members as fixed at the both ends.
- Calculate fixed end moments due to external loads.
- At hinged supports (or) simply supported supports,release the points by applying equal and opposite moment.
- Calculate stiffness factor at each joint.
- Unbalanced moment at a point,is distributed to the adjacent spans according to their distribution factors.
- Proceed this process to get the required degree of precision.

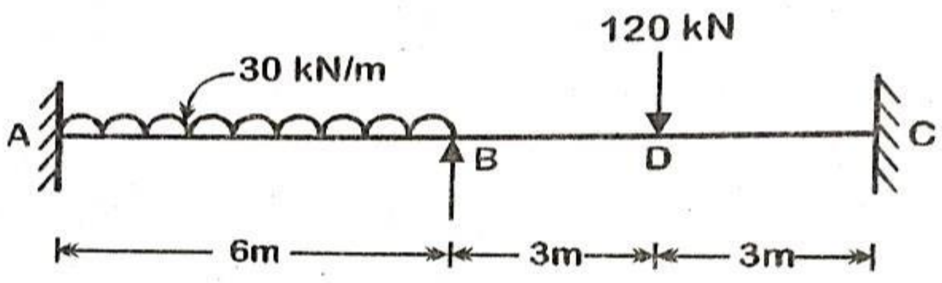
3.2 ANALYSIS OF CONTINUOUS BEAMS IN MOMENT DISTRIBUTION METHOD.

3.2.1 NUMERICAL EXAMPLES ON(CONTINUOUS BEAMS):

PROBLEM NO:01

For the continuous beam shown in figure,Calculate the support moments distribution method.Draw the SF and BM diagrams.

Solutions:



- **Fixed End Moments:**

MFAB = $-Wl^2/12 = -30x6^2/12 = -90$ kNm

MFBA = $Wl^2/12 = 30x6^2/12 = 90$ kNm

MFBC = -Wl/8 = -120X6/8 = -90 kNm

MFCB = Wl/8 = 120X6/8 = 90 kNm

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = 4EI/6	4EI/3	0.5
	BC	4EI/l = 4EI/6		0.5

- **Reactions:**

Moment About B,

MB = RAx6 - 30x6x6/2 + MA

-90 = 6RA – 540 – 90

6RA = 540; RA = 90KN

Moment About B,

MB = RC x 6 – 120 x 3 + MC

-90 = 6RC – 360 – 90

6RC = 360; RC = 60KN

RB = Total load – (RA + RC) = 30 x 6 + 120 – (90 + 60)

= 300 – 150 = 150KN

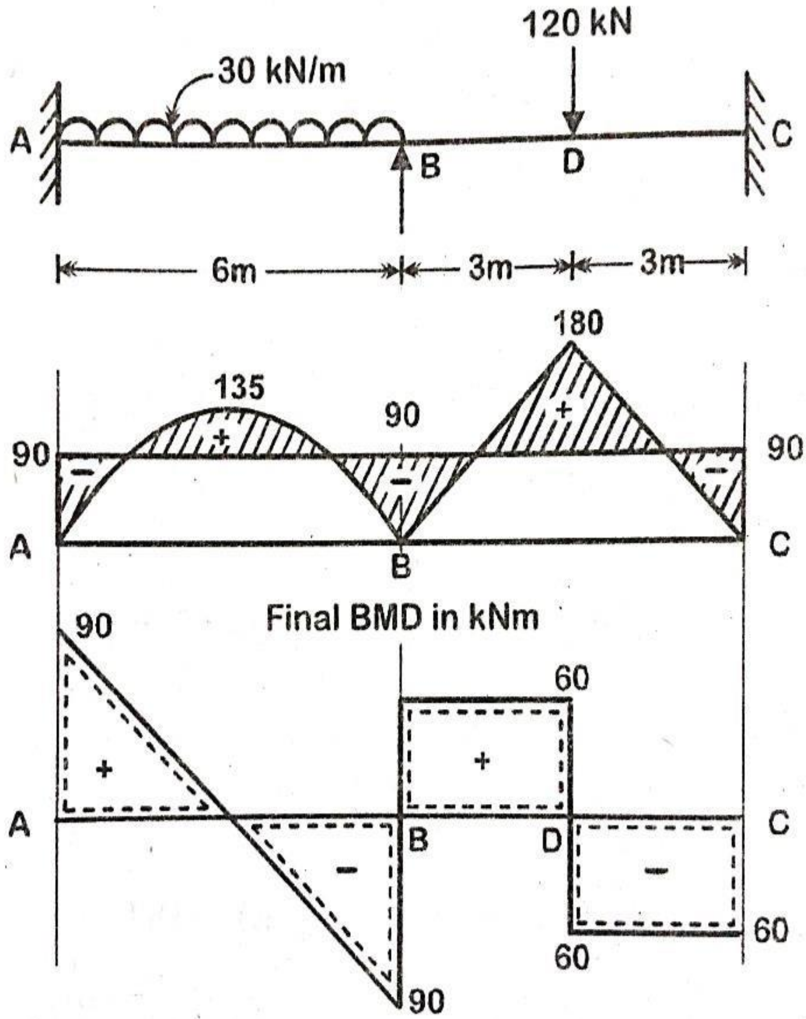
- **Moment Distribution Table**:

Joint	A	B		C
Member	AB	BA	BC	CB
D.F	-	0.5	0.5	-
F.E.M	-90	90	-90	90
Distribute	-	0	0	-
Final Moments	-90	90	-90	90
Conventional Moments	-90	-90	90	-90

- **Free BMD:**

MAB = $Wl^2/8 = 30 \times 6^2/8$ = 135kNm

MBC = Wl/4 = 120 x 6/4 = 180kNm



- **Final Bending Moments:**

MA = -90kNm; MB = -90kNm; MC = -90kNm

PROBLEM NO:02

For the continuous beam as shown in fig; Find the support moment carry out two cycles of distribution.

Solutions:

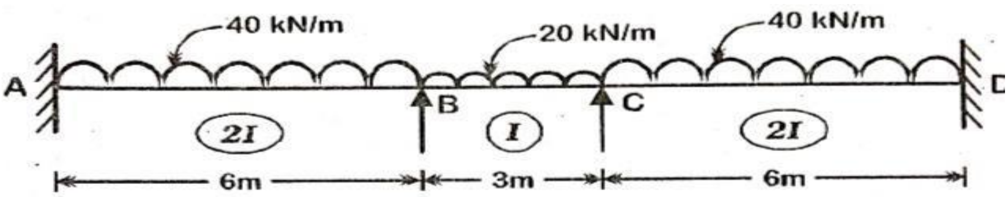


Fig. 3.9(a)

- **Fixed End Moments:**

MFAB = $-Wl^2/12 = -40x6^2/12 = -120$ kNm;

MFBA = $Wl^2/12 = 40x6^2/12 = 120$ kNm;

MFBC = $-Wl^2/12 = -20x3^2/12 = -15$ kNm;

MFBC = $Wl^2/12 = 20x3^2/12 = 15$ kNm;

MFCD = $-Wl^2/12 = -40x6^2/12 = -120$ kNm;

MFDC = $Wl^2/12 = 40x6^2/12 = 120$ kNm;

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4E(2I)/l = 4EI/3	7EI/3	0.57
	BC	3EI/l = 3EI/3		0.43
C	CB	3EI/l = 3EI/3	7EI/3	0.43
	CD	4E(2I)/l = 4EI/3		0.57

- **Free BMD:**

MAB = $Wl^2/8 = 40 \times 6^2/8 = 180$kNm;

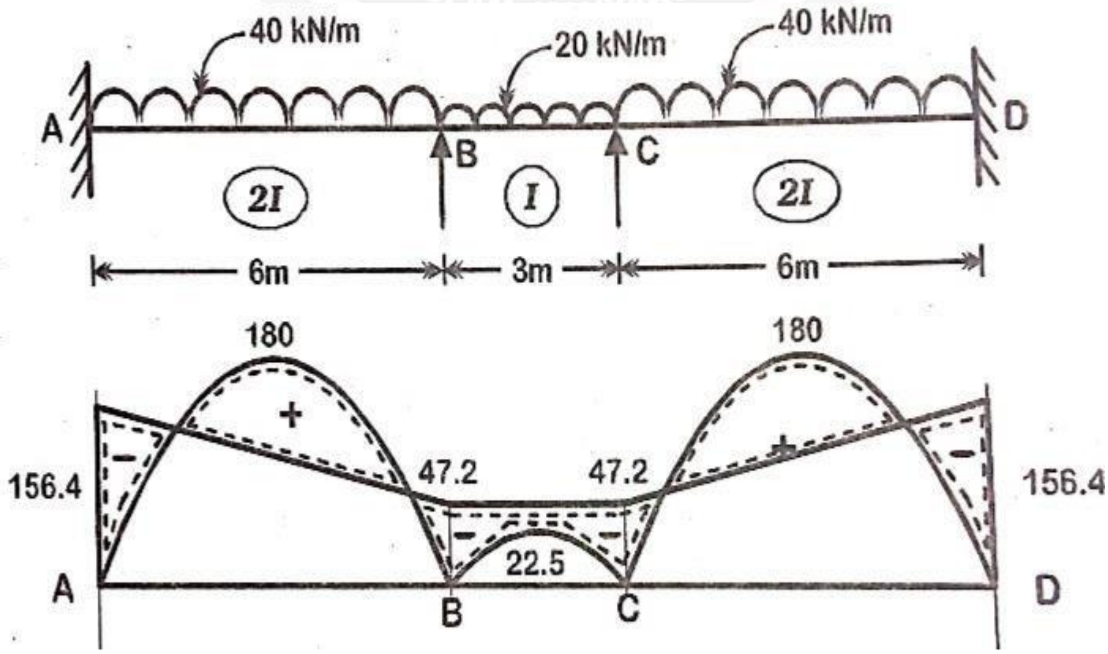
MBC = $Wl^2/8 = 20 \times 3^2/8 = 22$kNm;

MCD = $Wl^2/8 = 40 \times 6^2/8 = 180$kNm.

- **Moment Distribution Table:**

Joint	A	B		C		D
Member	AB	BA	CB	BC	CD	DC
D.F	-	0.57	0.43	0.43	0.57	-
F.E.M	-120	120	-15	15	-120	120
Distribute		-59.9	-45.1	45.1	59.9	
Carry over	-29.95		22.6	-22.6		29.95
Distribute		-12.9	-9.7	9.7	12.9	
Carry over	-6.45					6.45
Final Moments	-156.4	47.2	-47.2	47.2	-47.2	156.4
Conventional Moments	-156.4	-47.2	-47.2	-47.2	-47.2	-156.4

- **Moment Diagram:**



- **Final Moments:**

  MA = -156.40kNm; MB = -47.20kNm; MC = -156.40kNm

PROBLEM NO:03

A continuous beam ABCD, simply supported at A,B,C and D is loaded as shown in fig. EI is constsnt.

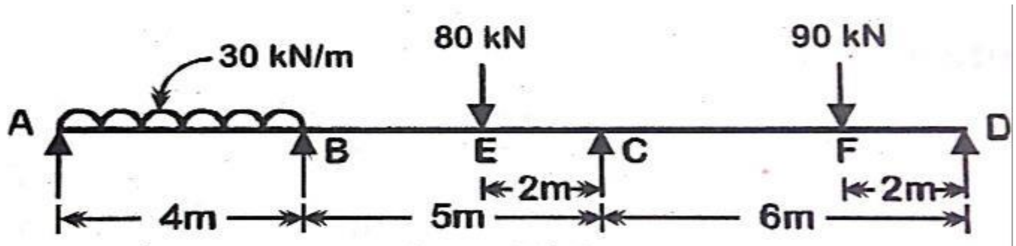


Fig. 3.11(a)

Solutions:

- **Fixed End Moments:**

  MFAB = $-Wl^2/12 = -30x4^2/12 = -40$ kNm;

  MFBA = $Wl^2/12 = 30x4^2/12 = 40$ kNm;

  MFBC = $-Wab^2/l^2 = -80x3x2^2/5^2 = -38.4$ kNm;

  MFBC = $Wa^2b/l^2 = 20x3^2x2/5^2 = 57.6$ kNm;

  MFCD = $-Wab^2/l^2 = -90x4x2^2/6^2 = -40$ kNm;

  MFDC = $Wa^2b/l^2 = 90x2^2x4/6^2 = 80$ kNm.

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	3EI/l = 3EI/4	27EI/20	0.56
	BC	3EI/l = 3EI/5		0.44
C	CB	3EI/l = 3EI/5	11EI/10	0.5
	CD	3EI/l = 3EI/6		0.5

- **Free BMD:**

  MAB = Wl^2/8 = 30 x 4^2/8 = 60kNm;

  MBC = Wab/l = 80 x 2 x 3/5 = 96kNm;

  MCD = Wab/l = 90 x 4 x 2/5 = 120kNm

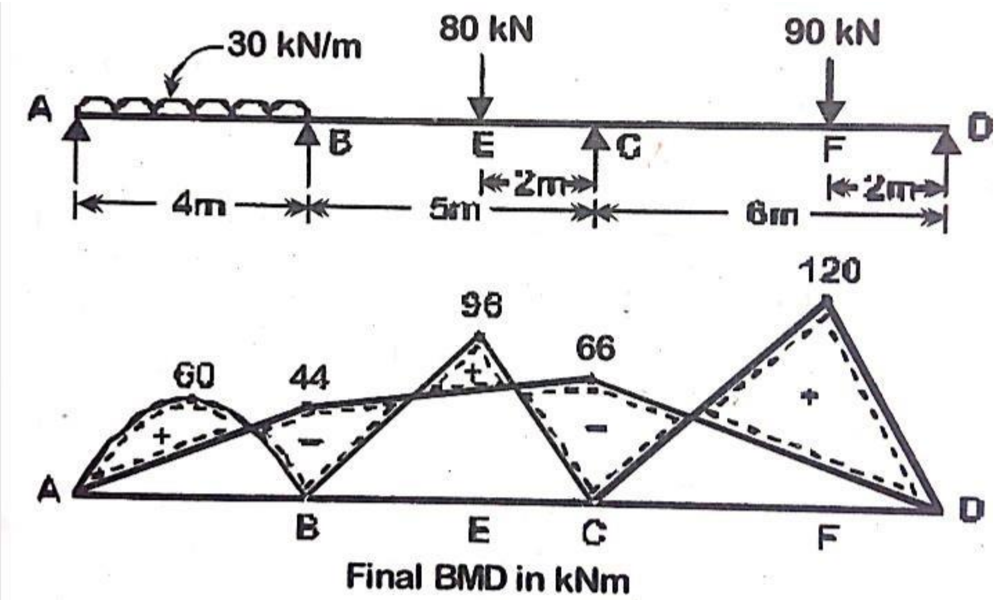
- **Moment Distribution Table:**

Joint	A	B		C		D
Member	AB	BA	CB	BC	CD	DC
D.F	-	0.56	0.44	0.5	0.5	-
F.E.M	-40	40	-38.4	57.6	-40	80
Release C & D	40					-80
carry over		20			-40	
Initial moments	0	60	-38.4	57.6	-80	0
Distribute		-12.096	-9.504	11.2	11.2	
Carry over		0	5.6	-4.552	0	
Distribute		-3.136	-2.464	2.376	2.376	
Carry over		0	1.188	-1.232	0	
Distribute		-0.66	-0.52	0.61	0.61	
Carry over			0.308	-0.26		
Final Moments	0	44	-44	66	-66	0
Conventional Moments	0	-44	-44	-66	-66	0

- **Final Moments:**

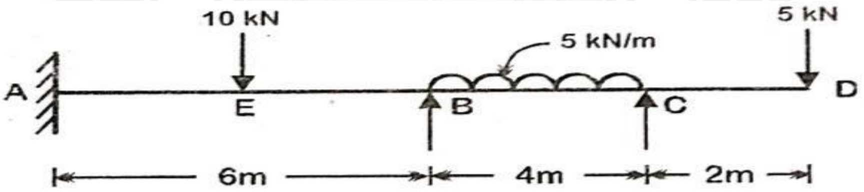
  MA = 0kNm; MB = -44kNm; MC = -66kNm; MD = 0kNm.

- **Final Moments Diagram:**



PROBLEM NO:04

Find the support moments for the continuous beam using moment distribution method. EI is constant.



Solutions:

- **Fixed End Moments:**

  MFAB = -Wl/8 = -10X6/8 = -7.5 kNm

  MFBA = Wl/8 = 10X6/8 = 7.5 kNm

  MFBC = -Wl^2/12 = -5x4^2/12 = -6.67 kNm

  MFCB = Wl^2/12 = 5x4^2/12 = 6.67 kNm

  MFCD = -Wxl= -5X2 = -10 kNm

- **Free BMD:**

  MAB = Wl/4 = 10 x 6/4 = 15kNm

  MBC = Wl^2/8 = 5 x 4^2/8 = 10kNm

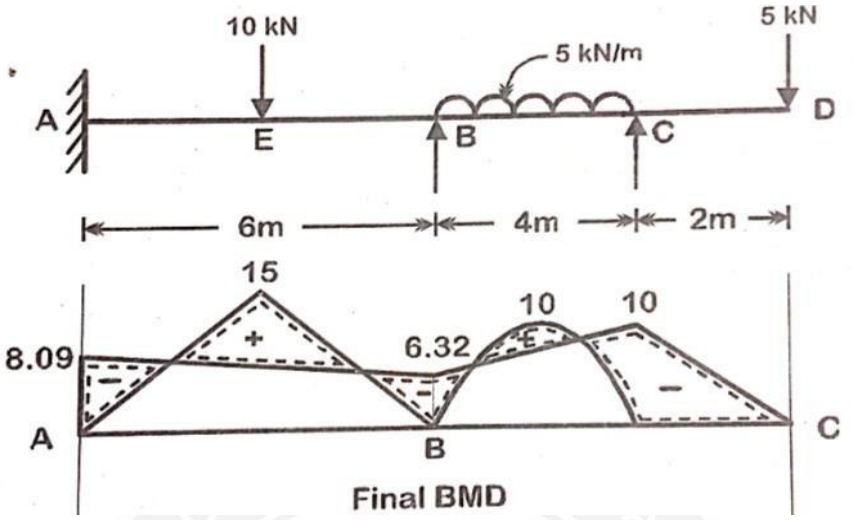
- **Distribution factor table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = 4EI/6	17EI/12	0.47
	BC	3EI/l = 3EI/4		0.53
C	CB	4EI/l = 3EI/4	3EI/4	1
	CD	0		0

- **Moment Distribution table:**

Joint	A	B		C	
Member	AB	BA	BC	CB	CD
D.F	-	0.47	0.53	1	0
F.E.M	-7.5	7.5	-6.67	6.67	-10
Release C & carry over	-	-	1.67	3.33	0
Initial moments	-7.5	7.5	-5	10	-10
Distribute	-	-1.18	-1.32	-	-
Carry over	-0.59	-	-	-	-
Final Moments	-8.09	6.32	-6.32	10	-10
Conventional Moments	-8.09	-6.32	-6.32	-10	-10

- **Final Moments Diagram:**



- **Final Moments:**

MA = -8.09kNm; MB = -6.32kNm; MC = -10kNm

3.3. ANALYSIS OF PLANE RIGID FRAMES WITH AND WITHOUT SWAY IN MOMENT DISTRIBUTION METHOD.

3.3.1. INTRODUCTION

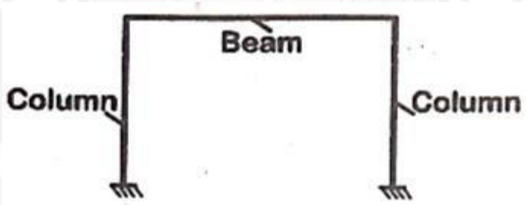
PLANE RIGID FRAMES

It is an indeterminate structure consisting of horizontal and inclined beams resting over columns.The nodes (or) joints of beams and columns behave like rigid joints.

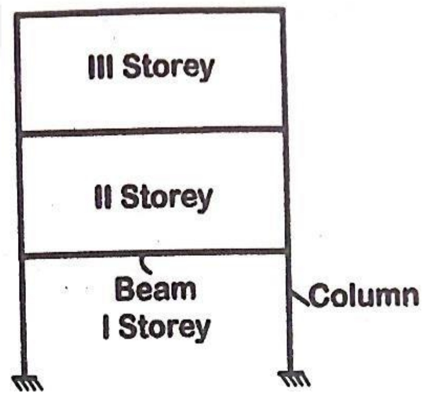
Depending on the number of bays and number of storeys these are classified as;

- Single bay single storey rigid frames
- Single bay multistorey rigid frames
- Multi bay single storey rigid frames and
- Multistorey rigid frames

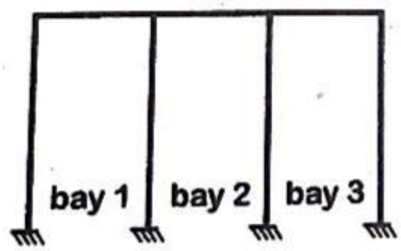
SKETCHES OF RIGID FRAMES



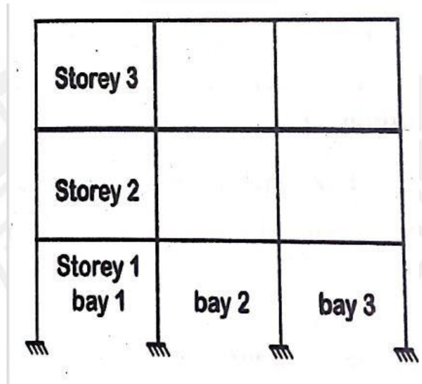
(a) Single bay single storey rigid frames



(b) Single bay multistorey rigid frames



(c) Multi bay single storey rigid frames

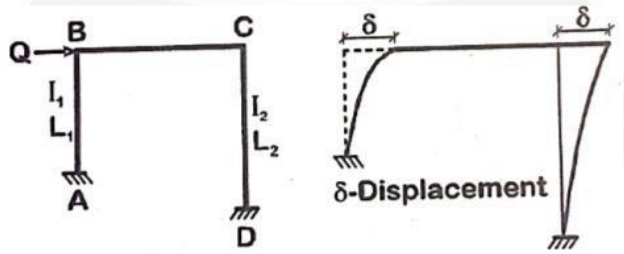


(d) Multistorey rigid frames

3.3.2. SWAY AND NON-SWAY FRAMES

With Sway Frames:

A portal frames is said to be a sway frame when it is subjected to unbalanced and instrument horizontal forces.



Without Sway Frames:

A portal frames is said to be a non-sway frame or without sway when it is subjected to vertical forces.

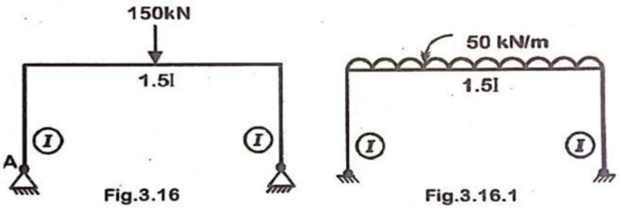


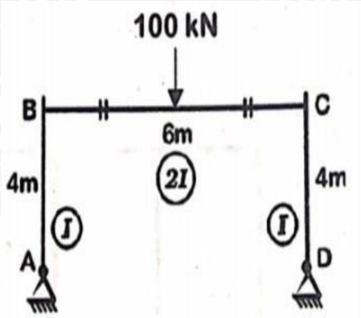
Fig.3.16 Fig.3.16.1

Non-Sway frames

3.3.3. NUMERICAL EXAMPLES ON(PLANE RIGID FRAMES):

PROBLEM NO:01

For the portal rigid frame compute the bending moments and draw the BMD



Solution:

- **Fixed End Moments:**

  MFAB = MFBA = 0

  MFBC = Wl/8 = 100X6/8 = -75 kNm

  MFCB = -Wl/8 = 100X6/8 = 75 kNm

  MFCD = MFDC = 0

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	3EI/l = 3EI/4	7EI/4	0.43
	BC	3E(2I)/l = 6EI/6		0.57
C	CB	3E(2I)/l = 6EI/6	7EI/4	0.57
	CD	3EI/l = 3EI/4		0.43

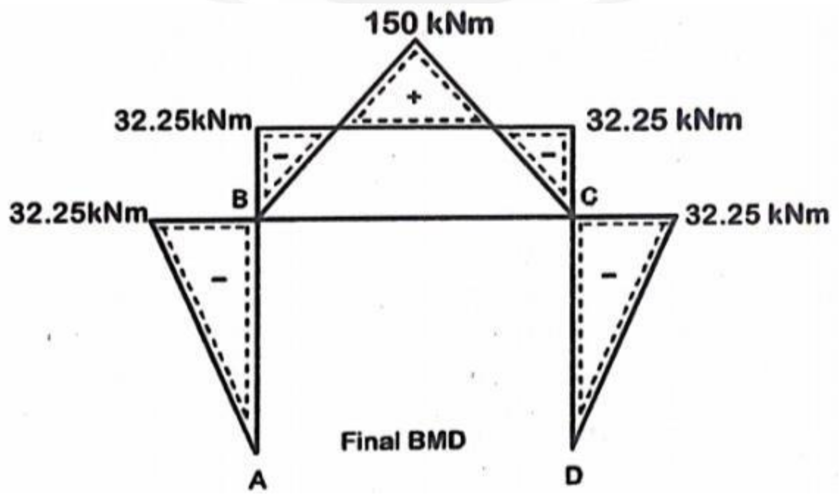
- **Free BMD:**

  MBC = Wl/4 = 100 x 6/4 = 150kNm

- **Moment Distribution Table**:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.43	0.57	0.57	0.43	-
F.E.M	0	0	-75	75	0	0
Distribute		32.25	42.75	-42.75	-32.25	
Final Moments	0	32.25	-32.25	32.25	-32.25	0
Conventional Moments	0	-32.25	-32.25	-32.25	-32.25	0

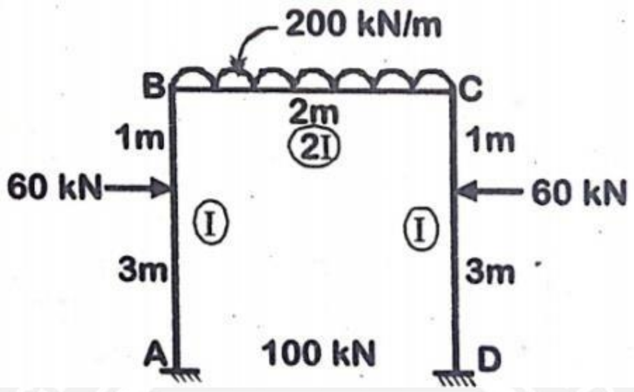
- **Moment Diagram:**



PROBLEM NO:02

Analyse the portal frames shown in fig by moment distribution method and draw the BMD.

Solution:



- **Fixed End Moments:**

  MFAB = $-Wab^2/l^2$ = $-60x3x1^2/4^2$ = -11.25 kNm;

  MFBA = Wa^2b/l^2 = $60x3^2x1/4^2$ = 33.76 kNm;

  MFBC = $-Wl^2/12$ = $-200x2^2/12$ = -66.67 kNm;

  MFCB = $Wl^2/12$ = $200x2^2/12$ = 66.67 kNm;

  MFCD = $-Wab^2/l^2$= $-60x3x1^2/6^2$ = -33.75 kNm;

  MFDC = Wa^2b/l^2 = $60x3^2x1/4^2$ = 11.25 kNm.

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = 4EI/4	4EI	0.25
	BC	3E(2I)/l = 6EI/2		0.75
C	CB	3E(2I)/l = 6EI/2	4EI	0.75
	CD	4EI/l = 4EI/4		0.25

- **Free BMD:**

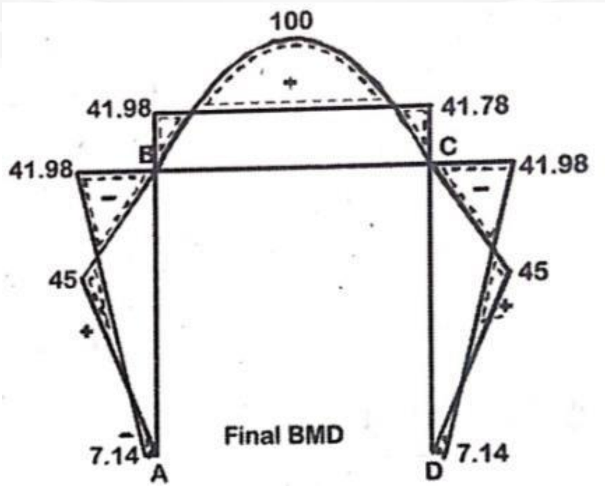
MAB = MCD = Wab/l= 60x3x1/4 = 45 kNm;

MBC = $Wl^2/8 = 200x2^2/8$ = 100 kNm.

- **Moment Distribution Table**:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.25	0.75	0.75	0.25	-
F.E.M	-11.25	33.75	-66.67	66.67	-33.75	11.25
Distribute		8.23	24.69	-24.69	-8.23	
Carry over	4.11					-4.11
Final Moments	-7.14	41.98	-41.98	41.98	-41.98	7.14
Conventional Moments	-7.14	-41.98	-41.98	-41.98	-41.98	-7.14

- **Moment Diagram:**

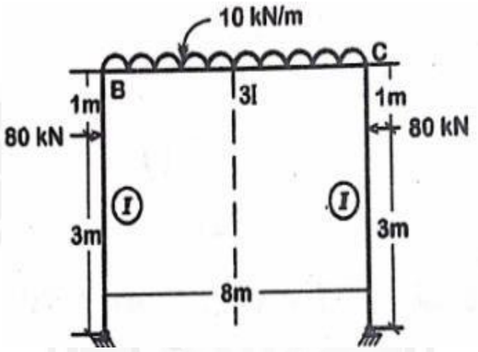


- **Result:**

  MA = MD = -7.14 kNm;   MB = MC = -41.98 kNm

PROBLEM NO:03

Analyze the symmetrical portal rigid frame shown below by moment distribution method and draw BMD



Solution:

- **Fixed End Moments:**

  MFAB = $-Wab^2/l^2 = -80x3x1^2/4^2 = -15$ kNm;

  MFBA = $Wa^2b/l^2 = 80x3^2x1/4^2 = 45$ kNm;

  MFBC = $-Wl^2/12 = -10x8^2/12 = -53.33$ kNm;

  MFCB = $Wl^2/12 = 10x8^2/12 = 53.33$ kNm;

  MFCD = $-Wab^2/l^2 = -80x1x3^2/4^2 = -45$ kNm;

  MFDC = $Wa^2b/l^2 = 80x1^2x3/4^2 = 15$ kNm;

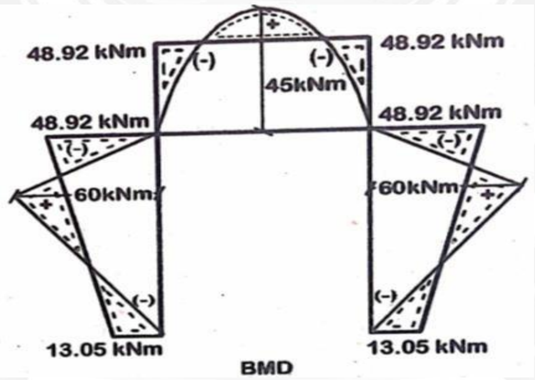
- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = 4EI/4	17EI/8	0.47
	BC	3E(3I)/l = 9EI/8		0.53
C	CB	3E(3I)/l = 9EI/8	17EI/8	0.53
	CD	4EI/l = 4EI/4		0.47

- **Moment Distribution Table**:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.47	0.53	0.53	0.47	-
F.E.M	-15	45	-53.33	53.33	-45	15
Distribute		3.92	4.41	-4.41	-3.92	
Carry over	1.95					-1.95
Final Moments	-13.05	48.92	-48.92	48.92	-48.92	13.05
Conventional Moments	-13.05	-48.92	-48.92	-48.92	-48.92	-13.05

- **Moment Diagram:**



- **Free BMD:**

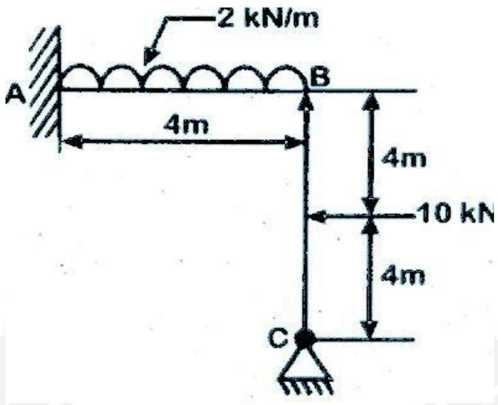
MAB = MCD = Wab/l= 80x3x1/4 = 60 kNm;
MBC = $Wl^2/8 = 10x6^2/8$ = 45 kNm.

- **Result:**

MA = MD = -13.05 kNm; MB = MC = -48.92 kNm

PROBLEM NO:03

Analyze the symmetrical portal rigid frame shown below by moment distribution method and draw BMD



Solution:

- **Fixed End Moments:**

  MFAB = -Wl^2/12 = -2x4^2/12 = -2.67 kNm;

  MFBA = Wl^2/12 = 2x4^2/12 = 2.67 kNm;

  MFBC = -Wl/8 = -10X8/8 = -10 kNm

  MFCB = Wl/8 = 10X8/8 = 10 kNm

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = EI	11EI/8	0.73
	BC	3EI/l = 3EI/8		0.27

- **Free BMD:**

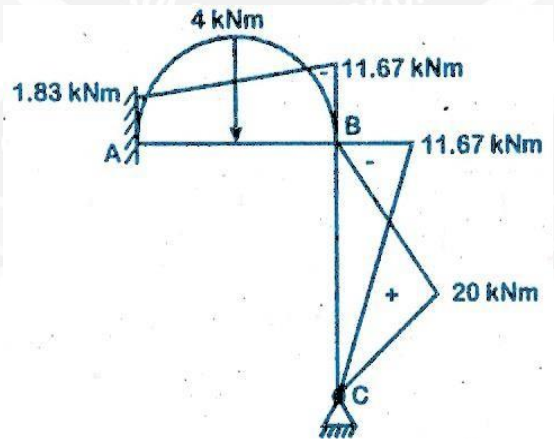
  MAB = Wl^2/8 = 2x4^2/8 = 4 kNm.

  MBC = Wl/4= 10x8/4 = 20 kNm;

- **Moment Distribution Table**:

Joint	A	B		C
Member	AB	BA	BC	CB
D.F	-	0.73	0.27	-
F.E.M	-2.67	2.67	-10	10
Release C and carry over	-	0	-5	-10
Initial Moments	-2.67	2.67	-15	0
Distribute	-90	9.00	3.329	-90
Carry over	4.5			
Final Moments	1.83	11.67	-11.67	0
Conventional Moments	-1.83	-11.67	-11.67	0

- **Moment Diagram:**



- **Result:**

MA = -1.83 kNm; MB = -11.67 kNm; MC = 0.

3.4. SUPPORT SETTLEMENTS IN MOMENT DISTRIBUTION METHOD.

3.4.1 SUPPORT SETTLEMENT IN STRUCTURAL ANALYSIS:

Support settlements may be caused by **soil erosion**, dynamic soil effects during earthquakes, or by partial failure or settlement of supporting structural elements. Supports could also potentially heave due to frost effects (this could be considered a negative settlement).

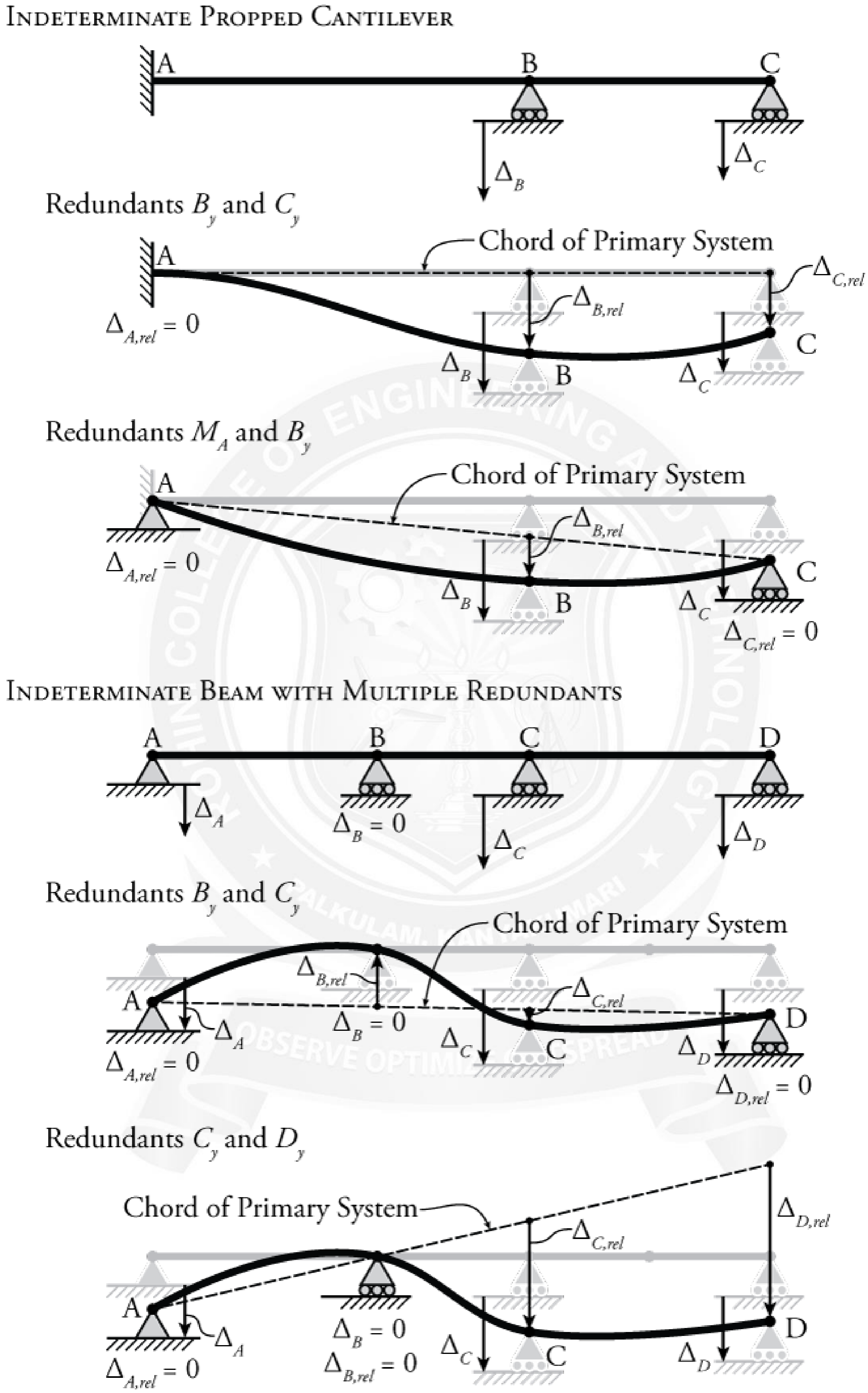
3.4.2. INTRODUCTION:

In the last lesson, the force method of analysis of statically indeterminate beams subjected to external loads was discussed. It is however, assumed in the analysis that the supports are unyielding and the temperature remains constant. In the design of indeterminate structure, it is required to make necessary provision for future unequal vertical settlement of supports or probable rotation of supports. It may be observed here that, in case of determinate structures no stresses are developed due to settlement of supports. The whole structure displaces as a rigid body. Hence, construction of determinate structures is easier than indeterminate structures.

The statically determinate structure changes their shape due to support settlement and this would in turn induce reactions and stresses in the system. Since, there is no external force system acting on the structures, these forces form a balanced force system by themselves and the structure would be in equilibrium. The effect of temperature changes, support settlement can also be easily included in the force method of analysis. In this lesson few problems, concerning the effect of support settlement are solved to illustrate the procedure.

3.4.3. SUPPORT DISPLACEMENTS:

The whole structure displaces as a rigid body. Hence, construction of determinate structures is easier than indeterminate structures. The statically determinate structure changes their shape due to support settlement and this would in turn induce reactions and stresses in the system.



Support settlements in continuous beams

3.4.4. NUMERICAL EXAMPLES ON(CONTINUOUS BEAMS):

PROBLEM NO:01

Analysis the continuous beam shown in fig.2.10,Calculate the support moments using moment distribution method.Support B settlememts by 10mm below the levels of A,C and D.Take E = 2 x 10^5 N/mm^2,I = 132 x 10^6 mm^4.Sketch the SF and BM diagrams.

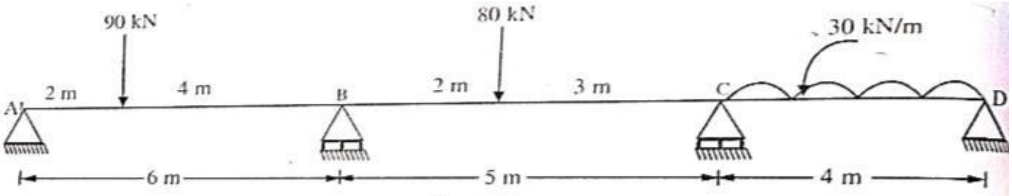
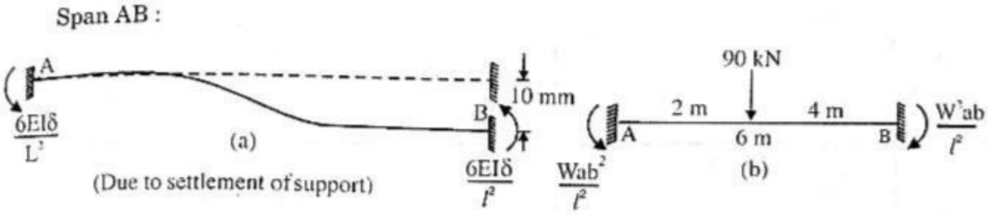


Fig. 3.42

Solution:

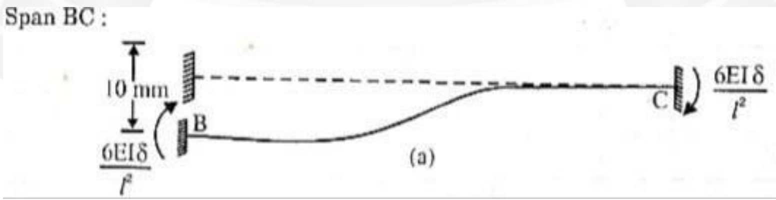
- **Fixed End Moments:**



MFAB = $-6EI\delta/l^2 - Wab^2/l^2 = -6x26400x10x10^{-3}/6^2 - 90x2x4^2/6^2 = -124$ kNm;

MFBA = $-6EI\delta/l^2 + Wa^2b/l^2 = -6x26400x10x10^{-3}/6^2 + 90x2x4^2/6^2 = -4$ kNm.

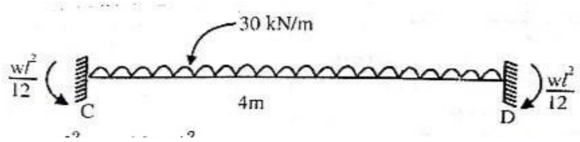
Span BC:



MFBC = $6EI\delta/l^2 - Wab^2/l^2 = 6x26400x10x10^{-3}/5^2 - 80x2x3^2/5^2 = 5.76$ kNm;

MFCB = $6EI\delta/l^2 + Wa^2b/l^2 = 6x26400x10x10^{-3}/5^2 + 80x2^2x3/5^2 = 101.76$ kNm.

Span CD:



MFCD = -Wl^2/12 = -30x4^2/12 = -40 kNm;

MFDC = Wl^2/12 = 30x4^2/12 = 40 kNm;

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	3/4xl/6 = I/8	13I/40	0.385
	BC	l/5 = I/5		0.615
C	CB	l/5 = I/5	31I/80	0.516
	CD	3/4xl/4 = 3I/16		0.484

- **Free BMD:**

MAB = MCD = Wab/l= 90x2x4/6= 120 kNm;

MBC = MCD = Wab/l= 80x2x3/4 = 96 kNm;

MCD = Wl^2/8 = 30x4^2/8 = 60 kNm.

- **Final Moments:**

MAB = 0

MBA = 35.841 kNm

MBC = - 35.841 kNm

MCB = 71.648 kNm

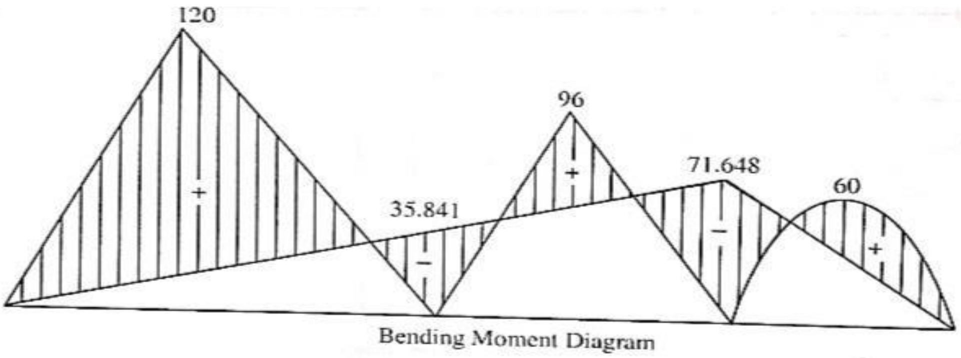
MCD = - 71.648 kNm

MDC = 0

- **Moment Distribution Table**:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.385	0.615	0.516	0.484	-
F.E.M	-124	-4	5.76	101.76	-40	40
Balance A & D	124					-40
Carry over		62			-20	
Initial Moments	0	58	5.76	101.76	-60	0
Balance B & C		-24.548	-39.212	-21.548	-20.212	
Carry Over			-10.774	-19.606		
Balance B & C		4.148	6.626	10.117	9.489	
Carry Over			5.059	3.313		
Balance B & C		-1.948	-3.111	-1.709	-1.604	
Carry Over			-0.855	-1.556		
Balance B & C		0.329	-0.526	0.803	0.753	
Carry Over			0.402	0.263		
Balance B & C		-0.155	-0.247	-0.136	-0.127	
Carry Over			-0.068	-0.124		
Balance B & C		0.026	0.042	0.064	0.06	
Carry Over			0.032	0.021		
Balance B & C		-0.012	-0.019	-0.011	-0.01	
Net Moment	0	35.841	-35.841	71.648	-71.648	0

- **Bending Moment Diagram**:

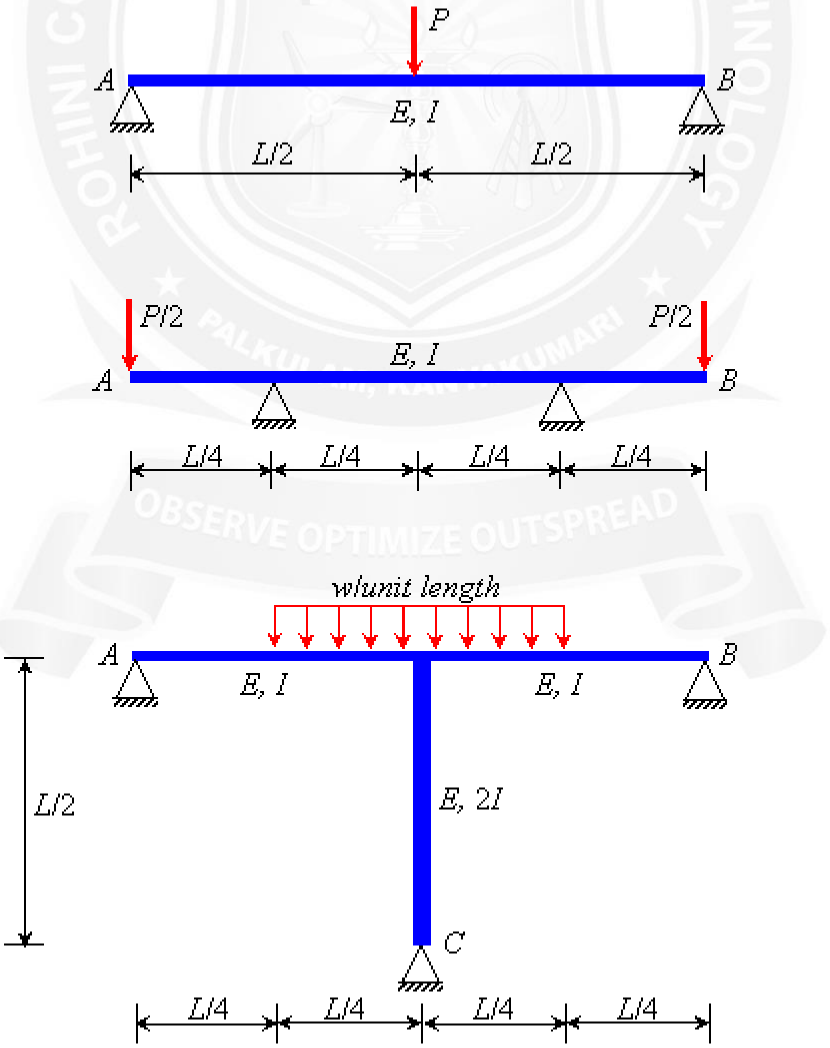


Bending Moment Diagram

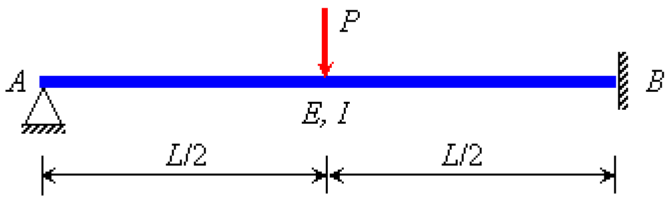
3.5 SYMMETRIC FRAMES WITH SYMMETRIC AND SKEW-SYMMETRIC LOADINGS

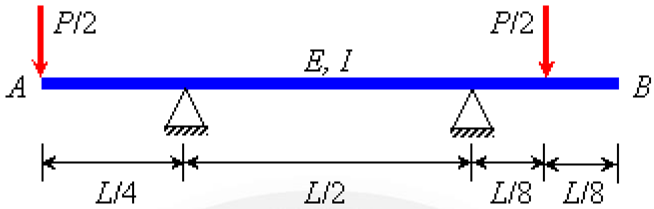
3.5.1 SYMMETRY AND ANTISYMMETRY

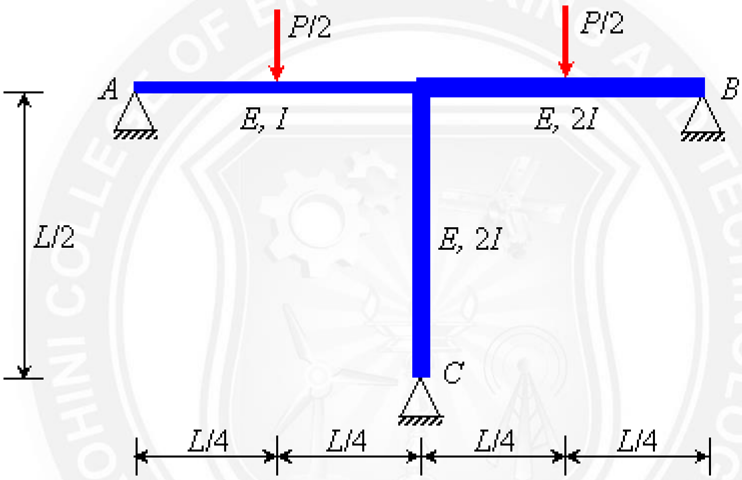
Symmetry or antisymmetry in a structural system can be effectively exploited for the purpose of analyzing structural systems. Symmetry and antisymmetry can be found in many real-life structural systems (or, in the idealized model of a real-life structural system). It is very important to remember that when we say symmetry in a structural system, it implies the existence of symmetry both in the structure itself including the support conditions and also in the loading on that structure. The systems shown in Fig. are symmetric because, for each individual case, the structure is symmetric and the loading is symmetric as well. However, the systems shown in Fig. are not symmetric because either the structure or the loading is not symmetric.



1.1. Symmetric structural systems



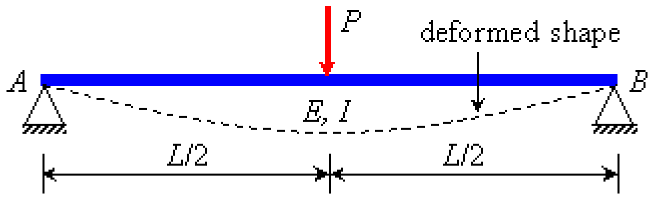


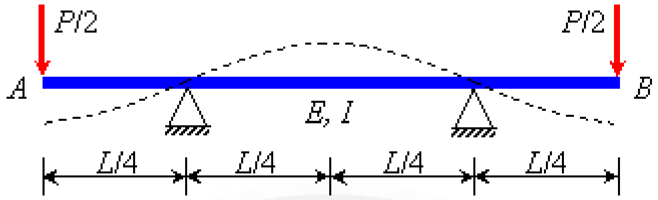


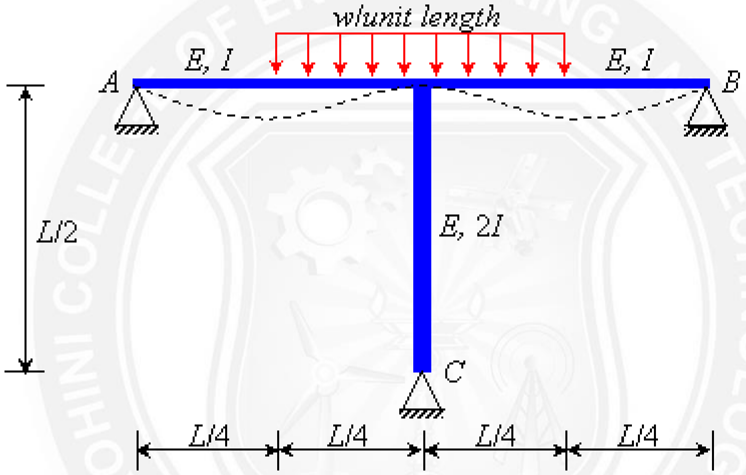
1.2. Non-symmetric (asymmetric) structural systems

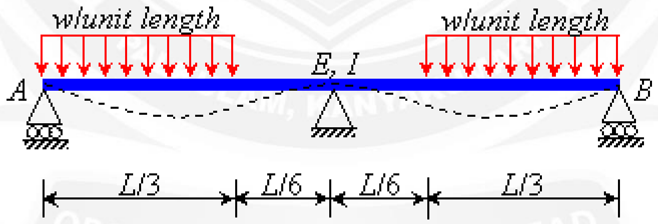
For an antisymmetric system the structure (including support conditions) remains symmetric,however, the loading is antisymmetric.The fig.1.2 ,shows the example of antisymmetric structural systems.

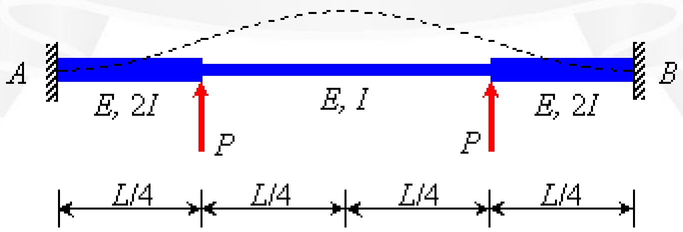
It is not difficult to see that the deformation for a symmetric structure will be symmetric about the same line of symmetry. This fact is illustrated in Fig. 1.3, where we can see that every symmetric structure undergoes symmetric deformation. It can be proved using the rules of structural mechanics (namely, equilibrium conditions, compatibility conditions and constitutive relations), that deformation for a symmetric system is always symmetric. Similarly, we always get antisymmetric deformation for antisymmetric structural systems



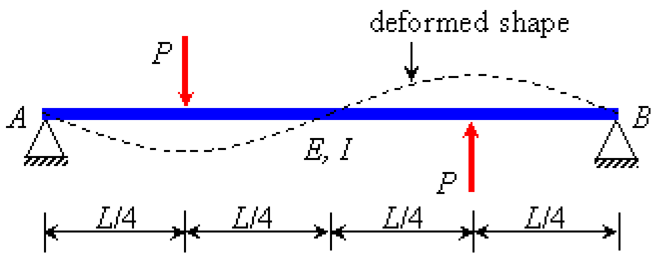


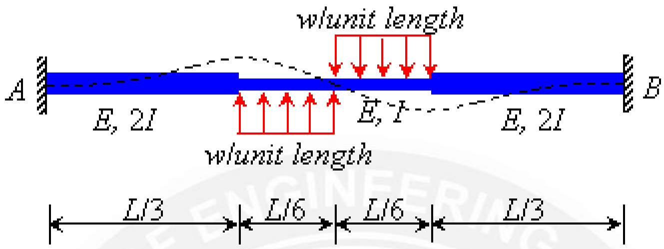


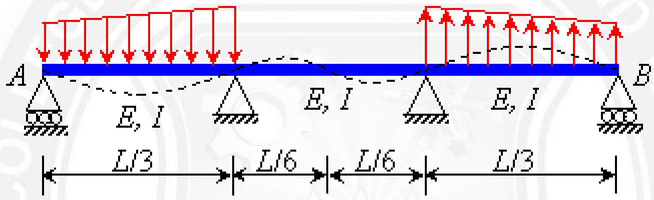




1.3. Deformation in symmetric systems

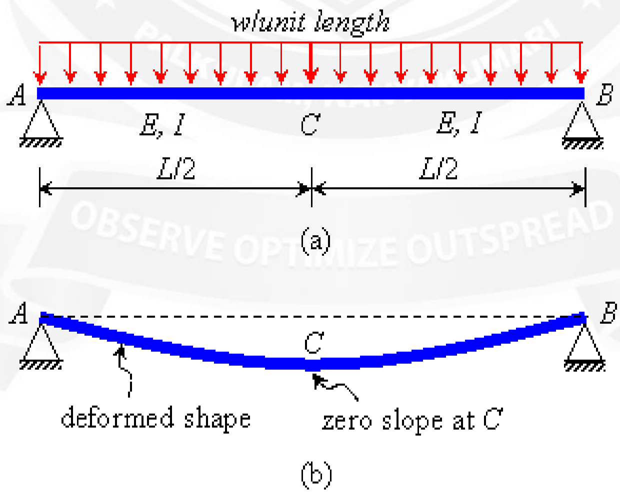






1.4. Deformation in antisymmetric systems

Let us look at beam *AB* in Fig. 1.20(a), which is symmetric about point C. The deformed shape of the structure will be symmetric as well (Fig. 1.20(b)). So, if we

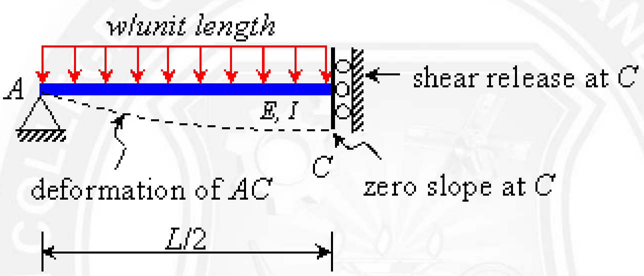


solve for the forces and deformations in part *AC* of the beam, we do not need to solve for part *CB* separately. The symmetry (or antisymmetry) in deformation gives us additional information prior to analyzing the structure and these information can be used to reduce the size of the structure that needs to be considered for analysis.

To elaborate on this fact, we need to look at the deformation condition at the point/line of symmetry (or antisymmetry) in a system. The following general rules about deformation can be deduced looking at the examples in Fig. 1.3 and Fig. 1.4:

- For a symmetric structure: slope at the point/line of symmetry is zero.
- For an antisymmetric structure: deflection at the point/line of symmetry is zero.

These information have to be incorporated when we reduce a symmetric (or antisymmetric) structure to a smaller one. If we want to reduce the symmetric beam in Fig. 1.20 to its one symmetric half *AC* , we have to integrate the fact the slope at point



1.6.Reduced system AC is adopted for analysis for beam AB

C for the reduced system AC will have to be zero. This will be a necessary boundary condition for the reduced system AC . We can achieve this by providing a support at C, which restricts any rotation, but allows vertical displacement, as shown in Fig. 1.6 (Note: this specific type of support is known as a “shear-release” or “shear-hinge”). Everything else (loading, other support conditions) remains unchanged in the reduced system. We can use this system AC for our analysis instead of the whole beam AB .

3.5.2 INTERNAL FORCE DIAGRAMS FOR A) A SYMMETRIC SYSTEM, AND B) AN ANTISYMMETRIC SYSTEM

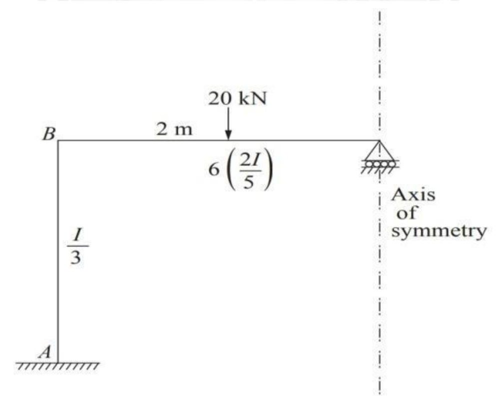
Having a priory knowledge about symmetry/antisymmetry in the structural system and in its deformed shape helps us know about symmetry/antisymmetry in internal forces in that system. (Symmetry in the system implies symmetry in equilibrium and constitutive relations, while symmetry in deformed shape implies symmetry in geometric compatibility.) Internal forces in a symmetric system are also symmetric about the same axis and similarly antisymmetric systems have antisymmetric internal forces. Detailed discussion on different types of internal forces in various structural systems and on internal force diagrams are provided in the next module (Module 2: Analysis of Statically Determinate Structures). Once we know about these diagrams we can easily see the following:

- A symmetric beam-column system has a symmetric bending moment diagram.
- A symmetric beam-column system has an antisymmetric shear force diagram.
- An antisymmetric beam-column system has an antisymmetric bending moment diagram.
- An antisymmetric beam-column system has a symmetric shear force diagram.

3.5.3 NUMERICAL EXAMPLES ON(SYMMETRIC AND SKEW-SYMMETRIC FRAMES):

PROBLEM NO:01

For the portal rigid frame compute the bending moments and draw the BMD



Solution:

- **Fixed end moments in skew symmetric case**

  MFBC = $-Wab^2/l^2 = -20x2x3^2/5^2 = -4.8$ kNm;

  MFCB = $Wa^2b/l^2 = 20x2^2x3/5^2 = 9.6$ kNm;

- **Distribution Factor Table for skews symmetric case**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	I/3 = 0.33I	2.73I	0.12
	BC	6(2I)/5 = 2.4I		0.88

- **Moment Distribution Table**:

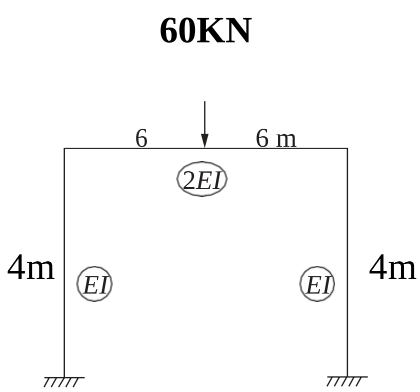
Joint	A	B	
Members	AB	BA	BC
DF	0	0.12	0.88
FEM			– 4.80
Balance		+0.58	+4.22
Carry over	– 0.58		
Final moment	– 0.58	+0.5	– 0.58

- **Result:**

MAB = -0.58 kNm; MBA = 0.5kNm; MBC = -0.58 kNm

PROBLEM NO:02

For the portal rigid frame compute the bending moments and draw the BMD



- **Fixed end moments in skew symmetric case**

  MFAB = MFBA = 0

  MFBC = -Wl/8 = -60X12/8 = -90 kNm

  MFCB = Wl/8 = 60X12/8 = 90 kNm

  MFCD = MFDC = 0

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	3E(2I)/l = 6EI/12	EI	0.43
	BC	3E(2I)/l = 6EI/12		0.57

- **Moment Distribution Table**:

Joint	A	B	
Members	A B	B A	B C
	0	0.12	0.88
FEM		-90	90
Balance		+0.58	+4.22
Carry over	– 0.58		
Final moment	– 0.58	+0.5	– 0.58

- **Result:**

MAB = -0.58 kNm; MBA = 0.5kNm; MBC = -0.58 kNm

4.1. FLEXIBLITY METHOD

4.1.1. INTRODUCTION

These are the two basic methods by which an indeterminate skeletal structure is analyzed. In these methods flexibility and stiffness properties of members are employed. These methods have been developed in conventional and matrix forms. Here conventional methods are discussed.

The number of releases required is equal to statically indeterminacy. Introduction of releases results in displacement discontinuities at these releases under the externally applied loads. Pairs of unknown by actions (forces and moments) are applied at these releases in order to restore the continuity or compatibility of structure.

The computation of these unknown by actions involves solution of linear simultaneous equations. The number of these equations is equal to statically indeterminacy. After the unknown by actions are computed all the internal forces can be computed in the entire structure using equations of equilibrium and free bodies of members. The required displacements can also be computed using methods of displacement computation.

In flexibility methods the unknowns are forces at the releases the method is also called force method. Since computation of displacement is also required at releases for imposing conditions of compatibility the method is also called compatibility method. In computation of displacements use is made of flexibility properties, hence, the method is also called flexibility method.

4.1.2. EQUILIBRIUM AND COMPATABILITY CONDITIONS

The three conditions of equilibrium are the sum of horizontal forces, vertical forces and moments at any joint should be equal to zero.

i.e., (H=0; V=0; M=0)

Forces should be in equilibrium

i.e., (F_X=0;F_Y=0;F_Z=0) i.e., (M_X=0;M_Y=0;M_Z=0)

Displacement of a structure should be compactable

The compatibility conditions for the supports

can be given as 1.Roller Support **(V=0)**

2.Hinged Support **(V=0, H=0)**

3.Fixed Support **(V=0, H=0, M=0)**

4.1.3. DETERMINATE AND INDETERMINATE STRUCTURAL SYSTEMS

If skeletal structure is subjected to gradually increasing loads, without distorting the initial geometry of structure, that is, causing small displacements, the structure is said to be stable. Dynamic loads and buckling or instability of structural system are not considered here. If forth stable structure it is possible to find the internal forces in all the members constituting the structure and supporting reactions at all the supports provided from statically equations of equilibrium only, the structure is said to be determinate.

If it is possible to determine all the support reactions from equations of equilibrium alone the structure is said to be externally determinate externally indeterminate. If structure is externally determinate but it is not possible to determine all internal forces then structure is said to be internally indeterminate. There are four structural system may be:

- Externally indeterminate but internally determinate
- Externally determinate but internally indeterminate
- Externally and Internally indeterminate
- Externally and Internally determinate

4.1.4. DETERMINATE Vs INDETERMINATE STRUCTURES.

Determinate structures can be solving using conditions of equilibrium alone (H=0; V=0; M=0). No other conditions are required.

Indeterminate structures cannot be solved using conditions of equilibrium because (H=0; V=0; M=0). Additional conditions are required for solving such structures. Usually matrix methods are adopted.

4.1.5. INDETERMINACY OF STRUCTURAL SYSTEM

The indeterminacy of a structure is measured as statically (s) or kinematical (k) Indeterminacy.

S= P (M - N + 1) - r = PR- r; K= P (N - 1) + r – s; K= PM - CP = 6 for space frames subjected to general loading

P = 3 for plane frames subjected to in plane or normal to plane loading. N = Number of nodes in structural system.

M=Numberofmembersofcompletelystiffstructurewhichincludesfoundationas singly connected system of members.

Incompletely stiff structure there is no release present. In singly connected system of rigid foundation members there is only one route between any two points in which tracks are not retraced. The system is considered comprising of closed rings or loops.

R = Number of loops or rings in completely stiff structure. r = Number of releases in the system.

C = Number of constraints in the system. R = (M -
N + 1) For plane and space trusses reduces S = M-
(NDOF) N+P

M = Number of members in completely stiffness.

P = 6 and 3 for space and plane truss respectively N= Number of nodes in truss.

NDOF = Degrees of freedom at node which is 2 for plane truss and 3 for space truss

For space truss = M- 3N+ 6

For plane truss = M- 2 N+ 3

Test for statically indeterminacy of structures system;

If S ≥ 0 structure is statically indeterminate

If S = 0 structure is statically determinate and

If S ≤ 0 structure is mechanism

It may be noted that structure may be mechanism even if S>0 if the releases are present in such away so as to cause collapse as mechanism. The situation of mechanism is unacceptable.

4.1.6. STATIC AND KINEMETIC INDETERMINACY

Statically Indeterminacy

It is difference of the unknown forces (internal forces plus external reactions) and the equations of equilibrium.

Kinematic Indeterminacy

It is the number of possible relative displacements of the nodes in the directions of stress resultants.

4.1.7. PRIMARY STRUCTURE

A structure formed by the removing the excess or redundant restraints from an indeterminate structure making it statically determinate is called primary structure. This is required for solving indeterminate structures by flexibility matrix method.

Indeterminate structure and Primary Structure

Indeterminatestructure **PrimaryStructure**

4.1.8. ANALYSIS OF INDETERMINATE STRUCTURES: (CONTINUOUS BEAMS)

Introduction

Solve statically indeterminate beams of degree more than one.

- To solve the problem in matrix notation.
- To compute reactions at all the supports.
- To compute internal resisting bending moment at any section of the continuous beam.

Beams which are statically indeterminate to first degree, were considered. If the structure is statically indeterminate to a degree more than one, then the approach presented in the force method is suitable.

Example Problems;

Problem 1.1

Calculate the support reactions in the continuous beam ABC due to loading as shown in Fig.1.1 Assume EI to be constant throughout.

Select two reactions vise, at B(R1) and C(R2) as redundant, since the given beam is statically indeterminate to second degree. In this case the primary structure is a cantilever beam AC. The primary structure with a given loading is shown in Fig. 1.2

In the present case, the deflections (L)1 and (L)2 of the released structure at B and C can be readily calculated by moment-area method. Thus

,

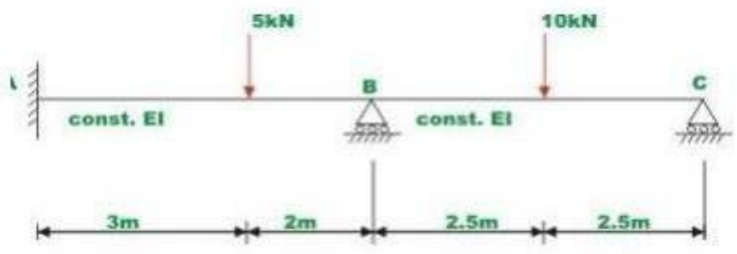


Fig 1.1

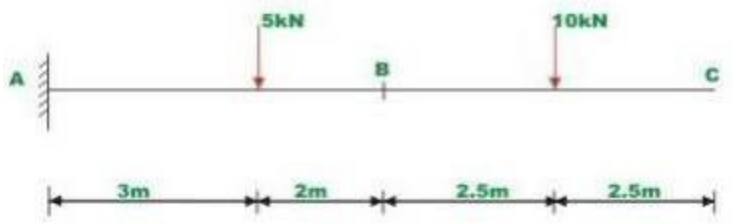


Fig 1.2

(L) 1= 819.16 / EI
(L) 2= 2311.875/ EI --- (1)

For the present problem the flexibility matrix is,

a11= 125/3EI ,a21= 625/6EI
a12= 625/6EI , a22 = 1000/3EI --- (2)

In the actual problem the displacements at B and Care zero. Thus the compatibility conditions for the problem may be written as,

a11 R1+ a12 R2 + (L) 1 = 0
a21 R1+ a22 R2+ (L) 2 = 0 --- (3)

Substituting the value of E and I in the above equation,

R1 = 10.609 KN and R2 = 3.620 KN

Using equations of static equilibrium,

R3 = 0.771 KN and R4 = 0.755KN

Problem 1.2

A Fixed beam AB of constant flexural rigidity is shown in Fig.1.3 The beam is subjected to auniform distributed load of *w* moment M=wL^2 kN.m. Draw Shear force and bending moment diagrams by force method.

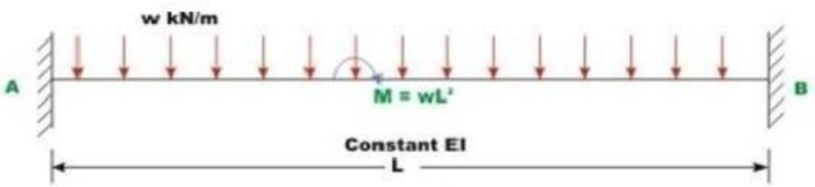


Fig 1.3 Fixed Beam

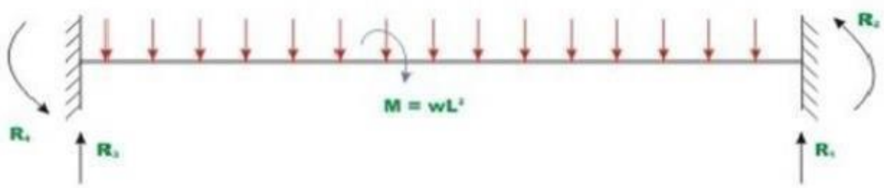


Fig 1.3 Fixed Beam with R_1 and R_2 as Redundant

Fig 1.3 Fixed Beam with R1 and R2 as Redundant;

Select vertical reaction (R1)and the support moment(R2) at B as the redundant. The primary structure in this case is acantilever beam which could be obtained by releasing the redundant R1 andR2.

The R1 is assumed to positive in the upward direction and R2 is assumed to be positive in the counterclockwise direction. Now, calculate deflection at B due to only applied loading. Let (L) be the transverse deflection at B and L be the slope at B due to external loading.The positive directions of the selected redundant are shown in Fig.8.3b

The deflection(L_1)and(L_2)of the released structure can be evaluated from unit load method. Thus,

$$(L_1) = wL^4/8EI - 3wL^4/8EI = wL^4/2EI \text{ ---- } (1)$$

$$(L_2) = wL^3/6EI - wL^3/2EI = 2wL^3/3EI \text{ ---- } (2)$$

The negative sign indicates that (L)is downwards and rotation(is 1 L2) clockwise.

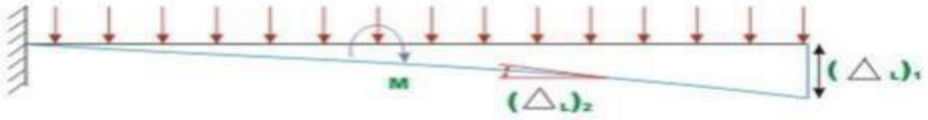


Fig 1.4 Primary Structure with external loading

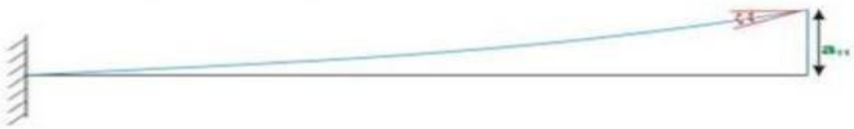


Fig 1.5 Primary Structure with unit load along R_1

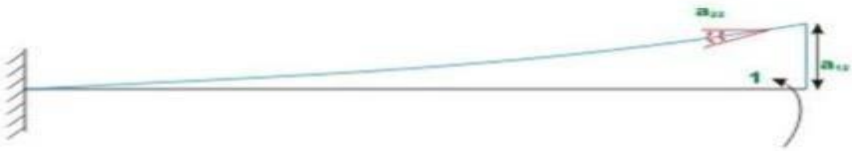


Fig 1.6 Primary Structure with unit Moment along R_2

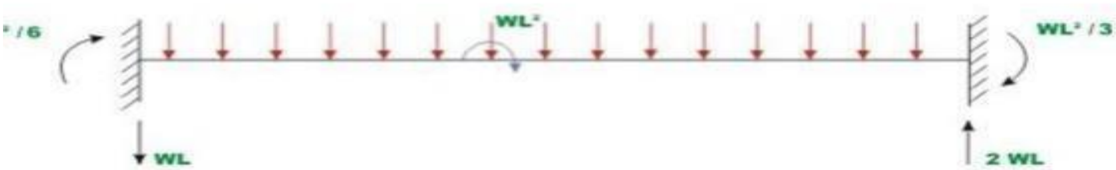


Fig 1.7 Reaction

Draw SFD & BMD:

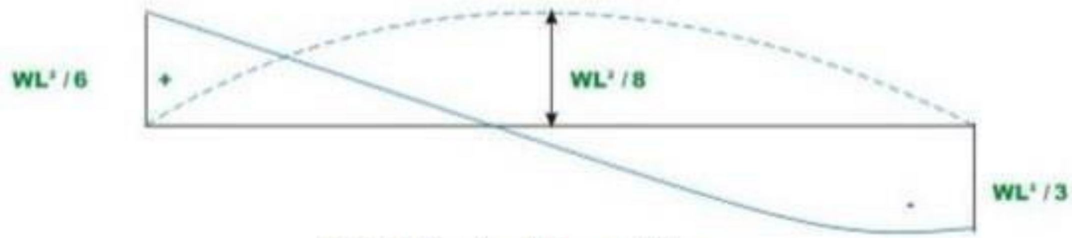


Fig1.8.Bending Moment Diagram

Fig1.9.Shear Force Diagram

4.2 FORMATION OF FLEXIBILITY MATRIX METHOD

For simple,determinate flexural members,developing the {F} matrix is only a process of finding deflections (and/ or rotations) due to a set of simple forces (including moments).We may have to invoke some of the known methods of comuting deflections (like Mohr´s theorems,conjugate beam method etc).

Again, as in the flexibility method, we see that the greater the degree of indeterminacy (kinematic in this case) the greater the number of equations requiring solution, so that a computer-based approach is necessary when the degree of indeterminacy is high.

Generally this requires that the force–displacement relationships in a structure are expressed in matrix form. We therefore need to establish force–displacement relationships for structural members and to examine the way in which these individual force–displacement relationships are combined to produce a force–displacement relationship for the complete structure. Initially we shall investigate members that are subjected to axial force only.

4.2.1.ANALYSING STRUCTURES BY FLEXIBILITY METHOD

Using Flexibility Method for analyzing simple indeterminate structures is like using a helicoptor to go to the next street.The real value of the method can be realized only when we deal with a large number of redundancies.This method is specifically meant for use with computers.

However, the following examples (of manually solving simple structures) are aimed at reinforcing the fundamental principles and procedures involved.

Example 4.21

For the beam shown in fig.,find the deflection at the mid span.

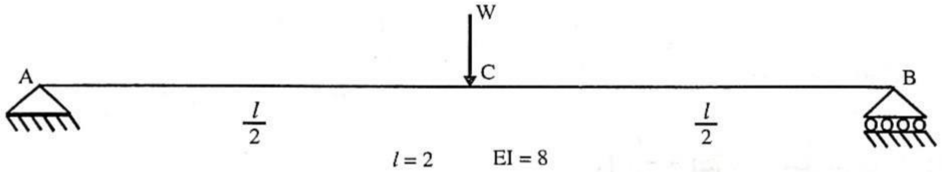
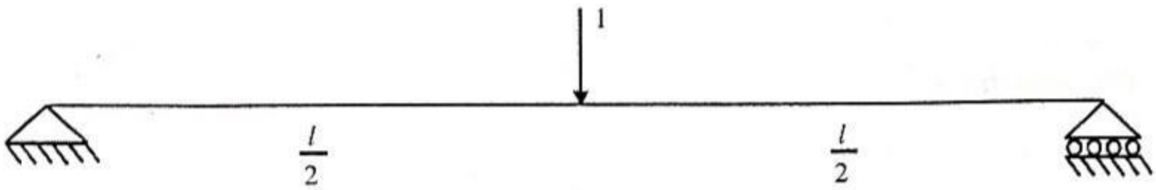


Fig. 4.26

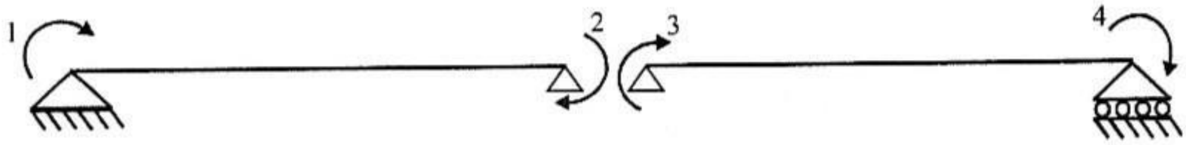
Solution:

Step 1:The given structure, being determinate, is also the primary structure.Hence the system co-ordinate is as under

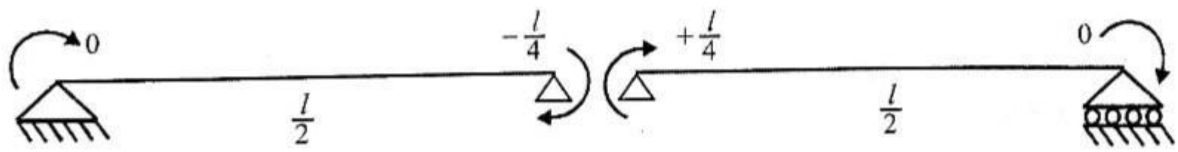


System co-ordinate

Step 2. Let us choose AC and CB as elements and define element co-ordinates as under



Step 3. To generate the [b] matrix relating system force vector, {F} and element force vector {P}, Let us get the equilibrium conditions for the 2 elements, when $F_1 = 1$



$$[b] = [0\ \ -l/4\ \ +l/4\ \ 0]^T$$

$$= [0\ -1/2\ \ +1/2\ \ 0]^T$$

This equation simply means that the bending moment at mid span is Wl/4,where W = 1

Step 4:Element flexibility matrix [α] is given by

$$\frac{l/2}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \frac{1}{48}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

Step 5. Compute [a]

$$[a] = [b]^T \quad [\alpha] \quad [b]$$

$$= \begin{bmatrix} 0 & -\frac{1}{2} & +\frac{1}{2} & 0 \end{bmatrix} \frac{1}{48}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ -\frac{1}{2} \\ +\frac{1}{2} \\ 0 \end{bmatrix}$$

$$[a] = \frac{1}{48}\ [1]$$

Step 6:Find the form {u} = [a] {F}

{u} = 1 x W/48 = W/48

The ludicrous fact is that we have used a lio ofelaborate steps to arrive at the conclusion that the mid span deflection is $Wl^3/48EI$.

Example 4.22

A cantilever of length 15 m is subjected to a single concentrated load of 50 kN at the middle of the span. Find the deflection at the free end using flexibility matrix method. EI is uniform throughout.

Solution:

Step1:

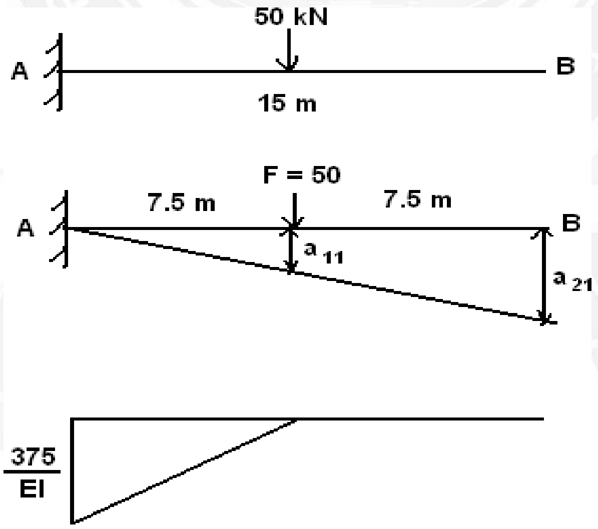
Static Indeterminacy :

Degree of redundancy = 3-3= 0

It is static determinate structures.

Step 2: Deflection at B:

Apply a unit force at given load.



The deflection is calculated by M/EI

Deflection at (a) = (1/2x7.5x375/EI)

Deflection at B = 17578.125/EI

4.2.3.PROCEDURE FOR FLEXIBILITY METHOD:

There are eight steps to find the support moments in continuous beams,frames and trusses;they are

- Decide on the primary structure;indicate redundants $\{F\}^0$
- Select {P} co-ordinates for internal forces.
- Compile external forces $\{F\}^*$,This would include the effects of off node forces.

Then,

$$\{F\} = \begin{Bmatrix} \{F\}^* \\ \cdots\cdots \\ \{F\}^0 \end{Bmatrix} \quad \text{and} \quad \{u\} = \begin{bmatrix} \{u\}^* \\ \cdots\cdots \\ \{u\}^0 \end{bmatrix}, \{u\}^0 = \{0\}$$

Generate [b] matrix such that $\{P\} = [b]\ \{F\}$; $[b] = [b]^*\ [b]^0$

- Synthesize element flexibility matrix $[\alpha]$
- Compute [a] using $[a] = [\alpha]\ [b]$; $[a] = \begin{bmatrix} [a]_{11} & [a]_{12} \\ [a]_{21} & [a]_{22} \end{bmatrix}$
- Isolate $\{F\}^0$,the redundant forces, from the condition

  $\{P\}^0 = \{0\}$

  $\{F\}^0 = -\ [a]_{22}{}^{-1}\ \ [a]_{21}\ \ \{F\}^*$
- Isolate $[a]^*$ $\quad [a]^* = [a]_{11} - [a]_{22}{}^{-1}\ [a]_{21}$
- Get {P} from $\{P\} = [b]\ \ \{F\}$ and $\{P\} - \{P\}^e$

  $\{P\}^f$ are the final member forces.

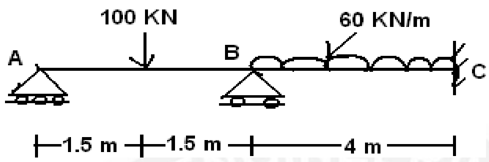
  $\{P\}^f$ are also the support moments at all joints.

4.3. ANALYSIS THE CONTINUOUS BEAM BY FLEXIBILITY METHOD.

4.3.1. NUMERICAL PROBLEMS ON CONTINUOUS BEAMS;

PROBLEM NO:01

Analysis the continuous beam shown in fig,by using Flexibility method.



Solution:

- **Static indeterminacy:**

  Degree of redundancy = (1 + 1 + 3) – 3 = 2

  Release at B and C by apply hinge

- **Fixed End Moments:**

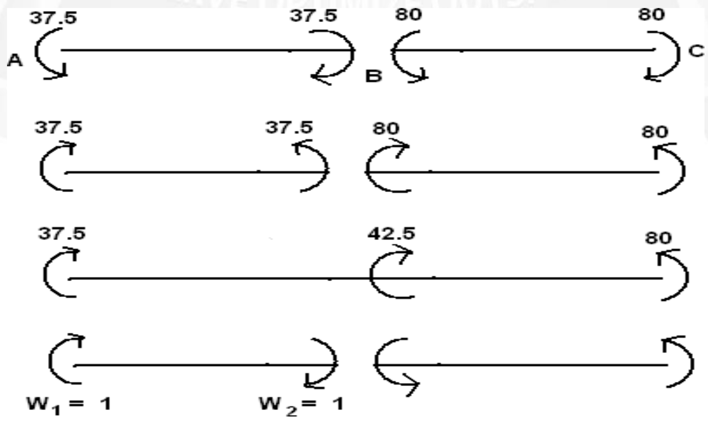
  MFAB = -Wl/8 = -100X3/8 = -37.5 kNm

  MFBA = Wl/8 = 100X3/8 = 37.5 kNm

  MFBC = $-Wl^2/12 = -60x4^2/12$ = -80 kNm

  MFCB = $Wl^2/12 = 60x4^2/12$ = 80 kNm

- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

  B = B_W . B_X

$$B_W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and } B_X = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI}\begin{bmatrix} 1 & -0.5 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & -0.67 & 1.33 \end{bmatrix}$$

$$\mathbf{F_X = B_X^T \cdot F \cdot B_X}$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -0.5 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & -0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_x = \frac{1}{EI}\begin{bmatrix} 2.33 & -0.67 \\ -0.67 & 1.33 \end{bmatrix}$$

$$F_x^{-1} = EI\begin{bmatrix} 0.502 & 0.253 \\ 0.253 & 0.879 \end{bmatrix}$$

$$\mathbf{F_W = B_X^T \cdot F \cdot B_W}$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -0.5 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & -0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_W = \frac{1}{EI}\begin{bmatrix} 0.5 & -1 \\ 0 & 0 \end{bmatrix}$$

- **Displacement Matrix (X):**

X = - F_X^{-1} . F_W . W

$$= -\frac{EI}{EI}\begin{bmatrix} 0.502 & 0.253 \\ 0.253 & 0.879 \end{bmatrix}\begin{bmatrix} 0.5 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 37.5 \\ 42.5 \end{bmatrix}$$

$$= -\begin{bmatrix} 0.251 & -0.502 \\ 0.127 & -0.253 \end{bmatrix}\begin{bmatrix} 37.5 \\ 42.5 \end{bmatrix}$$

$$= -\begin{bmatrix} -11.923 \\ -\ 5.99 \end{bmatrix}$$

$$X = \begin{bmatrix} 11.923 \\ 5.99 \end{bmatrix}$$

- **Internal Force (P):**

P = [ ]
X

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 37.5 \\ 42.5 \\ 11.923 \\ 5.99 \end{bmatrix}$$

$$P = \begin{bmatrix} 37.5 \\ 30.58 \\ 11.923 \\ 5.99 \end{bmatrix}$$

- **Final Moments (M):**

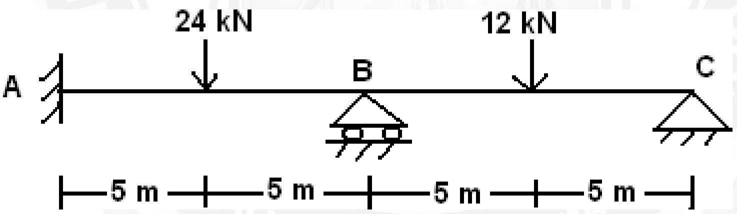
M = μ + P

$$= \begin{bmatrix} -37.5 \\ 37.5 \\ -80 \\ 80 \end{bmatrix} + \begin{bmatrix} 37.5 \\ 30.58 \\ 11.923 \\ 5.99 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 \\ 68.08 \\ -68.08 \\ 95.99 \end{bmatrix}$$

PROBLEM NO:02

Analysis the continuous beam ABC shown in fig,by using Flexibility method.And sketch the bending moment diagram.



Solution:

- **Static indeterminacy:**

  Degree of redundancy = (3 + 1 + 1) – 3 = 2

  Release at A and B by apply hinge

- **Fixed End Moments:**

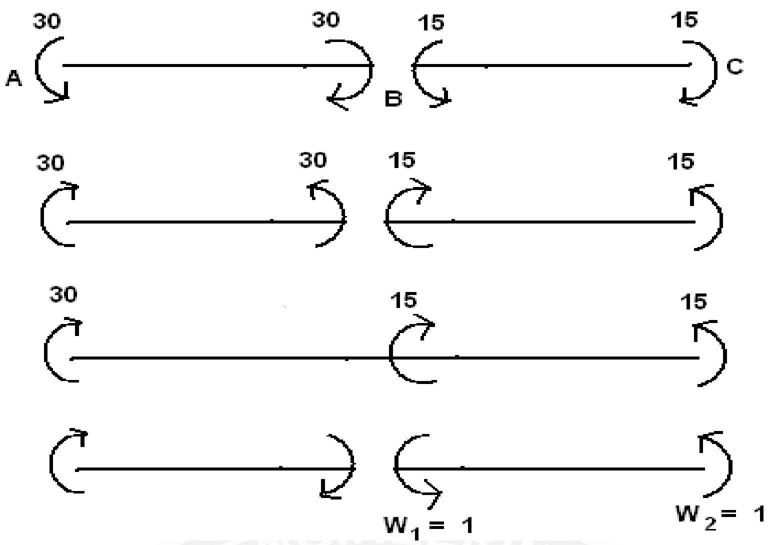
  MFAB = -Wl/8 = -24X10/8 = -30 kNm

  MFBA = Wl/8 = 24X10/8 /8 = 30 kNm

  MFAB = -Wl/8 = -12X10/8 = -15 kNm

  MFBA = Wl/8 = 12X10/8 /8 = 15 kNm

- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

B = BW . BX

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI}\begin{bmatrix} 3.33 & -1.67 & 0 & 0 \\ -1.67 & 3.33 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}$$

$F_X = B_X^T . F . B_X$

$$= \frac{1}{EI}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 3.33 & -1.67 & 0 & 0 \\ -1.67 & 3.33 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \frac{1}{EI}\begin{bmatrix} 3.33 & 1.67 \\ 1.67 & 6.66 \end{bmatrix}$$

$$F_x^{-1} = EI\begin{bmatrix} 0.3435 & -0.086 \\ -0.086 & 0.1717 \end{bmatrix}$$

$$\mathbf{F_W = B_X^T \,.\, F \,.\, B_W}$$

$$= \frac{1}{EI}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 3.33 & -1.67 & 0 & 0 \\ -1.67 & 3.33 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_W = \frac{1}{EI}\begin{bmatrix} 0 & 0 \\ 3.33 & -1.67 \end{bmatrix}$$

- **Displacement Matrix (X):**

  $$\mathbf{X = -\,F_X^{-1} \,.\, F_W \,.\, W}$$

$$= -\frac{EI}{EI}\begin{bmatrix} 0.3435 & -0.086 \\ -0.086 & 0.1717 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 3.33 & -1.67 \end{bmatrix}\begin{bmatrix} -15 \\ -15 \end{bmatrix}$$

$$= -\begin{bmatrix} -0.286 & 0.144 \\ 0.144 & -0.286 \end{bmatrix}\begin{bmatrix} -15 \\ -15 \end{bmatrix}$$

$$= -\begin{bmatrix} 2.13 \\ -4.29 \end{bmatrix}$$

$$X = \begin{bmatrix} -2.13 \\ 4.29 \end{bmatrix}$$

- **Internal Force (P):**

  $$\mathbf{P = [\ ]_X}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -15 \\ -15 \\ -2.13 \\ 4.29 \end{bmatrix}$$

$$P = \begin{bmatrix} -2.13 \\ -4.29 \\ -10.71 \\ -15 \end{bmatrix}$$

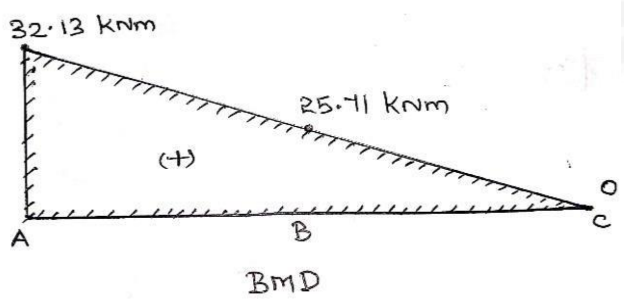
- **Final Moments (M):**

$$\mathbf{M = \mu + P}$$

$$= \begin{bmatrix} -30 \\ 30 \\ -15 \\ 15 \end{bmatrix} + \begin{bmatrix} -2.13 \\ -4.29 \\ -10.71 \\ -15 \end{bmatrix}$$

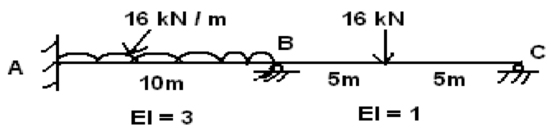
$$M = \begin{bmatrix} -32.13 \\ 25.71 \\ -25.71 \\ 0 \end{bmatrix}$$

- **Bending Moment Diagram:**



PROBLEM NO:03

Analysis the continuous beam shown in fig,by using Flexibility method.



Solution:

- **Static indeterminacy:**

  Degree of redundancy = (3 + 1 + 1) – 3 = 2

  Release at A and B by apply hinge

- **Fixed End Moments:**

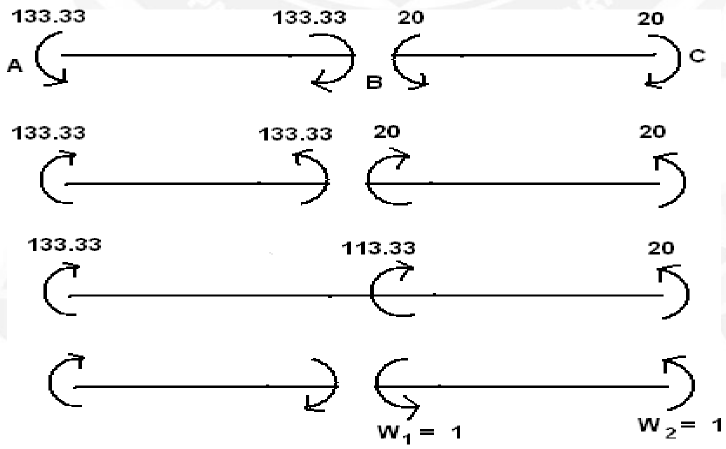
  MFAB = $-Wl^2/12 = -16x10^2/12 = -133.33$ kNm

  MFBA = $Wl^2/12 = 16x10^2/12 = 133.33$ kNm

  MFBC = -Wl/8 = -16X10/8 = -20 kNm

  MFCB = Wl/8 = 16X10/8 = 20 kNm

- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

  $B = B_W \cdot B_X$

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \begin{bmatrix} 1.11 & -0.56 & 0 & 0 \\ -0.56 & 1.11 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}$$

$F_X = B_X^T . F . B_X$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1.11 & -0.56 & 0 & 0 \\ -0.56 & 1.11 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \begin{bmatrix} 1.11 & 0.56 \\ 0.56 & 4.44 \end{bmatrix}$$

$$F_x^{-1} = \begin{bmatrix} 0.962 & -0.121 \\ -0.121 & 0.241 \end{bmatrix}$$

$F_W = B_X^T . F . B_W$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1.11 & -0.56 & 0 & 0 \\ -0.56 & 1.11 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_W = \begin{bmatrix} 0 & 0 \\ 3.33 & -1.67 \end{bmatrix}$$

- **Displacement Matrix (X):**

  X = - F_X^{-1} . F_W . W

$$= -\begin{bmatrix} 0.962 & -0.121 \\ -0.121 & 0.241 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 3.33 & -1.67 \end{bmatrix}\begin{bmatrix} -113.33 \\ -20 \end{bmatrix}$$

$$= -\begin{bmatrix} 41.62 \\ -82.90 \end{bmatrix}$$

$$X = \begin{bmatrix} -41.62 \\ 82.90 \end{bmatrix}$$

- **Internal Force (P):**

$$P = [\ ]_X$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} -113.33 \\ -20 \\ -41.62 \\ 82.90 \end{bmatrix}$$

$$P = \begin{bmatrix} -41.62 \\ -82.90 \\ -30.43 \\ -20 \end{bmatrix}$$

- **Final Moments (M):**

$$M = \mu + P$$

$$= \begin{bmatrix} -133.33 \\ 133.33 \\ -20 \\ 20 \end{bmatrix} + \begin{bmatrix} -41.62 \\ -82.90 \\ -30.43 \\ -20 \end{bmatrix}$$

$$M = \begin{bmatrix} -174.95 \\ 50.43 \\ -50.43 \\ 0 \end{bmatrix}$$

4.4. ANALYSIS OF INDETERMINATE RIGID FRAMES BY FLEXIBILITY METHOD.

4.4.1. NUMERICAL PROBLEMS ON RIGID FRAMES;

PROBLEM NO:01

Analysis the rigid portal frame ABCD shown in fig,by using Flexibility method.

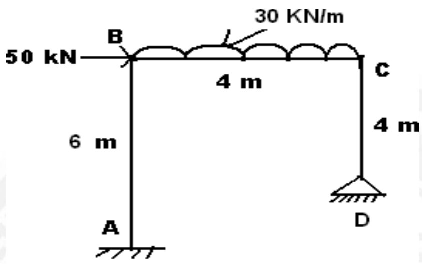


Fig.

Solution:

- **Static indeterminacy:**

  Degree of redundancy = (3 + 2) – 3 = 2

  Release at B and C by apply hinge

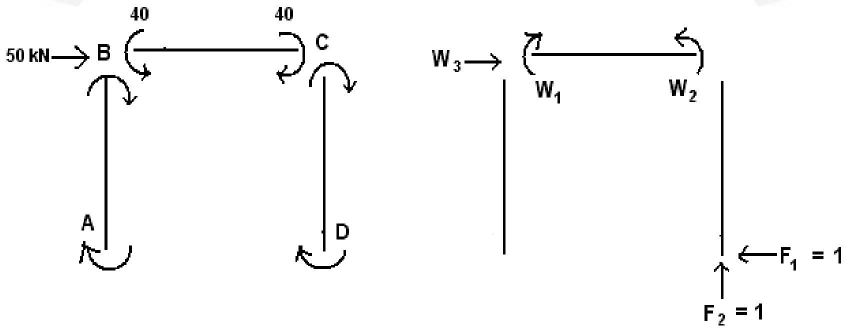
  Apply a unit force at B joint.

- **Fixed End Moments:**

  MFBC = $-Wl^2/12 = -30x4^2/12 = -40$ kNm

  MFCB = $Wl^2/12 = 30x4^2/12 = 40$ kNm

- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

  B = $B_W . B_X$

$$B = \begin{bmatrix} 0 & 0 & -6 & -2 & 4 \\ 0 & 0 & 0 & 4 & -4 \\ 1 & 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 & 0 & 0 \\ 0 & 0 & -0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & 0 & 0 & -0.67 & 1.33 \end{bmatrix}$$

$$\mathbf{F_X = B_X^T \cdot F \cdot B_X}$$

$$= \frac{1}{EI}\begin{bmatrix} -2 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 & 0 & 0 \\ 0 & 0 & -0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & 0 & 0 & -0.67 & 1.33 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 4 & -4 \\ -4 & 4 \\ 4 & 0 \\ -4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \frac{1}{EI}\begin{bmatrix} 141.28 & -104 \\ -104 & 117.28 \end{bmatrix}$$

$$F_x^{-1} = EI\begin{bmatrix} 0.0204 & 0.0181 \\ 0.0181 & 0.0246 \end{bmatrix}$$

$F_W = B_X^T . F . B_W$

$$= \frac{1}{EI}\begin{bmatrix} -2 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 & 0 & 0 \\ 0 & 0 & -0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & 0 & 0 & -0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 0 & 0 & -6 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$F_W = \frac{1}{EI}\begin{bmatrix} -8 & -5.32 & 48 \\ 5.32 & 0 & -72 \end{bmatrix}$$

- **Displacement Matrix (X):**

  $X = - F_X^{-1} . F_W . W$

$$= -\frac{EI}{EI}\begin{bmatrix} 0.0204 & 0.0181 \\ 0.0181 & 0.0246 \end{bmatrix}\begin{bmatrix} -8 & -5.32 & 48 \\ 5.32 & 0 & -72 \end{bmatrix}\begin{bmatrix} 40 \\ -40 \\ 50 \end{bmatrix}$$

$$= -\begin{bmatrix} 0.0669 & 0.1085 & 0.3240 \\ 0.0139 & 0.0963 & 0.9024 \end{bmatrix}\begin{bmatrix} 40 \\ -40 \\ 50 \end{bmatrix}$$

$$= -\begin{bmatrix} -14.536 \\ -41.824 \end{bmatrix}$$

$$X = \begin{bmatrix} 14.536 \\ 41.824 \end{bmatrix}$$

- **Internal Force (P):**

  P = [ ] ***X***

$$= \begin{bmatrix} 0 & 0 & -6 & -2 & 4 \\ 0 & 0 & 0 & 4 & -4 \\ 1 & 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 40 \\ -40 \\ 50 \\ 14.536 \\ 41.824 \end{bmatrix}$$

$$P = \begin{bmatrix} -161.776 \\ -109.152 \\ 149.152 \\ 58.144 \\ -98.144 \\ 0 \end{bmatrix}$$

- **Final Moments (M):**

$$\mathbf{M = \mu + P}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -40 \\ 40 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -161.776 \\ -109.152 \\ 149.152 \\ 58.144 \\ -98.144 \\ 0 \end{bmatrix}$$

$$M = \begin{bmatrix} -161.776 \\ -109.152 \\ 109.152 \\ 98.144 \\ -98.144 \\ 0 \end{bmatrix}$$

PROBLEM NO:02

Analysis the rigid portal frame ABCD shown in fig,by using Flexibility method.And sketch the bending moment diagram.

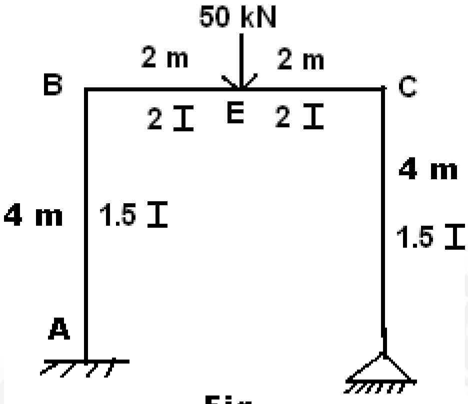


Fig.

Solution:

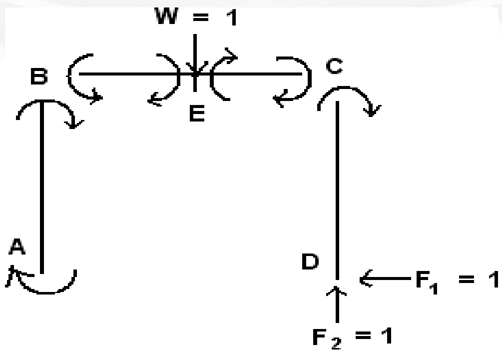
- **Static indeterminacy:**

  Degree of redundancy = (3 + 2) – 3 = 2

  Release at D by apply horizontal and vertical supports

  Apply a unit force at E joint.

- **Fixed End Moments:**
  MFAB = MFBA = MFBC = MFCB = MFCD = MFDC = 0
- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

  $B = B_W \cdot B_X$

$$B = \begin{bmatrix} -2 & 0 & 4 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \\ 0 & 4 & -2 \\ 0 & -4 & 2 \\ 0 & 4 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI}\begin{bmatrix} 0.89 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.44 & 0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & -0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.17 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.33 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.17 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.89 & -0.44 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.44 & 0.89 \end{bmatrix}$$

$F_X = B_X^T . F . B_X$

$$= \frac{1}{EI}\begin{bmatrix} 0 & 4 & -4 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & -2 & 2 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0.89 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.44 & 0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & -0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.17 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.33 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.17 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.89 & -0.44 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.44 & 0.89 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 4 & -4 \\ -4 & 4 \\ 4 & -2 \\ -4 & 2 \\ 4 & 0 \\ -4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \frac{1}{EI}\begin{bmatrix} 60.48 & -37.28 \\ -37.28 & 53.2 \end{bmatrix}$$

$$F_x^{-1} = EI\begin{bmatrix} 0.0291 & 0.0203 \\ 0.0203 & 0.033 \end{bmatrix}$$

$$\mathbf{F_W = B_X^T \cdot F \cdot B_W}$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & 4 & -4 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & -2 & 2 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0.89 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.44 & 0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & -0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.17 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.33 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.17 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.89 & -0.44 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.44 & 0.89 \end{bmatrix}\begin{bmatrix} -2 \\ 2 \\ -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F_W = \frac{1}{EI}\begin{bmatrix} 14.64 \\ -24.60 \end{bmatrix}$$

- **Displacement Matrix (X):**

  $$\mathbf{X = - F_X^{-1} \cdot F_W \cdot W}$$

$$= -\frac{EI}{EI}\begin{bmatrix} 0.0291 & 0.0203 \\ 0.0203 & 0.033 \end{bmatrix}\begin{bmatrix} 14.64 \\ -24.60 \end{bmatrix} 50$$

$$= -\begin{bmatrix} -0.0734 \\ -0.5146 \end{bmatrix} 50$$

$$= -\begin{bmatrix} -3.67 \\ -25.73 \end{bmatrix}$$

$$X = \begin{bmatrix} 3.67 \\ 25.73 \end{bmatrix}$$

- **Internal Force (P):**

P = [ ] ***X***

$$= \begin{bmatrix} -2 & 0 & 4 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \\ 0 & 4 & -2 \\ 0 & -4 & 2 \\ 0 & 4 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 50 \\ 3.67 \\ 25.73 \end{bmatrix}$$

$$P = \begin{bmatrix} 2.92 \\ 11.76 \\ -11.76 \\ -36.78 \\ 36.78 \\ 14.68 \\ -14.68 \\ 0 \end{bmatrix}$$

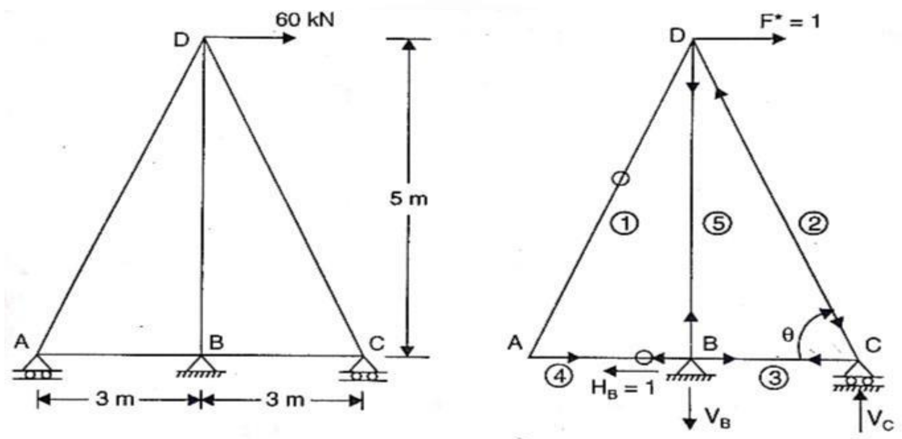
The final moments also same, since there are no external forces acting on the members.

ANALYSIS OF INDETERMINATE PIN-JOINTED FRAMES BY FLEXIBILITY METHOD.

NUMERICAL PROBLEMS ON PIN-JOINTED FRAMES;

PROBLEM NO:01

Analysis the truss loaded as shown in fig,by using Matrix Flexibility method.And find the member forces.A and E are the same for all members.



Solution:

- **Static indeterminacy:**

Degree of internal indeterminacy; I = m - (2j - 3) = 5 – (2 x 4 – 3) = 0

Degree of external indeterminacy; E = r – R = 4 – 3 = 1

The structure is externally indeterminate to one degree.Treating support A as redundant,the primary structure is shown in fig,

To get the B matrix :

The B matrix would be in 2 parts,B_X and B_W.To get the B_X matrix let us apply a unit force (F_X = 1) at D as shown and get the member forces by method of joints.

Reactions: HB = 1

Equating moments about C to zero,

VB x 3 = 1 x 5 ; VB = 1.667

Hence, VC = 1.667

At joint A, FAB = FAD = 0

At joint B, FBA = VB = 1.667 (tension)

At joint C, FCD sin θ = VC = 1.667

FCD = 1.667/sin θ =1.944 (comp)

FBC = FCD cos θ = 1.00 (tallies)

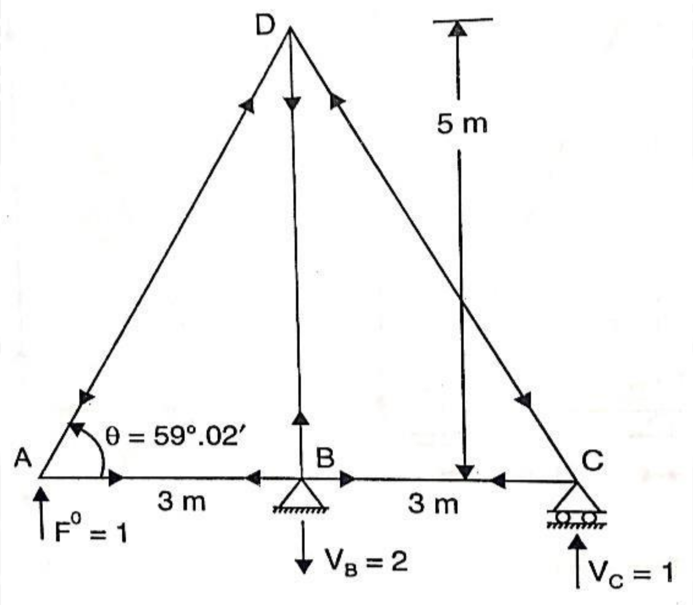
Thus, $B_X = [0 \quad -1.944 \quad 1.00 \quad 0 \quad 1.667]^T$

B_W is obtained by applying a unit force at the location of the redundant F_X

Equating to zero,moments about B,

VC x 3 = 1 x 3; VC = 1

From $\Sigma V = 0$; VB = 2



At joint A, FCD sin θ = 1 ; FAD = 1.667 (comp)

FAB = FAD cos θ ; FAB = 0.60 (tension)

At joint B, FAB = FBC = 0.60 (tension)

FBD = 2 (tension)

At joint C, FCD sin θ = 1 ; FCD = 1.667 (comp)

$FBC = FCD \cos\theta = 0.60$ (tallies)

Hence $B_W = [-1.166 \quad -1.166 \quad 0.60 \quad 0.60 \quad 2.00]^T$

$$= \begin{bmatrix} 0 & -1.166 \\ -1.944 & -1.166 \\ 1.00 & 0.60 \\ 0 & 0.60 \\ 1.667 & 2.00 \end{bmatrix}$$

$$\mathbf{F_X = B_X^T \cdot F \cdot B_X}$$

$$= [-1.166 \quad -1.166 \quad 0.6 \quad 0.6 \quad 2.0] \frac{1}{AE} \begin{bmatrix} 5.83 & & & & \\ & 5.83 & & & \\ & & 3.0 & & \\ & & & 3.0 & \\ & & & & 5.0 \end{bmatrix} \begin{bmatrix} -1.166 \\ -1.166 \\ 0.60 \\ 0.60 \\ 2.00 \end{bmatrix}$$

$$= [-6.798 \quad -6.798 \quad 1.8 \quad 1.8 \quad 10.0] \frac{1}{AE} \begin{bmatrix} -1.166 \\ -1.166 \\ 0.60 \\ 0.60 \\ 2.00 \end{bmatrix} = \frac{38.01}{AE}$$

$$\mathbf{F_W = B_X^T \cdot F \cdot B_W}$$

$$= [-1.166 \quad -1.166 \quad 0.6 \quad 0.6 \quad 2.0] \frac{1}{AE} \begin{bmatrix} 5.83 & & & & \\ & 5.83 & & & \\ & & 3.0 & & \\ & & & 3.0 & \\ & & & & 5.0 \end{bmatrix} \begin{bmatrix} 0 \\ -1.944 \\ 1.00 \\ 0 \\ 1.667 \end{bmatrix}$$

$$= [-6.798 \quad -6.798 \quad 1.80 \quad 1.80 \quad 10.00] \frac{1}{AE} \begin{bmatrix} 0 \\ -1.944 \\ 1.00 \\ 0 \\ 1.667 \end{bmatrix} = \frac{31.59}{AE}$$

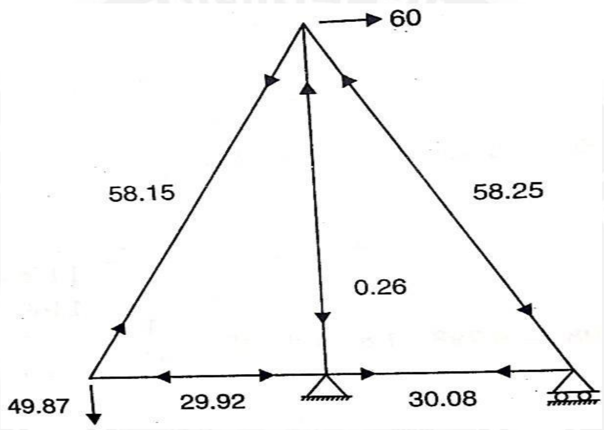
- **Displacement Matrix (X):**

$$= -\frac{AE}{38.01} \times \frac{31.59}{AE} \times 60 = -49.87 \text{ kN}$$

- **Final Moments (P):**

P = μ + F

$$\{P\} = \begin{bmatrix} 0 & -1.667 \\ -1.944 & -1.166 \\ 1.00 & 0.6 \\ 0 & 0.6 \\ 1.667 & 2.0 \end{bmatrix} \begin{Bmatrix} 60 \\ -49.87 \end{Bmatrix} = \begin{Bmatrix} 58.15 \\ -58.25 \\ 30.08 \\ -29.92 \\ -0.26 \end{Bmatrix}$$



5.1. STIFFNESS METHOD.

5.1.1. INTRODUCTION:

Stiffness method is the more popular younger brother of Flexibility method.Although the two methods are the opposites to each other,they are akin to each other in several respects.

Like in Flexibility method,this also involves generating element matrices,assembling them to get the system matrix and inverting the same to solve for nodal displacements,member displacements and eventually member forces.In the solution of the structure the important result is the member forces.Displacements are generally of lesser importance.

However in stiffness method we get to the displacements first and thence to member forces.

5.1.2. PROPERTIES OF THE STIFFNESS MATRIX:

It is a symmetric matrix and the sum of elements in any columns must be equal to zero.It is an unstable element therefore the the determinate is equal to zero.

The given indeterminate structure is first made kinematically determinate by introducing constraints at the nodes. The required number of constraints is equal to degrees of freedom at the nodes that is kinematic indeterminacy k. The kinematic ally determinate structure comprises of fixed ended members, hence, all nodal displacements are zero. These results in stress resultant discontinuities at these nodes under the action of applied loads or in other words the clamped joints are not in equilibrium.

Inorder to restore the equilibrium of stress resultants at the nodes the nodes are imparted suitable unknown displacements. The number of simultaneous equations representing joint equilibrium of forces is equal to kinematic indeterminacy k. Solution of these equations gives unknown nodal displacements. Using stiffness properties of members the member end forces are computed and hence the internal forces through out the structure.

Since nodal displacements are unknowns, the method is also called displacement method. Since equilibrium conditions are applied at the joints the method is also called equilibrium method. Since stiffness properties of members are used the method is also called stiffness method.

5.1.3. ELEMENT AND GLOBAL STIFFNESS MATRICES

Local co ordinates

In the analysis for convenience we fix the element coordinates coincident with the member axis called element (or) local coordinates (coordinates defined along the individual member axis)

Global co ordinates

It is normally necessary to define a coordinate system dealing with the entire structure is called system on global coordinates (Common coordinate system dealing with the entire structure)

Transformation matrix

The connectivity matrix which relates the internal forces Q and the external forces R is known as the force transformation matrix. Writing it in a matrix form,

$$\{Q\} = [b]\,\{R\}$$

Where; Q = member force matrix/vector,

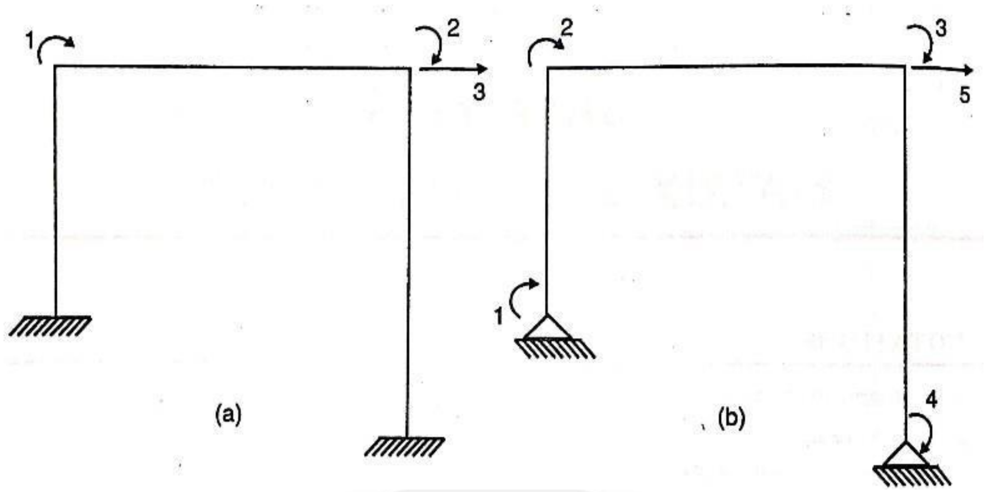
b = force transformation matrix

R = external force/load matrix/ vector

5.1.4. RESTRAINED STRUCTURE

In the Flexibility methods the difficulty of solving a structure increases with the static indeterminacy of a structure.

In stiffness methods the difficulty increases with its kinematic indeterminacy.Thus, structures with more constraints(supports,fixities etc.) are more easily solved then structures with more freedom.Strangely,the structure in fig,(a) is easier to tackle than the structure in fig,(b).Thus we have to get familiar with kinematic indeterminacies or freedoms.



5.1.5. PIN JOINTED FRAMES

In the case of pin jointed plane frames,we have to assign two degrees of freedom to each node.

The elements in trusses are very distinct.Each element shall have one degree of freedom for each end except the ends that are restrained.Normally the questions of forces not at co-ordinates will not arise in trusses.

An introduction to the stiffness method was given in the previous Page. The basic principles involved in the analysis of beams, trusses were discussed. The problems were solved with hand computation by the direct application of the basic principles.

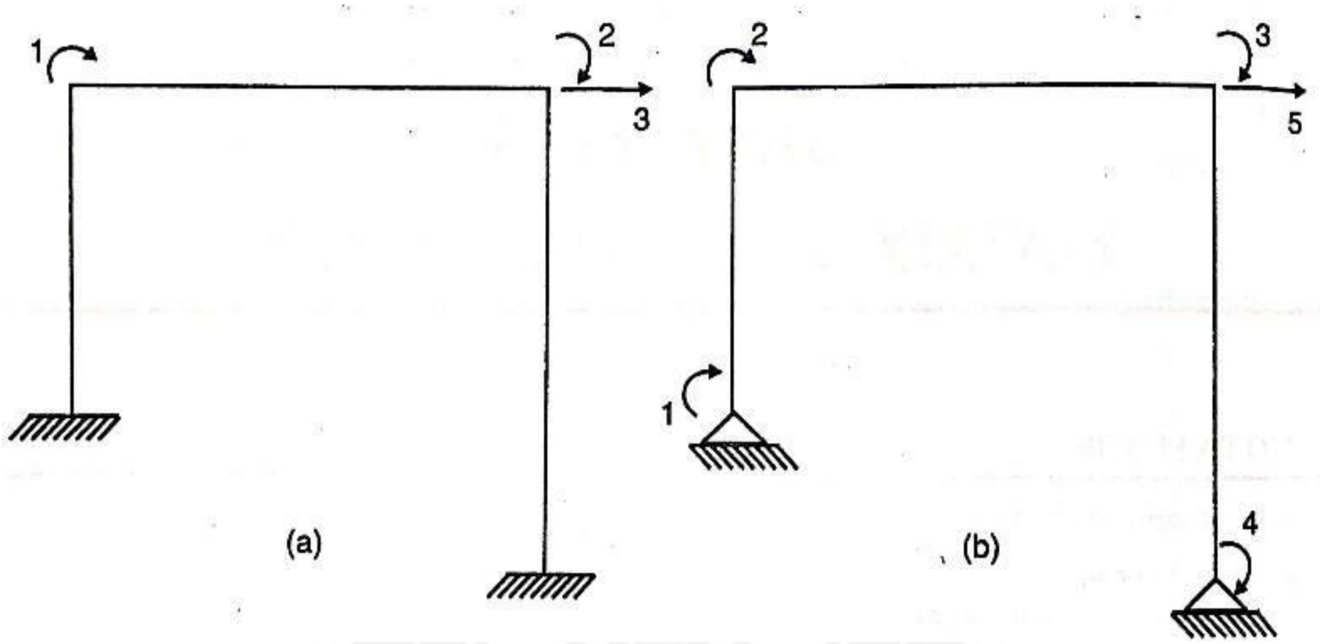
In this session a formal approach has been discussed which may be readily programmed on a computer. In this less on the direct stiffness method as applied to planar truss structure is discussed.

Planetrusses are made up of short thin members inter connected a thin gesto form triangulated patterns. Ahinge connection can only transmit forces from one member to another member but not the moment. For analysis purpose, the truss is loaded at the joints. Hence, a truss member is subjected to only axial forces and the forces remain constant along the length of the member. The forces in the member at its two ends must be of the same magnitude but actin the opposite directions for equilibrium.

5.2. FORMATION OF STIFFNESS MATRICES

The *n x n* stiffness matrix of a structure with a specified set of *n* co-ordinates is determined by applying one unit displacement at a time and determining the forces at each co-ordinate to sustain that displacement.

For example if we want to determine the 3 x 3 stiffness matrix for the structure in this fig.5.1,.



- Find the forces at 1,2 and 3 when displacements at 1 is unity and displacements at 2 and 3 are zero i.e., find P1,P2 and P3 when $\delta_1 = 1$ and $\delta_2 = \delta_3 = 0$.These 3 Forces constitute the first column of the stiffness matrix $[k_1]$.
- Find the 3 forces at 1,2 and 3 when $\delta_2 = 1$ and $\delta_1 = \delta_3 = 0$.These 3 Forces constitute the second column of the stiffness matrix $[k_1]$.
- Find forces at 1,2 and 3 when $\delta_3 = 1$and $\delta_1 = \delta_2 = 0$.These 3 forces make the third column of $[k_1]$.

Example 5.2.1

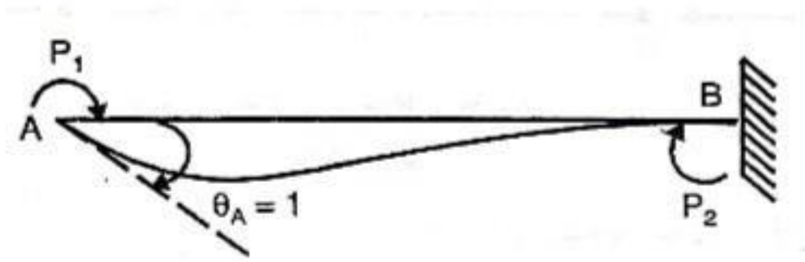
Determine the 2 x 2 stiffness matrix of the beam system shown in fig.5.7

$l = 3$, A = 1, E = 2, I = 3

Fig. 5.7

Solution:

Step 1.To find the first column of [k] apply a unit displacement at 1 only and restrained 2 from rotating



If $\theta = 1$,

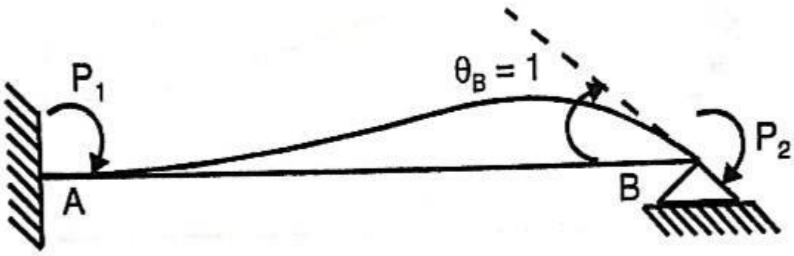
$P_1 = 4EI\theta_A / L = 4 \times 2 \times 3 / 3 = 8$

$P_2 = 2EI\theta_A / L = 2 \times 2 \times 3 / 3 = 4$

Hence;

$$\begin{Bmatrix} k_{11} \\ k_{21} \end{Bmatrix} = \begin{Bmatrix} 8 \\ 4 \end{Bmatrix}$$

Step 2.To get the second column of [k] apply a unit rotation at B and restrain A



$P_2 = 4Ei\theta_B / L = 4 \times 2 \times 3 / 3 = 8$

$P_1 = 2Ei\theta_B / L = 2 \times 2 \times 3 / 3 = 4$

Hence;

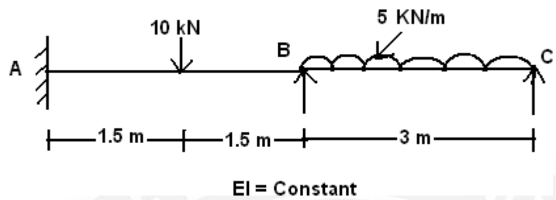
$$\begin{Bmatrix} k_{12} \\ k_{22} \end{Bmatrix} = \begin{Bmatrix} 4 \\ 8 \end{Bmatrix} \quad \text{and } [k] = \begin{bmatrix} 8 & 4 \\ 4 & 8 \end{bmatrix}$$

5.3. ANALYSIS OF CONTINUOUS BEAMS BY STIFFNESS METHOD

5.3.1.NUMERICAL PROBLEMS ON CONTINUOUS BEAMS;

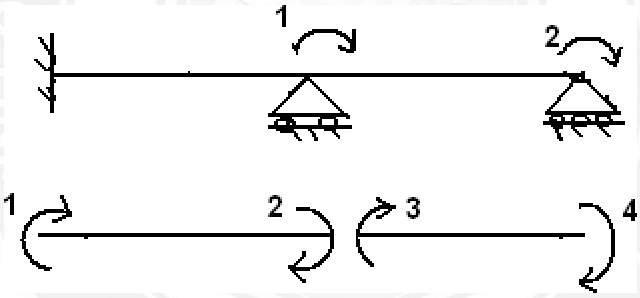
PROBLEM NO:01

Analysis the continuous beam by Stiffness Method.And find the final moments.



Solution:

- **Assigned co-ordinates:**



- **Fixed End Moments:**

  MFAB = -Wl/8 = -10X3/8 = -3.75 kNm

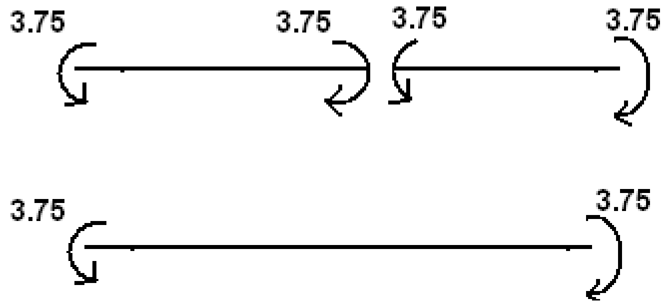
  MFBA = Wl/8 = 10X3/8 = 3.75 kNm

  MFBC = $-Wl^2/12 = -5x3^2/12 = -3.75$ kNm

  MFCB = $Wl^2/12 = 5x3^2/12 = 3.75$ kNm

- **Fixed End Moments Diagrams:**

$$W^O = \begin{bmatrix} 0 \\ 3.75 \end{bmatrix}$$



- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Stiffness Matrix (K):**

$$K = \frac{EI}{L}\begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$K = EI\begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix}$$

- **System Stiffness Matrix (J):**

$J = A^T . K . A$

$$= EI \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= EI \begin{bmatrix} 0.67 & 1.33 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$J = EI \begin{bmatrix} 2.6 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI} \begin{bmatrix} 0.431 & -0.217 \\ -0.217 & 0.861 \end{bmatrix}$$

- **Displacement Matrix (Δ):**

  $\Delta = J^{-1} \cdot W$

  $= J^{-1} [W^{*} - W^{0}]$

$$= \frac{1}{EI} \begin{bmatrix} 0.431 & -0.217 \\ -0.217 & 0.861 \end{bmatrix} \left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 3.75 \end{Bmatrix} \right]$$

$$\Delta = \frac{1}{EI} \begin{bmatrix} 0.814 \\ -3.228 \end{bmatrix}$$

- **Element Force (P):**

  $P = K \cdot A \cdot \Delta$

$$= \frac{EI}{EI} \begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{EI}{EI}\begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.814 \\ -3.228 \end{bmatrix}$$

$$= \begin{bmatrix} 0.67 & 0 \\ 1.33 & 0 \\ 1.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 0.814 \\ -3.228 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.545 \\ 1.082 \\ -1.081 \\ -3.75 \end{bmatrix}$$

- **Final Moments (M):**

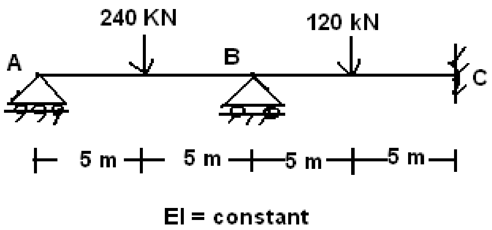
  M = μ + P

$$= \begin{bmatrix} -3.75 \\ 3.75 \\ -3.75 \\ 3.75 \end{bmatrix} + \begin{bmatrix} 0.545 \\ 1.082 \\ -1.081 \\ -3.75 \end{bmatrix}$$

$$M = \begin{bmatrix} -3.205 \\ 4.832 \\ -4.832 \\ 0 \end{bmatrix}$$

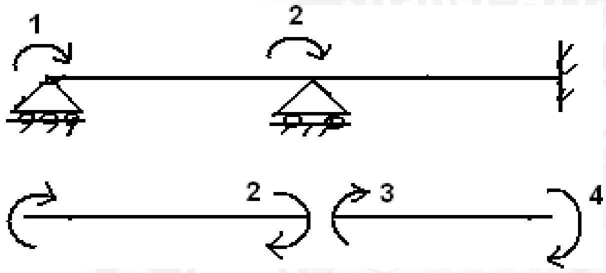
PROBLEM NO:02

Analysis the continuous beam by Stiffness Method.And find the final moments.



Solution:

- **Assigned co-ordinates:**



- **Fixed End Moments:**

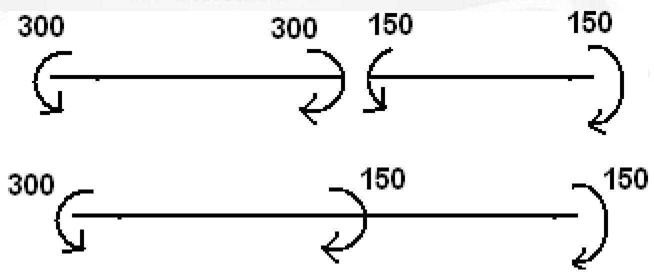
MFAB = -Wl/8 = -240X10/8 = -300 kNm

MFBA = Wl/8 = 240X10/8 = 300 kNm

MFBC = -Wl/8 = -120X10/8 = -150 kNm

MFCB = Wl/8 = 120X10/8 = 150 kNm

- **Fixed End Moments Diagrams:**



$$W^{O} = \begin{bmatrix} -300 \\ 150 \end{bmatrix}$$

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

- **Stiffness Matrix (K):**

$$K = \frac{EI}{L}\begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$K = EI\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0.2 & 0.4 \end{bmatrix}$$

- **System Stiffness Matrix (J):**

J = A^T . K . A

$$= EI\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0.2 & 0.4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= EI\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0.4 & 0.2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$J = EI\begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI}\begin{bmatrix} 2.86 & -0.71 \\ -0.71 & 1.43 \end{bmatrix}$$

- **Displacement Matrix (Δ):**

$\Delta = J^{-1} . W$

$= J^{-1} [W^* - W^0]$

$$= \frac{1}{EI}\begin{bmatrix} 2.86 & -0.71 \\ -0.71 & 1.43 \end{bmatrix}\left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -300 \\ 150 \end{Bmatrix}\right]$$

$$\Delta = \frac{1}{EI}\begin{bmatrix} 964.5 \\ -427.5 \end{bmatrix}$$

- **Element Force (P):**

$P = K . A . \Delta$

$$= \frac{EI}{EI}\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0.2 & 0.4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 964.5 \\ -427.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.4 \\ 0 & 0.4 \\ 0 & 0.2 \end{bmatrix}\begin{bmatrix} 964.5 \\ -427.5 \end{bmatrix}$$

$$P = \begin{bmatrix} 300 \\ 21.9 \\ -171 \\ -85.5 \end{bmatrix}$$

- **Final Moments (M):**

  M = μ + P

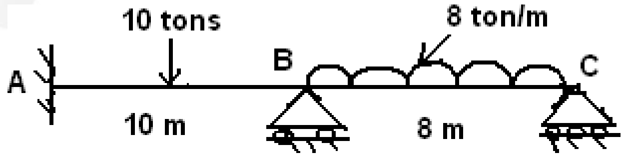
$$= \begin{bmatrix} -300 \\ 300 \\ -150 \\ 150 \end{bmatrix} + \begin{bmatrix} 300 \\ 21.9 \\ -171 \\ -85.5 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 \\ 321.9 \\ -321 \\ 64.5 \end{bmatrix}$$

PROBLEM NO:03

A two span continuous beam ABC is fixed at A and simply supported over the supports B and C. AB = 10 m and BC = 8 m. moment of inertia is constant throughout. A single central concentrated load of 10 tons acts on AB and a uniformly distributed load of 8 ton/m acts over BC. Analyse the beam by stiffness matrix method.

Solution:



- **Fixed End Moments:**

  MFAB = -Wl/8 = -10X10/8 = -12.5 kNm

  MFBA = Wl/8 = 10X10/8 = 12.5 kNm

  MFBC = $-Wl^2/12 = -8x8^2/12$ = -42.67 kNm

  MFCB = $Wl^2/12 = 8x8^2/12$ = 42.67 kNm

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Stiffness Matrix (K):**

$$K = \frac{EI}{L}\begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$K = EI\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix}$$

- **System Stiffness Matrix (J):**

  J = A^T . K . A

$$= EI\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI}\begin{bmatrix} 1.29 & -0.65 \\ -0.65 & 2.32 \end{bmatrix}$$

- **Displacement Matrix (Δ):**

$\Delta = J^{-1} . W$

$= J^{-1} [W^{*} - W^{0}]$

$$= \frac{1}{EI}\begin{bmatrix} 1.29 & -0.65 \\ -0.65 & 2.32 \end{bmatrix}\left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -30.17 \\ 42.67 \end{Bmatrix}\right]$$

$$\Delta = \frac{1}{EI}\begin{bmatrix} 66.65 \\ -118.60 \end{bmatrix}$$

- **Element Force (P):**

$P = K . A . \Delta$

$$= \frac{EI}{EI}\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 66.65 \\ -118.60 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 0 \\ 0.4 & 0 \\ 0.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix}\begin{bmatrix} 66.65 \\ -118.60 \end{bmatrix}$$

$$P = \begin{bmatrix} 13.33 \\ 26.66 \\ 3.68 \\ -42.64 \end{bmatrix}$$

- **Final Moments (M):**

  M = μ + P

$$= \begin{bmatrix} -12.5 \\ 12.5 \\ -42.67 \\ 42.67 \end{bmatrix} + \begin{bmatrix} 13.33 \\ 26.66 \\ 3.68 \\ -42.64 \end{bmatrix}$$

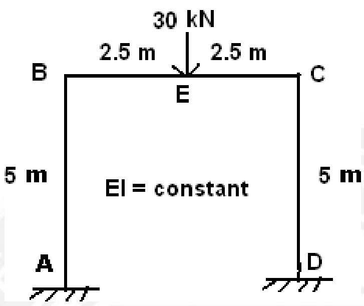
$$M = \begin{bmatrix} 0.83 \\ 39.16 \\ -39 \\ 0 \end{bmatrix}$$

5.4. ANALYSIS OF RIGID FRAMES BY STIFFNESS MATRICES METHOD

5.4.1.NUMERICAL PROBLEMS ON RIGID FRAMES;

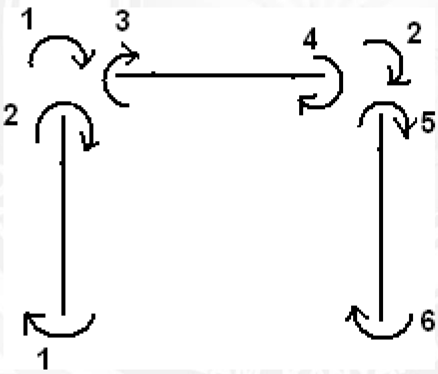
PROBLEM NO:01

Analysis the rigid portal frame ABCD shown in fig,by using Stiffness method.



Solution:

- **Assigned Co-Ordinates:**



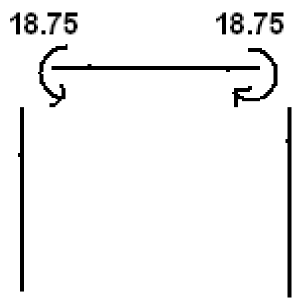
- **Fixed End Moments:**

  MFBC = -Wl/8 = -30x5/8 = -13.75 kNm

  MFCB = Wl/8 = 30x5/8 = 13.75 kNm

  MFAB = MFBA = MFCD = MFDC = 0

- **Fixed End Moments Diagrams:**



$$W^{O} = \begin{bmatrix} -18.75 \\ 18.75 \end{bmatrix}$$

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- **Stiffness Matrix(K):**

$$K = \frac{EI}{L}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix} = EI\begin{bmatrix} 0.8 & 0.4 & 0 & 0 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \end{bmatrix}$$

- **System Stiffness Matrix(J):**

J = A^{T} . K . A

$$= EI\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0.8 & 0.4 & 0 & 0 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$J = EI\begin{bmatrix} 1.6 & 0.4 \\ 0.4 & 1.6 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI}\begin{bmatrix} 0.67 & -0.17 \\ -0.17 & 0.67 \end{bmatrix}$$

- **Displacement Matrix(Δ):**

$$\boldsymbol{\Delta = J^{-1} \,.\, W}$$

$$\boldsymbol{= J^{-1}\,[W^{*} - W^{0}]}$$

$$= \frac{1}{EI}\begin{bmatrix} 0.67 & -0.17 \\ -0.17 & 0.67 \end{bmatrix}\left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -18.75 \\ 18.75 \end{Bmatrix}\right]$$

$$\Delta = \frac{1}{EI}\begin{bmatrix} 15.75 \\ -15.75 \end{bmatrix}$$

- **Element Force (P):**

$$\boldsymbol{P = K \,.\, A \,.\, \Delta}$$

$$= \frac{EI}{EI}\begin{bmatrix} 0.8 & 0.4 & 0 & 0 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 15.75 \\ -15.75 \end{bmatrix}$$

$$P = \begin{bmatrix} 6.3 \\ 12.6 \\ 6.3 \\ -6.3 \\ -12.6 \\ -6.3 \end{bmatrix}$$

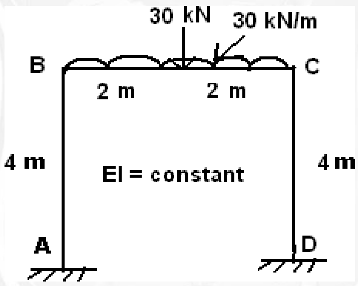
- **Final Moments (M):**

M = μ + P

$$= \begin{bmatrix} 0 \\ 0 \\ -18.75 \\ 18.75 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6.3 \\ 12.6 \\ 6.3 \\ -6.3 \\ -12.6 \\ -6.3 \end{bmatrix} = \begin{bmatrix} 6.3 \\ 12.6 \\ -12.5 \\ 12.5 \\ -12.6 \\ -6.3 \end{bmatrix}$$

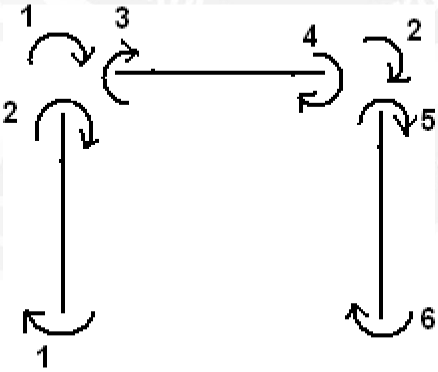
PROBLEM NO:02

Analysis the portal rigid frame ABCD using stiffness method and find the support moments.



Solution:

- **Assigned Co-Ordinates:**



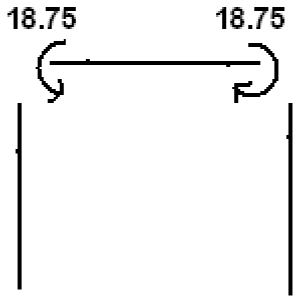
- **Fixed End Moments:**

  MFBC = -[Wl/8 + Wl^2/12] = -[30 x 4/8 + 30 x 4^2/12]= -55 kNm

  MFCB = [Wl/8 + Wl^2/12] = [30 x 4/8 + 30 x 4^2/12] = 55 kNm

  MFAB = MFBA = MFCD = MFDC = 0

- **Fixed End Moments Diagrams:**



$$W^{O} = \begin{bmatrix} -55 \\ 55 \end{bmatrix}$$

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- **Stiffness Matrix(K):**

$$K = \frac{EI}{L}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix} = EI\begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 1 \end{bmatrix}$$

- **System Stiffness Matrix(J):**

$J = A^{T} . K . A$

$$= EI \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$J = EI \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI} \begin{bmatrix} 0.53 & -0.13 \\ -0.13 & 0.53 \end{bmatrix}$$

- **Displacement Matrix(Δ):**

$$\Delta = J^{-1} . W$$

$$= J^{-1} [W^* - W^0]$$

$$= \frac{1}{EI} \begin{bmatrix} 0.53 & -0.13 \\ -0.13 & 0.53 \end{bmatrix} \left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -55 \\ 55 \end{Bmatrix} \right]$$

$$\Delta = \frac{1}{EI} \begin{bmatrix} 36.3 \\ -36.3 \end{bmatrix}$$

- **Element Force (P):**

$$P = K . A . \Delta$$

$$= \frac{EI}{EI} \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 36.3 \\ -36.3 \end{bmatrix}$$

$$P = \begin{bmatrix} 18.15 \\ 36.3 \\ 18.15 \\ -18.15 \\ -36.3 \\ -18.15 \end{bmatrix}$$

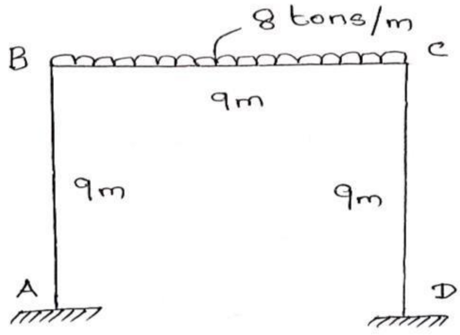
- **Final Moments (M):**

M = μ + P

$$= \begin{bmatrix} 0 \\ 0 \\ -55 \\ 55 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 18.15 \\ 36.3 \\ 18.15 \\ -18.15 \\ -36.3 \\ -18.15 \end{bmatrix} = \begin{bmatrix} 18.15 \\ 36.3 \\ -36.3 \\ 36.45 \\ -36.3 \\ -18.15 \end{bmatrix}$$

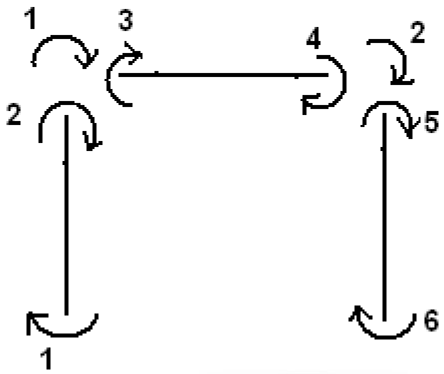
PROBLEM NO:03

A portal frame ABCD with supports A and D are fixed at same level carries a uniformly distributed load of 8 tons/m on the span AB. Span AB = BC = CD = 9 m. EI is constant throughout. Analyse the frame by stiffness matrix method.



Solution:

- **Assigned Co-Ordinates:**



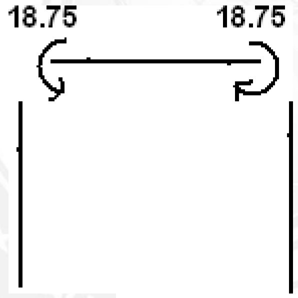
- **Fixed End Moments:**

MFBC = -Wl^2/12 = -8 x 9^2/12 = -54 ton.m

MFCB = Wl^2/12 = -8 x 9^2/12 = 54 ton.m

MFAB = MFBA = MFCD = MFDC = 0

- **Fixed End Moments Diagrams:**



$$W^O = \begin{bmatrix} -54 \\ 54 \end{bmatrix}$$

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1\,1\,0 & 0 & 0 \\ 0 & 0 & 0\,1 & 1 & 0 \end{bmatrix}$$

- **Stiffness Matrix(K):**

$$K = \frac{EI}{L}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix} = EI\begin{bmatrix} 0.44 & 0.22 & 0 & 0 & 0 & 0 \\ 0.22 & 0.44 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.44 & 0.22 & 0 & 0 \\ 0 & 0 & 0.22 & 0.44 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.44 & 0.22 \\ 0 & 0 & 0 & 0 & 0.22 & 0.44 \end{bmatrix}$$

- **System Stiffness Matrix(J):**

$J = A^T . K . A$

$$= EI\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0.44 & 0.22 & 0 & 0 & 0 & 0 \\ 0.22 & 0.44 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.44 & 0.22 & 0 & 0 \\ 0 & 0 & 0.22 & 0.44 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.44 & 0.22 \\ 0 & 0 & 0 & 0 & 0.22 & 0.44 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$J = EI\begin{bmatrix} 0.88 & 0.22 \\ 0.22 & 0.88 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI}\begin{bmatrix} 1.212 & -0.303 \\ -0.303 & 1.212 \end{bmatrix}$$

- **Displacement Matrix(Δ):**

$\Delta = J^{-1} . W$

$= J^{-1} [W^* - W^0]$

$$= \frac{1}{EI}\begin{bmatrix} 1.212 & -0.303 \\ -0.303 & 1.212 \end{bmatrix}\left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -54 \\ 54 \end{Bmatrix}\right]$$

$$\Delta = \frac{1}{EI}\begin{bmatrix} 81.81 \\ -81.81 \end{bmatrix}$$

- **Element Force (P):**

P = K . A . Δ

$$= \frac{EI}{EI}\begin{bmatrix} 0.44 & 0.22 & 0 & 0 & 0 & 0 \\ 0.22 & 0.44 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.44 & 0.22 & 0 & 0 \\ 0 & 0 & 0.22 & 0.44 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.44 & 0.22 \\ 0 & 0 & 0 & 0 & 0.22 & 0.44 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 81.81 \\ -81.81 \end{bmatrix}$$

$$= \begin{bmatrix} 0.22 & 0 \\ 0.44 & 0 \\ 0.44 & 0.22 \\ 0.22 & 0.44 \\ 0 & 0.44 \\ 0 & 0.22 \end{bmatrix}\begin{bmatrix} 81.81 \\ -81.81 \end{bmatrix}$$

$$P = \begin{bmatrix} 18 \\ 36 \\ 18 \\ -18 \\ -36 \\ -18 \end{bmatrix}$$

- **Final Moments (M):**

M = μ + P

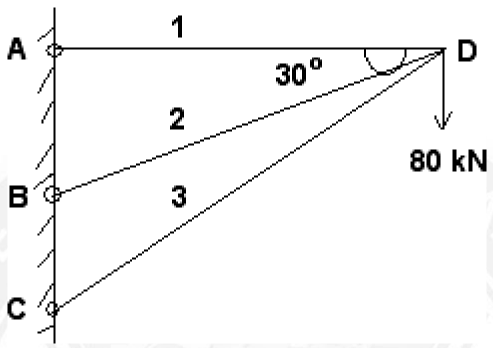
$$= \begin{bmatrix} 0 \\ 0 \\ -54 \\ 54 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 18 \\ 36 \\ 18 \\ -18 \\ -36 \\ -18 \end{bmatrix} = \begin{bmatrix} 18 \\ 36 \\ -36 \\ 36 \\ -36 \\ -18 \end{bmatrix}$$

5.5. ANALYSIS OF RIGID FRAMES BY STIFFNESS MATRICES METHOD

5.5.1. NUMERICAL PROBLEMS ON PIN JOINTED FRAMES;

PROBLEM NO:01

Using matrix stiffness method, analyze the truss for the member forces in the truss loaded as shown in figure. AE and L are tabulated below for all the three members.

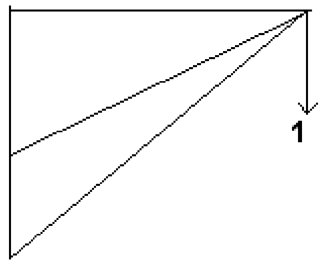


Member	AE	L
AD	400	400
BD	461.9	461.9
CD	800	800

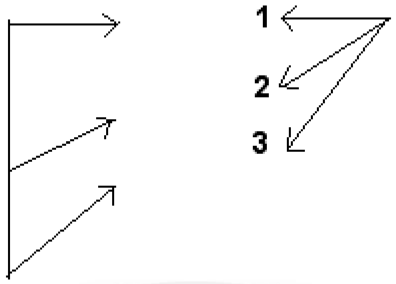
Solution:

- **Assigned Co-Ordinates:**

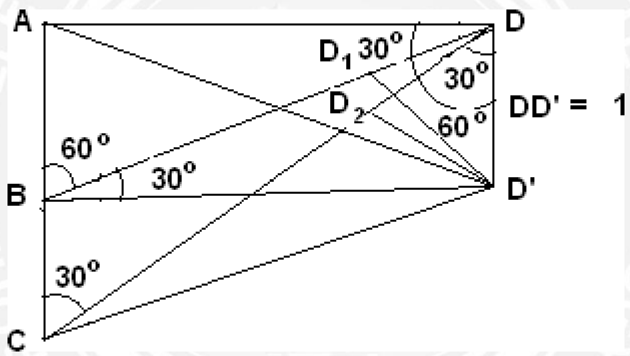
  Global Co-Ordinates



Local Co-Ordinates



- **Displacement Diagrams:**



- **Formation of [A] Matrix:**

  Apply unit displacement in DD'. Displacement along 1, AD = 0

  Displacement along 2 and 3,

  $DD_1 = \cos 60^o = 0.5$ and $DD_2 = \cos 30^o = 0.866$

$$A = \begin{bmatrix} 0 \\ -0.5 \\ -0.866 \end{bmatrix}$$

- **Stiffness Matrix [K]:**

$$K = \frac{AE}{L}\begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- **System Stiffness Matrix(J):**

J = A^T . K . A

$$= \begin{bmatrix} 0 & -0.5 & -0.866 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -0.5 \\ -0.866 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -0.5 & -0.866 \end{bmatrix} \begin{bmatrix} 0 \\ -0.5 \\ -0.866 \end{bmatrix}$$

$$J = 1$$

$$J^{-1} = 1$$

- **Displacement Matrix(Δ):**

Δ = J^{-1} . W

= 1 x 80 = 80 mm

- **Element Force (P):**

P = K . A . Δ

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -0.5 \\ -0.866 \end{bmatrix} 80$$

$$= \begin{bmatrix} 0 \\ -0.5 \\ -0.866 \end{bmatrix} 80$$

- **Final Force (P):**

$$= \begin{bmatrix} 0 \\ -40 \\ -69.28 \end{bmatrix}$$

