// EC3491 - Communication Systems //

Unit - 1 Amplitude modulation

Review of signals:

Signals:

→ A function of one or more independent Variables which contain some information is called signal.

Eg:

→ Electric Voltage or current, such as radio signal, TV signal, telephone signal.

→ pressure signal, sound signal - Non electric signals.

Systems:

→ A system is a set of elements or functional block that are connected together and produces an output in response to an input signal.

Eg:

→ An audio amplifier, attenuator

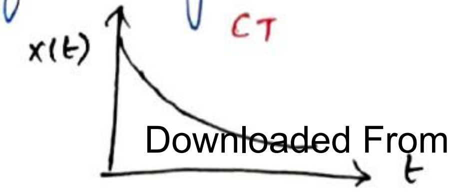
→ Any machine or engine are also systems.

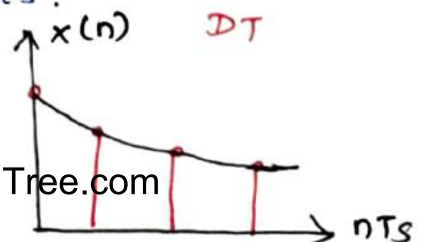
Classification of signals:

- → Continuous time (CT) and discrete time signals(DT
- → periodic and non periodic signals.
- → Even and odd signals
- → Energy and power signals
- → Deterministic and Random signals

Continuous Time and Discrete time signals

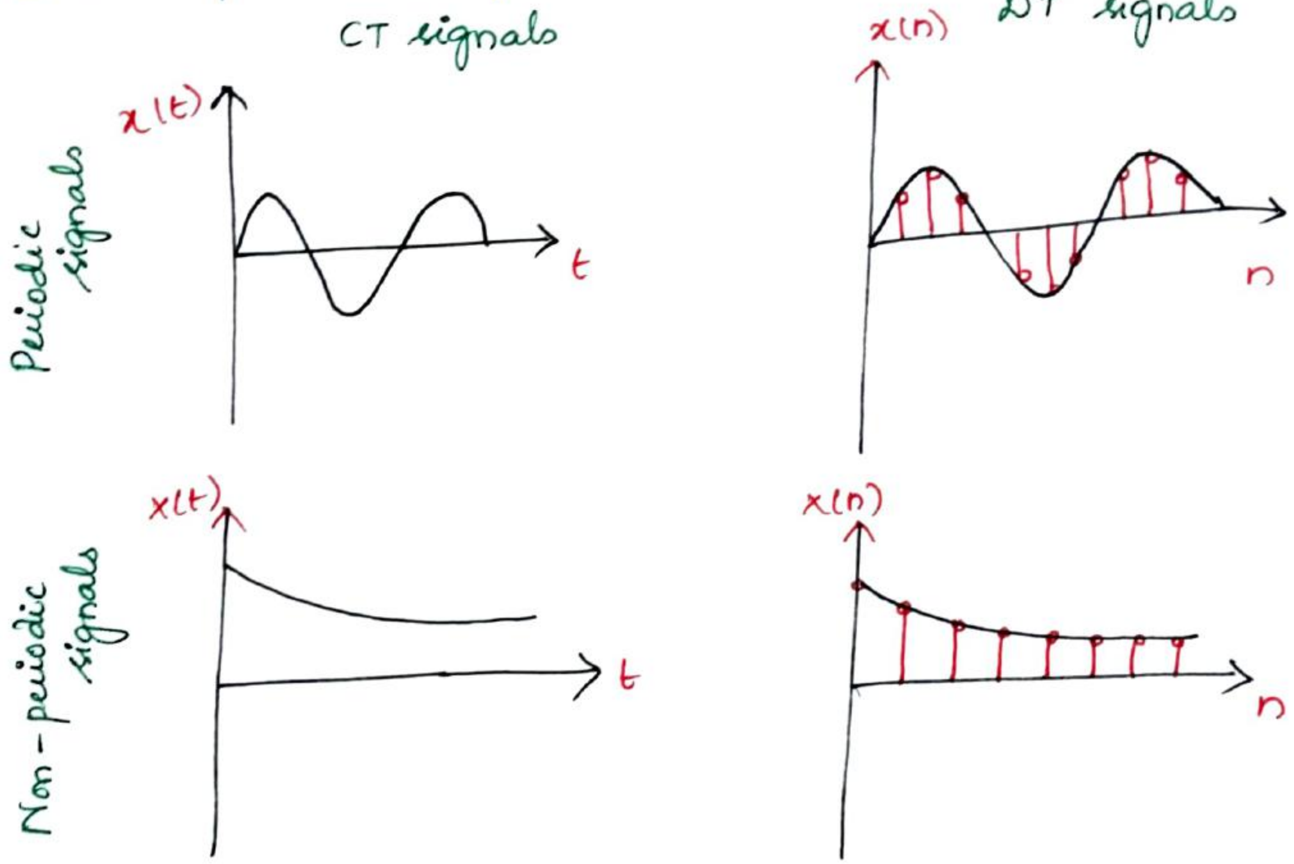
→ A CT signal is defined continuously with respect to time. A DT signal is defined only at Specific or regular time instants.





Periodic and Non - periodic signals

→ A signal is said to be periodic if it repeats at regular intervals. Non - periodic signals do not repeat at regular intervals.



Even and odd signals

→ A signal is said to be even signal if inversion of time axis does not change the amplitude.

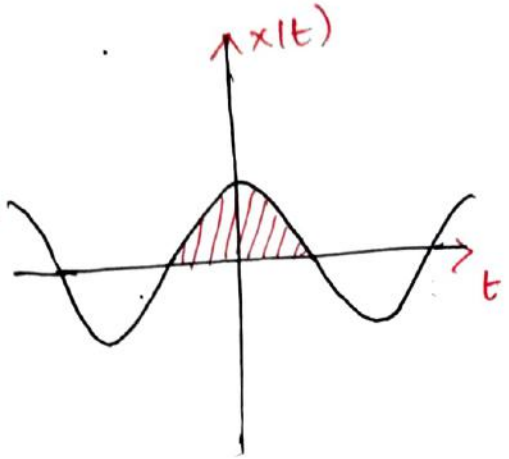
$$x(t) = x(-t)$$
$$x(n) = x(-n)$$

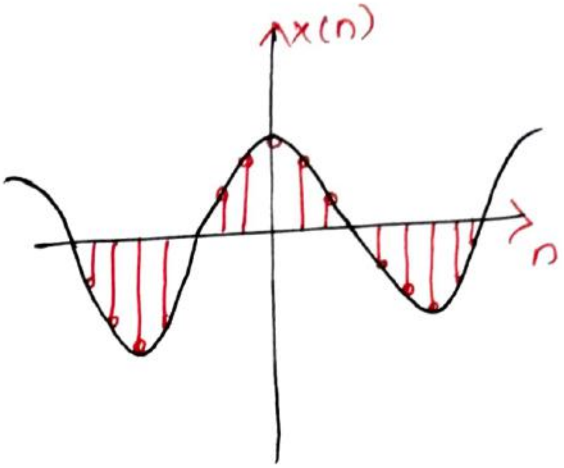
→ A signal is said to be odd signal if inversion of time axis also inverts amplitude of the signal.

$$x(t) = -x(-t)$$
$$x(n) = -x(-n)$$

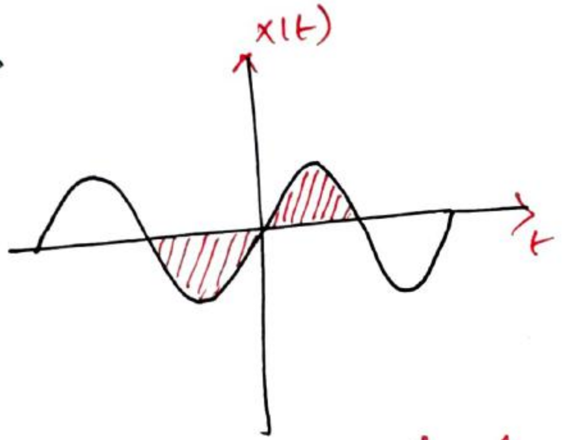
CT signals   DT signals

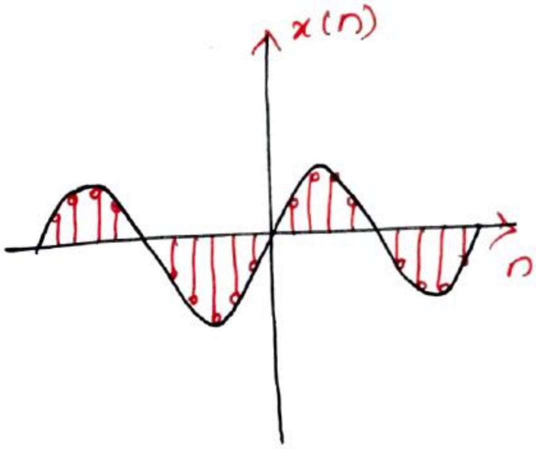
Even signal





odd signal





Energy and power signals

→ A signal is said to be an energy signal if its normalized energy is non zero and finite.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \rightarrow CT$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \rightarrow DT$$

→ A signal is said to be power signal if its normalized power is non zero and finite.

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \rightarrow CT$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \rightarrow DT$$

Deterministic and Random signals

→ A deterministic signal can be completely represented by mathematical equation at any time.

$$x(t) = \cos \omega t$$
$$x(n) = \cos 2\pi f n$$

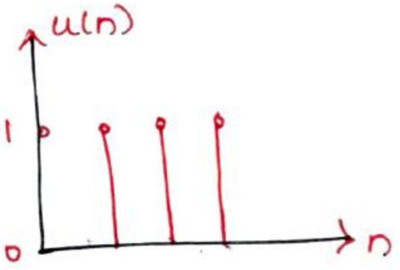
→ A signal which cannot be represented by any mathematical equation is called random signal.

Eg: → Noise generated in electronic components, transmission channels.

Unit step function

→ The unit step signals has amplitude of "1" for positive values of independent variables and "0" for negative values of independent variable.

$$u(t) = \begin{cases} 1 ; & \text{for } t \geq 0 \\ 0 ; & \text{for } t < 0 \end{cases} \qquad u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

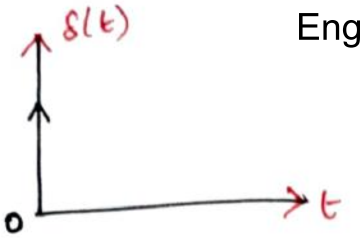


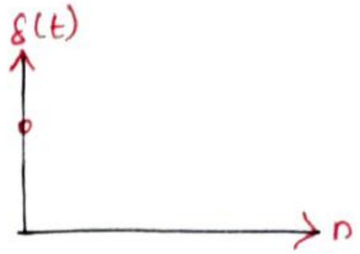
Unit Impulse or Delta function

→ Area under unit impulse approaches "1" width is zero. amplitude becomes

$$\int_{-\infty}^{\infty} \delta_t = 1 \text{ and } t \to 0 \quad \Big| \quad \delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

CT DT





Properties of Impulse function

Shifting property

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

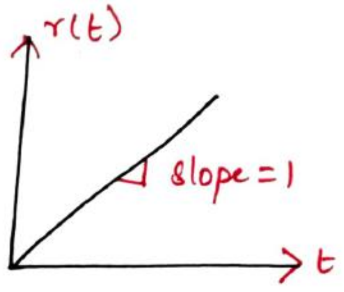
Replication property

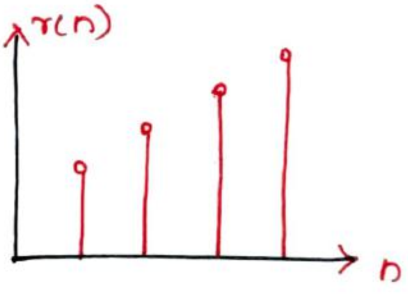
$$\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(t)$$

Unit Ramp function

→ Linearly growing function for positive values of independent variable.

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} \qquad r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

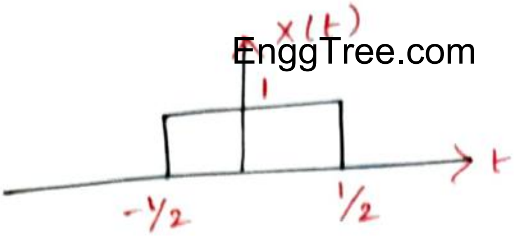




Rectangular pulse

→ Rectangular pulse centered at $t = 0$ represents the double sided frequency response of Low pass filter

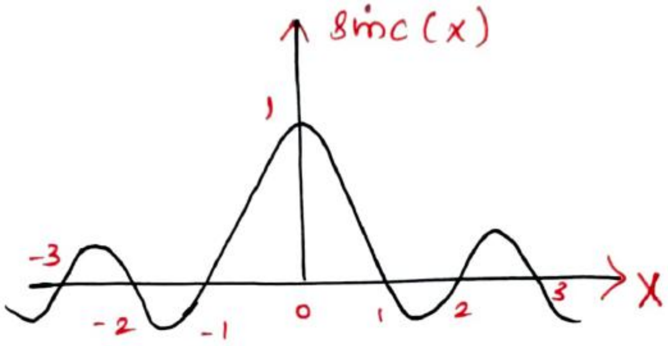
$$\text{rect}(t) = 1 \quad \text{for } -\tfrac{1}{2} \leq t < \tfrac{1}{2}$$
$$0 \quad \text{elsewhere}$$



Sinc pulse

→ sinc pulse or sinc function is very important mathematical model. it is used extensively in mathematical analysis of communication systems.

$$\text{Sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$



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Review of systems

→ A system is a set of elements or functional blocks that are connected together and produce an output in response to an input signal.

Static and Dynamic systems

→ The continuous time system is said to be static or if its output depends upon the present input only.

$$v(t) = Ri(t)$$

→ Consider Voltage drop in the resistance depends up on current through it.

→ Thus output v(t) depends upon present current i(t).

→ Thus the voltage across the capacitor depends upon present as well as past current values.

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

Time Invariant and Time Variant systems

→ A continuous time system is time invariant if the time shift in the input signal results in corresponding time shift in the output.

$$f[x(t-t_1)] = y(t-t_1)$$

$$f[x(n-k)] = y(n-k)$$

Linear and Non-Linear systems

→ A system is said to be linear if it satisfies the superposition principle.

$$y_1(t) = f[x_1(t)]$$

$$y_2(t) = f[x_2(t)]$$

$$f[a_1 x_1(t) + a_2 x_2(t)] = a_1 y_1(t) + a_2 y_2(t)$$

a_1, a_2 are the arbitary constants.

Causal and Non-Causal systems

→ The system is said to be causal if its output at any time depends upon present and past input only.

$$y(t_0) = f[x(t)\ ;\ t \le t_0]$$

→ The system is non-causal if its output depends upon future inputs.

$$x(n+1),\ x(n+2),\ x(n+3) \ldots.$$

Stable and unstable systems

→ when every bounded input produces bounded output, then the system is called Bounded Input Bounded Output (BIBO) stable.

$$\left.\begin{array}{ll} |x(t)| \leq M_x \leq \infty & \rightarrow CT \\ |x(n)| \leq M_x \leq \infty & \rightarrow DT \end{array}\right\} \text{input}$$

$$\left.\begin{array}{ll} |y(t)| \leq M_y \leq \infty & \rightarrow CT \\ |y(n)| \leq M_y \leq \infty & \rightarrow DT \end{array}\right\} \text{output}$$

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LTI systems and Impulse Response

→ The system which is linear and time invariant is called Linear Time Invariant (LTI) system.

→ when input to the system is an impulse function, its output is called impulse response.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

→ LTI system is to be causal $h(t)=0$

→ if LTI system is stable $\int_{-\infty}^{\infty} |h(t)dt < \infty$

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Time and Frequency Domain Representation of signals

→ signals are analysed in time domain and frequency domain. some features are found in time domain, whereas some features are found in frequency domain.

→ signals are often converted from time domain to frequency domain to obtain more information about the signals.

→ fourier series gives time domain representation of the signal, where as fourier transform and Laplace transform gives frequency domain representation of signals.

Fourier series:

→ Fourier series is used to represent any periodic signal in terms of complex exponentials.

Fourier transform:

→ fourier transform is used to represent any non-periodic as well as periodic signal in terms of complex exponential functions.

Fourier series Representation

$$x(t) = \sum_{k=-\infty}^{\infty} x(t)\, e^{jk\omega_0 t}$$

$$x(k) = \frac{1}{T} \int_{T} e^{-jk\omega_0 t}$$

Properties:

- → Time shift
- → Frequency shift
- → Time differentiation
- → Convolution in time
- → persevals theorem

Fourier transform representation

$$FT \rightarrow \quad x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \quad (\text{or}) \quad x(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$IFT \rightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega)\, e^{j\omega t}\, d\omega = \int_{-\infty}^{\infty} x(f) \cdot e^{j2\pi ft}\, df$$

Properties

- → Time shift
- → frequency shift
- → Time scaling
- → Frequency differentiation
- → Time differentiation
- → convolution
- → Modulation
- → Duality
- → parseval's theorem

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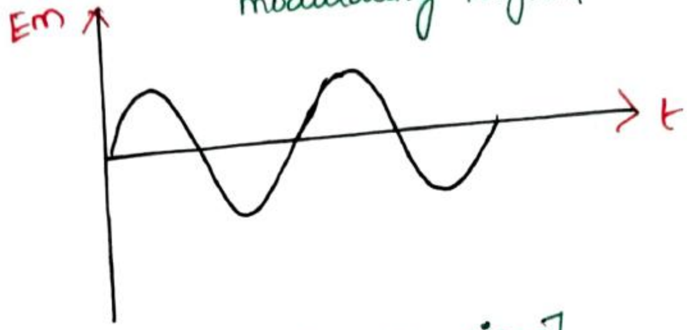
Amplitude Modulation

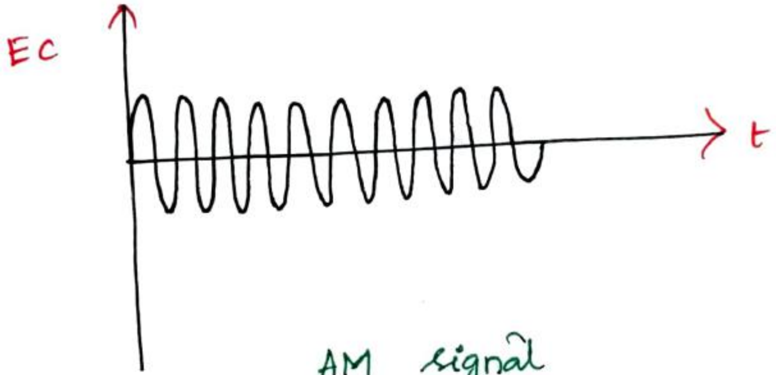
→ The amplitude of a carrier signal is varied according to the amplitude of modulating signal.

AM Envelope and Equation

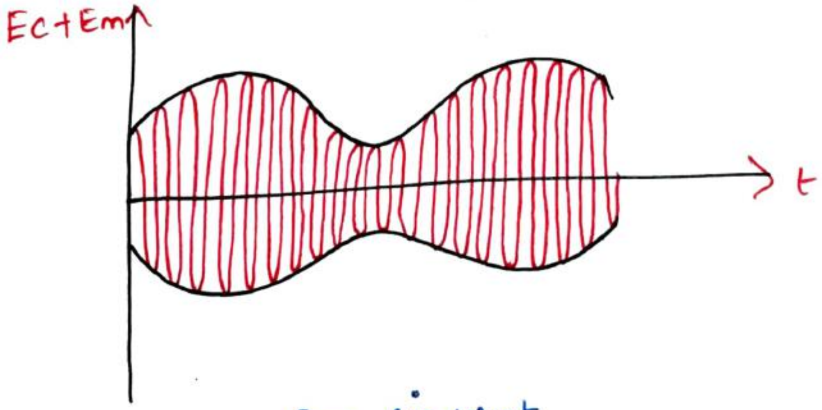
modulating signal



Carrier signal



AM signal



$e_m = E_m \sin \omega_m t$

$e_c = E_c \sin \omega_c t$

E_m → maximum amplitude of modulating signal

~~Em~~ E_c → maximum amplitude of carrier signal

ω_m → frequency of modulating signal

ω_c → frequency of carrier signal

$$E_{AM} = E_c + e_m$$

$$= E_c + E_m \sin w_m t$$

$$e_{AM} = E_{AM} \sin \theta$$

$$= E_{AM} \sin w_c t$$

$$\boxed{e_{AM} = (E_c + E_m \sin w_m t) \sin w_c t}$$

Modulation Index and Percentage modulation

→ it is defined as the ratio of maximum amplitude of modulating signal to maximum amplitude of carrier signal is called modulation Index.

$$m = \frac{E_m}{E_c}$$

$$E_m = \frac{E_{max} - E_{min}}{2} \quad ; \quad E_c = \frac{E_{max} + E_{min}}{2}$$

$$m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \times 100\%.$$

Frequency Spectrum and Bandwidth

$f_{USB} = f_c + f_m$

$f_{LSB} = f_c - f_m$

f_m → modulating signal frequency

f_{LSB} → Lower sideband frequency

f_{USB} → upper sideband frequency

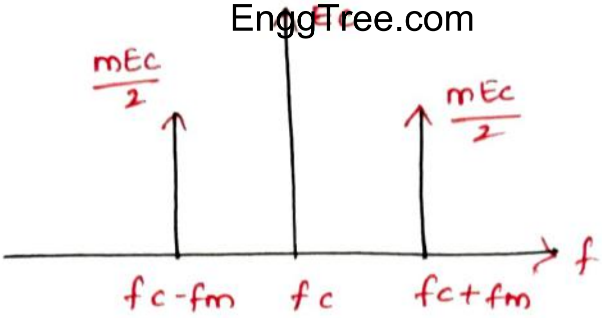
$$e_{AM} = (E_c + E_m \sin w_m t) \sin w_c t$$

$$= E_c \sin w_c t + E_m \sin w_m t \cdot \sin w_c t$$

$$= E_c [1 + m \sin w_m t) \sin w_c t$$

$$= E_c \sin w_c t + m E_c \sin w_m t \cdot \sin w_c t$$

$$e_{AM} = \underbrace{E_c \sin w_c t}_{\text{Carrier}} + \underbrace{\frac{mE_c}{2} \cos (w_c - w_m) t}_{\text{LSB}} - \underbrace{\frac{mE_c}{2} \cos (w_c + w_m) t}_{\text{USB}}$$



→ This contains full carrier and both the sidebands hence it is also called Double sideband full carrier (DSBFC).

$$BW = f_{USB} - f_{LSB}$$
$$= (fc+fm) - (fc-fm)$$
$$BW = 2fm$$

→ As we know that Bandwidth of the signal is to be the difference between highest and lowest frequencies.

AM power Distribution

$$P_{Total} = Pc + P_{USB} + P_{LSB}$$
$$= \frac{Ec^2}{R} + \frac{E_{LSB}^2}{R} + \frac{E_{USB}^2}{R}$$
$$Pc = \frac{Ec^2}{R} = \frac{(Ec/\sqrt{2})^2}{R} = \frac{Ec^2}{2R}$$
$$E_{LSB} = E_{USB} = \frac{mEc/2}{\sqrt{2}}$$
$$P_{LSB} = \left(\frac{mEc/2}{\sqrt{2}}\right)^2 / R = \frac{m^2 Ec^2}{8R}$$
$$P_{Total} = \frac{Ec^2}{2R} + \frac{m^2 Ec^2}{8R} + \frac{m^2 Ec^2}{8R}$$
$$= \frac{Ec^2}{2R}\left[1 + \frac{m^2}{4} + \frac{m^2}{4}\right] = \frac{Ec^2}{2R}\left[1 + \frac{m^2}{2}\right]$$
$$\boxed{P_{Total} = Pc\left[1 + \frac{m^2}{2}\right]}$$
$$\frac{P_{Total}}{Pc} = 1 + \frac{m^2}{2} \; ; \; \frac{m^2}{2} = \frac{P_{Total}}{Pc} - 1$$
$$\boxed{m = \sqrt{2\left(\frac{P_{Total}}{Pc} - 1\right)}}$$

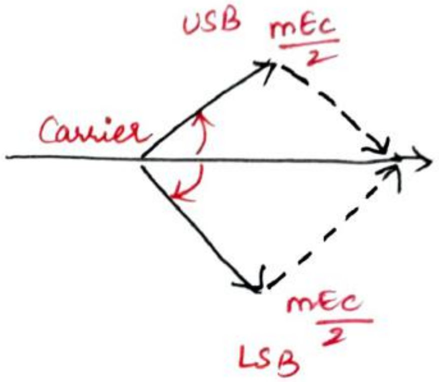
AM current Calculations

$$P = I^2 R$$

$$P_{Total} = I^2_{total} \cdot R$$

$$P_c = I_c^2 R$$

Phasor Diagram.



we know that

$$P_{Total} = P_c \left(1+\frac{m^2}{2}\right)$$

$$I^2_{total} R = I_c^2 \cdot R \left(1+\frac{m^2}{2}\right)$$

$$I^2_{total} \cancel{R} = I_c^2 \cancel{R} \left(1+\frac{m^2}{2}\right)$$

$$I^2_{total} = I_c^2 \left(1+\frac{m^2}{2}\right)$$

$$I_{total} = I_c \sqrt{1+\frac{m^2}{2}}$$

$$m = \sqrt{2\left(\frac{I^2_{total}}{I_c^2} - 1\right)}$$

Transmission Efficiency

→ it is defined as the ratio of power contained in both sidebands to total transmitted power.

$$\eta = \frac{P_{USB} + P_{LSB}}{P_t}$$

$$\eta = \frac{m^2 P_c}{4} + \frac{m^2 P_c}{4} \Big/ P_c\left(1+\frac{m^2}{2}\right) = \frac{m^2}{m^2+2}$$

Advantages:

- → Inexpensive
- → Can travel for long distance
- → Very simple modulators and demodulators

Disadvantages:

- → wastage in Bandwidth
- → poor performance of noise presence
- → Inefficient use of transmitter power

Applications:

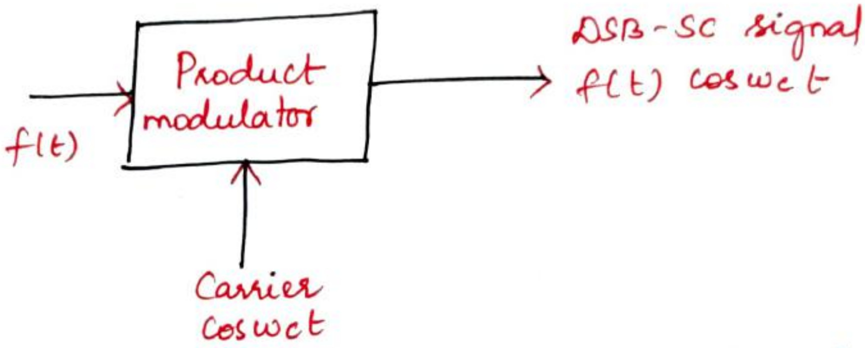
- → Two way mobile radio communications
- → used for commercial Broadcasting of both audio and video signals.

Double side Band suppressed carrier system (DSB-SC)

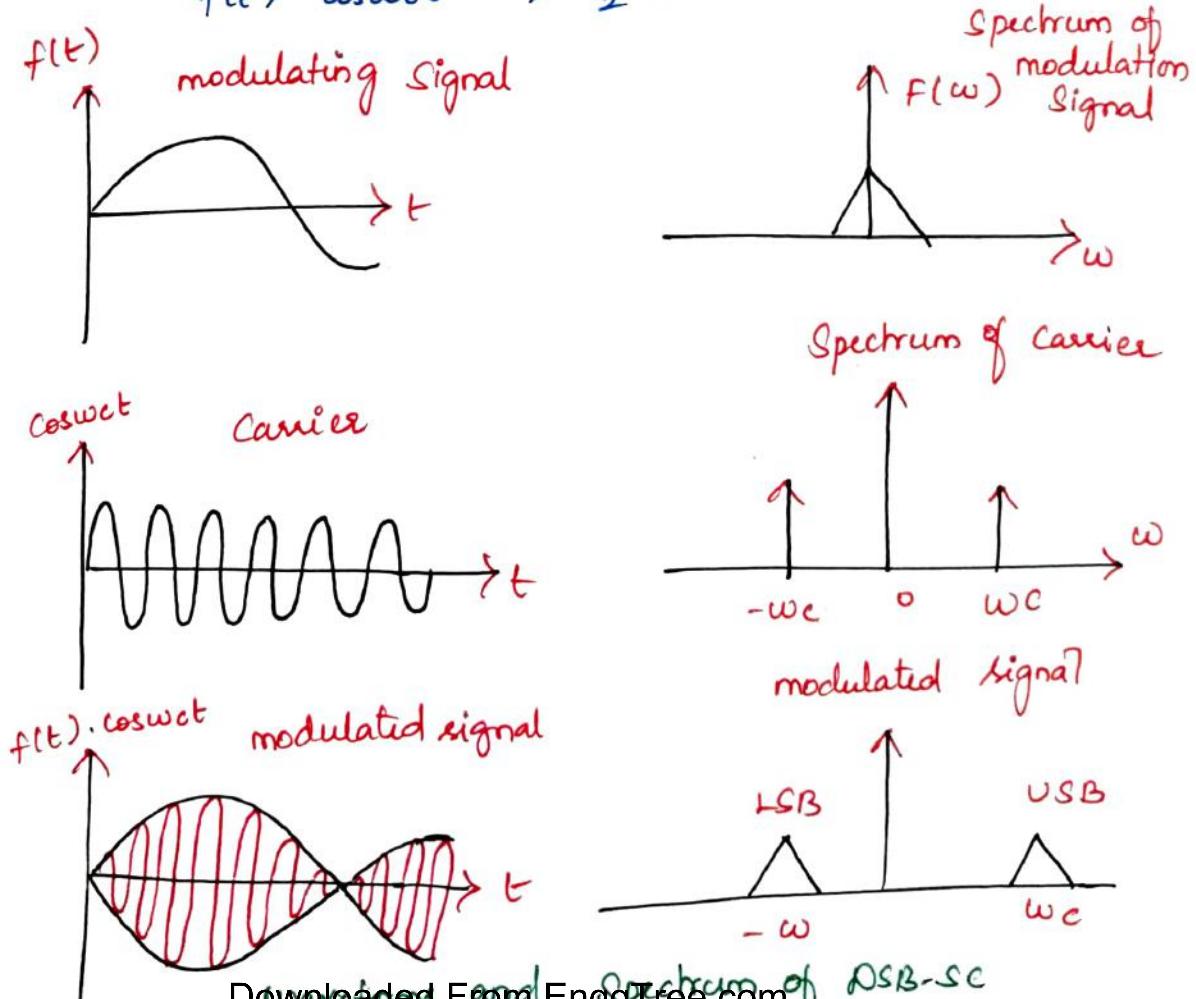
→ we know that in AM, the carrier and two sidebands are transmitted. The carrier does not contain any information.

→ Hence most of the energy in the transmitted signal is unused. The suppressed carrier system do not transmit carrier. only one carrier (sideband) are transmitted. This saves lot of power.

DSB-SC Modulator



$$f(t)\cdot\cos\omega_c t \longrightarrow \frac{1}{2}\,F(\omega+\omega_c)+F(\omega-\omega_c)]$$



Waveform and Spectrum of DSB-SC

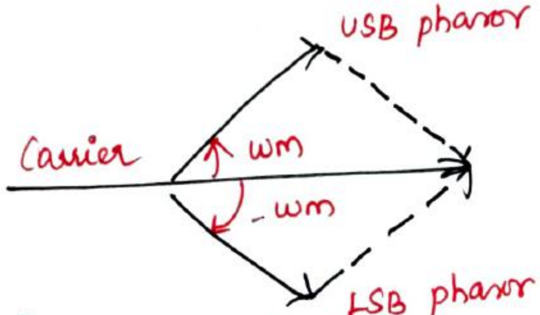
$$e_{AM} = \underbrace{E_c \sin \omega_c t}_{\text{Carrier}} + \underbrace{\frac{mE_c}{2} \cos(\omega_c - \omega_m)t}_{\text{LSB}} - \underbrace{\frac{mE_c}{2} \cos(\omega_c + \omega_m)t}_{\text{USB}}$$

Phasor Representation

$$e(t) = f(t) \cdot E_c \cos \omega_c t$$

$$= E_m E_c \cos \omega_m t \cdot \cos \omega_c t$$

$$= \frac{1}{2} E_m E_c \left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t \right]$$



Effect of Frequency and phase Error

→ The carrier is generated locally at the receiver and it is used for detection. if there is frequency or phase error in the locally generated carrier, then the output is distorted.

$$f_o(t) = \frac{1}{2} f(t) \cdot \cos(\Delta\omega t + \phi)$$

i) $\Delta\omega = 0$; $\phi = 0$ (frequency and phase error)

$$f_o(t) = \frac{1}{2} f(t)$$

ii) $\Delta\omega = 0$; $\phi \neq 0$ (only phase error)

$$f_o(t) = \frac{1}{2} f(t) \cos \Delta\omega$$

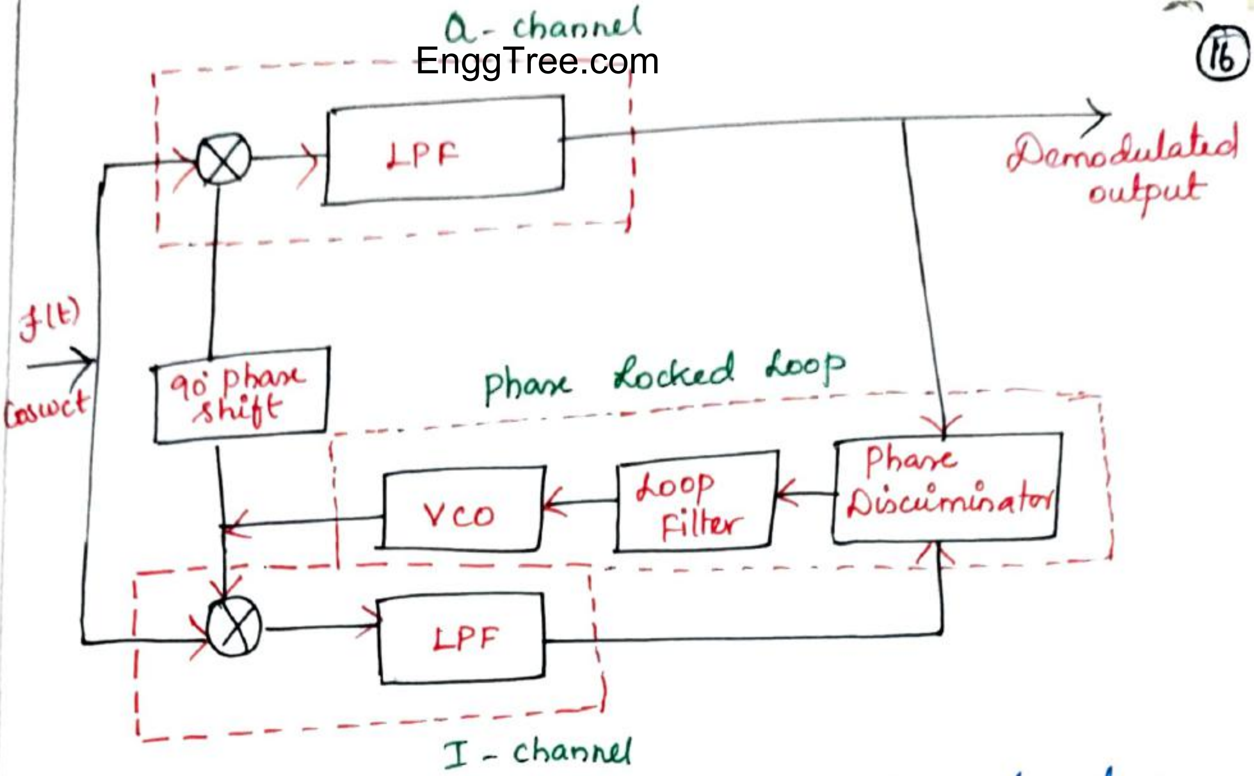
iii) $\Delta\omega \neq 0$; $\phi = 0$ (only frequency error)

$$f_o(t) = \frac{1}{2} f(t) \cos \Delta\omega t$$

Demodulation of DSB-SC signal using costas Loop

→ VCO outputs generates the carrier, which in phase with the transmitter carrier. The carrier is given to Coherent detector (I - channel)

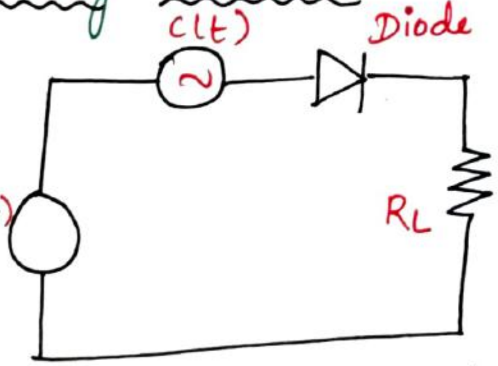
→ Quadrature carrier generated by 90° phase shift and given to another Coherent detector (Q - channel)

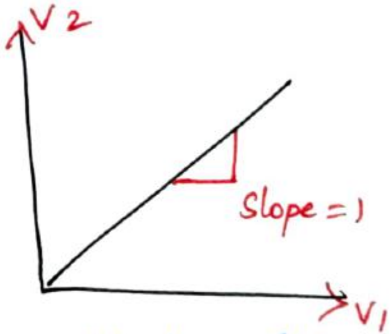


→ The output of I - channel and Q - channel are given to the phase discriminator of phase locked loop. phase difference $(\phi - \theta)$ indicates the difference between VCO frequency and input carrier frequency.

→ phase locked loop, tries to adjust its frequency so as to minimise the difference $\phi - \theta$. Hence variations due to $\cos(\phi - \theta)$ can be filtered out.

Switching modulator





→ Diode is assumed to act as a ideal switch that is, it presents zero impedance when it is forward biased and infinite impedance when it is reversed biased.

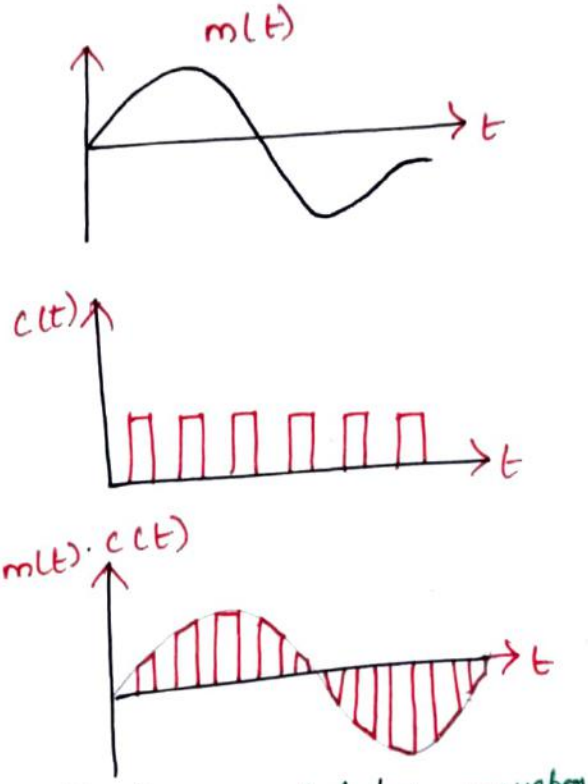
→ Diode is controlled by carrier wave c(t).

→ c(t) > 0 ; Diode (f(w)

c(t) < 0 ; Diode (R.B)

$$E_2(t) = \begin{cases} e(t) & , c(t) > 0 \\ 0 & , c(t) < 0 \end{cases}$$

$E_2(t) \rightarrow$ output Voltage.



switching modulator waveform

power and Efficiency of DSB-SC

$$e_{DSB-SC}(t) = e_m(t)\cdot e_c(t) = e_m(t)\cdot E_c \cos\omega_c t$$

$$= E_m \cos\omega_m t \cdot E_c \cos\omega_c t$$

$$e_{DSB-SC}(t) = \frac{E_m E_c}{2}\cos(\omega_c+\omega_m)t + \frac{E_m E_c}{2}\cos(\omega_c-\omega_m)t$$

power in one side Band

$$P_{LSB} = \frac{E_c^2 \cdot E_m^2}{4}$$

useful power $= \dfrac{E_c^2 E_m^2}{8}$

Hence power efficiency $\eta = \dfrac{E_c^2 E_m^2/8}{E_c^2 E_m^2/4} = \dfrac{1}{2}$ (or) $= 50\%$.

Advantages:

→ DSB-SC more efficient compared to DSB-FC

→ Better signal to noise ratio.

Disadvantage

→ Eventhough the carrier is suppressed the Bandwidth of DSB-SC remains same as DSB-FC.

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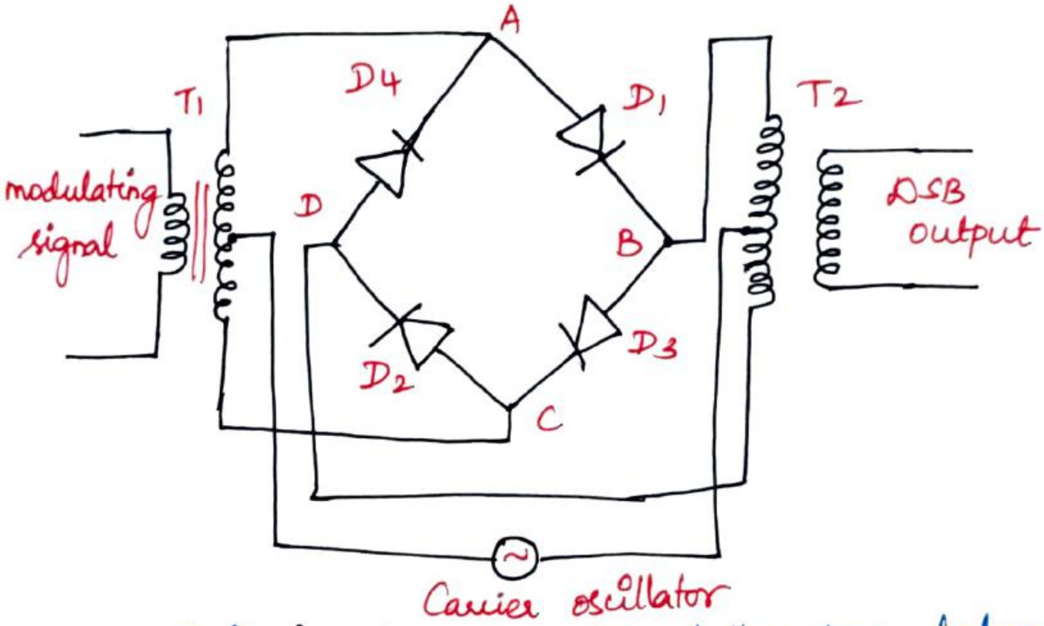
Balanced modulator or Ring modulator using diodes



→ it is also called ~~the~~ lattice type balanced modulator. The modulator consist of input Transformer T_1, output transformer T_2 and four diodes.

→ The modulating signal is applied to the input of transformer T_1, carrier signal is applied to center tap of two transformers T_1 and T_2.

→ The DSB output is collected at the secondary of transformer T_2.

→ when positive half cycle is applied

D_1, D_2 (forward bias)

D_3, D_4 (Reverse bias)

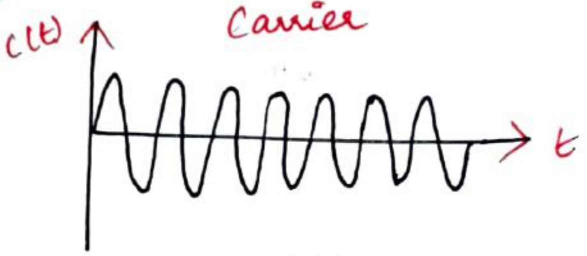
→ The current in the upper part of the winding produces a magnetic field that is equal and opposite to the magnetic field produced by the current in the lower half of the secondary.

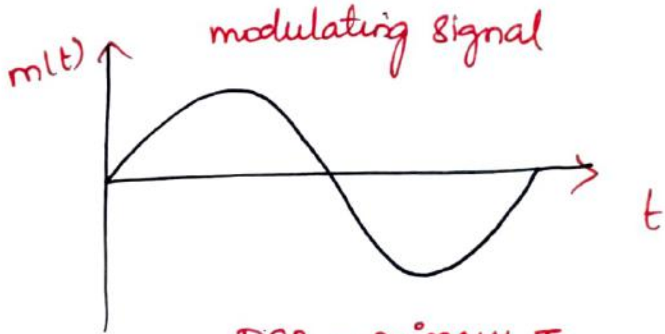
→ As magnetic fields are equal and opposite, they cancel each other, producing no output at the secondary of T_2. Carrier is suppressed.

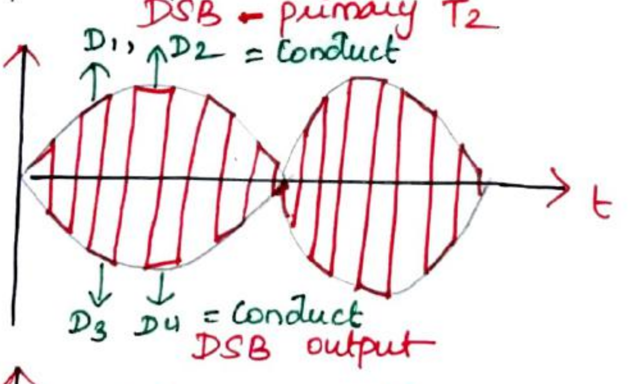
→ when negative half cycle is applied

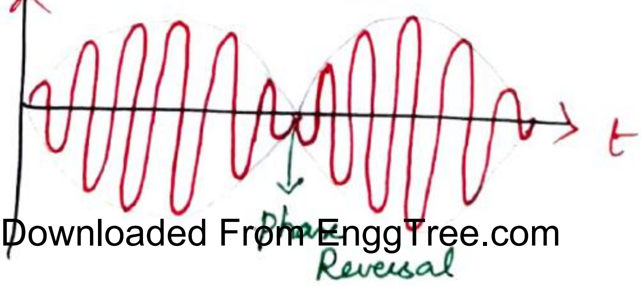
D_1, D_2 (Reverse bias)
D_3, D_4 (forward bias)

→ This inverts the polarity of modulating signal when it is applied to primary of T_2. Thus the carrier is totally suppressed.

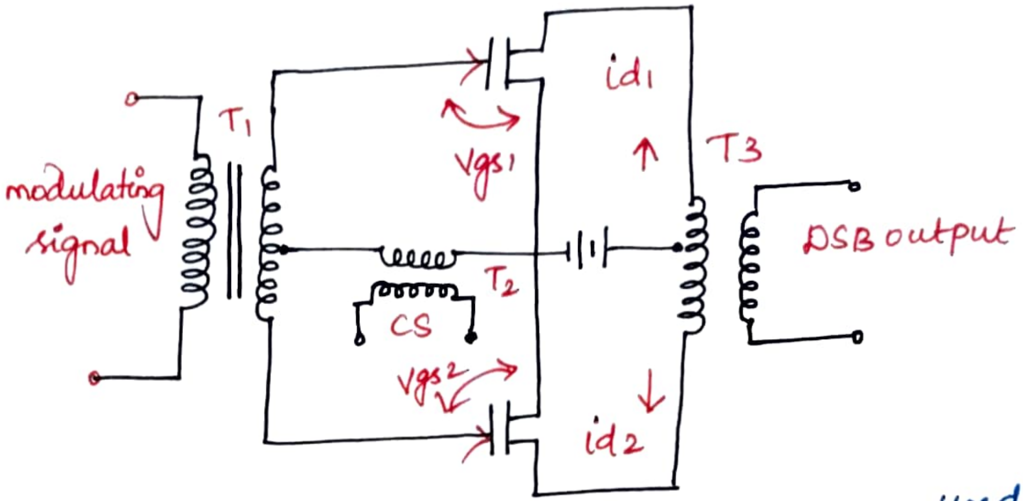








Balanced Modulator using FETs



→ Three transformers are T1, T2, T3 are used to the Balanced modulator. The carrier signal is applied to the center taps of input transformer T1 and output T3 through transformer T2.

→ The modulating signal is applied to the input primary Transformer transformer T1. Carrier signal is applied to primary Transformer T2. The modulating voltage appears 180° out of phase at gates. → since these are the opposite ends of the center tapped transformer. These opposite and equal currents cancel each other.). at the primary of the output transformer There is no output in T3. Carrier

→ Hence There is no output in T3. Carrier is suppressed.

primary current

$$i_p = i_{d1} - i_{d2} = aV_{gs1} + bV_{gs1}^2 - aV_{gs2} - bV_{gs}^2$$

$$= a(V_{gs1} - V_{gs2}) + b(V_{gs1}^2 - V_{gs}^2)$$

$$= a(V_{gs1} - V_{gs2}) + b(V_{gs1} + V_{gs2})(V_{gs1} - V_{gs2})$$

Apply KVL to input circuit

$$V_{gs1} = \frac{1}{2} e_m + e_c$$

$$V_{gs2} = -\frac{1}{2} e_m + e_c$$

$$i_p = a\left[\left(\tfrac{1}{2}e_m + e_c\right) - \left(-\tfrac{1}{2}e_m + e_c\right)\right] + b\left[\left(\tfrac{1}{2}e_m + e_c\right) + \left(-\tfrac{1}{2}e_m + e_c\right)\right]\left[\left(\tfrac{1}{2}e_m + e_c\right) - \left(-\tfrac{1}{2}e_m + e_c\right)\right]$$

$$= a e_m + 2b\, e_c \cdot e_m$$

e_m has low frequency

$$i_p = 2b\, e_c \cdot e_m$$

$$e_c = E_c \sin w_c t \; ; \; e_m = E_m \sin w_m t$$

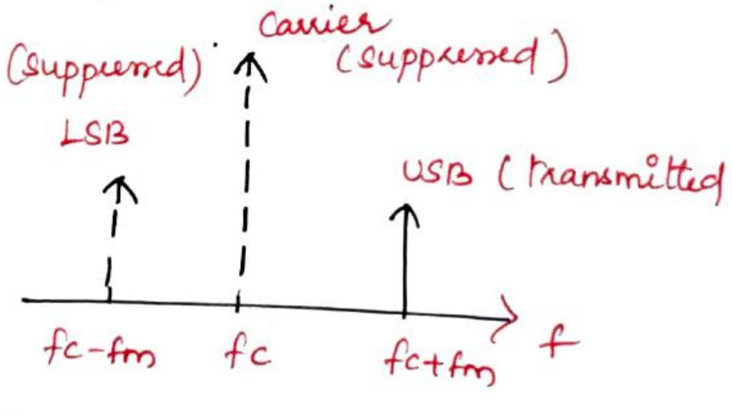
$$i_p = 2b\, E_c \cdot E_m \sin w_c t \cdot \sin w_m t$$

$$i_p = b\, E_c \cdot E_m \left\{ \cos(w_c - w_m)t - \cos(w_c + w_m)t \right\}$$

Single SideBand Modulation

→ The two sidebands are carry the same information that would be carried by DSBFC system.

→ Carrier and one of the sideband in AM is suppressed, then only one sideband remains. It is called single sideband suppressed carrier (SSBSC).



$$P_{Total} = P_c\left(1 + \frac{m^2}{2}\right) = \underbrace{P_c}_{\text{Carrier}} + \underbrace{\frac{m^2 P_c}{2}}_{\text{Sideband power}}$$

$$m = 1 \; ; \; P_{Total} = P_c + \frac{P_c}{2}$$

→ This power of the carrier which does not carry any information need to be suppressed.

$$e_{SSB}(t) = \cos(\omega_c \pm \omega_m)t$$
$$= \cos\omega_m t \cdot \cos\omega_c t \pm \sin\omega_m t \cdot \sin\omega_c t$$

Advantages :

→ Bandwidth is half of that required by DSBFC system.

→ power consumption is Low

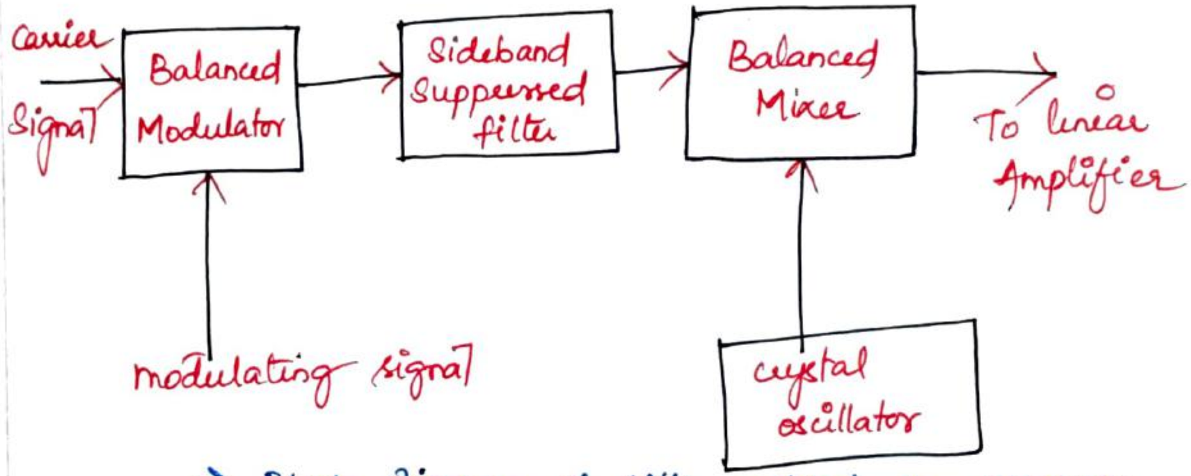
→ effect of noise at the receiver circuit is reduced.

Disadvantages :

→ SSB systems are costly.

→ They are complex, since carrier of one side band need to be suppressed.

Filter Method to produce SSB



→ Block diagram of filter method to suppress one side band. Balanced modulator produce DSB output.

→ DSB signal contains both the sidebands and it is given to sideband suppression filter to remove unwanted sideband.

→ The filter must have a flat passband and extremely high attenuation outside the passband.

→ The required value of Q factor increases as the difference between modulating frequency and carrier frequency increases.

→ High frequency required value of Q is so high.

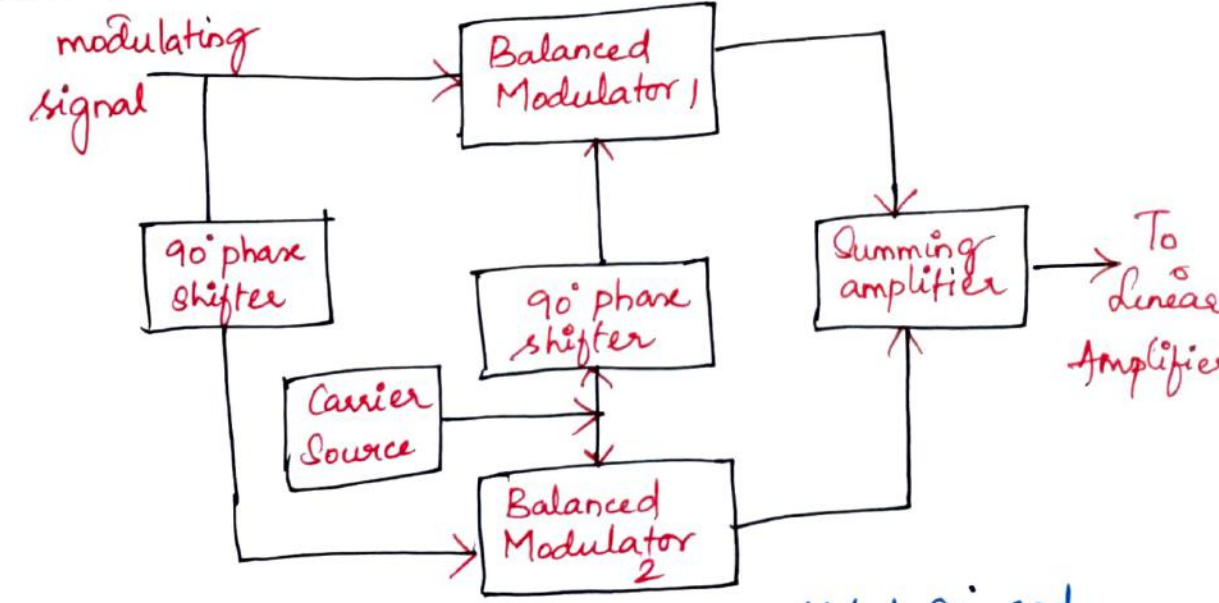
→ Then the filter suppresses one of the sidebands.

→ The frequency of the SSB signal generated at output of filter is very low as compared to the transmitter frequency.

→ frequency is boosted up to the transmitter frequency by the balanced mixer and crystal oscillator.

→ This process of frequency of frequency boosting is also called as up conversion. SSB signal having frequency equal to the transmitter frequency is then amplified by the linear amplifiers.

Phase shift method to generate SSB



→ The carrier signal is shifted 90° and applied to the balanced modulator M1.

→ The modulating signal is also directly applied to this balanced modulator.

→ The carrier signal is directly applied to the balanced Modulator M2.

→ The modulating signal is phase shifted by 90° and applied to balanced modulator M2.

→ Both the modulators produce an output consisting of only sidebands.

→ The upper balanced modulator (M1) generates upper sideband and lower sideband, but each one is shifted by +90°.

→ The lower sideband and upper sideband of (M2) +90°, −90°

→ Lower sidebands of the balanced modulator are (+90°, −90°) 180° out of phase and hence cancel each other.

→ The output of summing amplifier contains only upper sideband SSB signal. The carrier is already suppressed by balanced modulators.

$$\text{output } M_1 = \cos[(w_ct+90°)-w_mt] - \cos[(w_ct+90°)+w_mt]$$

$$\text{output } M_2 = \cos[w_ct-(w_mt+90°)] - \cos[w_ct+(w_mt+90°)]$$

Weaver's Method or Third method of SSB generation

→ it uses of 2 carriers one is audio subcarrier at frequency f_o and other is RF carrier at frequency f_c.

→ The input modulator $s(t) = \cos(2\pi f_m t)$

$$x_1(t) = \cos(2\pi f_m t) \cdot \cos(2\pi f_o t)$$

$$x_1(t) = \frac{1}{2}\left\{\cos 2\pi(f_o+f_m)t - \cos 2\pi(f_o-f_m)t\right\}$$

$$x_2(t) = \cos(2\pi f_m t) \cdot \sin(2\pi f_o t)$$

$$x_2(t) = \frac{1}{2}\left\{\sin 2\pi(f_o+f_m)t + \sin 2\pi(f_o-f_m)t\right\}$$

$$\text{upper Lowpass filter} = \frac{1}{2}\cos 2\pi(f_o-f_m)t$$

$$\text{lower Lowpass filter} = \frac{1}{2}\sin 2\pi(f_o-f_m)t$$

$$y_1(t) = \frac{1}{2} \cos 2\pi (f_o - f_m) t \cdot \cos(2\pi f_c t)$$

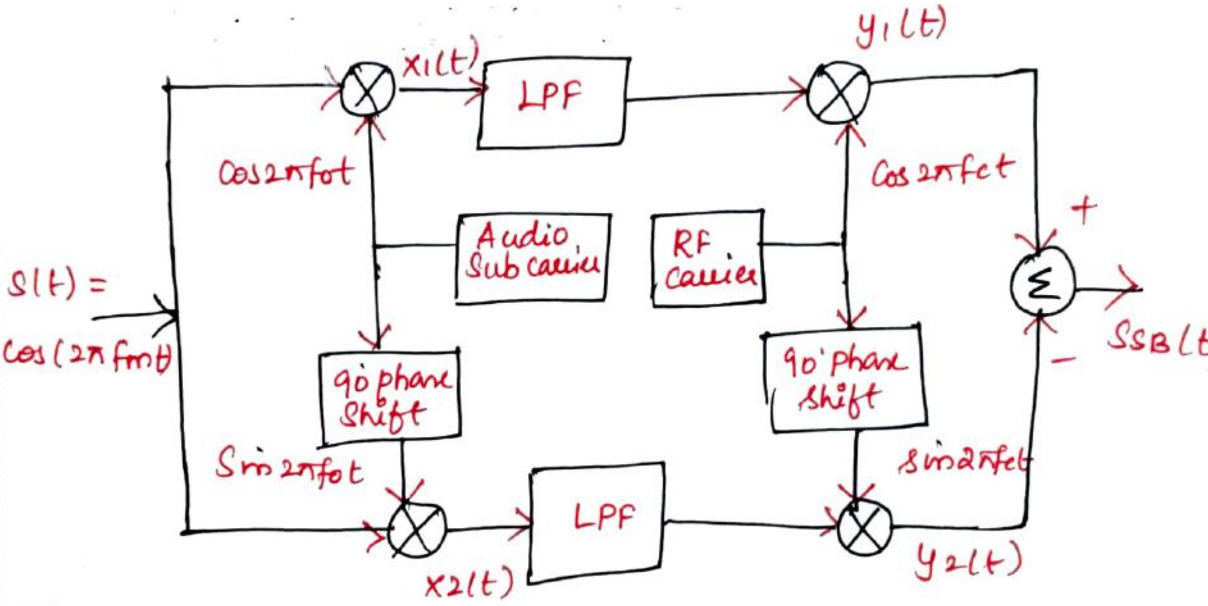
$$y_1(t) = \frac{1}{4} \{ \cos 2\pi (f_c + f_o - f_m) t - \cos 2\pi (f_c - f_o + f_m) t \}$$

$$y_2(t) = \frac{1}{2} \sin 2\pi (f_o - f_m) t \cdot \sin(2\pi f_c t)$$

$$y_2(t) = \frac{1}{4} \{ \cos 2\pi (f_c - f_o + f_m) t - \cos 2\pi (f_c + f_o - f_m) t \}$$

$$SSB(t) = y_1(t) - y_2(t)$$

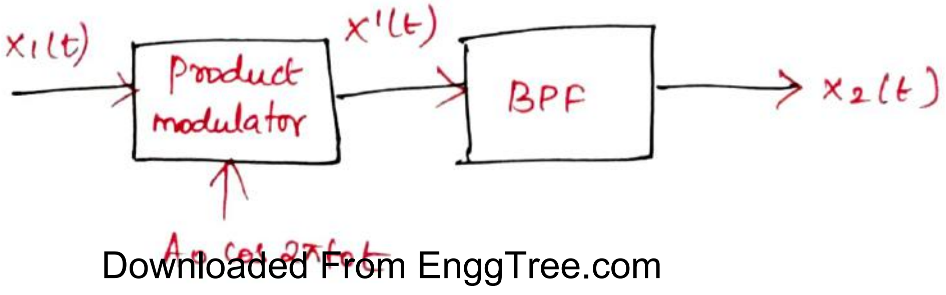
$$= \frac{1}{2} \cos 2\pi (f_c + f_o - f_m) t$$

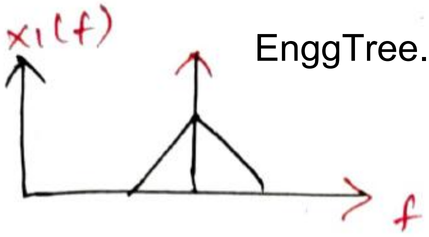


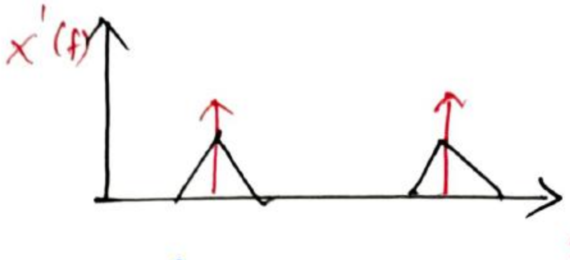
Frequency Translation

→ single sideband modulation basically performs frequency translation. It is also called frequency changing, mixing or heterodyning.

→ SSB, the message spectrum is shifted by an amount equal to the carrier frequency fc.







up conversion: $f_o = f_2 - f_1$

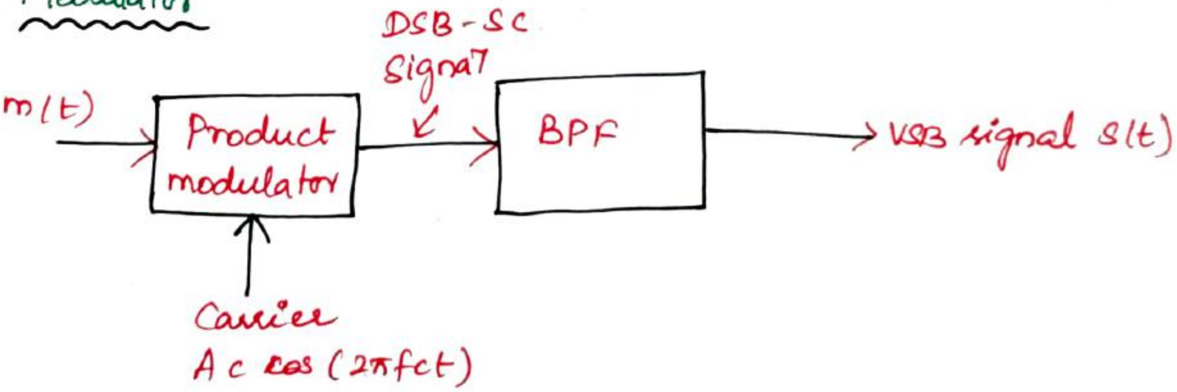
down conversion: $f_o = f_1 - f_2$

* * *

Vestigial sideband Transmission

→ one of the sideband is partially suppressed and vestige (portion) of the other sideband is transmitted. This vestige (portion) compensates the suppression of the sideband. it is called vestigial sideband transmission.

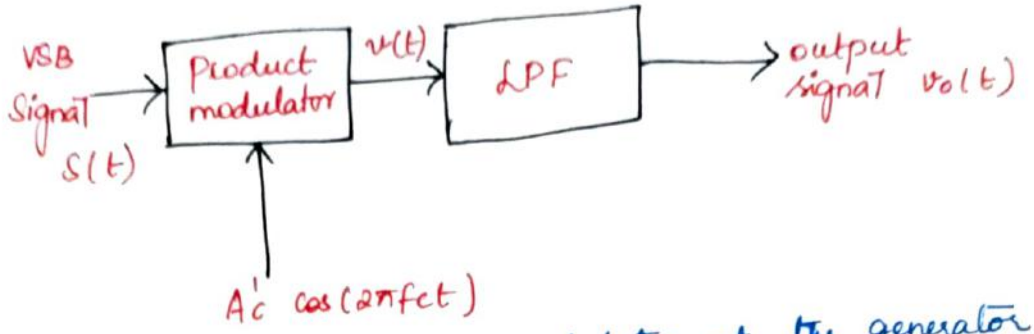
VSB Modulator



→ The product modulator generates DSB-SC signal from the message and carrier. The bandpass filter is designed in such a way that it suppresses one sideband partially and passes a portion (Vestige) of other sideband.

→ The output of the bandpass filter is VSB signal

Demodulation of VSB



→ The product modulator at the generator gives DSB-SC signal.

→ fourier transform will be obtained with the help of modulation theorem.

$$\frac{A_c}{2}\left\{M(f-f_c)+M(f+f_c)\right\}$$

$$S(f)=\frac{A_c}{2}\left\{M(f-f_c)+M(f+f_c)\right\}H(f)$$

The output of product modulator in demodulator is given by,

$$v(t)=s(t)\cdot A_c'\cos(2\pi f_c t)=A_c'\, s(t)\cos(2\pi f_c t)$$

By modulation theorem, the fourier transform of above equation becomes,

$$V(f)=\frac{A_c'}{2}\left\{S(f-f_c)+S(f+f_c)\right\}$$

put S(f) in V(f) equation

$$V(f)=\frac{A_cA_c'}{4}\left\{M(f-2f_c)+M(f)\right\}H(f-f_c)+\frac{A_cA_c'}{4}\left\{M(f)+M(f+2f_c)\right\}H(f+f_c)$$

$$=\frac{A_cA_c'}{4}\left\{H(f-f_c)+H(f+f_c)\right\}M(f)+\frac{A_cA_c'}{4}\left\{M(f-2f_c)\,H(f-f_c)+M(f+2f_c)\,H(f+f_c)\right\}$$

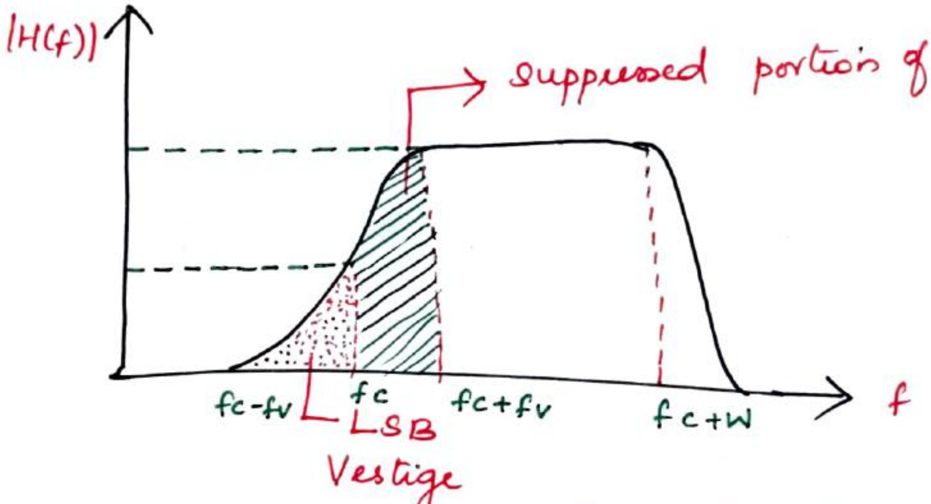
→ The first term represents the frequency spectrum of modulating signal.

→ The second term represents the frequency spectrum of VSB signal having carrier frequency 2fc.

→ Carrier frequency 2fc can be removed by low pass filter.

$$v_0(f) = \frac{A_c A_c'}{4}\{H(f-f_c)+H(f+f_c)\}M(f)$$

Magnitude response of VSB filter.



→ fc to fc+W is USB. its portion from fc to fc+fv is suppressed partially.

→ fc to fc-W is LSB. its portion from fc-fv to fc is transmitted as Vestige

→ $|H(f)| = \frac{1}{2}$

→ sum of any two frequency components in the range $f_c - f_v \le f \le f_c + f_v$ is equal to unity

$$H(f-f_c) + H(f+f_c) = 1$$

Transmission Bandwidth

$$B_T = f_v + W$$

Advantages:

→ Low frequencies, near fc are transmitted without any attenuation.

→ Bandwidth is reduced compared to DSB.

Applications:

→ VSB is mainly used for TV transmission, since low frequencies near fc represent significant picture details. They are unaffected due to VSB.

—— × ——

Hilbert Transform

→ Hilbert transform of signal x(t) is given by,

$$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$$

→ Here $\hat{x}(t)$ is the Hilbert transform.

Inverse Hilbert transform is given as,

$$x(t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{x}(\tau)}{t-\tau} d\tau$$

The fourier transform of the Hilbert transform is given as,

$$\hat{X}(f) = -j\ sgn(f) \cdot X(f)$$

Here $X(f)$ is the fourier transform of x(t)

Pre-envelope

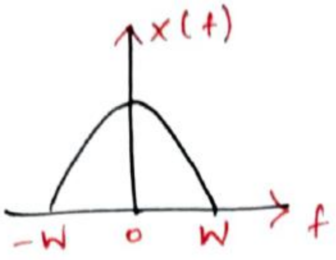
→ pre-envelope of the signal x(t) is defined as,

$$x_p(t) = x(t) + j\,\hat{x}(t)$$

→ Thus the signal x(t) is real part of pre-envelope and Hilbert transform $\hat{x}(t)$ is the imaginary part of pre-envelope.

→ The fourier transform of the pre-envelope is given

$$X_p(f) = X(f) + j[-j\ sgn(f) \cdot X(f)]$$



$$Xp(f) = X(f) + sgn(f) \cdot X(f)$$

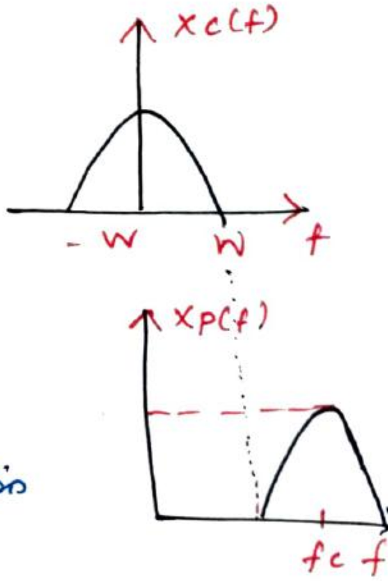
$$Sgn(f) = \begin{cases} 1 & \text{for } f > 0 \\ 0 & \text{for } f = 0 \\ -1 & \text{for } f < 1 \end{cases}$$

$$Xp(f) = \begin{cases} 2X(f) & \text{for } f > 0 \\ X(0) & \text{for } f = 0 \\ 0 & \text{for } f < 0 \end{cases}$$

→ pre-envelope of the signal has no frequency content for negative frequencies.

Complex Envelope

→ The complex envelope of the Bandpass signal $x(t)$



$$xc(t) = xp(t) \cdot e^{-j2\pi fct}$$

$xc(t)$ → complex envelope

$xp(t)$ → pre-envelope

fc → center frequency

$$xp(t) = xc(t) \cdot e^{j2\pi fct}$$

Fourier transform of above Equation

$$Xp(f) = Xc(f - fc)$$

→ Complex envelope of the bandpass signal is low pass spectrum.

Angle Modulation / Phase and frequency modulation

→ when frequency or phase of the carrier is varied by the modulating signal, then it is called angle modulation.

→ There are two types of angle modulation

Frequency modulation

→ when frequency of carrier varies as per amplitude variations of modulating signal, then it is called frequency modulation (FM). Amplitude and phase remains constant.

Phase modulation

→ when phase of the carrier varies as per amplitude variations of modulating signal, then it is called phase modulation (PM).

angle modulated wave is mathematically Expressed as,

$$e(t) = E_c \sin[\omega_c t + \theta(t)]$$

$e(t)$ = angle modulated wave

E_c → peak amplitude of carrier

ω_c → carrier frequency

$\theta(t)$ → instantaneous phase deviation

$$\theta(t) \propto e_m(t)$$

Relationship / Difference between FM and PM

→ Instantaneous phase $= \omega_c t + \theta(t)$ rad

→ Instantaneous frequency $= \frac{d}{dt}[\omega_c t + \theta(t)]$

$= \omega_c + \theta'(t)$ rad/sec

$\theta(t) = k\, e_m(t)$ rad

$\theta'(t) = k_1\, e_m(t)$ rad/sec

k_1 → deviation sensitivity of frequency

k → " " " phase

$$\theta(t) = \int \omega(t)\, dt$$

$$= \int k_1\, e_m(t)\, dt$$

$$= k_1 \int e_m(t)\, dt$$

$$e_m(t) = E_m \cos \omega_m t$$

$$\theta(t) = k_1 \int E_m \cos \omega_m t\, dt$$

$$\theta(t) = k_1 \frac{E_m}{\omega_m} \sin \omega_m t$$

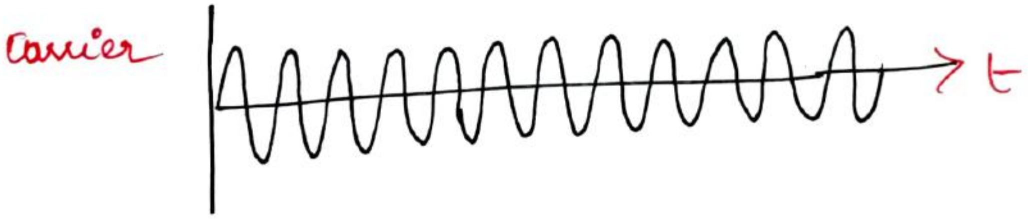
$$e(t) = E_c \sin [\omega_c t + \theta(t)]$$

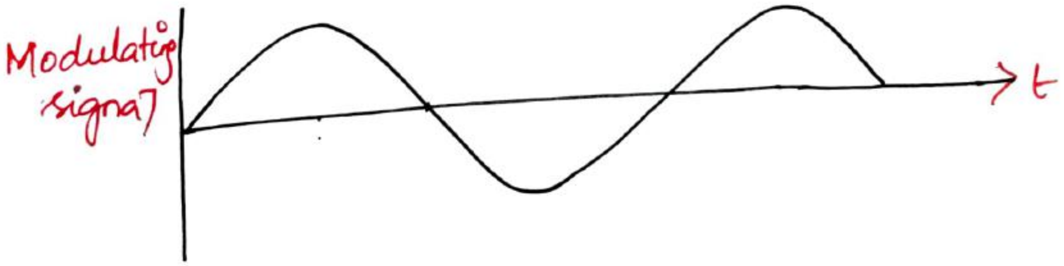
FM Equation : $e(t) = E_c \sin \left[\omega_c t + \frac{k_1 E_m}{\omega_m} \sin \omega_m t\right]$

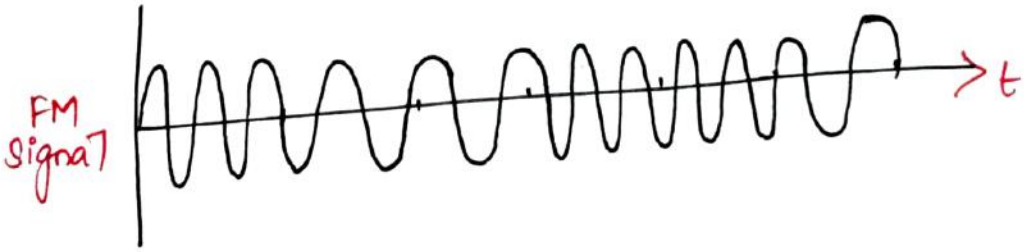
PM Equation : $e(t) = E_c \sin [\omega_c t + k E_m \cos \omega_m t]$

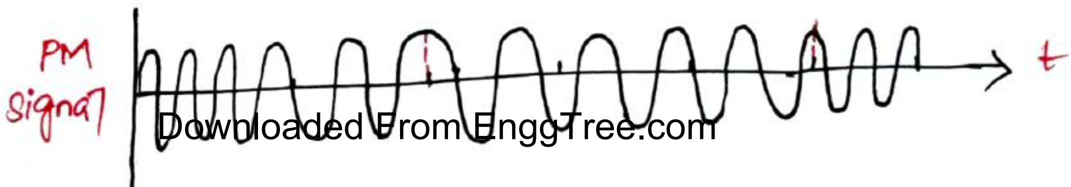
FM and PM waveforms:

→ for FM signal, the maximum frequency deviation takes place when modulating signal is at positive and negative peaks.









→ for PM signal the maximum frequency deviation takes place near zero crossing of the modulating signal.

Phase deviation, modulation index and frequency Deviation

→ FM signal, in general is expressed as,

$e_{FM}(t) = E_c \sin[w_c t + m \sin w_m t]$

→ PM signal expressed as

$e_{PM}(t) = E_c \sin[w_c t + m \cos w_m t]$

m → modulation Index

for PM : $m = K E_m$ rad

for FM : $m = \frac{\delta}{f_m} = \frac{\text{Maximum frequency deviation}}{\text{Modulating frequency}}$

$\%\text{ modulation} = \frac{\text{Actual frequency deviation}}{\text{Maximum allowable frequency deviation}}$

$DR = \frac{\text{Maximum frequency deviation}}{f_m(\max)}$.

Frequency Spectrum of Angle modulated waves

$$e(t) = E_c \{ J_0 \sin w_c t + J_1 [\sin(w_c+w_m)t - \sin(w_c-w_m)t]$$
$$+ J_2 [\sin(w_c+2w_m)t - \sin(w_c-2w_m)t]$$
$$+ J_3 [\sin(w_c+3w_m)t - \sin(w_c-3w_m)t]$$
$$+ J_4 [\sin(w_c+4w_m)t + \sin(w_c-4w_m)t] + \ldots$$

J_0, J_1, J_2 are the Bessel functions.

Bandwidth :

$BW = 2 f_m$ Hz

By Carson's rule

$$\boxed{BW = 2[\delta + f_m(\max)] \text{ Hz}}$$

Average power in FM and PM modulators

→ Total power in angle modulated wave is equal to power of an unmodulated carrier.

$$P_c = \frac{E_c^2}{2R} \quad ; \quad P_t = \frac{E_c^2}{2R}$$

$$P_t = P_0 + P_1 + P_2 + \dots + P_n$$

$$= \frac{E_0^2}{2R} + \frac{2E_1^2}{2R} + \frac{2E_2^2}{2R} + \dots + \frac{2E_n^2}{2R}$$

$$P_t = \frac{1}{R}\left[\frac{E_0^2}{2} + E_1^2 + E_2^2 + \dots + E_n^2\right]$$

Narrowband FM

$$e(t) = E_c \cos\left[\omega ct + \frac{k_1 E_m}{\omega m} \sin \omega mt\right]$$

$$\frac{k_1 E_m}{\omega m} = m$$

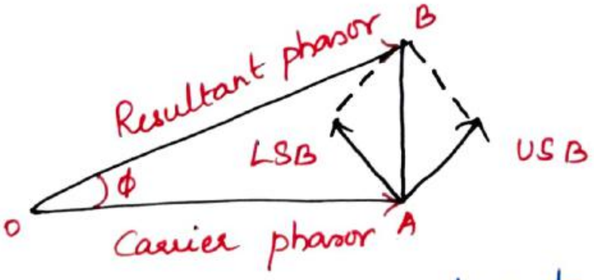
$$e(t) = E_c \cos\left[2\pi f ct + m \sin 2\pi f mt\right]$$

$$e(t) = E_c \cos(2\pi f ct) \cdot \cos \underbrace{[m \sin(2\pi f mt)]}_{1} -$$

$$E_c \sin(2\pi f ct) \cdot \underbrace{\sin [m \sin (2\pi f mt)]}_{m \sin 2\pi f mt}$$

$$e(t) = E_c \cos(2\pi f ct) - m E_c \sin(2\pi f ct) \cdot \sin(2\pi f mt)$$

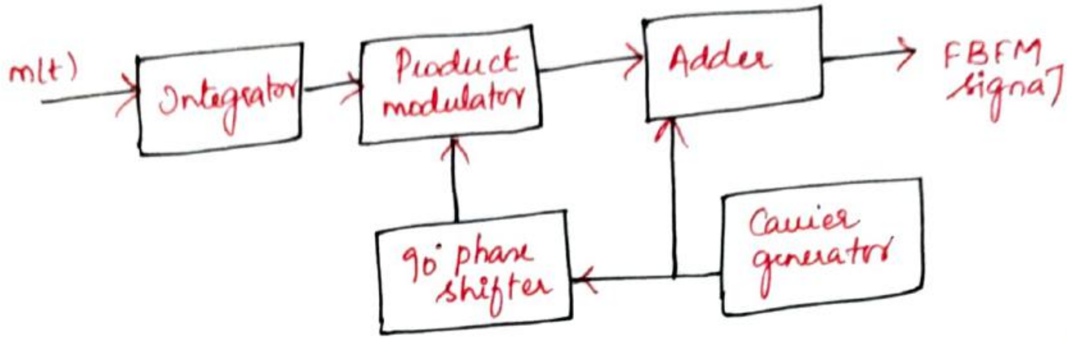
phasor Diagram of Narrowband FM



→ phasor corresponding to USB rotates in counter clockwise at ωm

→ phasor corresponding to LSB rotates in ~~ccw~~ clockwise direction at $-\omega m$.

Generation of Narrowband FM



→ one of the input of product modulator is 90° phase shifted carrier signal.

→ input of product modulator is integrated modulated signal $\int_0^t m(t)\,dt$.

→ output of product modulator $E_c \cos \omega_c t$.

WideBand FM

$$e(t) = E_c \cos\left[2\pi f_c t + m \sin 2\pi f_m t\right]$$

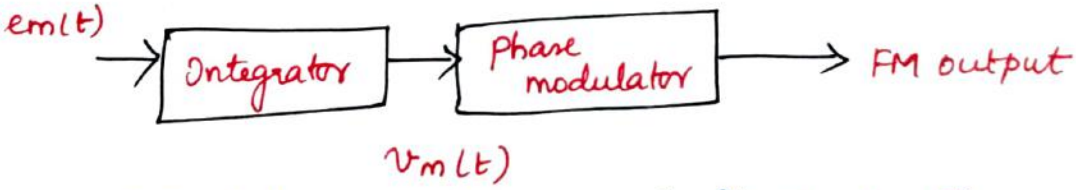
$$= Re\left[E_c\, e^{j(2\pi f_c t + m \sin 2\pi f_m t)}\right]$$

$$x(t) = E_c\, e^{jm \sin 2\pi f_m t}$$

$$e(t) = Re\left[x(t) \cdot e^{j2\pi f_c t}\right]$$

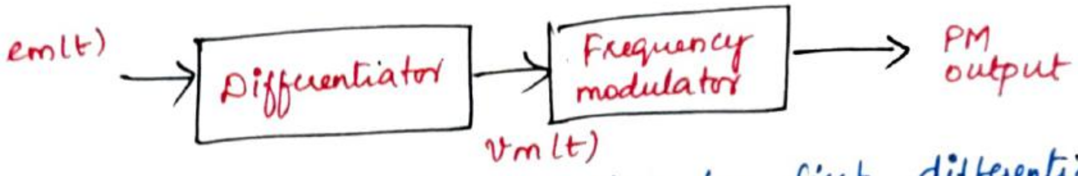
$$E(f) = \frac{E_c}{2} \sum_{n=-\infty}^{\infty} J_n(m) \left\{ \delta(f - f_c - n f_m) + \delta(f + f_c + n f_m) \right\}$$

Generation of FM from PM :



→ Instantaneous frequency is direct function of em(t). Thus FM can be obtained from PM by integrator before PM.

PM using FM modulator



→ PM can be generated by first differentiating the input signal and then passing through FM modulator.

$$\theta(t) = k_1 \, em(t)$$

Nonlinear Effects in FM

Strong Non linearity:

→ This type of nonlinearity is intentionally introduced in a controlled manner.

Weak Non linearity:

→ This type of non linearity is introduced because of imperfections in the communication channel.

— × —

Comparison Between AM and FM

Amplitude modulation

→ Amplitude of the carrier is varied according to amplitude of modulating signal

→ AM has poor fidelity due to narrow BW.

→ Most of the power is in carrier hence less efficient.

→ Noise interference is more

→ Adjacent channel interference is present

→ AM Broadcast operates in MF and HF range

→ AM only carrier and two sidebands are present.

→ Transmission equipment is simple

→ Transmitted power varies according to modulation index

→ Depth of modulation have limitation. It cannot be increased above 1.

Frequency modulation

→ frequency of the carrier is varied according to amplitude of the modulating signal.

→ since the BW is large, fidelity is better

→ All the transmitted power is useful

→ Noise inteference is minimum.

→ Adjacent channel inteference is avoided due to wide Bandwidth.

→ FM Broadcast operates VHF and UHF range

→ Infinite number of sidebands are present

→ Transmission equipment is complex

→ Transmitted power remains constant irrespective of modulation index.

→ Depth of modulation have no limitation. It can be increased by increasing frequency deviation.

* * *

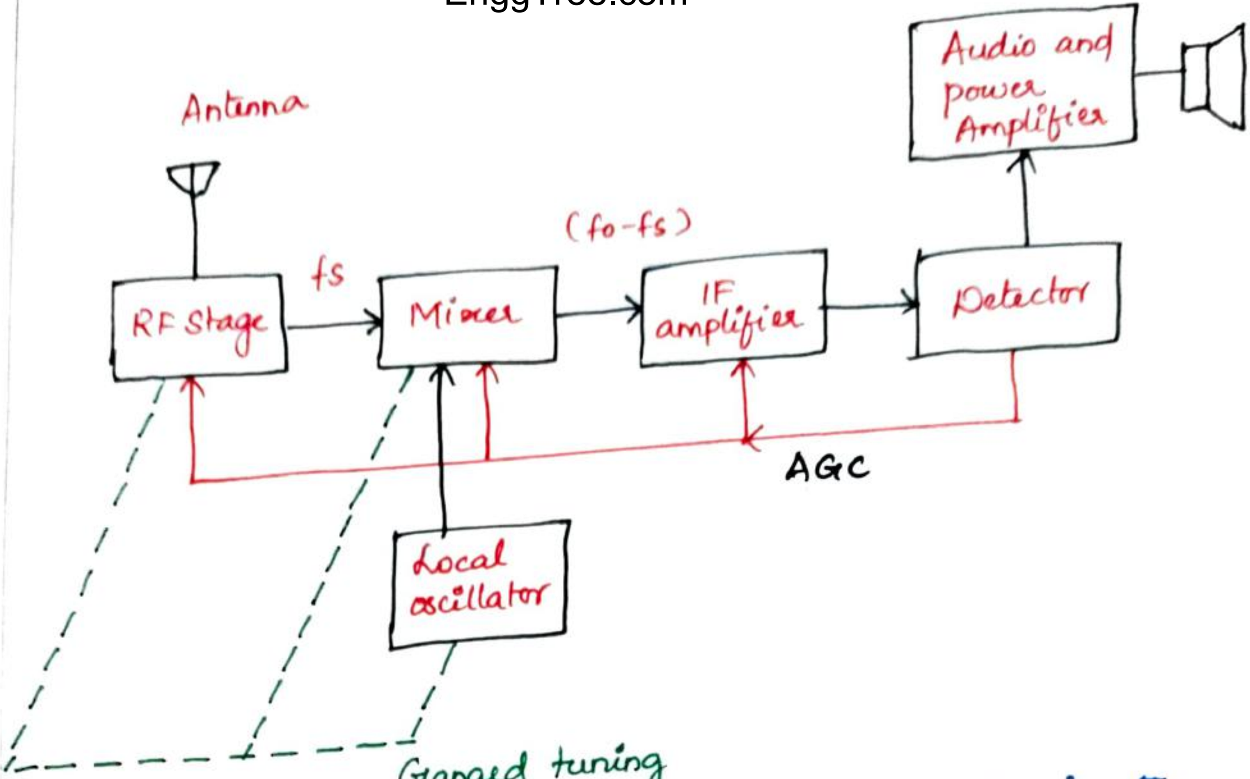
Superheterodyne Receiver

→ The problems of TRF receiver are overcome in this receiver. The superheterodyne receiver converts all incoming RF frequencies to a fixed lower frequency. Called Intermediate frequency (IF).

→ This IF is amplified and detected to get the original signal.

RF amplifier

→ The Antenna receives all the frequency signals and gives it to RF amplifier.



Ganged tuning

→ The RF stage amplifies the signals in the required range of frequencies. Thus it provides initial gain and Selectivity.

Mixer and Local oscillator

→ The output of the RF amplifier is given to the mixer Stage. The Local oscillator output is also applied to the mixer.

→ Let us assume that local oscillator frequency is fo and signal frequency is fs.

→ The signal frequency fs and Local oscillator fo are mixed in the mixer in such a way that frequency difference (fo-fs) is produced at the output of mixer.

→ This difference fo-fs is called Intermediate Frequency (IF). The signal at this IF Contains the Same modulation as the incoming signal.

IF amplifier

→ The IF is amplified by one or more IF amplifier stages and given to the detector. Most of the gain and selectivity is provided by these IF amplifiers.

→ Normally IF is fixed for the AM receivers. To change the particular station, the local oscillator frequency fo is changed in such a way that the frequency fs' of that station and fo has the difference equal to IF.

→ Hence the Bandwidth of the IF amplifiers is relatively narrow.

Automatic Gain Control

→ A part of output is taken from the detector and it is applied to RF amplifier, mixer and IF amplifier for gain control. This is called Automatic Gain Control or AGC.

→ This AGC maintains the constant output voltage level over a wide range of RF input signal levels.

Detector

→ The detector converts AM signal at IF to original modulating signal. Diode detector is mostly used for AM signals. The output of detector is low power audio or modulating signal.

Audio and power amplifier

→ The signal received from detector is very weak and hence needs amplification. Audio power amplifiers normally have one or more stages. The signal is amplified and given to Speaker.

Advantages

→ The selectivity of this receiver is better since its IF amplifiers are narrowband and they operate only at IF.

→ The design of IF amplifiers is relatively simple since they operate only at IF.

— × —

Unit I Two marks (Amplitude Modulation)

1) Limitations of AM

→ Amplitude of AM depends upon depth of modulation. Hence transmitter power is not constant.

→ Modulation index maximum value is 1. if the value exceeded, signal lost

— x —

2) What are the advantages of converting the low frequency signal into high frequency signal?

→ The antenna size is very large at low frequencies. such antenna is practically not possible to fabricate.

→ High carrier frequencies require reasonable antenna size for transmission and reception.

→ High frequencies can be transmitted using tropospheric scatter propagation, which is used to travel long distances.

→ it allows multiplexing of the signals.

— x —

3) Comparison of Various AM systems

Parameter	AM / DSBFC	DSB-SC	SSB-SC	VSB
Method	Carrier & Both side bands	only sidebands	only one sideband	one side Band & part of other sideband
Bandwidth	2fm	2fm	fm	fm < BW < 2fm
Generation	Easy	Easy	Complex	Complex
Transmission Efficiency	33.3 %	100 %	100 %	33.3 % < η ≤ 100 %

4) Define modulation index for AM.

→ Modulation index is the ratio of maximum amplitude of modulating signal (Em) to maximum amplitude of carrier signal (Ec)

$$m = \frac{Em}{Ec}$$

— × —

5) For television signal transmission vestigial sideband modulation is selected. Justify.

→ In TV transmission, significant picture details are represented by low frequencies. These frequencies are well preserved by vestigial sideband transmission.

— × —

6) Define heterodyning

→ The signal frequency fs and local oscillator frequency fo are mixed in the mixer in such a way that the frequency difference fo-fs is produced at the output.

→ This frequency difference always remains constant and it is called Intermediate Frequency (IF). it becomes easy to amplify and detect signals at single IF. This process is called heterodying.

— × —

7) Define carrier swing

→ carrier swing is defined as the total variation of frequency from lowest to the highest values.

Carrier swing = 2 × frequency deviation of FM carrier

— × —

8) why DSBFC - AM is Bandwidth inefficient when Compared with SSB AM?

→ DSB - FC AM, Both the sidebands are transmitted. Hence the Bandwidth required is 2fm.

→ SSB AM one side band and carrier is suppressed. only one side Band transmitted. Bandwidth required is fm.

→ Thus SSB-AM requires half of the Bandwidth compared to DSB-FC AM. Therefore DSB-FC AM is bandwidth inefficient.

—— × ——

9) Advantages of RF amplifier

→ RF amplifier improves the selectivity

→ it improves the senstivity

→ signal to noise ratio improved.

—— × ——

10) A carrier of 6kV is amplitude modulated by an audio signal of 3kV. Find ma.

$$m = \frac{E_m}{E_c} = \frac{3kV}{6kV} = 0.5$$

—— × ——

11) Define Degree of modulation

→ it depends on amplitude of the modulating signal to relative carrier amplitude.

Under modulation (ma<1)
Over modulation (ma>1)
Critical modulation (ma=1)

—— × ——

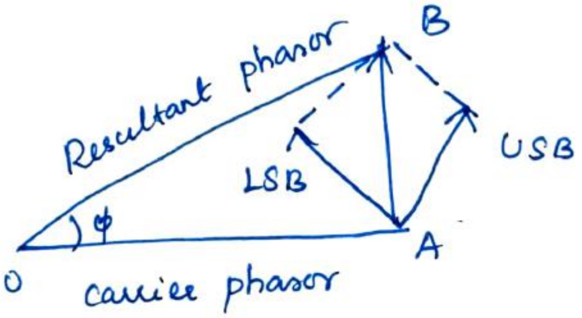
12) Carson's rule of FM Bandwidth

$$BW = 2(\delta + f_{m(max)})$$

$\delta \rightarrow$ maximum frequency deviation and $f_{m(max)}$ is the maximum signal frequency.

— × —

13) phasor Diagram of narrowband FM



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14) List of non linear effects in FM system.

→ The Spectrum of FM signal contains infinite number of sidebands.

→ FM signal equation is nonlinear function of modulating signal.

— × —

15) Define transmission Bandwidth

→ Transmission Bandwidth is the Bandwidth required to transmit AM or FM signal.

AM transmission BW = $2f_m$

FM transmission BW = $2[\delta + f_{m(max)}]$

— × —

16) Advantages of Vestigial side band?

→ Reduced Bandwidth

→ Frequency near f_c are not suppressed

— × —

17) Determine the hilbert transform of cos wt .

→ Hilbert transform shift the phase of positive frequency components by -90° and that of negative frequency components by +90°

Hence H [cos wt] = Sin wt .

* * *

18) what is the bandwidth required for an FM wave in which the modulating frequency signal is 2KHZ . maximum frequency deviation is 12KHZ?

$$f_m = 2KHZ \; ; \; \delta = 12KHZ$$

$$BW = 2 [\delta + f_m (max)] \; HZ$$

$$BW = 2 [12 + 2] = 28 \; KHZ$$

* * *

19) Applications of FM

→ used in mobile radio two way communication

→ cellular radio , microwave & satellite communica-tion

→ Used for broadcast radio transmission of sound signals.

* * *

20) why frequency modulation prefered for voice transmission

→ voice signal requires the frequencies upto 10KHZ. FM doesn't have wide bandwidth.

→ Hence FM is prefered for voice transmission

* * *

UNIT - 1 Big Questions

1) classifications of signals and system
2) Amplitude modulation
3) DSB-SC Modulator and Demodulator
4) Balanced modulator and ring modulator using diode DSB-SC method.
5) SSB-SC (filter and phase shift, weaver's method)
6) VSB
7) Hilbert Transform, pre-envelope, complex envelope
8) Angle Modulation
9) Super heterodyne Receiver

UNIT - 2 Random process and Sampling

Basics of probability

Experiment:

→ it is the process which is conducted to get some results.

Sample space:

→ set of all possible outcomes of an experiment is called sample space of the experiment.

Event: → The expected subset of the sample space or happening is called an event.

probability:

→ probability of event 'A' is defined as the ratio of number of possible favourable outcomes to total no. of outcomes.

$$P(A) = \frac{\text{Number of possible favourable outcomes}}{\text{Total number of outcomes}}$$

permutations and combinations

Combination of 'n' taken 'r' at a time

$$nCr = \frac{n!}{(n-r)!\,r!}$$

permutations of 'n' taken 'r' at a time

$$nPr = \frac{n!}{(n-r)!}$$

Axioms (properties of probability)

i) $P(A) = P(S) = 1$

2) $0 \le P(A) \le 1$

3) $P(A+B) = P(A) + P(B)$

Conditional probability :

$$P(B/A) = \frac{P(AB)}{P(A)} \quad ; \quad P(A/B) = \frac{P(AB)}{P(B)}$$

P(AB) is the joint probability of A & B

Joint probability has commutative property

$$P(AB) = P(BA)$$

Bayes Rule or Bayesian policy

→ Let $B_1, B_2, B_3 \dots B_n$ be mutually exclusive events and event A occurs only when any one $B_1, B_2, B_3 \dots B_n$ occurs.

$$P(B_i/A) = \frac{P(B_i)\,P(A/B_i)}{\sum_{i=1}^{n} P(B_i)\,P(A/B_i)}$$

→ This relation is called Bayes rule.

Probability of statiscally Independent Events :

→ A and B are the two events possible from an experiment, and possibility of occurance of B simply does not depend on occurance of event A then the events are called statiscally independent events.

$$P(B/A) = P(B)$$

$$P(AB) = P(A) \cdot P(B)$$

$$P(A/B) = P(A)$$

Random Variables :

→ A function which takes on any value from the sample space and its range is some set of real numbers is called a random variable of the experiment.

Discrete Random Variable:

→ The random variable x is a discrete random variable if x can take on only finite number of values in any finite observation interval.

$$X = \{1, 4, 9, 16, 25, 36\}$$

Continuous Random Variable:

→ if the random variable 'x' takes on any value of whole observation interval, x is called continuous random variable.

Cumulative Distribution function (CDF):

→ CDF of a random variable 'x' is the probability that a random variable 'x' takes a value less than or equal to x.

$$F_X(x) = P(X \leq x)$$

probability Density function: (PDF)

→ The Derivative of cumulative Distribution function with respect to dummy variable is called PDF.

$$f_X(x) = \frac{d}{dx} F_X(x)$$

Statistical Averages for Random Variables

Mean $= m_x = E[X]$ → Discrete random Variable

$$m_x = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$ → continuous random Variable

Variance $= \sigma_x^2$

Standard deviation $= \sigma_x = \sqrt{E[x^2] - m_x^2}$

— x —

Random process :

→ A random process or Stochastic process is an ensemble or sample space. such sample space Consists of sample points whose outcome is random function of time.

stationary process :

→ when the statistical properties of a process do not change with time, it is called stationary process.

Mean of Random process :

→ Mean of random process is denoted by

Mx(t) $$m_x(t) = E[x(t)] = \int_{-\infty}^{\infty} x f_{x(t)}(x)\, dx$$

Autocorrelation function :

→ Autocorrelation function is defined as the expectation of product of two random variables are obtained by observing the random process at different times.

$$R_x(t_1, t_2) = E[x(t_1)\ x(t_2)]$$

$$R_x(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_{x(t_1)x(t_2)}(x_1, x_2)\, dx\ dx_2$$

Wide sense Stationary:

→ The process may not be Stationary in Strict sense, still the mean and autocorrelation function are independent of shift of time origin.

power Spectral Density:

→ fourier transform is given as

$$X(f) = \int_{-T}^{T} x(t) \cdot e^{-j2\pi ft} dt$$

power density Spectrum is defined as

$$Sx(f) = \lim_{T \to \infty} \frac{E[|x(f)|^2]}{2T}$$

properties of PSD:

1) Einstein - wiener - Khintchine relations

→ power Spectral density Sx (f) and autocorrelation function Rx (τ) form a fourier transform pair.

$$Sx(f) = \int_{-\infty}^{\infty} Rx(\tau) \cdot e^{-j2\pi f\tau} d\tau$$

$$Rx(\tau) = \int_{-\infty}^{\infty} Sx(f) \cdot e^{j2\pi f\tau} df$$

2) Zero frequency value of PSD:

→ The Zero frequency Value of the PSD of a stationary process is equal to total area under the autocorrelation

$$Sx(0) = \int_{-\infty}^{\infty} Rx(\tau) d\tau$$

3) Mean Square Value of process

→ The mean square value of a stationary random process is equal to total area under the PSD Curve.

$$E[x^2(t)] = \int_{-\infty}^{\infty} Sx(f) df$$

4) PSD is always positive :

→ The PSD of the stationary process is always positive.

$$S_x(f) \geq 0$$

5) PSD is Even function :

→ The PSD of a real valued random process is even function of frequency.

$$S_x(-f) = S_x(f)$$

Ergodic process :

→ A random process is called ergodic process if time averages are equal to ensemble averages.

$$m_x(t) = m_x(T)$$

$$R_x(t_1 t_2) = R_x(\tau, T)$$

— x —

Noise Sources and their types :

Atmospheric Noise :

→ it is many times referred to as static noise. The static noise is mainly due to electrical disturbances, such as lightening.

→ Electric discharges that occur between clouds or between the earth and clouds.

Extraterrestrial Noise :

→ Extraterrestrial noise comes from sources in Space, which are again divided in two groups: solar noise, cosmic noise.

Industrial Noise:

→ Industrial noise is produced by automotive ignition systems, electric motors and generator brush contacts.

→ An electrical equipment when abruptly switched 'ON' or 'OFF' produces 'transients' that create noise.

Internal Noise:

→ The noise is generated internally in the circuit. Electric components such as resistors, diodes and transistors produce noise.

→ Although this is a low level noise, it can interfere with weak signal. The internal noise is found to obey certain laws, and hence it is possible to design the equipment in which the effects of noise are minimized.

Thermal or Thermal Agitation or Johnson Noise:

→ The electrons in a conductor posses varying amounts of energy by virtue of temperature of conductor.

→ The small fluctuation in energy are sufficient to produce small noise voltages in the conductor.

→ These random fluctuations produced by the thermal agitation of the electrons are called the thermal noise.

Shot Noise:

→ Normally it is assumed that the current in an electronic device, such as diode or transistor, under d.c condition is constant at every instant of time.

→ Actually, however the current consists of a stream of individual electrons and holes, and it is only the time average flow which is constant.

→ The fluctuations in the number of electrons (or holes) constitute the shot noise.

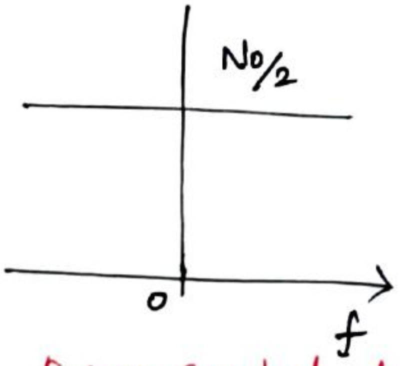
white Noise:

→ white noise is not the noise source.

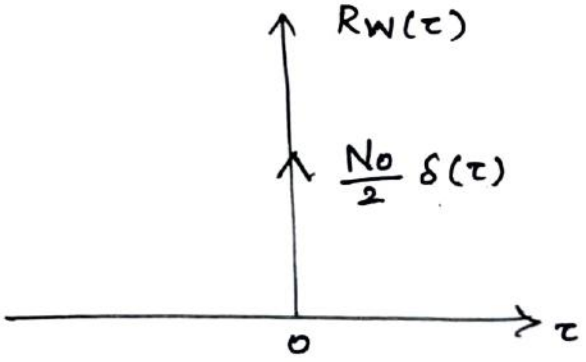
→ It is the classification of noise.

→ The noise which has Gaussian distribution and have flat spectral density over a wide range of frequencies is called white noise.

$$S_W(f) = \frac{N_0}{2}$$



power spectral density of white noise ;



Autocorrelation function of white Noise.

$N_0 = KT_e$

T_e → noise temperature , K → Boltzmann's constant

Output Signal to Noise Ratio (SNR)$_o$:

$$(SNR)_o = \frac{\text{Average power of message signal at the receiver output}}{\text{Average power of noise at the receiver output}}$$

Channel Signal to Noise Ratio : (SNR)c

$$(SNR)_c = \frac{\text{Average power of message signal at the receiver input}}{\text{Average power of noise in message bandwidth at the receiver input}}$$

Figure of merit :

$$FOM = \frac{(SNR)_o}{(SNR)_c} = \frac{(SNR)_o}{(SNR)_i}$$

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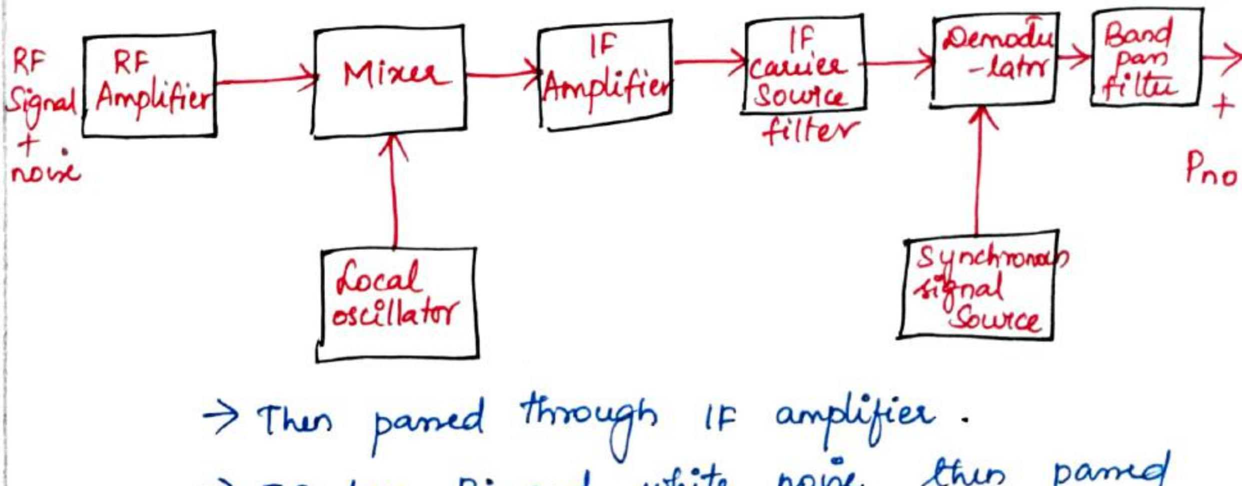
Receiver for AM signal

→ AM signal receiver which uses superheterodyne principle.

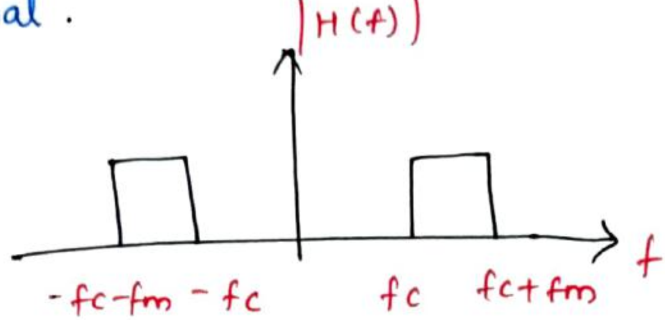
→ Mixer converts the incoming RF signal to IF.



→ Then passed through IF amplifier.

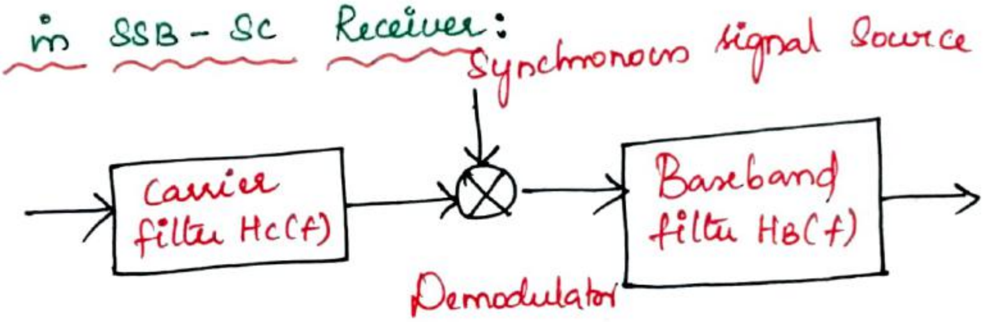
→ IF has Pi and white noise then passed through IF carrier filter and given to demodulator

→ The demodulator recovers the baseband signal.



Ideal characteristics of IF Filter

Noise in SSB-SC Receiver:



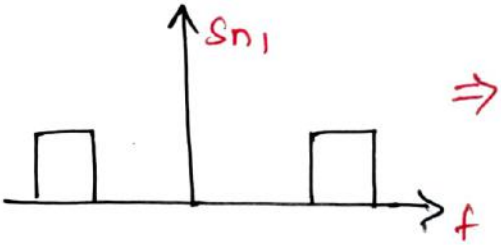
$$x_1(t) = A\cos[2\pi(f_c+f_m)t]$$

$$x_2(t) = x_1(t)\cdot x_c(t)$$

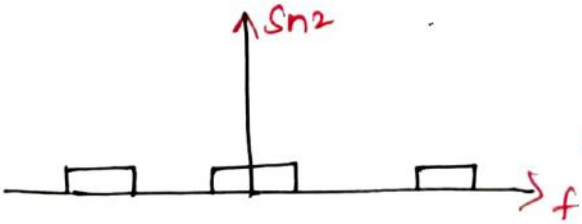
$$= A\cos[2\pi(f_c+f_m)t]\cos(2\pi f_c t)$$

$$P_i = \frac{A^2}{2}\ ;\ P_o = \frac{A^2}{8}\ ;\ \frac{P_o}{P_i} = \frac{1}{4}$$

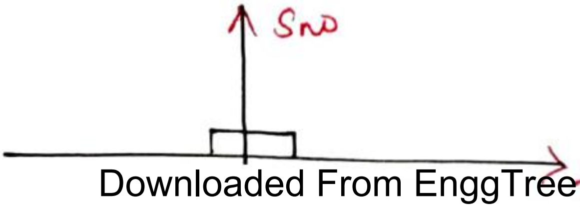
$$P_{no} = \frac{N_o f_M}{4}$$



⇒ power spectral density of noise at the output of carrier filter



⇒ Spectral density at the output of multiplier



⇒ Spectral density of noise at output of baseband filter

Calculation of signal to noise ratio :

$P_o = \frac{a^2}{8}$ ⇒ output signal power

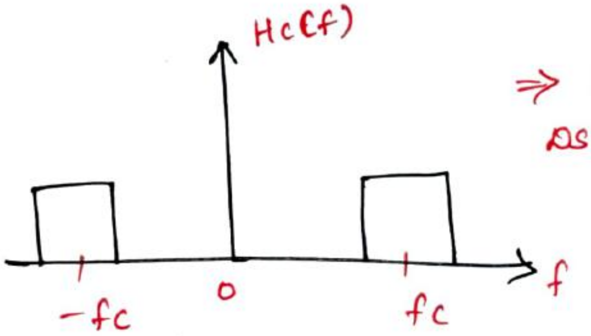
$P_{no} = \frac{N_o f_M}{4}$ ⇒ output noise power

$$\left(\frac{S}{N}\right)_{output} = \frac{A^2/8}{N_o f_M/4} = \frac{A^2}{2 N_o f_M}$$

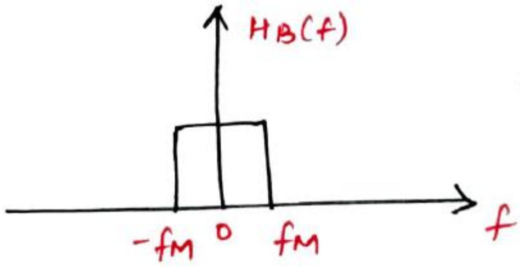
$$P_i = \frac{A^2}{2}$$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_i}{N_o f_M}$$

Noise in DSB-SC Receiver :



⇒ Carrier filter for DSB-SC transmission



⇒ Response of baseband Low pass filter.

$$P_{no} = \frac{N_o f_M}{2}$$

$$P_i = \frac{A^2}{2} \quad ; \quad P_o = \frac{A^2}{4}$$

Signal to Noise ratio

$$\left(\frac{S}{N}\right)_{output} = \frac{P_o}{P_{no}}$$

$$\left(\frac{S}{N}\right)_{output} = \frac{A^2/4}{N_o f_M/2} = \frac{A^2}{2 N_o f_M}$$

$$P_i = \frac{A^2}{2}$$

$$\left(\frac{S}{N}\right)_{output} = P_i / N_o f_M$$

Figure of merit :

$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{A^2}{2N_0 f_M} \Big/ \frac{A^2}{2N_0 f_M}$$

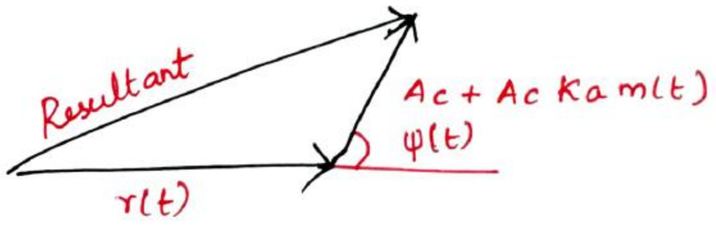
$$\gamma = 1$$

$$(SNR)_c = \frac{\text{Modulated signal power}}{\text{Average noise power}}$$

$$(SNR)_c = \frac{\frac{Ac^2}{2}(1+ka^2P)}{N_0 B} = \frac{Ac^2(1+ka^2P)}{2N_0 B}$$

Threshold Effect :

→ when the carrier to noise ratio reduces below certain value, the message information lost. The performance of envelope detector deteriorates rapidly and it has no proportion to carrier to noise ratio. This called threshold effect.



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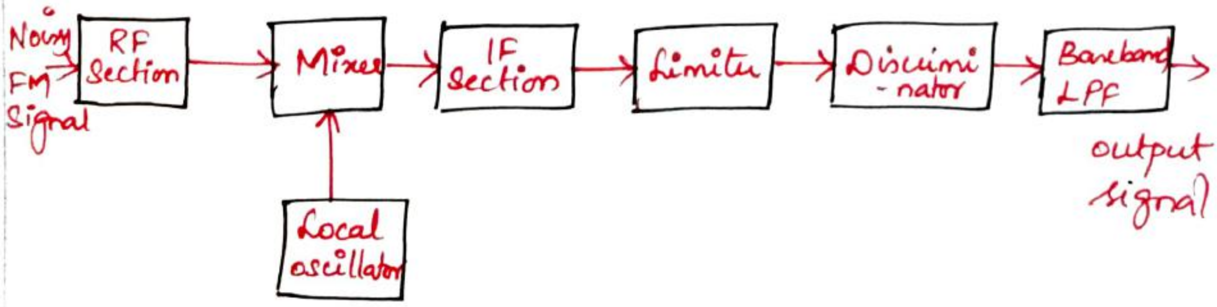
Noise performance Analysis of FM systems :



→ FM Receiver is also superheterodyne type.

→ it has limiter and discriminator as the different stages compared to AM superheterodyne receiver.

→ The amplitude limiter followed by IF section removes the amplitude variations in the signal.

→ These amplitude variations normally results from noise.

→ Discriminator produces hybrid modulated wave in which both the amplitude and frequency vary according to message signal.

→ The envelope detector inside the discriminator then obtains the message signal according to amplitude variations.

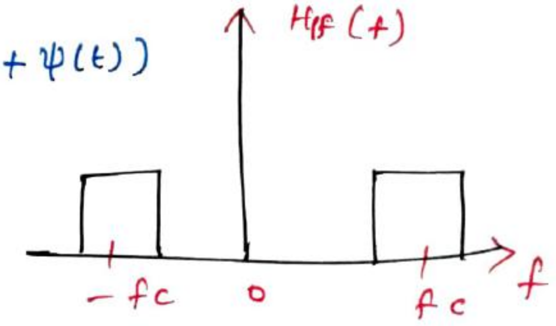
→ The baseband low pass filter has the bandwidth such that it passes only the message signal and all other out of band noise components are attenuated.

Noise in FM receivers

$$n(t) = r(t) \cdot \cos(2\pi f_c t + \psi(t))$$

$$r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

$$\psi(t) = \tan^{-1}\left[\frac{n_s(t)}{n_c(t)}\right]$$

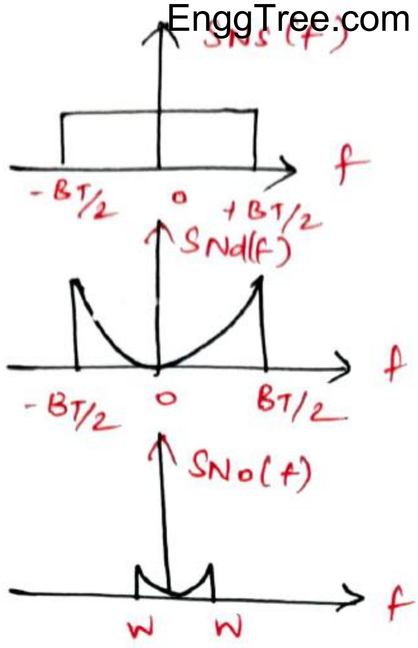


IF filter characteristic

Output signal power

$$P_o = k_f^2 P$$

$$S_{No}(f) = \frac{2 N_0 W^3}{3 A_c^2}$$



PSD of noise in FM Receiver

Output Signal to Noise ratio

$$(SNR)_0 = \frac{\text{Average output signal power}}{\text{Average output noise power}}$$

$$(SNR)_0 = \frac{K_f^2 P}{\frac{2N_0W^3}{3A_c^2}} = \frac{3A_c^2 \cdot K_f^2 \cdot P}{2N_0W^3}$$

Channel Signal to Noise ratio

$$(SNR)_c = \frac{A_c^2/2}{N_0 \cdot W} = \frac{A_c^2}{2N_0W}$$

Figure of merit

$$\frac{(SNR)_0}{(SNR)_c} = \frac{3K_f^2P}{W^2}$$

Capture effect :

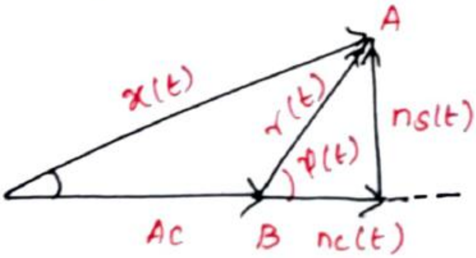
→ FM system minimizes the effects of noise interference. This can be effective when interference is weak compared to FM signal.

→ This suppresses FM signal, when noise and FM signal are equal, FM receiver locking fluctuates between them. This is known as capture effect.

FM threshold effect:

→ As the carrier to noise ratio is reduced further crackling or sputtering sound appears at the receiver output. Near the breaking point the theoretically calculated output signal to noise ratio becomes large, but the actual value is very small.

→ This phenomenon is called threshold effect.



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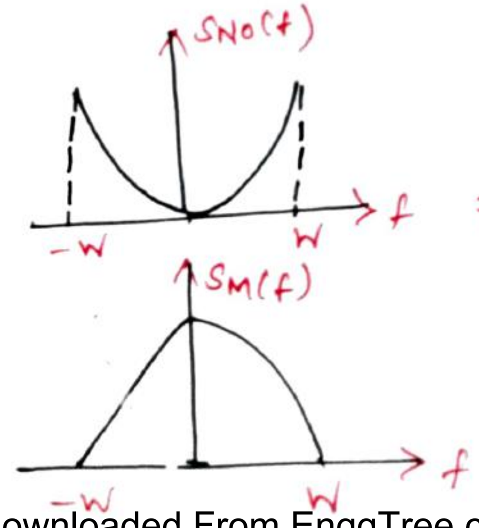
Pre-Emphasis and De-emphasis:

→ power spectral density of noise increases rapidly at high frequencies

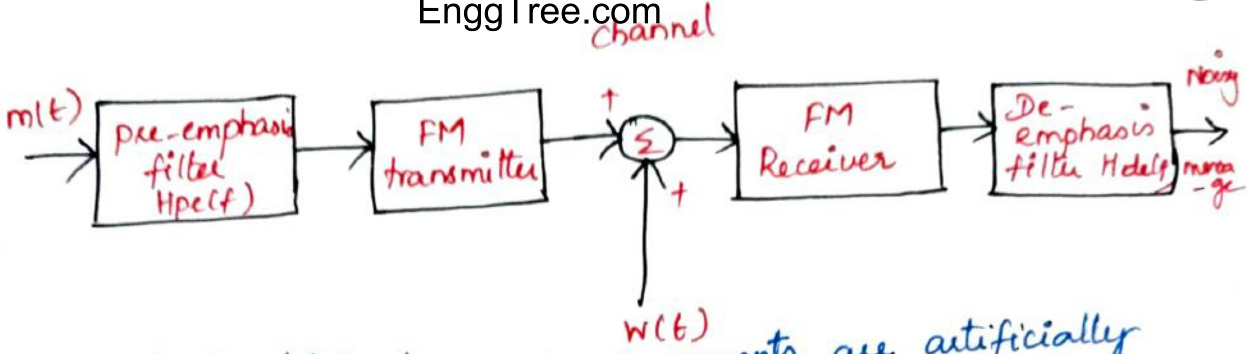
→ Efficient use of frequency band and improved noise performance can be obtained with the help of pre-emphasis and de-emphasis circuits.

→ it is used to improve the threshold

→ Simple RC networks are used to boost the high frequencies at the transmitter and attenuate them at the receiver side.



⇒ PSD of noise and message.



→ The high frequency components are artificially emphasised by pre-emphasis filter before modulation.

→ This equalizes the low frequency and high frequency portions of PSD and complete band is occupied.

→ The FM signal is then transmitted, Noise adds to this signal before it reaches the receiver.

→ At the receiver de-emphasis is performed on high frequency components.

→ This restores the power distribution of original signal.

→ Because of de-emphasis at the receiver, the high frequency Components of noise are also reduced. This improves the signal to noise ratio.

→ In order to obtain the original signal back, the transfer functions of pre-emphasis and de-emphasis filter must be inverse of each other.

$$H_{de}(f) = \frac{1}{H_{pe}(f)}$$

— × —

Low pass sampling theorem:

→ A continuous time signal can be completely represented in its samples and recovered back if sampling frequency is twice of the highest frequency content of the signal.

$$f_s \geq 2W$$

f_s → sampling frequency

W → higher frequency content.

proof of Sampling theorem

i) Define $x_\delta(t)$

ii) fourier transform of $x_\delta(t)$ (i.e) $X_\delta(f)$

iii) Relation between $X(f)$ and $X_\delta(f)$

iv) Relation between $x(t)$ and $x(nT_s)$

i) Define $x_\delta(t)$

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\, \delta(t-nT_s)$$

$$\boxed{x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t-nT_s)}$$

$x(nT_s)$ is basically $x(t)$ sampled at $t = nT_s$

$n = 0, \pm 1, \pm 2, \pm 3 \ldots$

ii) FT of $x_\delta(t)$ (i.e) $X_\delta(f)$

$$X_\delta(f) = FT\left\{ \sum_{n=-\infty}^{\infty} x(t)\, \delta(t-nT_s) \right\}$$

$$x(t) \overset{FT}{\longleftrightarrow} X(f)$$

$$\delta(t-nT_s) \overset{FT}{\longleftrightarrow} f_s \cdot \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

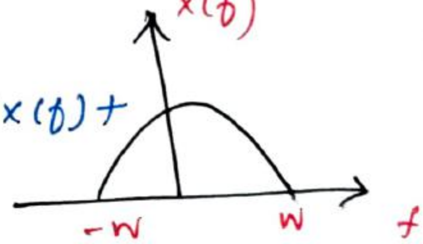
$$X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) \cdot \delta(f-nf_s)$$

$$\boxed{X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)}$$ By shifting property

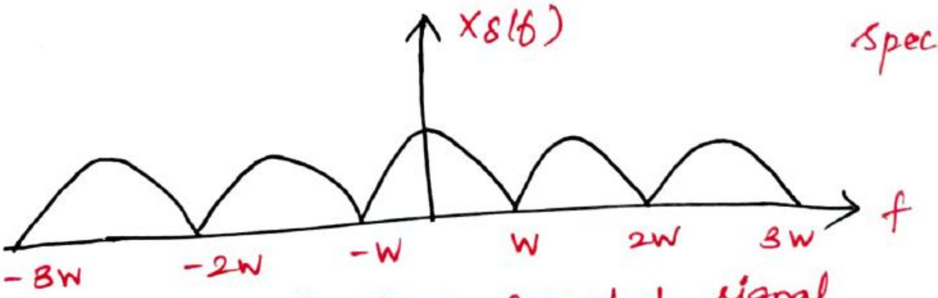
Spectrum of sampled signal

$= \cdots f_s X(f-2f_s) + f_s X(f-f_s) + f_s(X(f) + f_s X(f-f_s) + f_s \cdot X(f-2f_s) + \cdots$



$X_\delta(f)$

Spectrum of original signal X(f)



Spectrum Sampled signal

iii) Relation between X(f) and Xδ(f)

$$X_\delta(f) = f_s \; X(f)$$

$$\boxed{X(f) = \frac{1}{f_s} X_\delta(f)}$$

iv) Relation between x(t) and x(nTs)

$$X(f) = \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) \, e^{-j2\pi f n T_s}$$

$$x(t) = \text{IFT} \left\{ \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) \, e^{-j2\pi f n T_s} \right\}$$

$$x(t) = \int_{-W}^{W} \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) \, e^{-j2\pi f n T_s} \cdot e^{j2\pi f t} \, df$$

$$x(t) = \sum_{n=-\infty}^{\infty} x(nTs) \frac{1}{fs} \int_{-W}^{W} e^{j2\pi f(t-nTs)} \, df$$

$$= \sum_{n=-\infty}^{\infty} x(nTs) \cdot \frac{1}{fs} \left[\frac{e^{j2\pi f(t-nTs)}}{j2\pi(t-nTs)} \right]_{-W}^{W}$$

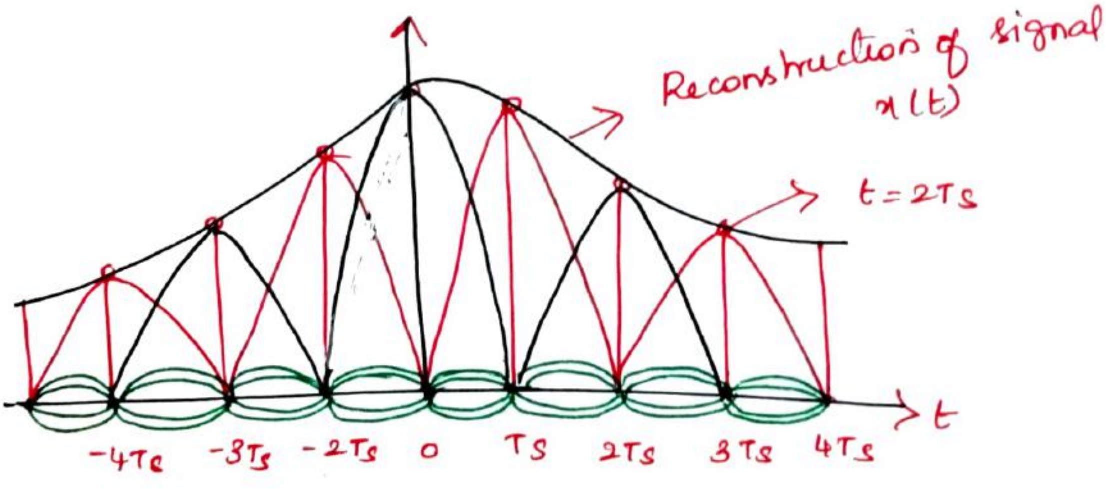
$$= \sum_{n=-\infty}^{\infty} x(nTs) \cdot \frac{1}{fs} \left\{ \frac{e^{j2\pi W(t-nTs)} - e^{-j2\pi W(t-nTs)}}{j2\pi(t-nTs)} \right\}$$

$$= \sum_{n=-\infty}^{\infty} x(nTs) \cdot \frac{1}{fs} \cdot \frac{\sin 2\pi W(t-nTs)}{\pi(t-nTs)}$$

$$= \sum_{n=-\infty}^{\infty} x(nTs) \cdot \frac{\sin[\pi(2Wt-n)]}{\pi(2Wt-n)}$$

$$fs = 2W \; ; \quad Ts = \frac{1}{fs} = \frac{1}{2W}$$

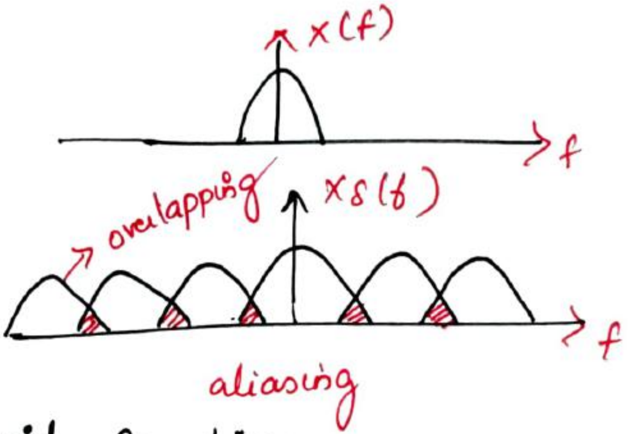
$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} x(nTs) \cdot \operatorname{sinc}(2Wt-n)}$$



Reconstruction of x(t) from its samples

Effect of under Sampling (Aliasing)

→ when the high frequency interferes with low frequency and appears as low frequency, then the phenomenon is called aliasing.



To avoid Sampling:

→ sampling rate $f_s \geq 2W$

→ Strictly bandlimit the signal to 'W'

Nyquist rate

→ when the sampling rate becomes exactly equal to 2W. then it is called Nyquist rate.

$$\text{Nyquist rate} = 2W \text{ HZ}.$$

Nyquist interval:

→ it is the time interval between any two adjacent samples when sampling rate is Nyquist rate.

$$\text{Nyquist interval} = \frac{1}{2W} \text{ Seconds}$$

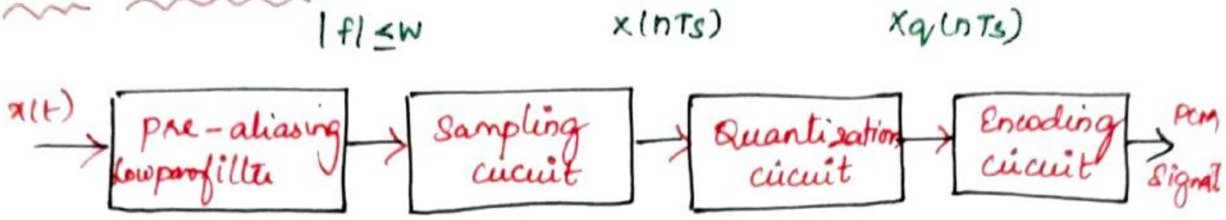
Sampling Bandpass signals

→ The bandpass signal x(t) whose maximum bandwidth is 2W can be completely represented into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth.

* * *

Pulse code Modulation (PCM) Generation and Reconstruction :

PCM Generator :



→ x(t) is passed through pre-aliasing low pass filter of cut off frequency 'W' HZ. all frequency components higher than 'W' Hz are blocked.

→ sampling frequency $f_s \geq 2W$

→ Quantization circuit quantizes the sampled signal to finite quantization levels.

→ The quantized signal takes any one of the 'q' quantization levels.

→ Quantized signal is then represented by finite number of digits. For Quantization level of '7' will be encoded as 111 by 3 bits. The encoded signal is called PCM signal.

PCM Receiver :

→ The PCM signal is given to regenerator circuit. The regenerator reshapes the pulses and removes the noise.

Transmission Bandwidth in PCM:

→ Let the quantizer use 'v' number of binary digits to represent each level. Then the number of levels that can be represented by 'v' number of digits will be

$$q = 2^v$$

q → total number of digital levels of quantizer

signaling rate in PCM : $r = v f_s$

Transmission Bandwidth = $\begin{cases} B_T \geq \frac{1}{2} r \\ B_T \geq \frac{1}{2} v f_s & \text{since } f_s \geq 2W \\ B_T \geq v W \end{cases}$

Advantages:

→ Effect of channel noise and interference is reduced.

→ Multiplexing of various PCM signals is easily possible.

→ Encryption or decryption can be easily incorporated for security purpose.

Disadvantages:

→ PCM system are complex compared to analog pulse modulation methods.

→ The channel bandwidth is also increased.

Applications:

→ Telephone communication

→ Multimedia Communication

→ Space Communication

→ Speech processing in raw a format

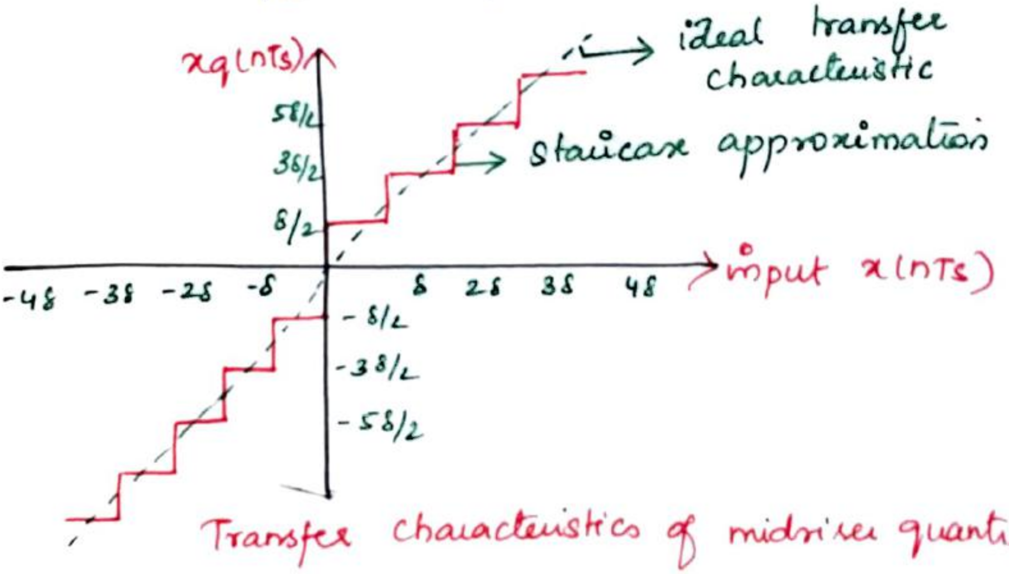
→ Satellite communication

Uniform Quantization and Quantization Noise:

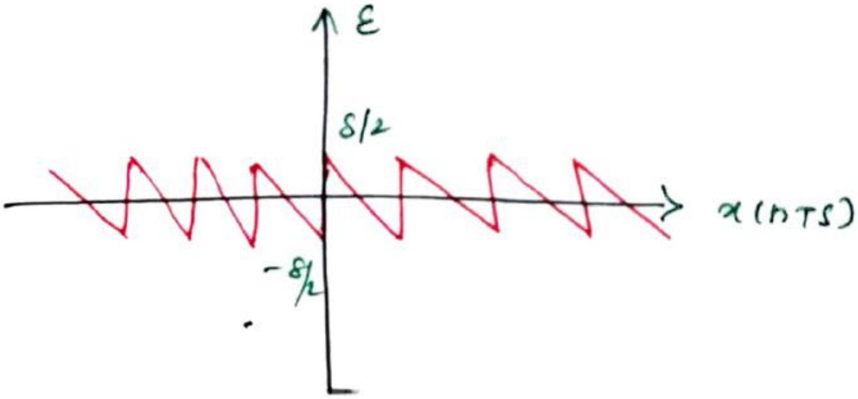
→ sample value is quantized to nearest digital level. This quantization can be uniform or non uniform.

→ In uniform Quantization step or difference between two quantization levels remains constant over the complete amplitude range.

$$\varepsilon = x_q(nT_s) - x(nT_s)$$
$$= \delta/2 - 0 = \delta/2$$



Transfer characteristics of midriser quantizer



Quantization error

$$\varepsilon_{max} = \left|\delta/2\right|$$

$$-\delta/2 \le \varepsilon \le \delta/2$$

Derivation of Quantisation Error for Uniform Qunatization :

$$\varepsilon = x(nTs) - x(nTs)$$

$$\text{Total amplitude range} = x_{max} - (-x_{max})$$
$$= 2\,x_{max}$$

$$\delta = \frac{x_{max} - (-x_{max})}{q}$$

$$= \frac{2x_{max}}{q}$$

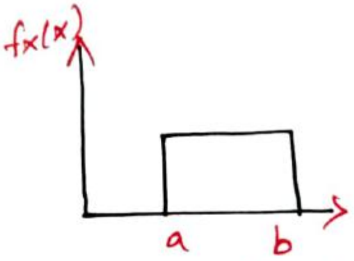
$x_{max} = 1$

$-x_{max} = -1 \quad ; \quad \delta = \frac{2}{q}$

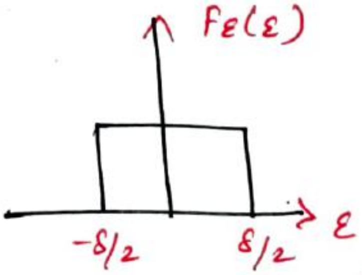
$\delta \rightarrow$ Step size

$\varepsilon \rightarrow$ Quantization error

$$\varepsilon_{max} = \left|\delta/2\right|$$



Uniform distribution



Uniform distribution for Quantization error

$$f_x(x) = 0 \qquad \text{for } x \le a$$
$$= \frac{1}{b-a} \qquad \text{for } a < x \le b$$
$$= 0 \qquad \text{for } x \ge b$$

Normalised noise power $= \frac{\delta^2}{12}$

(or)

Quantization error

Derivation of maximum signal to Quantization Noise ratio for Linear Quantization;

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

$$\delta = \frac{2x_{max}}{2^{v}}$$

$$= \text{Normalized signal power} \,/\, (\delta^2/12)$$

$$\frac{S}{N} = \frac{P}{\left(\frac{2x_{max}}{2v}\right)^2/12} = \frac{3P}{x^2_{max}} \cdot 2^{2v}$$

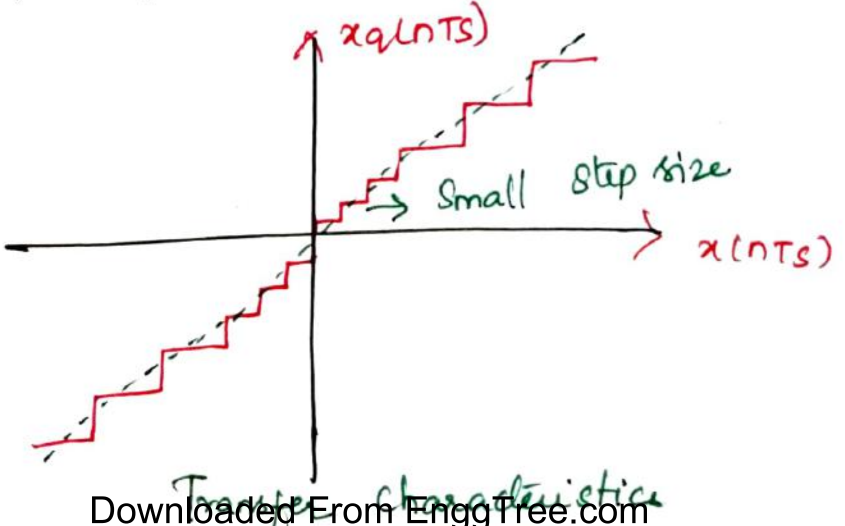
$$\boxed{\frac{S}{N} = \frac{3P}{x^2_{max}} \cdot 2v}$$

signal to Quantization noise ratio for normalized values of power : $\left(\frac{S}{N}\right) dB \leq (4.8 + 6v)\, dB$

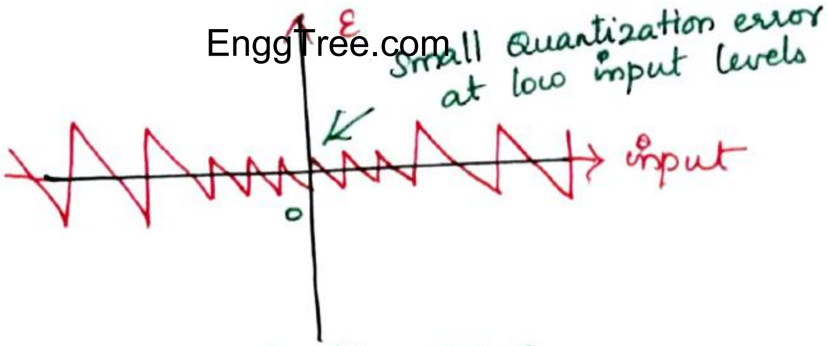
—— × ——

Non Uniform Quantization and Logarthemic Companding

→ For Uniform Quantization, the Step size is not fixed. it varies according to certain Law or as per input signal amplitude.



Transfer characteristics



Quantization error

→ speech and music signals are characterised by large crest factor

$$\text{crest factor} = \frac{\text{Peak Value}}{\text{RMS Value}}$$

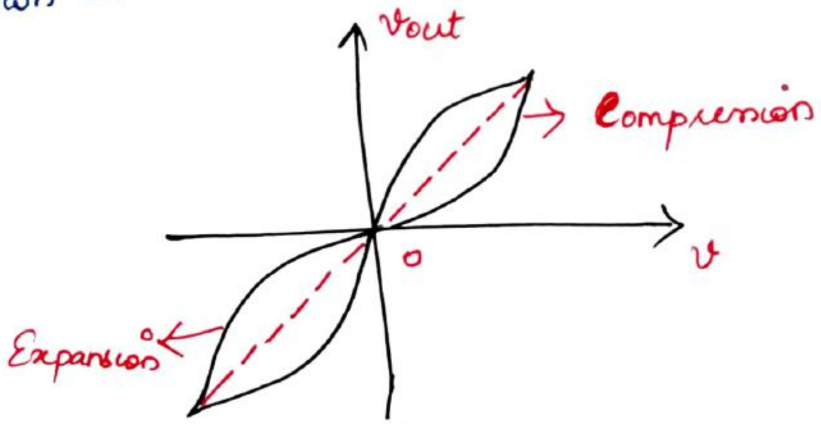
$$= \frac{x_{max}}{\sqrt{P}} = \frac{1}{\sqrt{P}}$$

$$\left(\frac{S}{N}\right) dB \geq (4.8 + 6v)\, dB$$

Normalize the signal power

Logarithmic Companding:

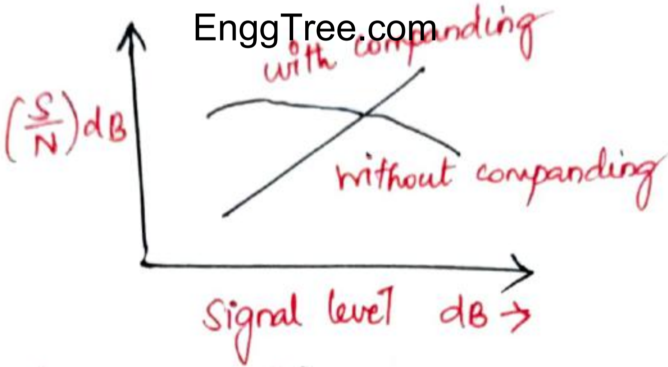
→ Compression of signal at transmitter and expansion at receiver is called combinely as companding.



Companding curve.

μ - Law Companding for Speech signals.

$$z(x) = (\text{sgn}\, x) \frac{\ln(1 + \mu|x|)}{\ln(1+\mu)} \quad |x| \leq 1$$



A - Law for companding :

→ A Law provides piecewise compressor characteristic.

→ A = 1

$$z(x) = \begin{cases} \frac{A|x|}{1+\ln A} & \text{for } 0 \le |x| \le \frac{1}{A} \\ \frac{1+\ln(A|x|)}{1+\ln A} & \text{for } \frac{1}{A} \le |x| \le 1 \end{cases}$$

Signal to noise ratio of companded PCM

for μ - Law companding

$$\frac{S}{N} = \frac{3q^2}{[\ln(1+\mu)]^2}$$

for A Law companding

$$\left(\frac{S}{N}\right) = \frac{3q^2 \cdot A^2}{(1+\ln A)^2}$$

for μ - Law companding, $G_c = \frac{\mu}{\ln(1+\mu)}$

for A - Law companding, $G_c = \frac{A}{1+\ln A}$

—— × ——

pulse Analog Modulation:

→ The modulating signal can modulate amplitude, width (duration) or position of the pulse.

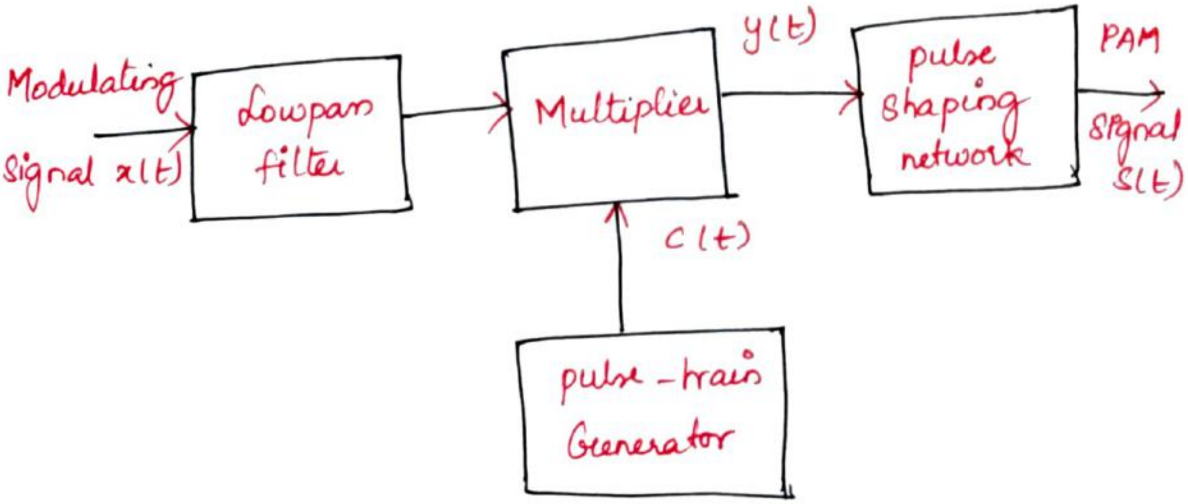
→ PAM (pulse Amplitude Modulation)
→ PWM (pulse width Modulation)
→ PPM (pulse position Modulation)

Pulse Amplitude Modulation (PAM)

→ The amplitude of the pulse is directly proportional to amplitude of the modulating signal at the sampling instant.

→ width and position of the pulse remains unchanged.

Generation of PAM:

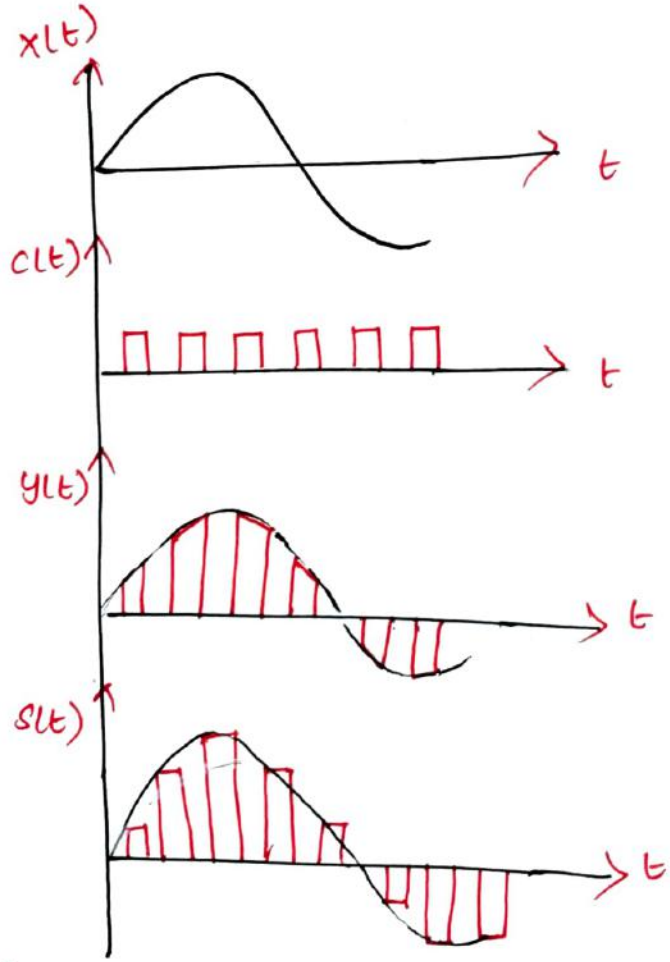


→ modulating signal x(t) is the signal to be transmitted and given to low pass filter.

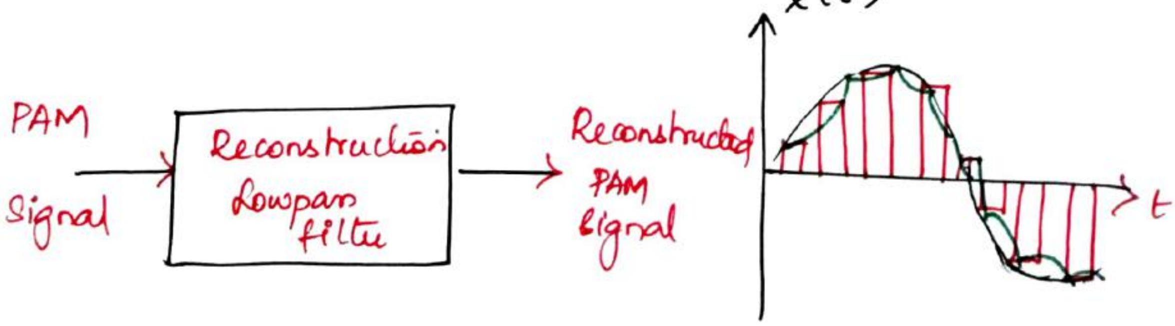
→ Lowpass filter perfoms band limiting. which avoids aliasing.

→ Band limited signal is then sampled at the multiplier. The multiplier samples x(t) with the help of a pulse train generator.

→ Multiplication of x(t) and c(t) produces y(t). → pulse shaping network produces flat top pulses.



Detection :



→ PAM Signal is passed through a lowpass reconstruction filter.

Transmission Bandwidth:

$$f_s \geq 2W$$

$$T_s \leq \frac{1}{2W} \quad \text{since } f_s = \frac{1}{T_s}$$

$$t << T_s \leq \frac{1}{2W}$$

$$f_{max} = \frac{1}{\tau + \tau} = \frac{1}{2\tau}$$

$$B_T \geq f_{max} >> W$$

$$B_T \geq \frac{1}{2\tau} \quad ; \quad B_T >> W$$

→ transmission bandwidth of PAM signal is very large compared to highest frequency.

Advantages:

→ PAM can be easily generated and detected

→ PAM forms the basis of many other pulse modulation techniques such as PCM, DM & ADM.

Disadvantages:

→ Interference of noise is maximum

→ PAM Varies, peak power required by the transmitter with modulating signal.

Applications:

→ PAM is used in instrumentation systems.

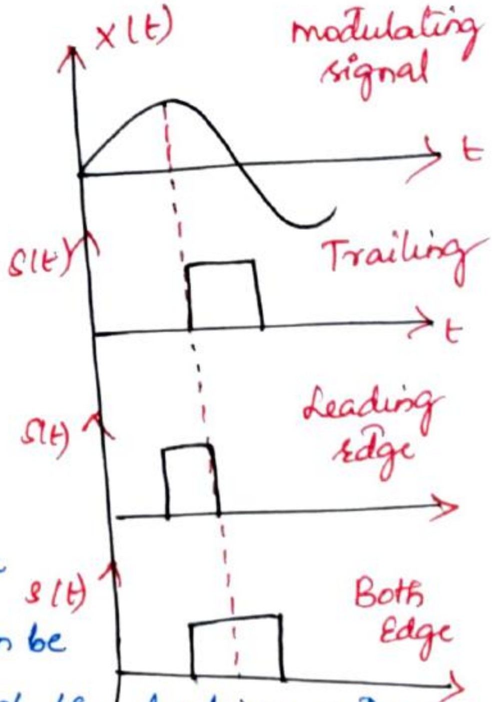
→ it is used in Analog to Digital Converters for computer interfacing.

—— X ——

Pulse width Modulation (PWM) :

→ pulse width modulation is also called as pulse duration modulation (PDM).

→ width of the pulse is directly proportional to amplitude of the modulating signal at the Sampling instant.



Trailing edge modulation:

→ position of leading edge fixed. position of trailing edge is changed according to required width of the pulse.

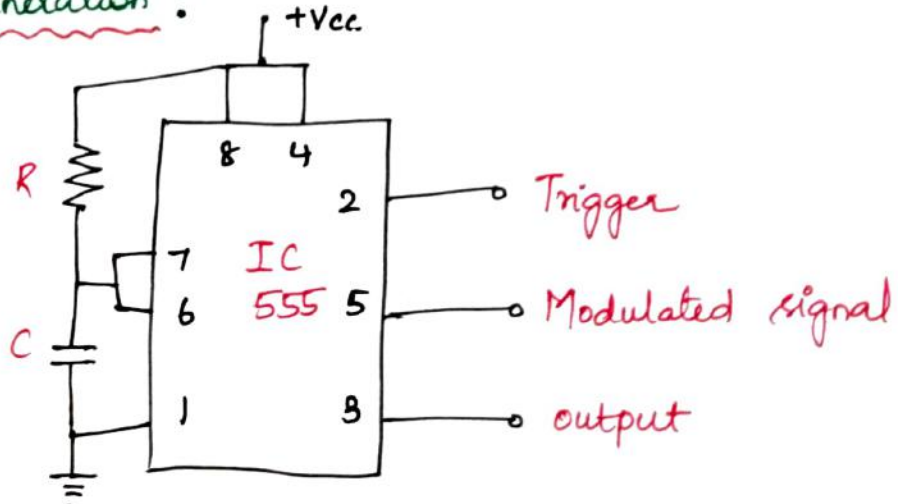
Leading edge modulation:

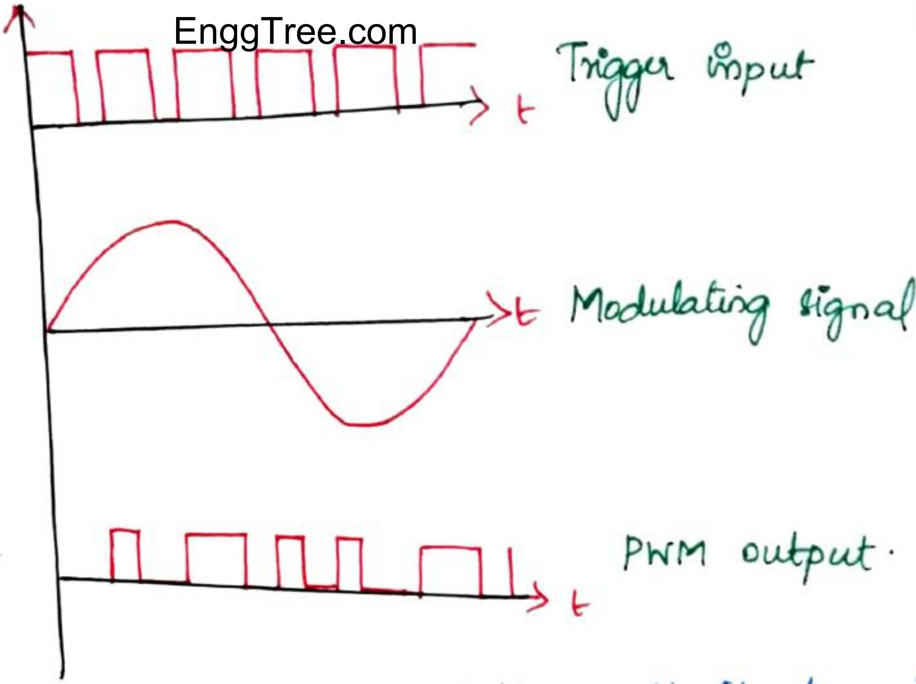
→ position of trailing edge is fixed. width of the pulse can be changed by varying the position of the leading edge.

Leading and Trailing edge modulation:

→ In this method center of the pulse is kept fixed at Sampling instant.

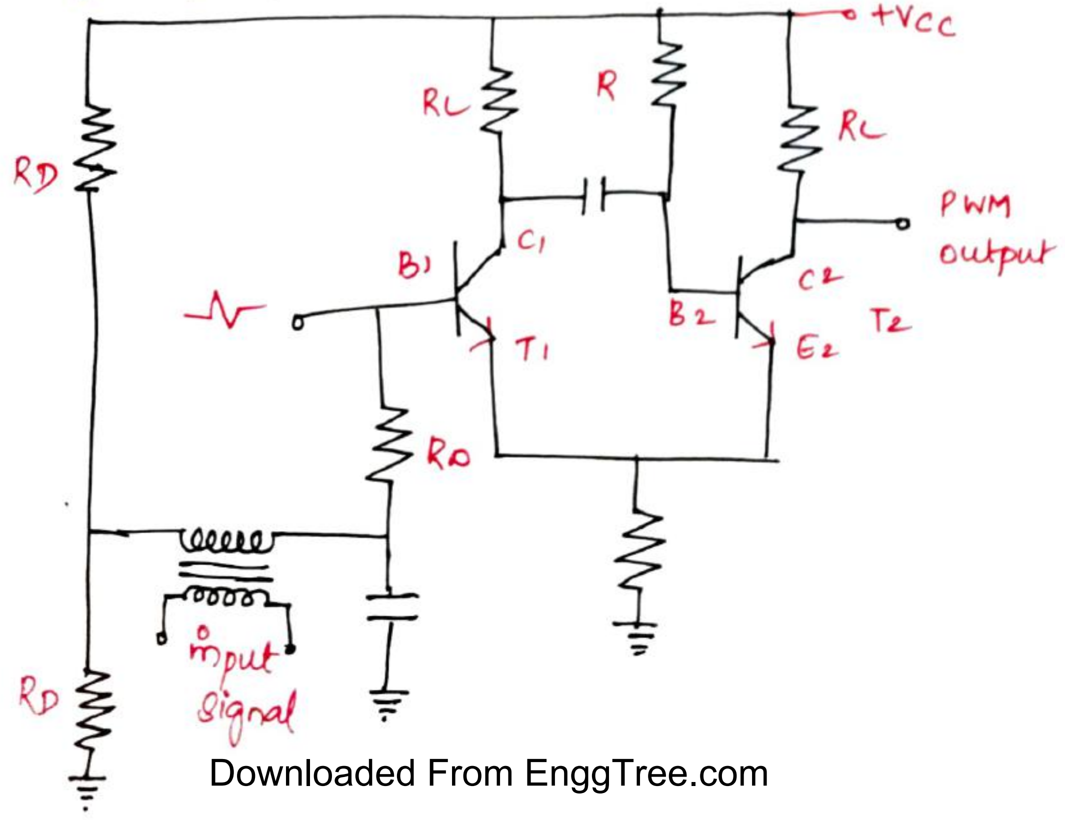
PWM Generation:





→ it is basically monostable multivibrator, with a modulating input signal applied at the control Voltage input.

→ Externally applied modulating signal changes the control voltage. As the result, time period required to charge the capacitor up to threshold voltage level changes, giving pulse modulated signal at the output.

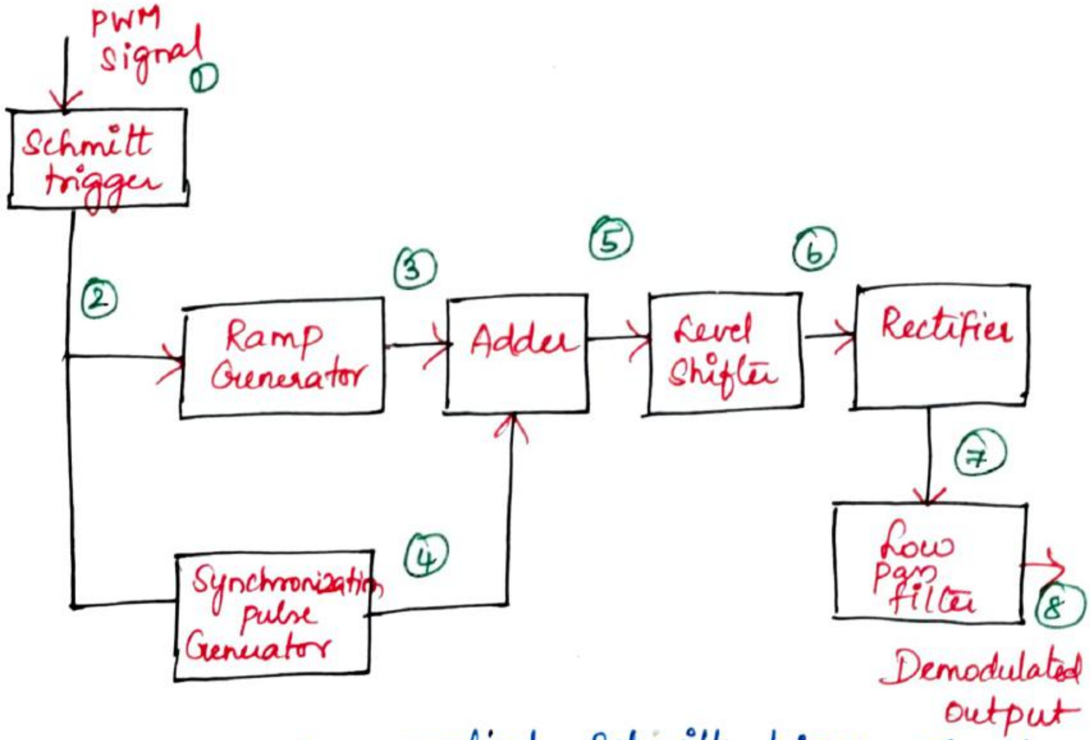


→ The Stable state for the positive trigger pulse at B1 switches T1 ON. B2 falls T2 switched OFF. RC becomes constant.

→ T1 is simultaneously switched OFF by regenerative action and stays OFF until the arrival of the next trigger pulse.

→ To make T2 ON, slightly more positive than voltage across RE. when the signal voltage is maximum Capacitor should charge to turn ON. T2 is also maximum.

Demodulation of PWM signal:



→ PWM pulse applied Schmitt trigger circuit. The Schmitt trigger circuit removes the noise in the PWM waveform.

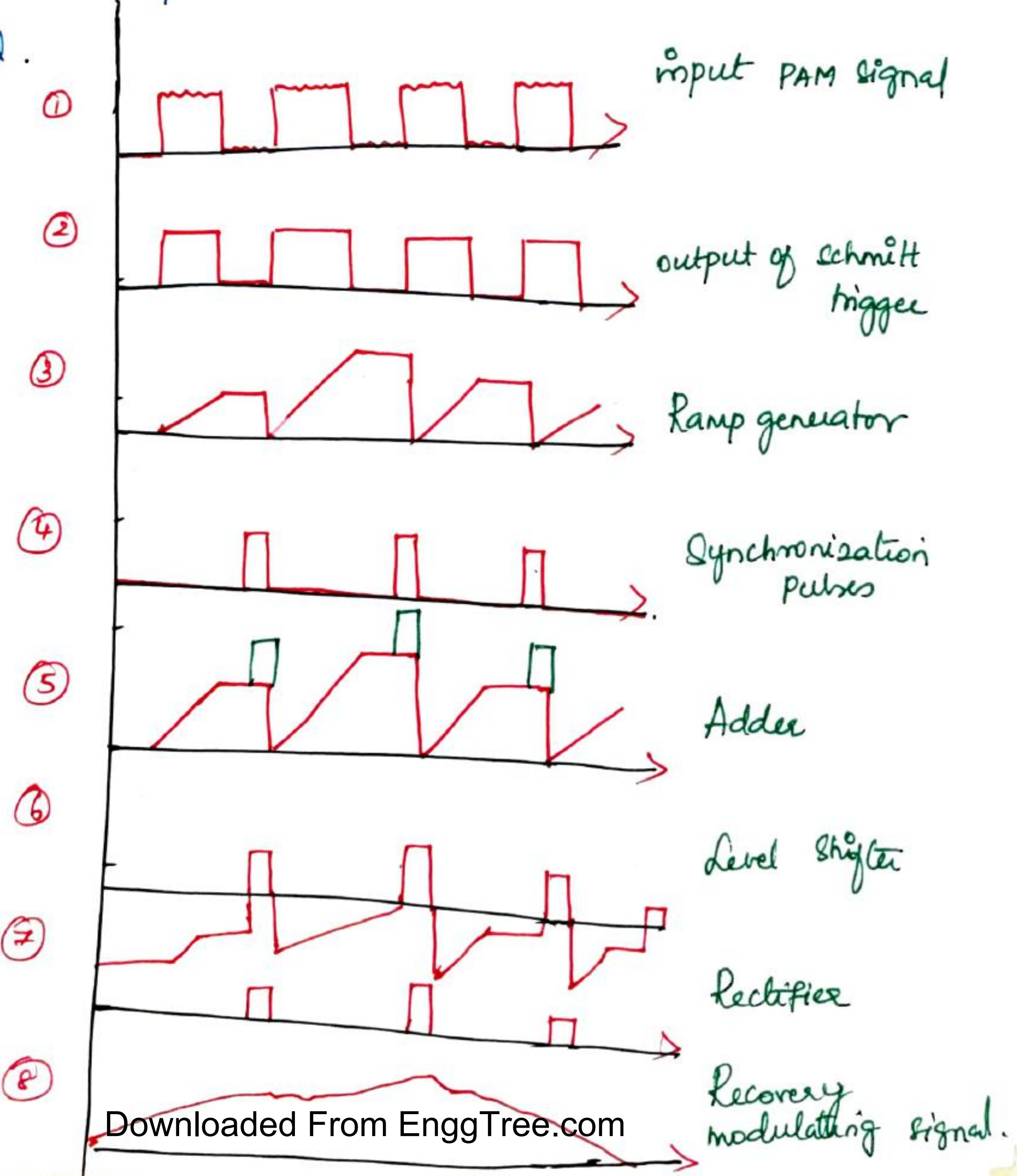
→ The regenerated PWM is then applied to the ramp generator and the Synchronization pulse Generator.

→ The ramp generator produces ramps for the duration of pulses such that height with constant amplitude and pulse width.

→ The delayed reference pulses and the output of ramp generator is added with the help of adder.

→ The output of adder is given to the level shifter. Negative offset waveform is clipped by rectifier.

→ finally the output of rectifier is passed through low pass filter to recover the modulating signal.



Advantages:

- → less noise
- → PWM communication does not require synchronization between transmitter and receiver.

Disadvantages:

- → pulses are varying in width and power content variable.
- → Large bandwidth required.

Applications:

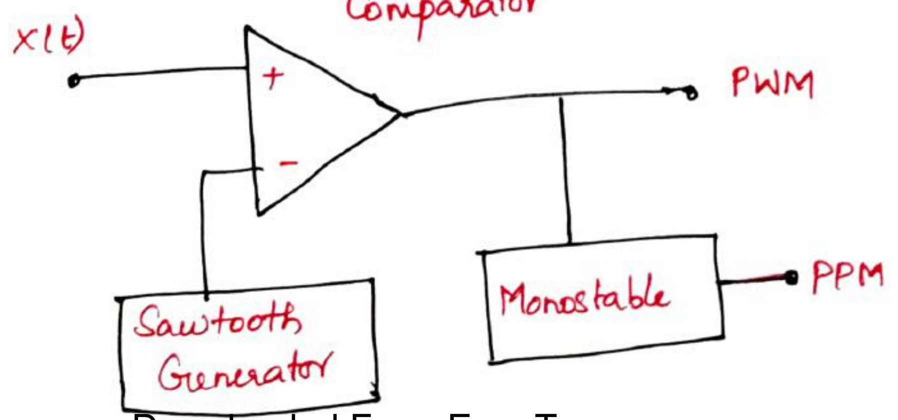
- → Used for asynchronous transmission over noisy channel.
- → PWM is used for generate PPM.
- → Motor control.

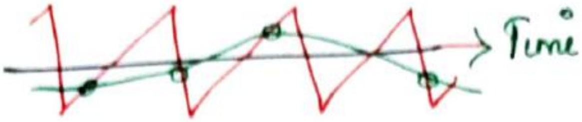
— × —

pulse position Modulation (PPM)

→ The amplitude and width of the pulse are kept constant, while the position of each pulse, with reference to the position of a reference pulse is changed according to the instantaneous sampled value of the modulating signal.

Generation of PPM:





→ Sawtooth generator generates the sawtooth signal of frequency fc. The Sawtooth signal, also called sampling signal is applied to the inverting input of comparator.

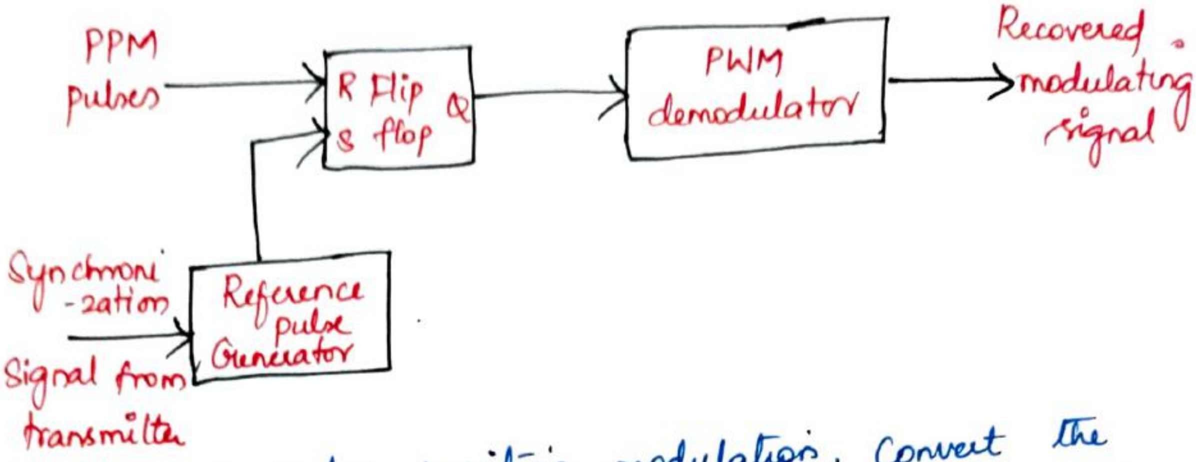
→ modulating signal x(t) is applied to the inverting input of the comparator.

→ The output of comparator is high when instantaneous Value of x(t) higher than the Sawtooth waveform. The leading edge of PDM signal occurs at the fixed time period.

→ when the sawtooth voltage is greater than x(t), output of Comparator remains zero.

→ PDM signal is nothing but output of Comparator. To generate pulse position modulation trigger input to one monostable multivibrator. Monostable output is zero until it is triggered

Demodulation of PPM signal :

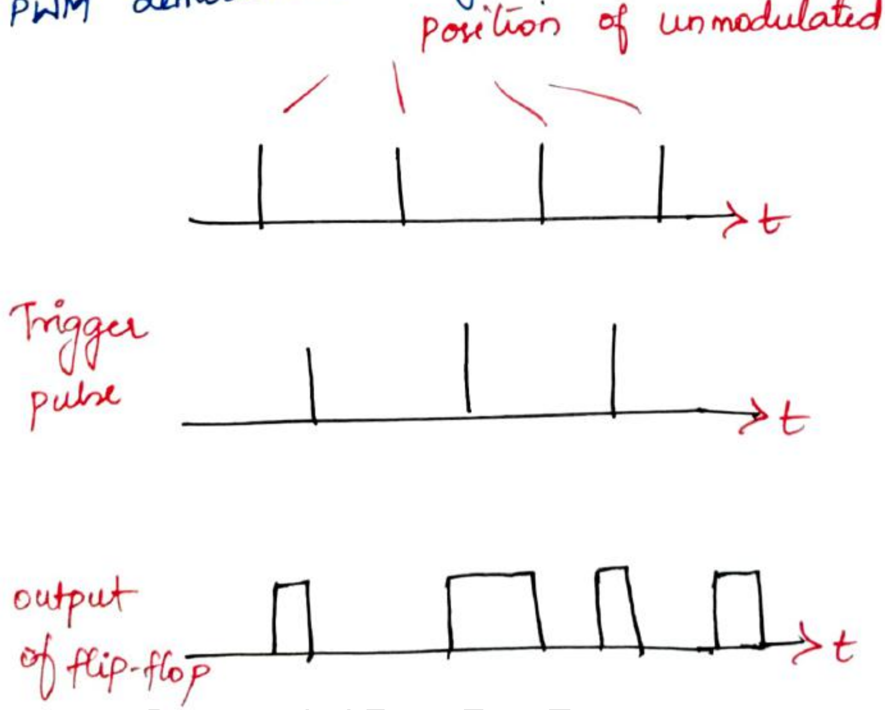


→ pulse position modulation, Convert the received pulses that vary in position to pulses that vary in length.

→ flip-flop circuit is set or turned ON when the reference pulse arrives.

→ Reference pulse generated from pulse generator. flip flop circuit is reset or turned OFF edge of the position modulated pulse.

→ PWM pulses are then demodulated by PWM demodulator to get original modulating signal.



Advantages:

→ Less noise interference
→ signal to noise separation is very easy.

Disadvantages:

→ synchronization between transmitter and receiver is required.
→ Large bandwidth is required as compared to PAM.

Applications:

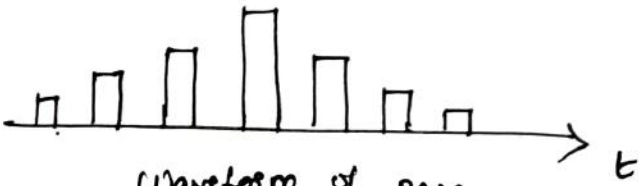
→ Synchronous Communication of analog pulses over short distances.

— × —

Comparison between pulse modulation methods

pulse amplitude modulation:

→ Amplitude of the pulse is proportional to amplitude of modulating signal.
→ Bandwidth of the transmission channel depends on width of the pulse.
→ Instantaneous power of the transmitter varies.
→ Noise interference is high. System is complex.
→ Similar to amplitude modulation



Waveform of PAM

Pulse width / Duration modulation :

→ width of the pulse is proportional to amplitude of modulating signal.

→ Bandwidth of transmission channel depends on rise time of the pulse.

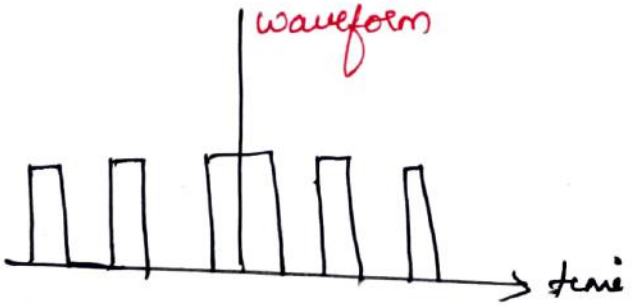
→ Instantaneous power of the transmitter varies.

→ Noise interference is minimum.

→ Simple to implement.

→ similar to frequency modulation

waveform



pulse position modulation

→ The relative position of the pulse is proportional to the amplitude of modulating signal.

→ Bandwidth of transmission channel depends on rising time of the pulse.

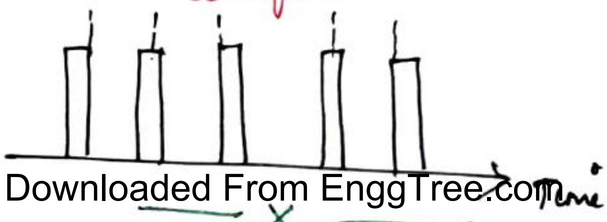
→ Instantaneous power of the transmitter remains constant.

→ Noise interference is minimum.

→ simple to implement.

→ Similar to phase modulation

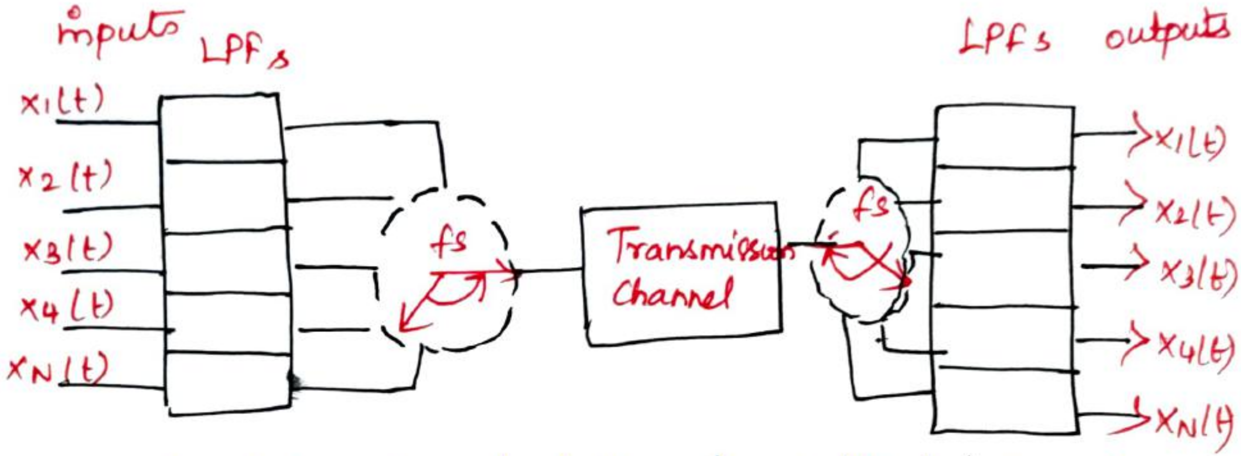
waveform



Time Division Multiplexing (TDM) :

→ The signals from different channels are transmitted over a common transmission line. The time spacing between the samples of one channel is occupied by other channels to improve utilization of the channel.

→ It is called Time division multiplexing.



→ Each channel to be transmitted is passed through the low pass filter. The outputs of the lowpass filter are connected to the rotating sample which is commutator.

→ it takes sample from each channel per revolution and rotates at the rate of fs.

→ signal is composed due to multiplexing of input channels is given to the transmission channel.

→ At the receiver decommutator separates the time multiplexed input channels.

→ Highest frequency content removed by Low pass filter.

$f_s \geq 2W$, $T_s = \frac{1}{f_s}$

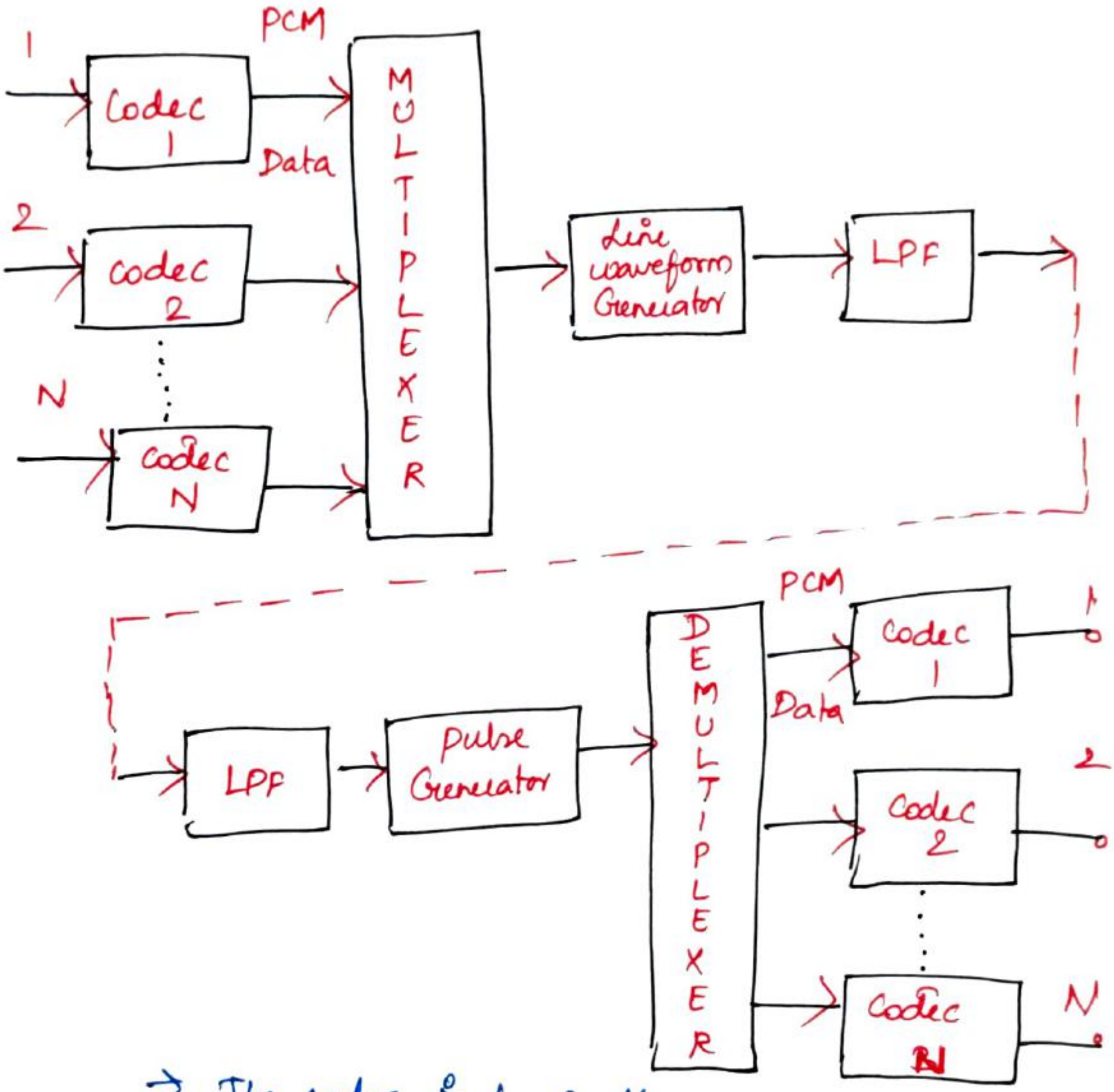
$T_2 \leq \frac{1}{2W}$

$B_T = NW$

N → No. of channel

W → Band limited to W

TDM system :



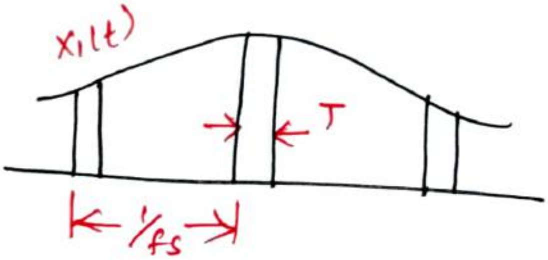
→ The coder is basically PCM encoder and decoder. At the receiver side, codec receives serial PCM data and generates analog signal. sampling frequency of PCM can be selected by External clock.

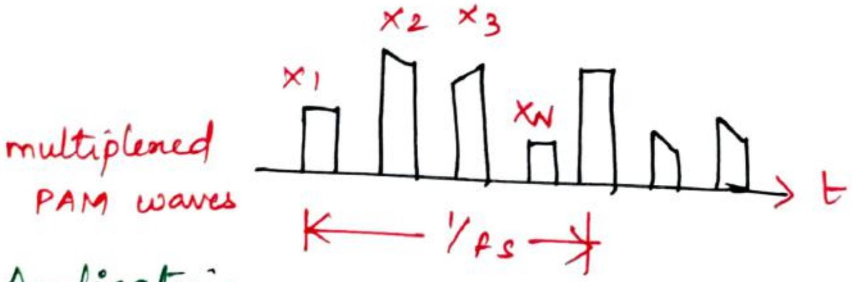
→ output of [illegible] codec is combined by the multiplexer into single bit stream.

→ Bit stream is converted to baseband waveform by line waveform generator.

→ LPF bandlimits the baseband signal

The waveform regenerator is used at the receiver to construct the input noisy waveform to clear digital signal.





Applications :

→ Transmission of telephone calls over 4-wire copper cable.

→ GSM Mobile systems

→ TDMA and data links

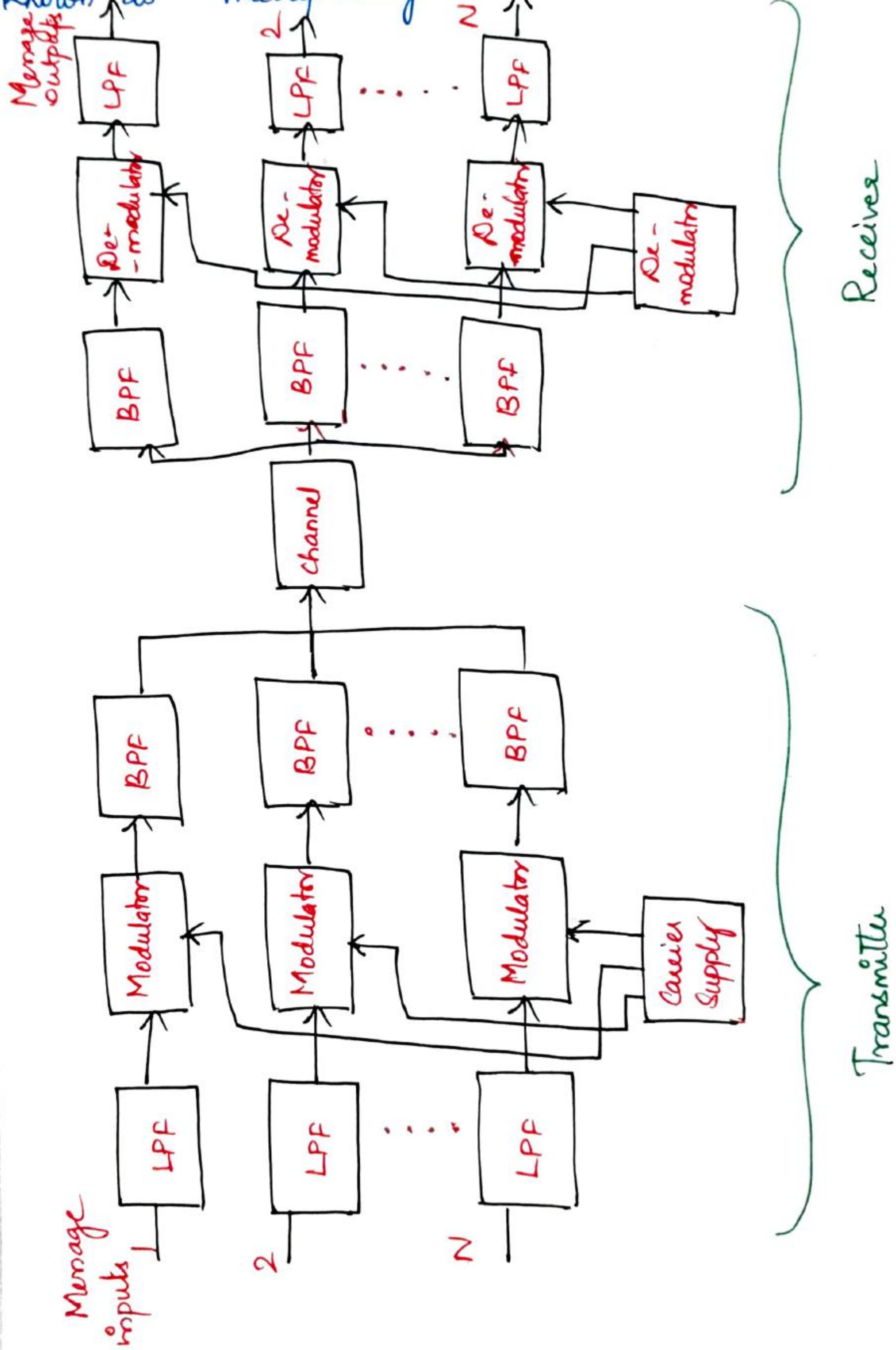
→ Rate interface for ISDN

→ SDM and SONET standards use TDM

— X —

Frequency Division Multiplexing : (FDM)

→ Number of transmitters are transmitting simultaneously on the same channel. This process is known as multiplexing.



→ Multiplexing techniques can be separating by frequency is called frequency division multiplexing.

→ input message signals, passed to Low pass filters. it removes high frequency signals.

→ filtered signal modulated by modulator. The Band pass filter restrict the band of each modulated wave to its prescribed ranged.

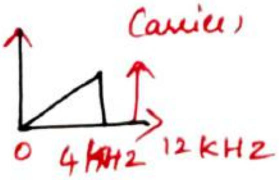
→ The outputs of bandpass filter are Combined in parallel which form the input to the Common channel.

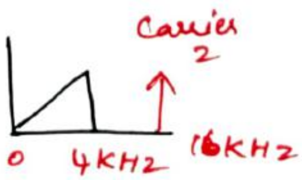
→ At the receiving end, Band pass filter Connected to the common channel separate the message signals on the frequency occupancy basis.

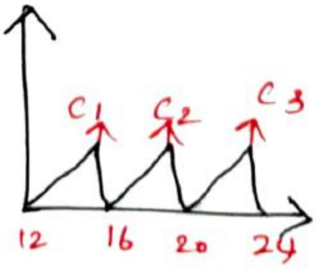
→ finally, the original signals are recovered by individual demodulators.

Transmission Bandwidth:

$$B = 24 \times 4 = 96\ KHZ.$$







3 - Channels forming pre-group.

→ Each pregroup modulator provides three carrier oscillators at 12, 16, 20 KHZ.

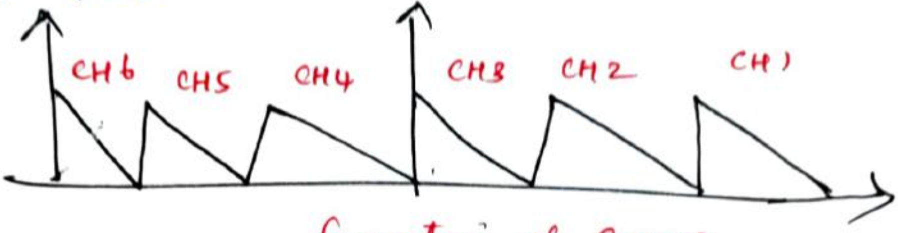
→ 0-4 KHZ ⇒ channel 1

→ 12 KHZ -16 ⇒ channel 2

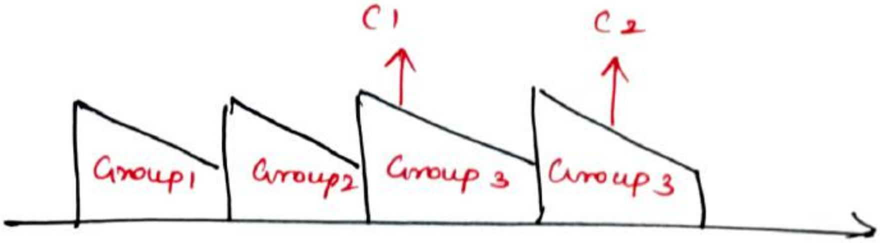
→ 20 - 24 KHZ ⇒ channel 3

→ All carrier are suppressed and output from four modulator added in group.

→ Each master group modulator provides 16 carrier frequencies.

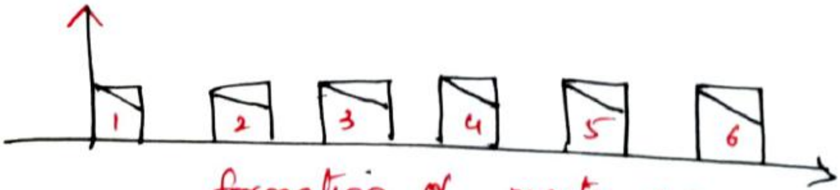


formation of group



formation of super group

→ output of 16 supergroups are added to master group



formation of master group

→ Therefore complete signal contains 960 FDM channels each 4KHz wide in frequency range from 60 to 4028 KHZ.

—— X ——

UNIT - 2 Random process and Sampling

2 Marks

1) when the random process is said to be Strict sense Stationary?

→ The statistical properties do not change with shift of time origin

→ stationary process should start at t = -∞ and should not stop till t = ∞ and t = -∞ and

—— × ——

2) write Einstein - wiener - khintchine relation

→ wiener - khintchine relations give the relationship between autocorrelation function and power Spectral density

—— × ——

3) what is mean by ergodic process?

→ A random process is called ergodic process if time averages are equal to ensemble averages.

$$m_x(t) = m_x(\tau)$$

$$R_x(t_1 t_2) = R_x(\tau, T)$$

—— × ——

4) what is capture effect in FM?

→ when the noise interference is stronger than FM signal, FM Receiver locks to interference.

→ This suppresses FM signal. when the noise interference as well as FM signal are of equal strength, then FM receiver locking fluctuates between them.

→ This phenomenon is called capture effect

—— × ——

5) what is the purpose of pre-emphasis and de-emphasis in FM?

→ The PSD of noise at the output of FM receiver rapidly at high frequencies but the PSD of message signal falls off at higher frequencies.

→ Message signal does't utilise the frequency band in efficient manner.

→ Efficient use of frequency band and improved noise performance can be obtained with the help of pre-emphasis and de-emphasis of FM.

——×——

6) when the carrier to noise ratio is high, how will you get figure of merit of FM system?

→ In case of high carrier to noise ratio the transmission bandwidth BT is increased.

→ it provides quadratic increase in figure of merit of the FM system.

——×——

7) what is mean by aliasing effect?

→ Aliasing effect takes place when sampling frequency is less than Nyquist rate, under such condition, the Spectrum of the sampled signal overlaps with itself.

→ Hence, high frequencies take the form of lower frequencies. This interface of the frequency components is called aliasing effect.

——×——

8) State Sampling theorem

→ A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency $fs \geq 2W$.

→ fs → sampling frequency

→ W → maximum frequency content.

— × —

9) what is mean by Quantization?

→ while converting the signal value from analog to digital, Quantization is performed. Analog value is assigned to the nearest digital level. This is called Quantization.

→ Quantized value is then converted to equivalent binary value. The quantization levels are fixed depending upon the number of bits.

— × —

10) what is prefiltering done before sampling?

→ The signal must be limited to some highest frequency 'W' Hz before sampling. Then the signal is sampled at the frequency of $fs = 2W$ or higher.

→ Hence the signal should be prefiltered to eliminate any frequency components higher than 'W' Hz.

→ If the signal is not prefiltered, then frequency components higher than 'W' Hz will generate aliasing in the sampled spectrum.

— × —

11) what is companding?

→ Compression of the signal at the transmitter and expansion at the receiver is combinely called as Companding.

—— × ——

12) what do you mean by sampling rate?

→ Analog signal is sampled at regular time intervals. The number of samples taken per second is called Sampling rate or sampling frequency.

—— × ——

13) How the multiplexing of digital signals can be accomplished?

→ TDM : Time Spacing between the samples of one channel is occupied by samples of other channels. Thus multiple channels transmit over common transmission line. This improves utilization of the transmission line.

→ FDM : The frequency bands of different channels share the common transmission bandwidth. Frequency bands of individual channels are non-overlapping. This improves the utilization of bandwidth of the transmission line.

—— × ——

14) what is signal to noise ratio?

→ signal to noise ratio is defined as the ratio of average signal power to average noise power

$$SNR = \frac{\text{Average signal power}}{\text{Average noise power}}$$

15) Define Quantisation error

→ Because of quantisation, inherent error are introduced in the signal. This error is called quantisation error.

$$\varepsilon = xq(nTs) - x(nTs)$$

$xq(nTs)$ is quantised value of the signal

$x(nTs)$ is the value of the sample, before quantisation.

— × —

16) Mention the merits of DPCM - (UNIT-3)

→ Bandwidth ~~required~~ requirement of DPCM is less compared to PCM.

→ Quantisation error is reduced because of prediction filter.

→ Number of bits used to represent one sample value are also reduced compared to PCM

— × —

17) what is the main difference in DPCM and DM (UNIT-3)

→ DM encodes the input sample by only one bit. It sends the information about +δ or -δ. (i.e) step rise or fall.

→ DPCM can have more than one bit for encoding the sample. It sends the information about difference between actual sample value and predicted sample value.

— × —

18) what is the advantage of delta modulation over pulse modulation scheme? (UNIT - 3)

→ Delta modulation encodes one bit per sample. Hence signaling rate is reduced in DM.

→ Transmitter and receiver implementation is very much simple.

— x —

19) what is mean by delta modulation systems? (UNIT - 3)

→ Delta modulation transmits only one bit per sample. present sample value is compared with the previous sample value and the indication, whether the amplitude is increased or decreased is sent.

→ The input signal x(t) is approximated to step signal by the delta modulator. step size fixed.

— x —

20) Compare uniform and non - uniform Quantization.

Uniform Quantization	Non - uniform Quantization
→ Quantization step size remains same throughput the dynamic range of the signal.	→ Quantization step size varies according to specific law depending up on amplitude of the input signal
→ signal to noise ratio varies with input signal amplitude	→ signal to noise can be maintained constant with non uniform Quantization.

— x —

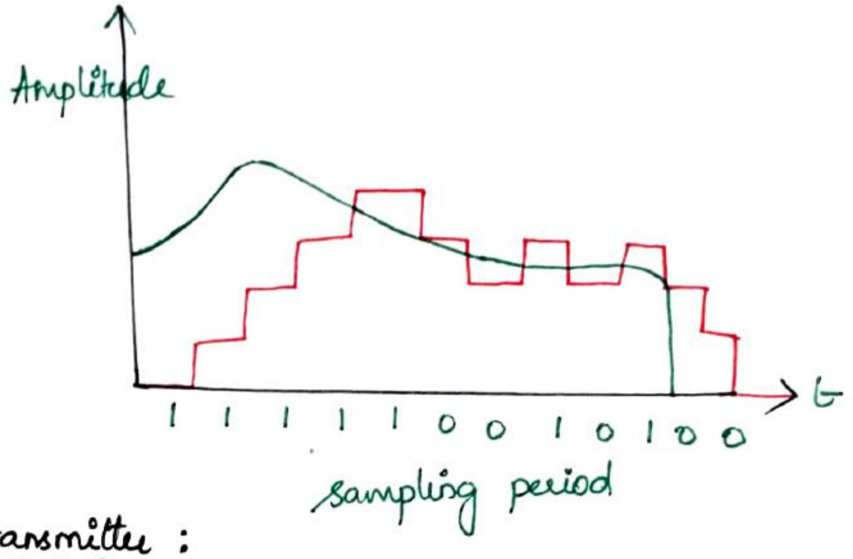
UNIT-2 Big Questions

1) Random process, Mean, Power spectral density and properties of PSD
2) Types of noise sources
3) AM - Receiver model
4) FM - Receiver model
5) pre-emphasis and de-emphasis
6) Low pass Sampling theorem
7) PCM
8) Uniform Quantization and Quantization noise
9) Non-uniform Quantization and logarithmic companding
10) PAM, PWM, PPM
11) TDM, FDM

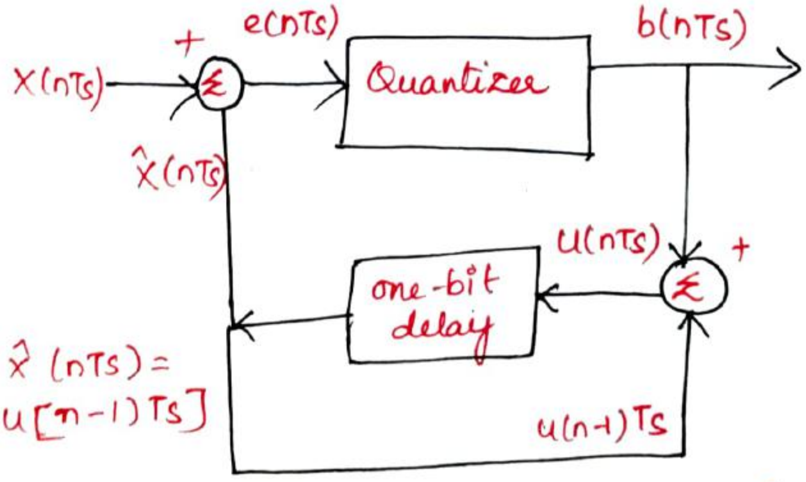
UNIT - 3 Digital Techniques

Delta Modulation :

→ Delta modulation transmits only one bit per sample. present sample value is compared with the previous sample value and the indication, whether the amplitude is increased or decreased.



DM Transmitter :



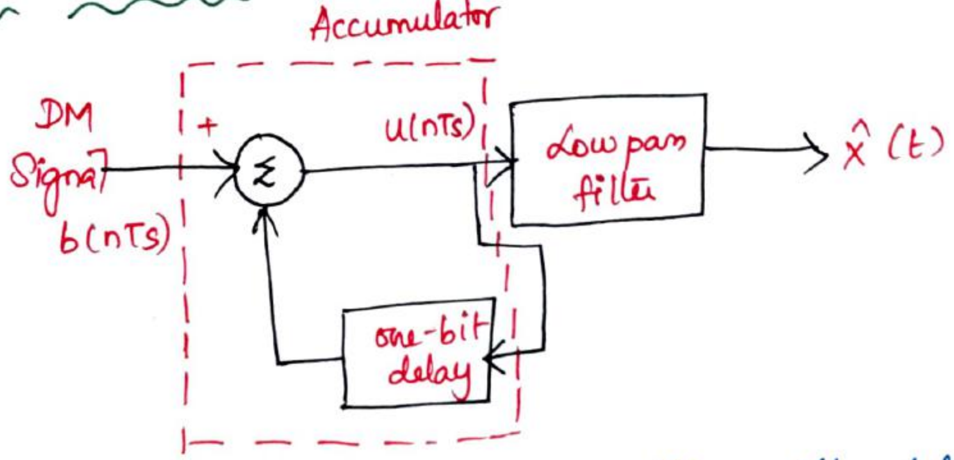
→ x(nTs) is the input signal and $\hat{x}$(nTs) is the reconstructed signal.

→ Error e(nTs) is the difference between input signal x(t) and reconstructed signal $\hat{x}$(nTs).

→ one bit quantizer provides $b(nTs) = +\delta$ if the error is positive and $b(nTs) = -\delta$ if the error is positive.

→ one bit delayed reconstructed signal $\hat{x}(nTs) = u[(n-1)Ts]$ is added with the DM signal. This provides the latest reconstructed signal.

DM Receiver:



→ DM signal is added with one bit delayed reconstructed signal $u(n-1)Ts$. This is accumulator operation.

→ The reconstructed signal is then passed through low pass filter for smoothing. The cutoff frequency of this low pass filter is equal to highest signal frequency in $x(t)$.

$$u(nTs) = u(nTs - Ts) + [\pm\delta]$$

$$= u[(n-1)Ts] + b(nTs)$$

→ The previous Sample approximation $u(n-1)Ts$ is restored by delaying one Sample period Ts.

→ The sampled input signal $x(nTs)$ and Staircase approximated signal $\hat{x}(nTs)$ are subtracted to get error signal $e(nTs)$.

→ At the receiver end accumulator and low pass filter are used.

→ Accumulator generates the staircase approximated signal output and is delayed by one Sampling period Ts.

→ if binary 1 then it add +δ step to the previous output. if the binary '0' step δ is subtracted from the delayed signal.

→ filter smoothen the staircase signal to reconstruct x(t).

$E_{max} = |\delta| \Rightarrow$ Quantization Error

Advantages :

→ Delta modulation transmits only one bit for one sample. Thus the signalling rate and transmission Channel bandwidth is quite small for delta modulation

→ Transmitter and receiver implementation is simple.

Disadvantages :

→ slope overload noise : The rate of rise of input signal x(t) is so high that the staircase signal cannot approximate it, Step size 'δ' becomes too Small for Staircase signal u(t) to follow the steep segment of x(t).

→ Large error between the Staircase approximated signal and original signal x(t). This error is called slope overload distortion. To reduce this error the step size should be increased when slope of signal of x(t) is high.

→ Granular Noise : Granular noise occurs when the step size is too large compared to small variations in the input signal. To make step size small we can reduce granular noise.

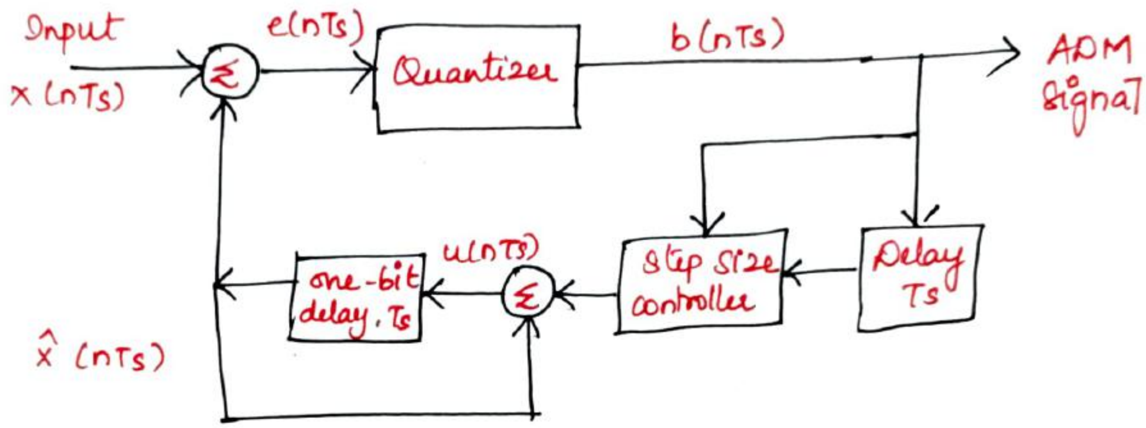
—— X ——

Adaptive Delta Modulation :

→ To overcome the quantization errors due to Slope overload and granular noise, the step size (δ) is made adaptive to variations in the input signal $x(t)$.

→ when the input is varying slowly, the step size is reduced. This method is called Adaptive Delta modulation (ADM).

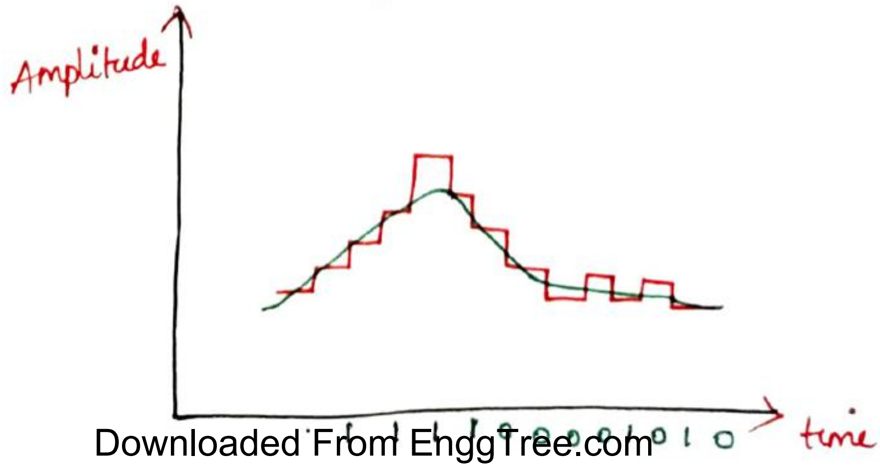
Transmitter :



→ Adaptive Delta modulators can take continuous changes in step or discrete changes in step size.

→ The step size increases or decreases according to certain rule depending on one-bit quantizer output.

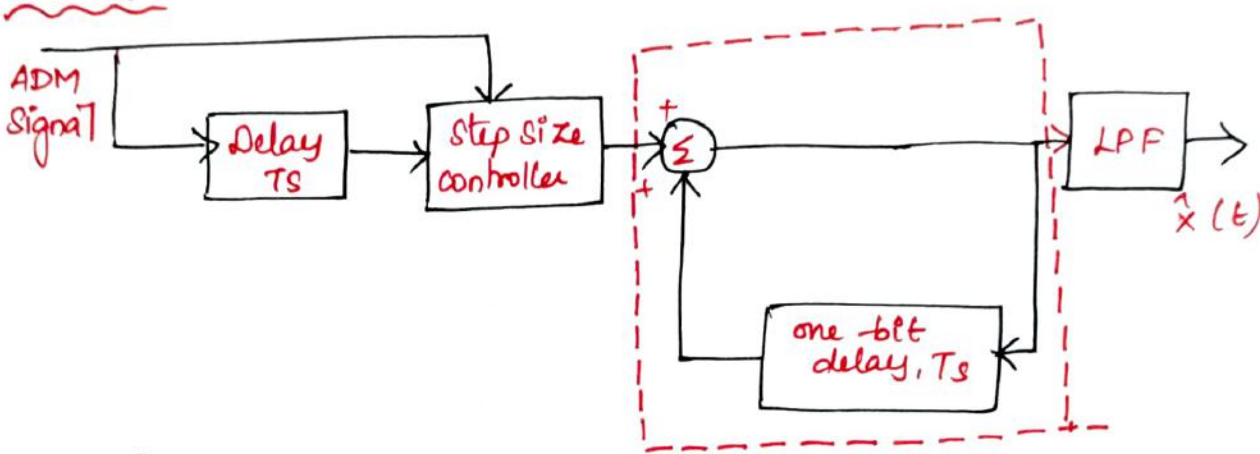
→ if one bit quantizer output is Low (0), then the step size may be reduced by one step. if output is high (1), step size may be doubled for next sample.



→ previous input and present input decides the step size.

→ Low pass filter then smoothens out the Staircase waveform to reconstruct the smooth signal.

Receiver :



Advantages :

→ signal to noise ratio is better than ordinary delta modulation because of the reduction in slope overload distortion and granular noise.

→ Because of the Variable step size, the dynamic range of ADM is wide.

→ utilisation of Bandwidth is better than delta modulation.

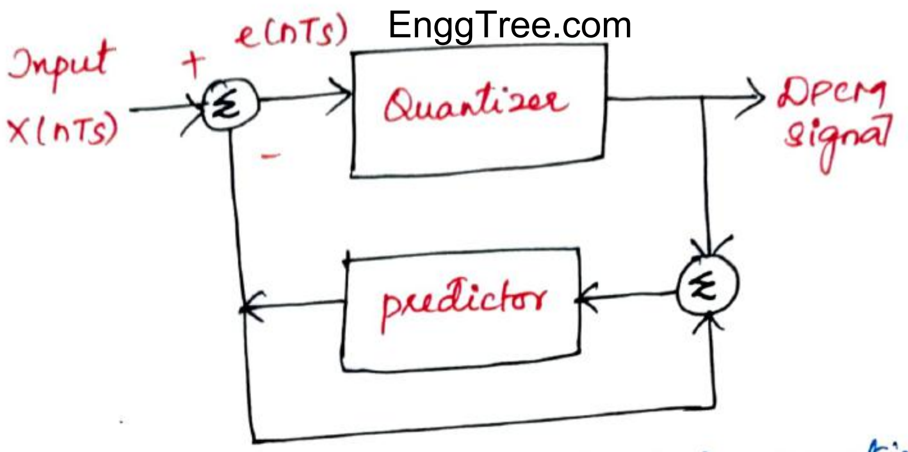
——— × ———

Differential pulse code Modulation (DPCM) :

→ The differential pulse code modulation works on the principle of prediction. The Value of the present Sample is predicted from the past samples. The prediction may not be exact but it is very close to the actual sample value.

DPCM Transmitter :

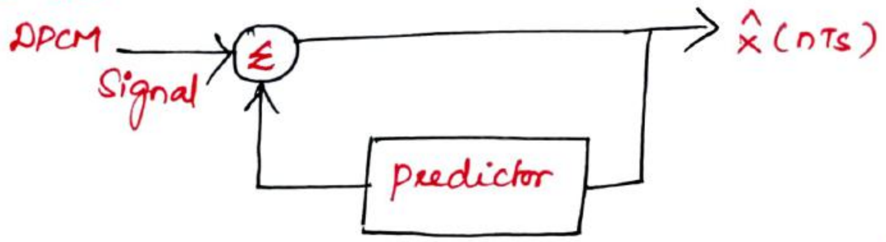
→ The difference between input signal x(nTs) and reconstructed signal $\hat{x}$(nTs) is Calculated.



→ This difference e(nTs) is quantized by the quantiser.

→ A predictor generator present estimate of the signal from quantized difference and previous estimates.

DPCM Receiver:



→ The predictor generates the present reconstructed signal $\hat{x}(nTs)$ from its previous values and incomming DPCM signal.

→ The reconstructed signal is more accurate since predictor considers complete history of the reconstructed signal $\hat{x}(nTs)$.

Signal to Noise Ratio in DPCM:

→ The noise power is basically mean square value $\{E[\varepsilon^2]\}$ of the noise. if the noise has zero mean, $m_\varepsilon = 0$;

$$\sigma_\varepsilon^2 = E[\varepsilon^2] - m_\varepsilon^2$$

$$\sigma_\varepsilon^2 = E[\varepsilon^2]$$

$$\sigma_x^2 = E[x^2] \Rightarrow \text{for variance}$$

$$\frac{S}{N} = \frac{\sigma_x^2}{\sigma_\varepsilon^2}$$

$$\frac{S}{N} = \frac{\sigma_x^2}{\sigma_e^2} \cdot \frac{\sigma_e^2}{\sigma_\varepsilon^2}$$

$$\left(\frac{S}{N}\right) = G_P \cdot \left(\frac{S}{N}\right)_{PCM}$$

$$G_P = \frac{\sigma_x^2}{\sigma_e^2}$$

$$\left(\frac{S}{N}\right)_{PCM} = \frac{\sigma_e^2}{\sigma_\varepsilon^2}$$

$$\left(\frac{S}{N}\right)_{dB} = G_P(dB) + \left(\frac{S}{N}\right)_{PCM}^{dB}$$

$$= G_P(dB) + (4.8 + 6v)$$

→ G_P is greater than unity

→ Signal to noise ratio is improved in DPCM.

—— × ——

Shannon's Theorems on Channel capacity:

→ The information is transmitted through channel with rate 'R' called information rate.

→ shannon's theorem says that it is possible to transmit information with an arbitrarily small probability of error provided that information rate 'R' is less than or equal to a rate 'C' called ~~cap~~ channel capacity.

$$R \le C.$$

$$C = B \log_2 \left(1 + \frac{S}{N}\right) \text{ bits/sec}$$

B → Bandwidth

S → Signal power

N → Total noise power

$N = 0$; $\frac{S}{N} = \infty$; $C = B \log_2 (1 + \infty) = \infty$

$B = \infty$; $C_\infty = \lim_{B \to \infty} C = 1.44\ S/N_0$

—— × ——

Error Control codes :

Block codes :

→ These codes consist of 'n' number of bits in one block or codeword. Codeword consists of 'k' message bits and (n-k) redundant bits.

Convolutional codes :

→ The coding operation is discrete time Convolution of input sequence with the impulse response of the encoder.

→ it can be classified Linear and Non Linear

Linear :

→ if two code words of the linear code are added by modulo-2 arithmetic, then it produces third codeword in the code.

Non Linear :

→ Addition of the non linear codewords does not necessarily produce third codeword.

Code word :

→ The encoded block of 'n' bits is called a code word.

Block length :

→ The number of bits 'n' after coding is called the block length of the code.

Code rate : → The ratio of message bits (k) and the encoder output bits (n) is called code rate.

$= k/n$

Channel data rate :

→ it is the bit rate at the output of Encoder. $R_o = \frac{n}{k} R_s$

Code Efficiency :

$$\text{Code Efficiency} = \frac{\text{Message bits in a block}}{\text{Transmitted bits for the block}}$$

—x—

Linear Block code problems

1) The Generator matrix for a (6,3) block code is given below. find all code vector of this code

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$$

solution :

i) Determine P submatrix

ii) check bits $C = MP$

iii) find Message Vector

$$G = [I_K : P_{K \times q}]$$

$$I_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{3 \times 3} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Message Vector

M_1	M_2	M_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Check bits

$$[c_1 \; c_2 \; c_3] = [m_1 \; m_2 \; m_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$c_1 = m_2 \oplus m_3$

$c_2 = m_1 \oplus m_3$

$c_3 = m_1 \oplus m_2$

Message Vector			Check bits			Complete code vector					
m_1	m_2	m_3	c_1	c_2	c_3	m_1	m_2	m_3	c_1	c_2	c_3
0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0	1	1	1	0
0	1	0	1	0	1	0	1	0	1	0	1
0	1	1	0	1	1	0	1	1	0	1	1
1	0	0	0	1	1	1	0	0	0	1	1
1	0	1	1	0	1	1	0	1	1	0	1
1	1	0	1	1	0	1	1	0	1	1	0
1	1	1	0	0	0	1	1	1	0	0	0

parity Matrix (H)

$$H = [P^T : I_q]_{q \times n}$$

$$H = \begin{bmatrix} 0 & 1 & 1 & : & 1 & 0 & 0 \\ 1 & 0 & 1 & : & 0 & 1 & 0 \\ 1 & 1 & 0 & : & 0 & 0 & 1 \end{bmatrix}$$

×

2) The parity check matrix of a particular (7,4) linear block code is given by

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) find the generator matrix (G); ii) code vectors

iii) minimum distance between code vectors

iv) How many error detected and corrected?

(OR)

Generate the codewords for (7,4) Hamming codes.

solution :

$n = 7 \; ; \; k = 4$

$q = 7 - 4 = 3$

i) $H = [P^T : I_q]$

$$P^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$G = [\, I_k : P_{k \times q} \,]_{k \times n}$

$G = [\, I_4 : P_{4 \times 3} \,]_{4 \times 7}$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

ii) code vectors

$C = MP$

$$[C_1 \; C_2 \; C_3] = [M_1 \; M_2 \; M_3 \; M_4] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$C_1 = m_1 \oplus m_2 \oplus m_3$

$C_2 = m_1 \oplus m_2 \oplus m_4$

$C_3 = m_1 \oplus m_3 \oplus m_4$

Message vector							Code vector							weight of CW
m1	m2	m3	m4	c1	c2	c3	m1	m2	m3	m4	c1	c2	c3	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	1	0	0	0	1	0	1	1	3
0	0	1	0	1	0	1	0	0	1	0	1	0	1	3
0	0	1	1	1	1	0	0	0	1	1	1	1	0	4
0	1	0	0	1	1	0	0	1	0	0	1	1	0	3
0	1	0	1	1	0	1	0	1	0	1	1	0	1	4
0	1	1	0	0	1	1	0	1	1	0	0	1	1	4
0	1	1	1	0	0	0	0	1	1	1	0	0	0	3
1	0	0	0	1	1	1	1	0	0	0	1	1	1	4
1	0	0	1	1	0	0	1	0	0	1	1	0	0	3
1	0	1	0	0	1	0	1	0	1	0	0	1	0	3
1	0	1	1	0	0	1	1	0	1	1	0	0	1	3
1	1	0	0	0	0	1	1	1	0	0	0	0	1	3
1	1	0	1	0	1	1	1	1	0	1	0	1	1	4
1	1	1	0	1	0	0	1	1	1	0	1	0	0	4
1	1	1	1	1	1	1	1	1	1	1	1	1	1	7

iv) Error detection

$d_{min} = 3$

$d_{min} \geq s+1$ Two errors detected

$3 \geq s+1$

$s \leq 2$

Error Correction

$d_{min} \geq 2t+1$

$3 \geq 2t+1$ one error will be corrected.

$t \leq 1$

—— X ——

3) Syndrome decoding $= S = YH^T$

For a systematic linear block code, the three parity check digits, c_4, c_5, c_6 is given by

$$c_4 = d_1 \oplus d_2 \oplus d_3$$

$$c_5 = d_1 \oplus d_2$$

$$c_6 = d_1 \oplus d_3$$

i) Construct generator matrix
ii) Construct code generated by this matrix
iii) Determine error correcting capability
iv) prepare a suitable decoding table
v) Decode the receiver words 101100 and 000110

Solution:

$$[C_4 \ C_5 \ C_6] = [d_1 \ d_2 \ d_3] \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

$$C_4 = d_1 P_{11} \oplus d_2 P_{21} \oplus d_3 P_{31}$$

$$C_5 = d_1 P_{12} \oplus d_2 P_{22} \oplus d_3 P_{32}$$

$$C_6 = d_1 P_{13} \oplus d_2 P_{23} \oplus d_3 P_{33}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Message vector			Check bits			Code vector						weight
d_1	d_2	d_3	c_4	c_5	c_6	d_1	d_2	d_3	c_4	c_5	c_6	
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	1	0	0	1	1	0	1	3
0	1	0	1	1	0	0	1	0	1	1	0	3
0	1	1	0	1	1	0	1	1	0	1	1	4
1	0	0	1	1	1	1	0	0	1	1	1	4
1	0	1	0	1	0	1	0	1	0	1	0	3
1	1	0	0	0	1	1	1	0	0	0	1	3
1	1	1	0	0	1	1	1	1	1	0	0	4

$d_{min} = 3$

$d_{min} \geq s+1$

$3 \geq s+1$

$s \leq 2$

$d_{min} \geq 2t+1$

$3 \geq 2t+1$

$t \leq 1$

$$H^T = \begin{bmatrix} P \\ \cdots \\ I_q \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Syndrome Vector $S = EH^T$

Let us calculate 2nd bit in error

E will be $= [0\,1\,0\,0\,0\,0]$

$$S = [0\,1\,0\,0\,0\,0] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$S = [1\,1\,0]$

4) Consider a linear block code with Generator matrix

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

i) Determine the parity check matrix

ii) Determine the error detecting and capability of the code.

iii) Draw the encoder and syndrome calculation circuits

iv) Calculate the syndrome for the received vector $r = [1101010]$

Solution:

$n = 7$; $k = 4$; $q = 4$

i)

$$G = [I_k ; P_{k \times q}]$$

$$G = [P_{4\times3} : I_4]$$

$$P_{4\times3} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$H = [P^T : I_q]$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

ii) $C = MP$

$$c_1 c_2 c_3 = [m_1 \; m_2 \; m_3 \; m_4] \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$c_1 = m_1 \oplus m_3 \oplus m_4$$

$$c_2 = m_1 \oplus m_2 \oplus m_3$$

$$c_3 = m_2 \oplus m_3 \oplus m_4$$

Error vector E	syndrome	
0 0 0 0 0 0	0 0 0	
1 0 0 0 0 0	1 1 1	→ first row of H^T
0 1 0 0 0 0	1 1 0	→ 2nd H^T
0 0 1 0 0 0	1 0 1	→ 3rd H^T
0 0 0 1 0 0	1 0 0	→ 4th H^T
0 0 0 0 1 0	0 1 0	→ 5th H^T
0 0 0 0 0 1	0 0 1	→ 6th H^T

To decode 101100

$Y = [101100]$

$S = YH^T$

$$= [101100]\begin{bmatrix}1&1&1\\1&1&0\\1&0&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= [1\oplus0\oplus1\oplus1\oplus0\oplus0 \quad 1\oplus0\oplus0\oplus0\oplus0\oplus0 \quad 1\oplus0\oplus1\oplus0\oplus0\oplus0]$$

$= [110]$

$E = [010000]$

$X = Y \oplus E = [101100] \oplus [010000]$

$\boxed{X = [111100]}$

To decode 000110

$Y = 000110$

$$S = YH = [000110]\begin{bmatrix}1&1&1\\1&1&0\\1&0&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$E = [010000]$

$X = Y \oplus E = [000110] \oplus [010000]$

$\boxed{X = 010110}$

—X—

Note: fill code vector table as previous problem method.

$$d_{min} \geq s+1 \qquad d_{min} \geq 2t+1$$
$$3 \geq s+1 \qquad 3 \geq 2t+1$$
$$s \leq 2 \qquad t \leq 1$$

iii) $$[s_1\ s_2\ s_3] = [1\,1\,0\,1\,0\,1\,0] \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = YH^T$$

$$s = 010$$

——— X ———

cyclic codes problem

5) find (7,4) systematic and Non-systematic cyclic code words of the message word 1101. Assume the generator polynomial as $1+x^2+x^3$

Solution:

$$M = (m_3\ m_2\ m_1\ m_0) = (1101)$$

$$M(P) = P^3+P^2+1$$

$$n=7\ ;\ k=4\ ;\ q=3$$

Systematic form:

$$C(P) = \text{rem}\left[\frac{P^q\ M(P)}{G(P)}\right]$$

$$P^q \cdot M(P) = P^3\ (P^3+P^2+1)$$

$$\begin{array}{r|l} & P^3 \\ \hline P^3+P^2+1 & P^6+P^5+P^3 \\ & P^6+P^5+P^3 \\ \hline & 0\quad 0\quad 0 \end{array}$$

$$C = 000$$

$$X = \{m_3\ m_2\ m_1\ m_0 : c_2\ c_1\ 0\} = \{1101:000\}$$

Non systematic form:

$$x(P) = M(P) \cdot G(P)$$

$$= (P^3+P^2+1)(P^3+P^2+1)$$

$$= P^6+P^5+P^3+P^5+P^4+P^2+P^3+P^2+1$$

$$= P^6+0P^5+P^4+0P^3+0P^2+0P+1$$

$$X = \{1010001\}$$

— × —

6) The generator polynomial of a (7,4) linear systematic cyclic block code is $1+x+x^3$. Determine the correct code word transmitted, if the received word is i) 1011011 ii) 1101111

Solution:

Error vector E	Syndrome vector
0 0 0 0 0 0 0	0 0 0
1 0 0 0 0 0 0	1 0 1
0 1 0 0 0 0 0	1 1 1
0 0 1 0 0 0 0	1 1 0
0 0 0 1 0 0 0	0 1 1
0 0 0 0 1 0 0	1 0 0
0 0 0 0 0 1 0	0 1 0
0 0 0 0 0 0 1	0 0 1

i) $y = 1011011$

$$y(P) = P^6+P^4+P^3+P+1$$

$$S(P) = \text{rem}\left[\frac{y(P)}{G(P}\right]$$

$$G(P) = 1+P+P^3$$

$$p^3$$

$$p^3+p+1 \,\big)\, \cancel{p^6}+\cancel{p^4}+\cancel{p^3}+p+1$$

$$\cancel{p^6}+\cancel{p^4}+\cancel{p^3}$$

$$p+1$$

$S = 011$

$E = 0001000$

$x = y \oplus E$

$= [1011011] \oplus [0001000]$

$x = 1010011$

ii) decode 1101111

$y(p) = p^6+p^5+p^3+p^2+p+1$

$$p^3+p^2+p+1$$

$$p^3+p+1 \,\big)\, p^6+p^5+p^3+p^2+p+1$$

$$p^6+p^4\ p^3$$

$$p^5+p^4+p^2+p+1$$

$$p^5+p^3+p^2$$

$$p^4+p^3+p+1$$

$$p^4+p^2+p$$

$$p^3+p^2+1$$

$$p^3+p+1$$

$$p^2+p$$

$S = 110$

$E = 0010000$

$x = Y \oplus E = [1101111] \oplus [0010000]$

$x = [1\ 1\ 1\ 1\ 1\ 1\ 1]$

—— X ——

Convolutional codes problem

6) A rate 1/3 convolution encoder has generating vector

$g_1 = \{100\}$; $g_2 = \{111\}$; $g_3 = \{101\}$

i) Sketch the encoder configuration

ii) Draw the code tree, State diagram and Trellis diagram

iii) If input message sequence is 10110, determine the sequence of the encoder.

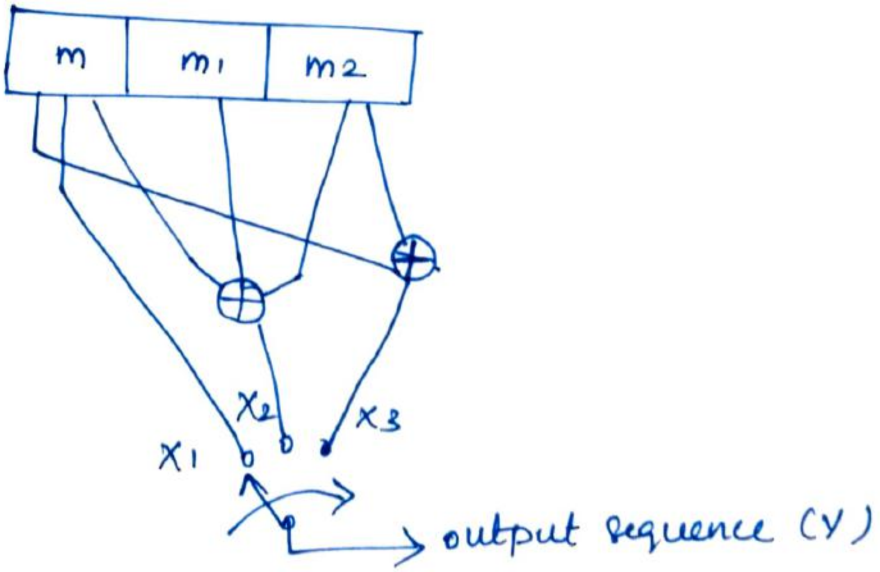
iv) Draw the Trellis diagram and extract the transmitted signal using viterbi algorithm for received sequence.

$r = \{101, 110, 101, 010, 101, 110, 011\}$

Solution:

$$\text{Rate} = \frac{k}{n} = \frac{1}{3} \; ;$$

i) Encoder configuration:



$g_1 = (100) = m$

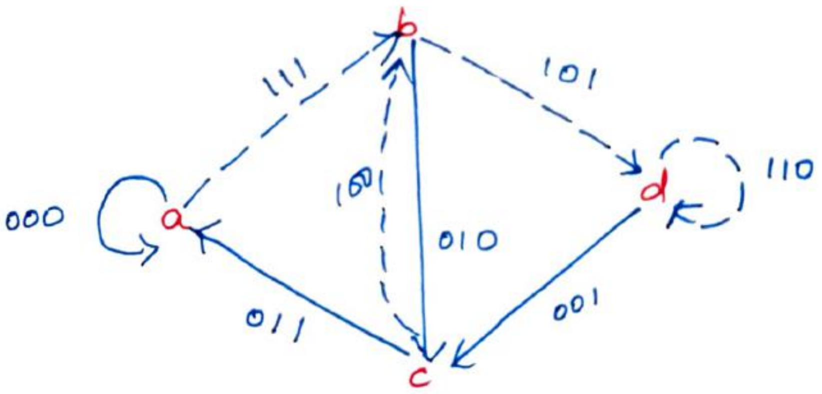
$g_2 = (111) = m \oplus m_1 \oplus m_2$

$g_3 = (101) = m \oplus m_2$

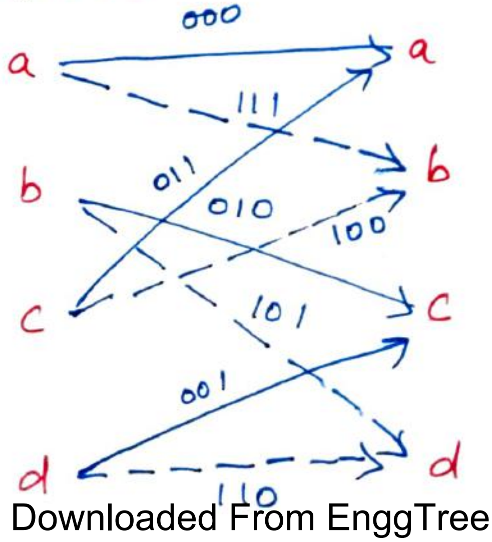
State Transition Table:

Current State	Input	Output			Next State	
m_2 m_1	m	x_1	x_2	x_3	m_1	m
a = 00	0	0	0	0	00 = a	
	1	1	1	1	01 = b	
b = 01	0	0	1	0	10 = c	
	1	1	0	1	11 = d	
c = 10	0	0	1	1	00 = a	
	1	1	0	0	01 = b	
d = 11	0	0	0	1	10 = c	
	1	1	1	0	11 = d	

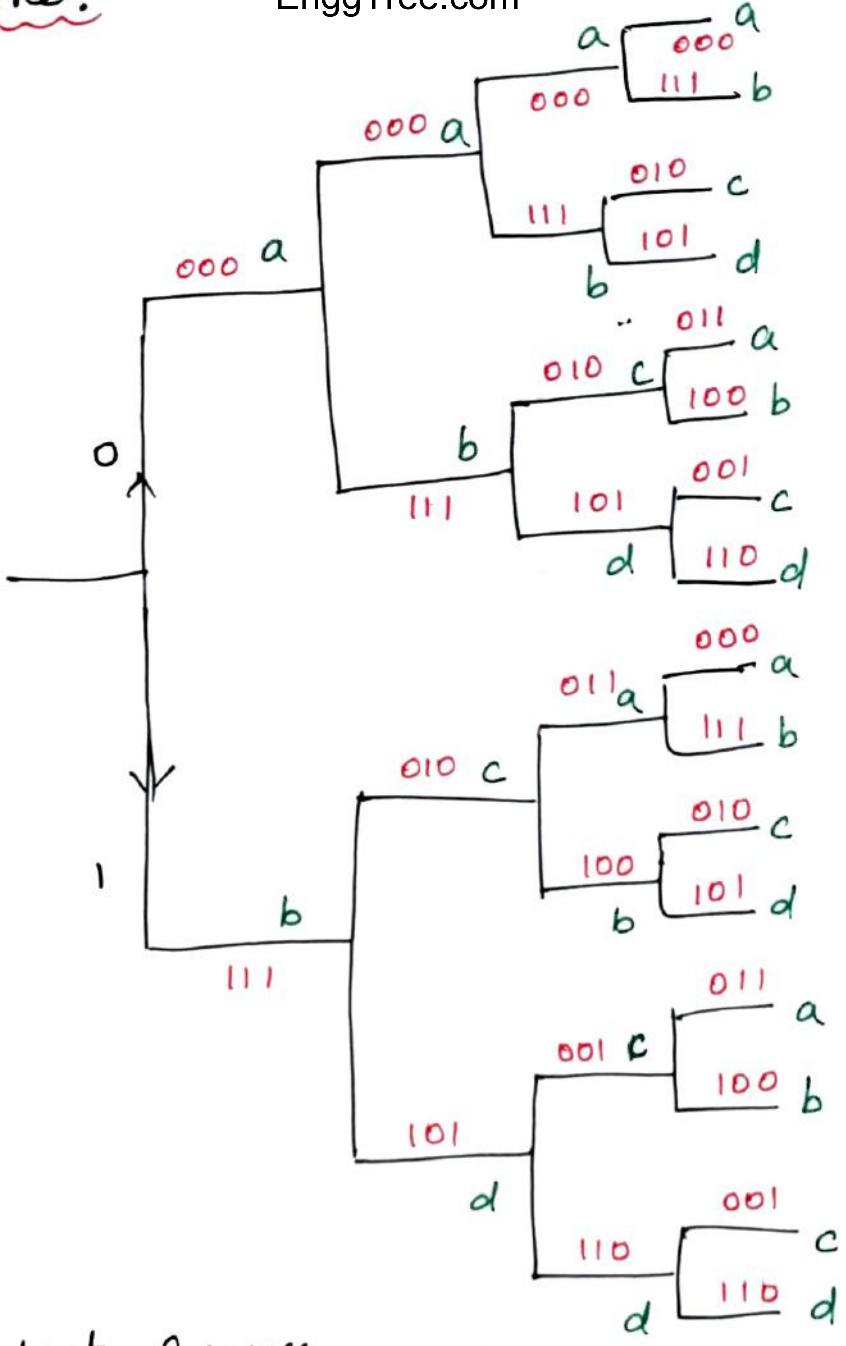
State Diagram:



Trellis Diagram:



Code Tree :



iii) Output Sequence $m = 10110$

$$\boxed{m(p) = 1 + p^2 + p^3}$$

$g_1 = \{1 0 0\} = 1$

$g_2 = \{1 1 1\} = 1 + p + p^2$

$g_3 = \{1 0 1\} = 1 + p^2$

$x_1 = g_1(p) . M(p)$

$x_1 = 1 (1 + p^2 + p^3)$

$x_1 = \{1011\}$

$x_2 = g_2(P) \cdot M(P)$

$= (1+P+P^2)(1+P^2+P^3)$

$= 1+P^2+P^3+P+P^3+P^4+P^2+P^4+P^5$

$= 1+P+0P^2+0P^3+0P^4+P^5$

$x_2 = \{1 1 0 0 0 1\}$

$x_3 = g_3(P) \cdot M(P)$

$= (1+P^2)(1+P^2+P^3)$

$= 1+P^2+P^3+P^2+P^4+P^5$

$= 1+P^3+P^4+P^5+0P^2$

$x_3 = \{1 0 0 1 1 1\}$

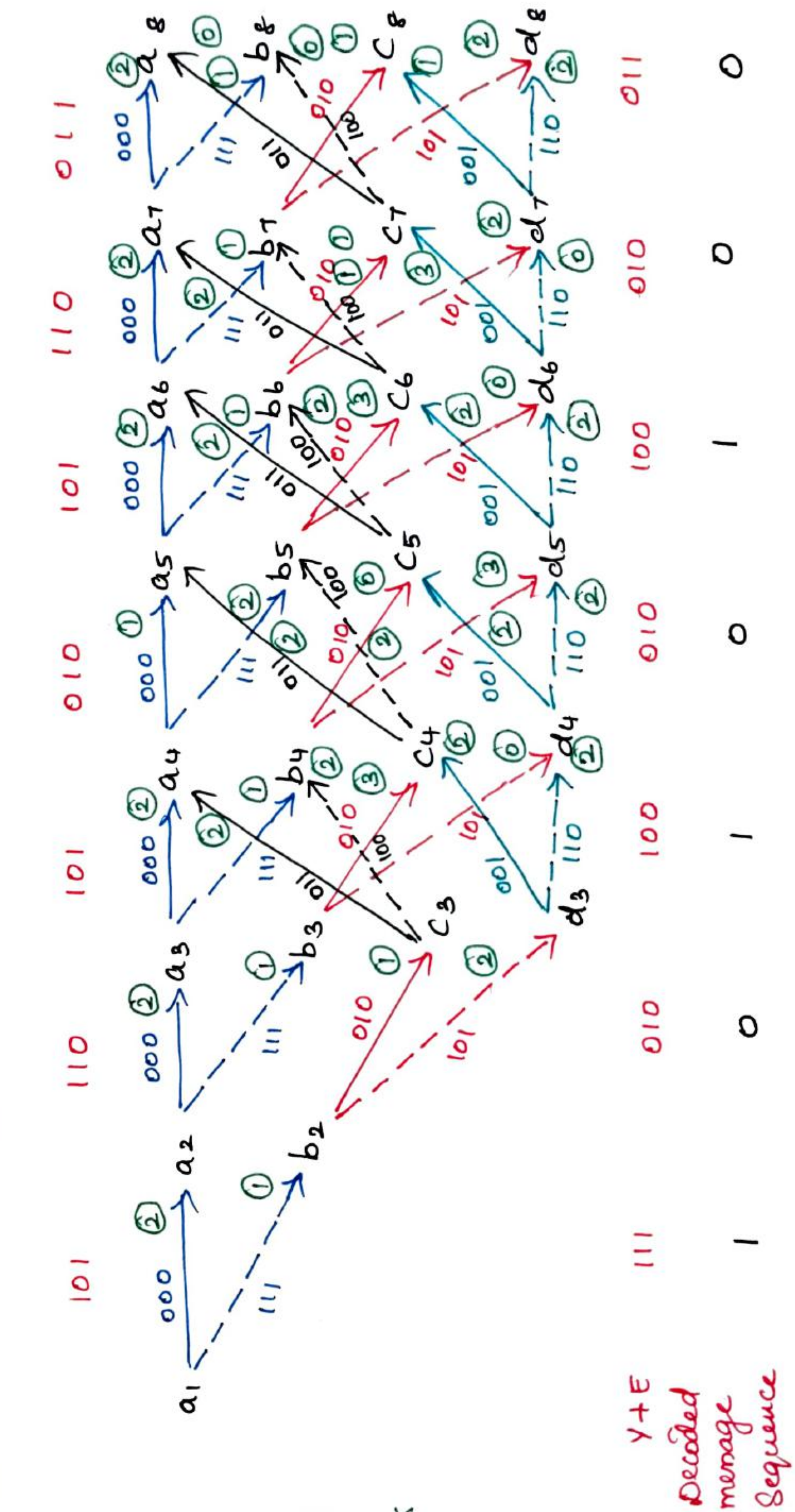
$x_1 = \{1 0 1 1 0 0\}$

$x_2 = \{1 1 0 0 0 1\}$

$x_3 = \{1 0 0 1 1 1\}$

$x_i = \{111, 010, 100, 101, 001, 011\}$

viterbi diagram:



— x —

UNIT — 3 Two marks

1. what are the two types of quantization errors that occur in delta modulation?

→ Slope overload error, The step size of quantization is not enough to follow the large changes in input signal.

→ Granular Noise, The step size is to large, hence approximated signal cannot follow the small variations in input signal.

—— × ——

2) List any four Speech encoding methods.

→ pulse code modulation
→ Differential pulse code modulation
→ Delta modulation
→ Linear predictive coding

—— × ——

3) what is linear predictor? on what basis predictor Coefficients determined?

→ Linear predictor provides the estimate to future Sample depending upon linear combination of past and present samples of the stationary process.

→ Basis for predictor co-efficients is to minimize the mean square value of the prediction error. The prediction error is the difference between actual sample value and predictor output.

—— × ——

4) what is the need of prediction filtering?

→ prediction filtering provides prediction error from present and past input Samples of the Stationary process.

—— × ——

5) How to overcome the slope overload?

→ Slope overload distortion occurs mainly due to fixed step size 'S' that can not follow rate of rise of input signal. Hence step size 'S' should be increased when the slope of the signal is high.

→ Step size is small, slope of the signal is low. This is done by adaptive delta modulation.

— × —

6) what is meant by delta modulation?

→ Delta modulation transmits only one bit per sample. The present sample value is compared with the previous sample value and the indication, whether the amplitude is increased or decreased. The step size is fixed.

— × —

7) A delta modulation system is tested with a 10KHZ, sinusoidal signal with 1V peak to peak at the input. It is sampled at 10 times the nyquist rate. what is the corresponding SQNR?

$f_m = 10KHZ.$

$W = f_m = 10KHz$

Nyquist rate $= 2W = 2 \times 10KHZ = 20KHZ$

sampling frequency $f_s = 10 \times$ Nyquist rate

$f_s = 10 \times 20KHZ = 200KHZ$

$$T_s = 1/f_s = \frac{1}{200KHZ} = \frac{1}{200 \times 10^3} sec$$

Delta modulation signal to noise ratio

$$\frac{S}{N} = \frac{3}{8\pi^2 W f_m^2 T_s^2} = 303.96$$

$$\frac{S}{N} = 10 \log_{10} 303.96 = 24.83dB$$

— × —

8) what is meant by syndrome of linear block code ?

→ The non-zero output of YH^T is called Syndrome and it is used to detect the errors in y .

→ Syndrome is denoted by 'S'

$$S = YH^T$$

— × —

9) what is Convolutional code ?

→ fixed number of input bits are stored in the shift register and they are combined with the help of mod-2 adders . This operation is equivalent to binary convolution and hence it is called convolution coding .

— × —

10) what is the need for error control codes / Need of channel coding ?

→ Error are introduced in the transmitted data when it is transmitted across the channel . These errors need to be detected and corrected .

→ Error control codes introduce redendency in the transmitted data to detected and correct errors .

— × —

11) State the significance of minimum distance of a block code .

→ The minimum distance 'dmin' of a block code is related to error detection and correction capabilities .

Error detection : $d_{min} \geq s+1$

Error correction : $d_{min} \geq 2t+1$

— × —

12) Define code rate of a block code .

→ The ratio of message bits (k) to encoder output bits (n) is called code rate . It is denoted by 'r' . $r = \frac{K}{n}$

—— × ——

13) List the properties of Syndrome .

→ The Syndrome depends only on the error pattern and not on the transmitted codeword .

→ All error patterns that differ at most by a codeword have the same Syndrome .

→ Syndromes represent different error patterns and they are the rows of H^T .

—— × ——

14) State channel coding theorem .

→ Given source of M equally likely messages, with M>>1, which is generating information at a rate R . Given channel with channel capacity C .

$$R \le C$$

—— × ——

15) Generate the cyclic code for (n, k) Syndrome Calculator .

→ for (n, k) syndrome calculator, Systematic (n, k) cyclic code is required .

$$\text{Systematic} = \text{rem}\left[\frac{P^q \; G(P)}{M(P)}\right]$$

$$\text{Non systematic} = M(P) \cdot G(P) .$$

—— × ——

16) what are the different methods of describing the Structure of a Convolutional code?

→ State diagram
→ Code tree
→ trellis diagram

→ encoding and decoding methods of convolutional code.

— × —

17) List the Various techniques used in encoding and decoding of convolutional code.

→ Encoding : Time domain encoding
Transform domain encoding
State diagram
Trellis diagram
Code tree

→ Decoding : Maximum likelihood decoding
Viterbi decoding
sequential decoding
Hard and soft decision decoding

— × —

UNIT-3 Big Questions

1) Delta modulation
2) Adaptive Delta modulation
3) DPCM (Differential pulse code modulation)
4) problems on Linear block codes
5) problems on Syndrome Calculation
6) problems on cyclic codes
7) problems on convolutional Encoding and decoding

16) what are the different methods of describing the Structure of a convolutional code?

- → State diagram
- → code tree
- → trellis diagram

→ encoding and decoding methods of convolutional code.

— × —

17) List the various techniques used in encoding and decoding of convolutional code.

→ Encoding : Time domain encoding
Transform domain encoding
State diagram
Trellis diagram
Code tree

→ Decoding : Maximum likelihood decoding
viterbi decoding
sequential decoding
Hard and soft decision decoding

— × —

UNIT - 4 - Digital Modulation Scheme

Digital Modulation schemes:

→ Digital data can modulate phase, frequency or amplitude of carrier.

passband Transmission:

Coherent (synchronous) detection:

→ In this method, the local carrier generated at the receiver is phase locked with the carrier at the transmitter. Hence it is also called synchronous detection.

Non Coherent (Envelope) detection:

→ In this method, the receiver carrier need not be phase locked with transmitter carrier. Hence it is also called envelope detection. Noncoherent detection is simple but it has higher probability of error.

Advantages of passband transmission:

→ signals generated by digital carrier systems are analog in nature. Hence all the types of analog channels can be used for transmission.

→ Because of modulation of analog carrier, multiplexing techniques can also be employed for bandwidth conservation

Drawbacks:

→ Modulation and demodulation equipments, transmitting/receiving antennas, interference problems make the system complex.

→ It is not suitable for short distance Communication.

Geomentric Representation of signals:

→ The messages, which are transmitted over the carrier from a signal space or vector space.

→ probability of error in transmission

→ Distance or separation between individual messages.

M number of energy signal

$$y_i(t) = \{ y_1(t), y_2(t), \dots y_M(t) \}$$

$$\phi j(t) = \{ \phi_1(t), \phi(t) \dots \phi_N(t) \}$$

N number of orthonormal basis functions.

Then the linear relationship between Si(t) and φj(t) can be written as,

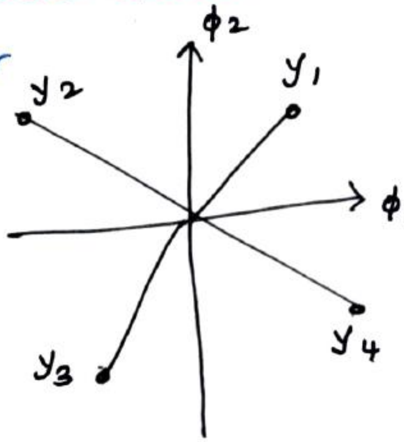
$$y_i(t) = y_{i1}\phi_1 + y_{i2}\phi_2 + \dots + y_{iN}\phi_N(t)$$

$$= \sum_{j=1}^{N} y_{ij}\phi_j(t)$$

$$y_{ij} = \int_0^T y_i(t)\,\phi_j(t)\,dt$$

T → Duration of the symbol yi(t)

Two dimensional signal space with three symbols:



→ Let us consider the vector space representation of M = 3 message symbols with the help of N=2 orthonormal basis functions.

$$y_i(t) = \sum_{j=1}^{2} y_{ij}(t)\,\phi_j(t)$$

Absolute Value or Norm of a Vector:

→ Consider the vector yi, which is completely determined by its coefficients.

$$y_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iN} \end{bmatrix} \quad \text{and} \quad i = 1, 2 \dots M$$

$$= \sum_{j=1}^{N} y_{ij}^2 \quad \text{and} \quad i = 1, 2 \dots M$$

Relationship between signal Energy and its vector:

→ The Energy of the signal $y_i(t)$ is given as,

$$E_i = \int_0^T y_i^2(t)\,dt$$

$$E_i = \int_0^T \left[\sum_{j=1}^{N} y_{ij}\,\phi_j(t)\right]\left[\sum_{k=1}^{N} y_{ik}\,\phi_k(t)\right] dt$$

$$E_i = y_i^T y_i$$

signal energy is equal to squared length of the signal vector.

Euclidean Distance:

$$d_{ik} = \| y_i - y_k \|$$

d_{ik} is the Euclidean between y_i and y_k and

Squared Euclidean distance

$$\| y_i - y_k \|^2 = \sum_{j=1}^{N} (y_{ij} - y_{kj})^2$$

——— × ———

Amplitude shift keying (ASK) or ON – OFF Keying (OOK):

→ ASK is the simplest digital modulation technique. In this method, there is only one unit energy carrier and it is switched on or off depending upon the input binary sequence.

ASK waveform can be represented as

$$s(t) = \sqrt{2P_s} \cos(2\pi f_0 t) \quad (\text{To transmit } 1)$$

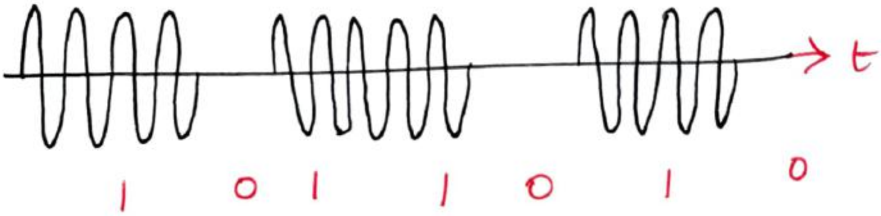
→ To transmit symbol 0, the signal $s(t) = 0$.

→ That is no signal is transmitted $s(t)$ contains some complete cycles of carrier frequency 'f'.

→ symbol 1 ⇒ pulse is transmitted

→ symbol 0 ⇒ no pulse is transmitted

→ Thus the ASK waveform looks like an ON-OFF of the signal. Hence it is also called ON-OFF keying.

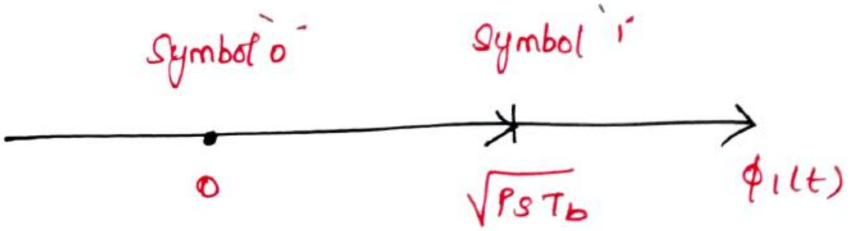


ASK waveform

Signal space diagram of ASK:

→ symbol 1 can be represented as,

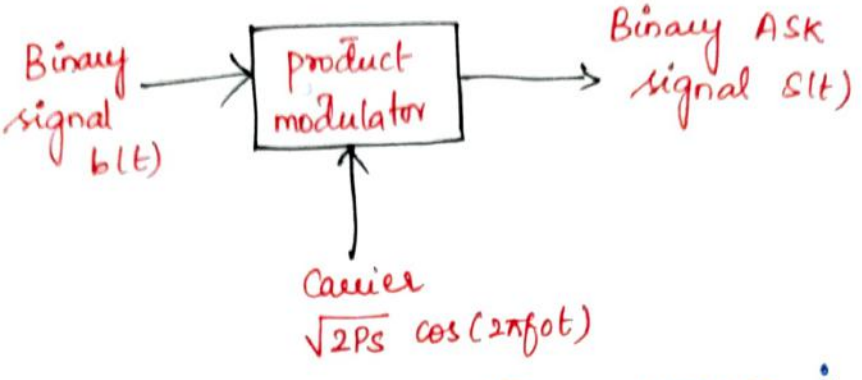
$$s(t) = \sqrt{P_s \cdot T_b} \cdot \sqrt{2/T_b} \cos(2\pi f_0 t) = \sqrt{P_s \cdot T_b}\, \phi_1(t)$$



→ The signal space diagram will have two points on $\phi_1(t)$. one will be at zero and other will be at $\sqrt{P_s \cdot T_b}$.

→ The distance between the two signal points will be $d = \sqrt{P_s T_b} = \sqrt{E_b}$

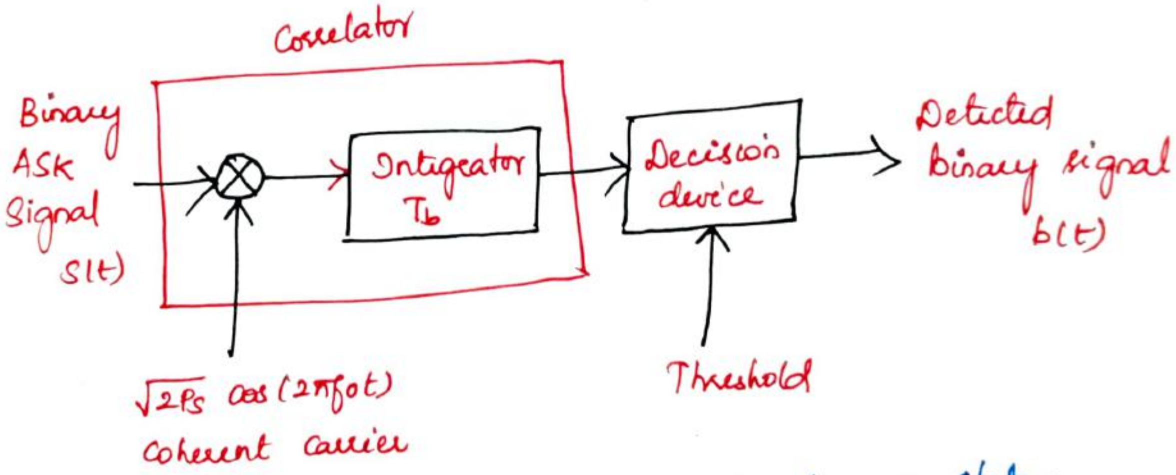
Generator :



→ The input binary sequence is applied to the product modulator. product modulator amplitude modulates the sinusoidal carrier.

→ it passes the carrier when input bit is '1'. it blocks the carrier when input bit is '0'.

ASK Detector :



→ ASK signal is applied to the correlator consisting of multiplier and integrator.

→ The locally generated coherent carrier is applied to the multiplier.

→ The output of multiplier is integrated over one bit period.

→ The decision device takes the decision at the end of every bit period.

→ it compares the output of integrator with the threshold.

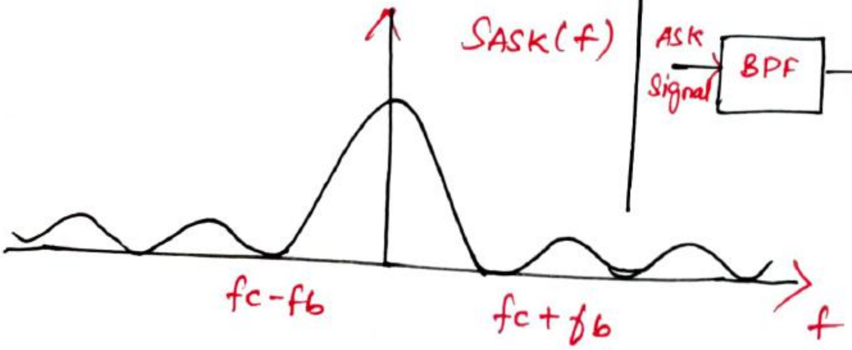
→ Decision is taken favour of '1' when threshold is exceeded. Decision is taken as '0' if threshold is not exceeded.

power Spectral Density of Binary ASK :

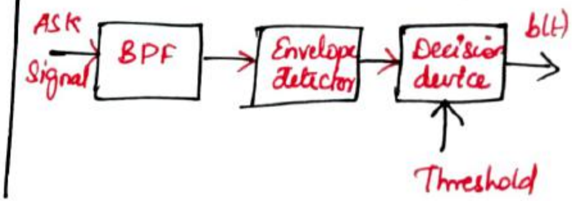
→ The ASK signal is simply obtained by switching on/off the carrier signal $\cos(2\pi f_0 t)$.

→ This carrier signal is multiplied with unipolar NRZ signal.

$$S_{ASK}(f) = \frac{T_b}{4} \operatorname{sinc}^2(fT_b) + \frac{1}{4}\delta(f)$$



Non coherent Detection of ASK



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Frequency shift keying (FSK) modulation / BFSK :

→ In binary frequency shift keying, the frequency of the carrier is shifted according to the binary symbol.

→ The phase of the carrier is unaffected. That is we have two different frequency signals according to binary symbols. Let there be a frequency shift by Ω.

if $b(t) = 1$; $S_H(t) = \sqrt{2Ps} \cos(2\pi f_0 + \Omega)t$

if $b(t) = 0$; $S_L(t) = \sqrt{2Ps} \cos(2\pi f_0 - \Omega)t$

when symbol 1 transmitted

$$f_H = f_0 + \frac{\Omega}{2\pi}$$

when symbol 0 transmitted

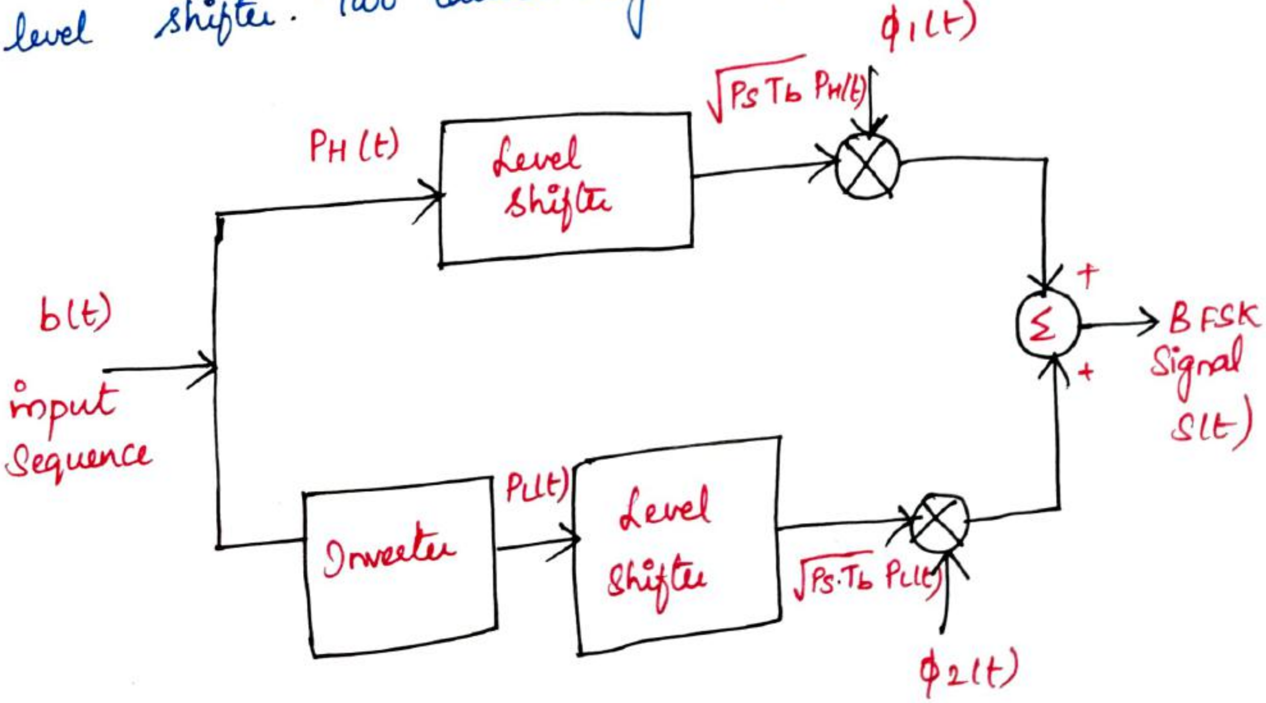
$$f_L = f_0 - \frac{\Omega}{2\pi}$$

BFSK Transmitter :

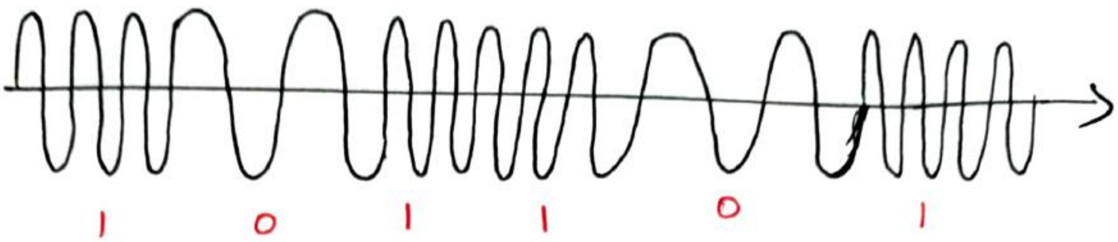
→ we know that input sequence b(t) is same as $P_H(t)$. An inverter is added after b(t) to get $P_L(t)$. $P_H(t)$ and $P_L(t)$ are unipolar signals.

→ The level shifter converts '+1' level to $\sqrt{P_S T_b}$. Zero level is unaffected.

→ further there are product modulator after level shifter. Two carrier signals $\phi_1(t)$ and $\phi_2(t)$.



BFSK signal



→ Output from both the multipliers are not possible at a time. This is because $P_H(t)$ and $P_L(t)$ are Complementary to each other.

Spectrum and Bandwidth of BFSK :

→ $s(t) = \sqrt{2P_s}\; P_H(t) \cos(2\pi f_H t) + \sqrt{2P_s}\; P_L \cos(2\pi f_L t)$

$$P_H(t) = \frac{1}{2} + \frac{1}{2} P_H'(t)$$

$$P_L(t) = \frac{1}{2} + \frac{1}{2} P_L'(t)$$

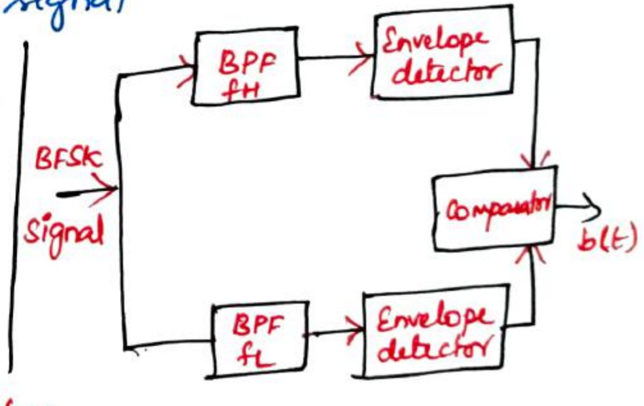
$$f_H - f_L = 2f_b$$

Bandwidth of BFSK signal

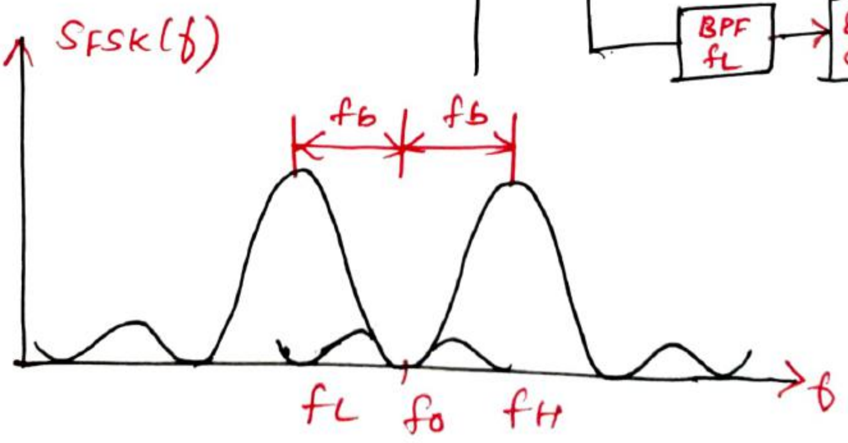
$$BFSK = 2f_b + 2f_b$$

$$BW = 4f_b$$

Non-coherent BFSK Receiver



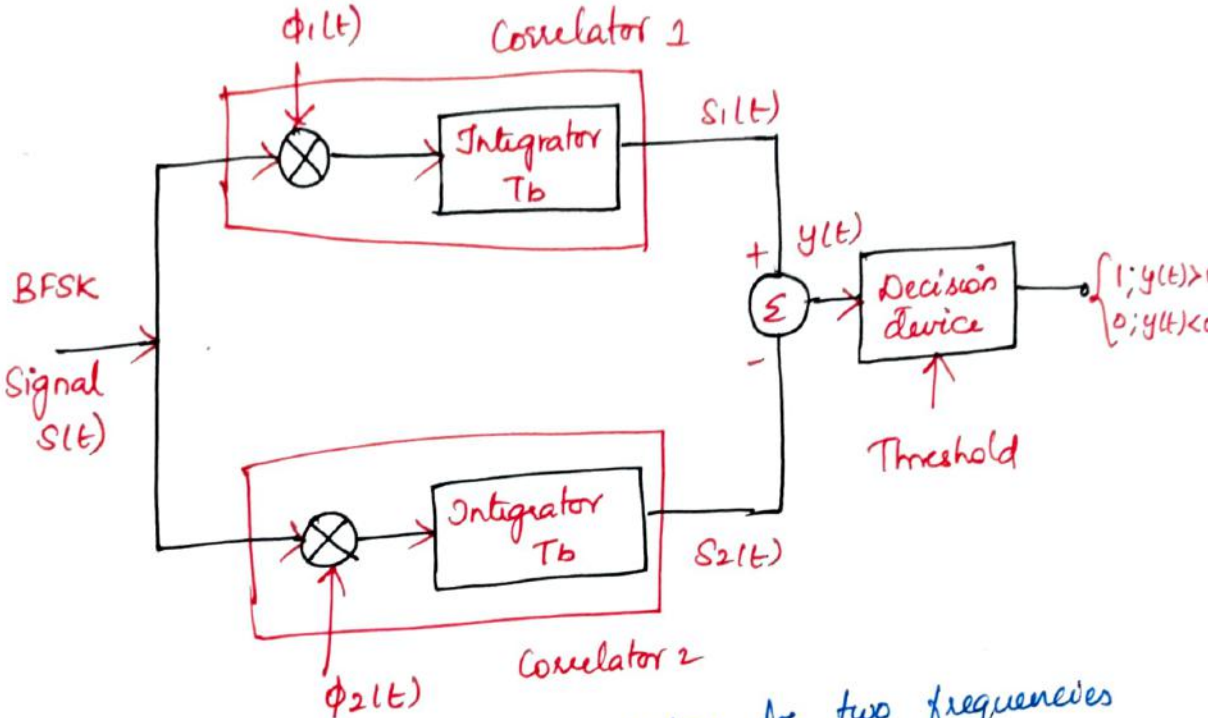
Power Spectral Density :



→ we compared this bandwidth of BPSK

(BFSK) BW = 2 BW (BPSK)

Coherent BFSK Receiver:



→ There are two corelators for two frequencies of FSK signal. These Corelators are supplied with locally generated carriers $\phi_1(t)$ and $\phi_2(t)$.

→ if transmitted frequency f_H then the $S_1(t)$ will be higher than $S_2(t)$. Hence $y(t)$ will be greater than zero.

→ The Decision device 1 ; $y>0$, if 0 ; $y<0$.

Geometrical Representation of orthogonal BFSK or signal space representation of orthogonal BFSK:

$$f_H = m f_b$$

$$f_L = n f_b$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi m f_b t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi n f_b t)$$

$$S_H(t) = \sqrt{P_S \cdot T_b} \cdot \phi_1(t)$$

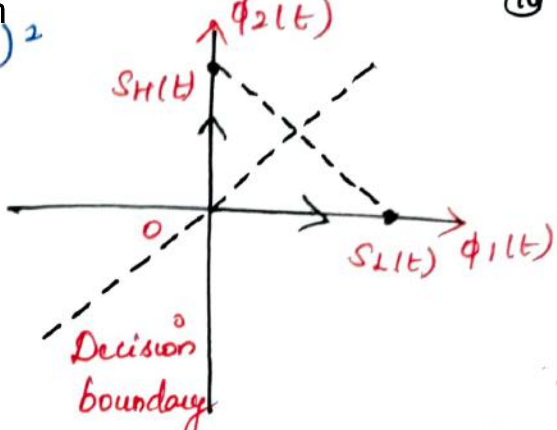
$$S_L(t) = \sqrt{P_S \cdot T_b} \cdot \phi_2(t)$$

$$d^2 = (\sqrt{P_s \cdot T_b})^2 + (\sqrt{P_s \cdot T_b})^2$$

$$= 2 P_s \cdot T_b$$

$$d = \sqrt{2 P_s \cdot T_b}$$

$$d = \sqrt{2E_b}$$



signal space representation

Advantages and Disadvantages of BFSK:

→ Eventhough the generation of orthogonal BFSK. of BFSK is easier it has many disadvantages compared to BPSK. → The distance between the signal points is less in BFSK. Error rate of BFSK is more compared to BPSK.

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Binary phase shift keying (BPSK) modulation:

→ Binary phase shift keying (BPSK), binary symbol '1' and '0' modulate the phase of the carrier.

$$S(t) = A \cos(2\pi f_0 t)$$

A → peak value of sinusoidal carrier.

$$P = \frac{1}{2} A^2$$

$$A = \sqrt{2P}$$

Symbol 1 ⇒ $S_1(t) = \sqrt{2P} \cos(2\pi f_0 t)$

Symbol 0 ⇒ $S_2(t) = \sqrt{2P} \cos(2\pi f_0 t + \pi)$

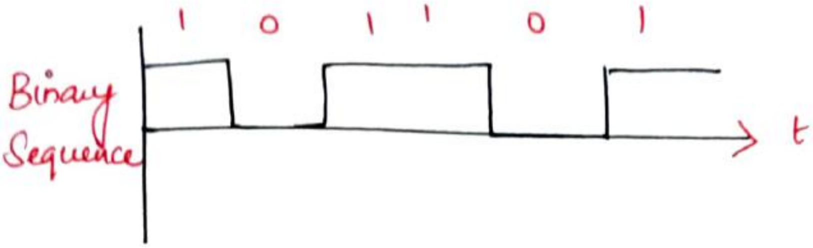
when we combine both the equation

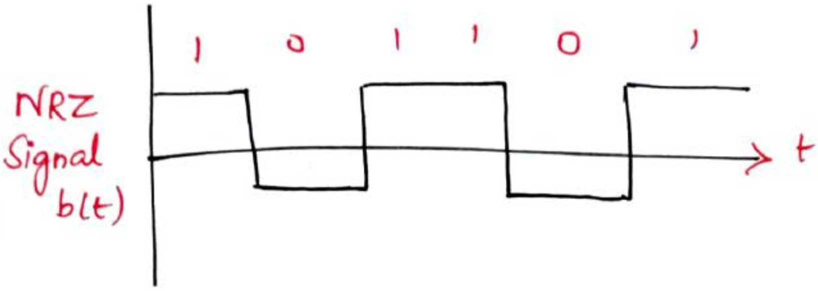
$$S(t) = b(t) \sqrt{2P} \cos(2\pi f_0 t)$$

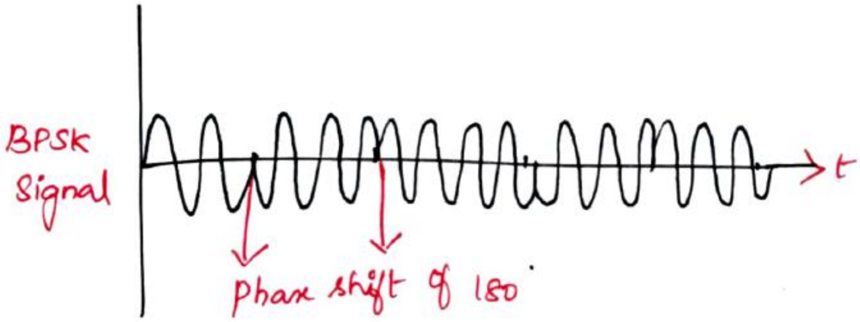
$b(t) = +1$ when binary '1' is to be transmitted

$b(t) = -1$ when binary '0' is to be transmitted.

BPSK signal :



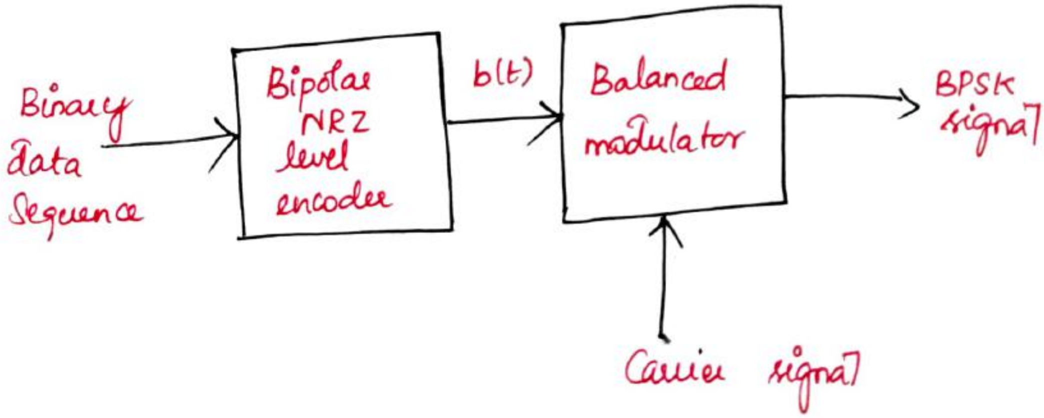




Generation of BPSK signal :

→ The BPSK signal can be generated by applying Carrier signal to the balanced modulator.

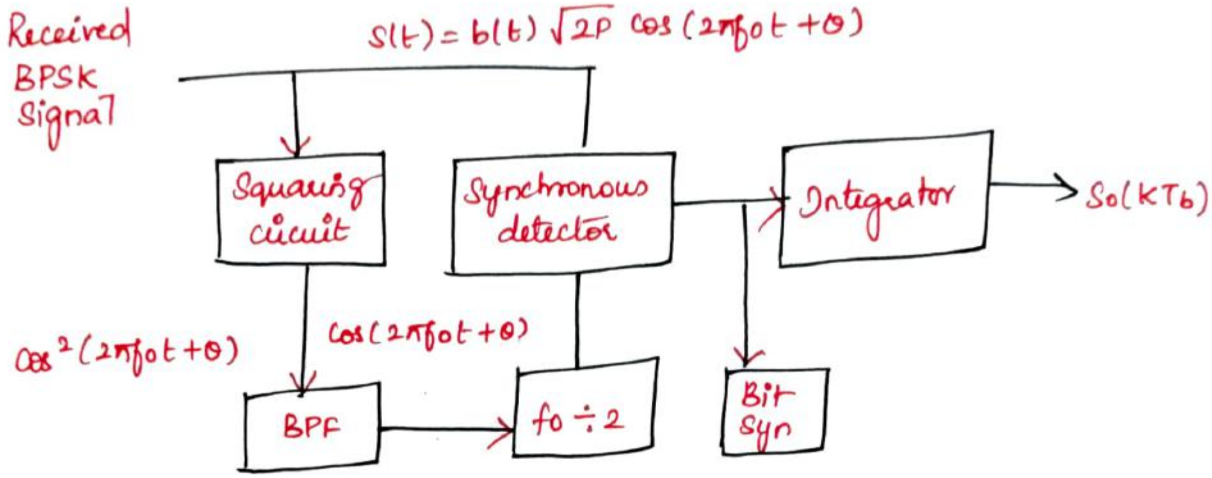
→ The NRZ level encoder converts the binary data sequence into bipolar NRZ signal.



Receiver :

→ The transmitted BPSK signal

$$S(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$



phase shift in received signal :

→ This signal undergoes the phase change depending upon the time delay from transmitter to receiver. This phase change is normally fixed phase shift in the transmitted signal. Let the phase shift be θ.

$$S(t) = b(t) \cdot \sqrt{2P}\cos(2\pi f_0 t + \theta)$$

Squaring circuit :

→ from the received signal, a carrier is separated since this is coherent detection.

→ Received signal is passed through a square law device. At the output of the square law device the signal will be, $\cos^2(2\pi f_0 t + \theta)$

BPF :

→ This signal is then passed through a bandpass filter whose passband is centered around $2f_0$. Bandpass filter removes the DC. level of ½ and at its output we get,

$$\cos 2(2\pi f_0 t + \theta)$$

Frequency divider :

→ The above signal passed through a frequency divider by two. Output of frequency divider we get a carrier signal whose frequency is f_0.

Synchronous demodulator :

→ The synchronous demodulator multiplies the input signal and the recovered carrier.

$$b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta) \times \cos(2\pi f_0 t+\theta)$$

Bit synchronizer and Integrator :

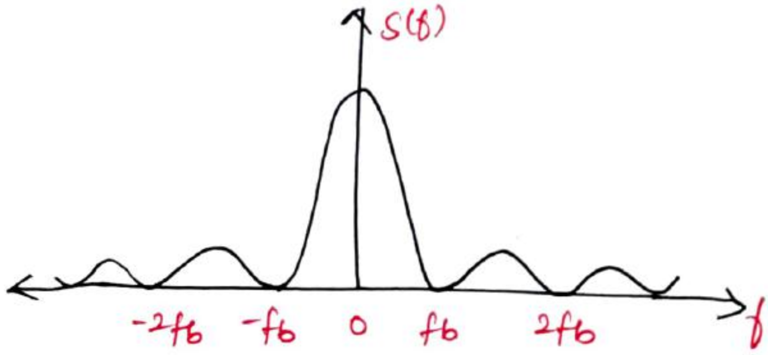
→ Integrator integrates the signal over one bit period. The bit synchronizer takes care of starting and ending times of a bit.

$$S_0(KT_b) = b(KT_b)\sqrt{P/2}\,T_b$$

Spectrum of BPSK signals :

$$S(t) = \pm\sqrt{2P}\cos(2\pi f_0 t)$$

$$\phi(t) = \sqrt{2}\cos(2\pi f_0 t)$$



ISI :

→ Individual bits mix with adjacent bits in the same channel. This effect is called intersymbol interference or ISI.

Bandwidth :

$BW = 2f_b$

Drawbacks :

→ Recovered carrier is unchanged even if the input signal has changed its sign. There is not possible to determine whether the receiver signal is equal to b(t) or -b(t). This result in ambiguity in the output signal.

→ This problem can be removed in differential phase shift keying.

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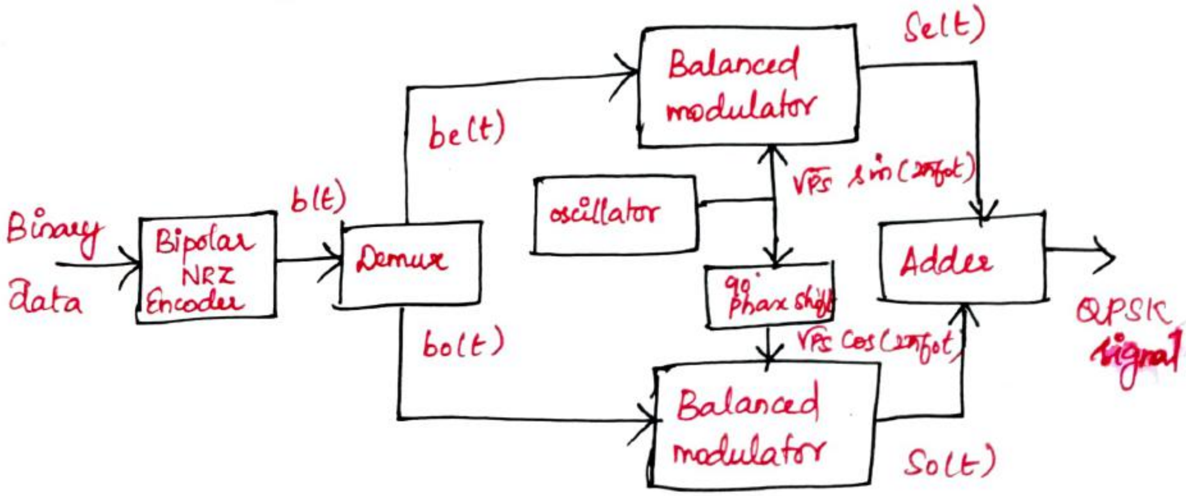
Quaternary phase shift keying (QPSK) / Quadrature phase shift keying :

→ Quadrature phase shift keying, two successive bits in the data sequence are grouped together. This reduces the bits rate of signalling rate. Hence reduces the bandwidth of the channel.

Input	successive bits	Symbol	phase shift
1 (+)	0 (-)	S_1	$\pi/4$
0 (-)	0 (-)	S_2	$3\pi/4$
0 (-)	1 (+)	S_3	$5\pi/4$
1 (+)	1 (+)	S_4	$7\pi/4$

QPSK Transmitter :



Input Sequence Converted to NRZ type:

→ The input binary sequence is first converted to a bipolar NRZ type of signal. This signal is called b(t).

→ it represents binary '1' by +1V and binary '0' by -1V.

Demultiplexing into odd and Even numbered sequences:

→ Demultiplexer divides b(t) into two separate bit streams of the odd numbered and even numbered bits. be(t) represents even numbered sequence and bo(t) represents odd numbered sequence.

Modulation of quadrature Carriers:

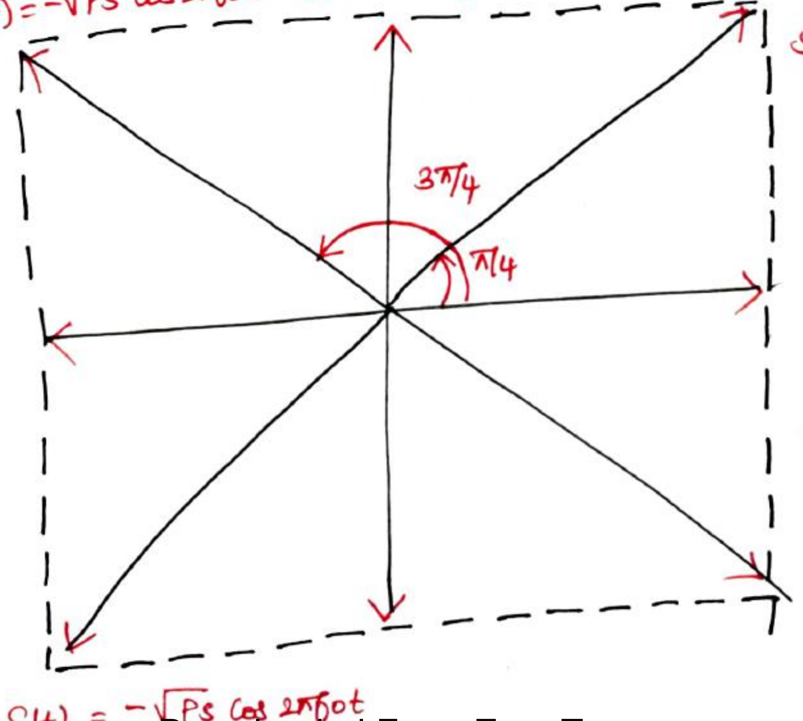
→ $S_e(t) = b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)$

$S_o(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t)$

$$S(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

Phasor diagram:

$S(t) = -\sqrt{P_s}\cos 2\pi f_0 t - \sqrt{P_s}\sin 2\pi f_0 t$

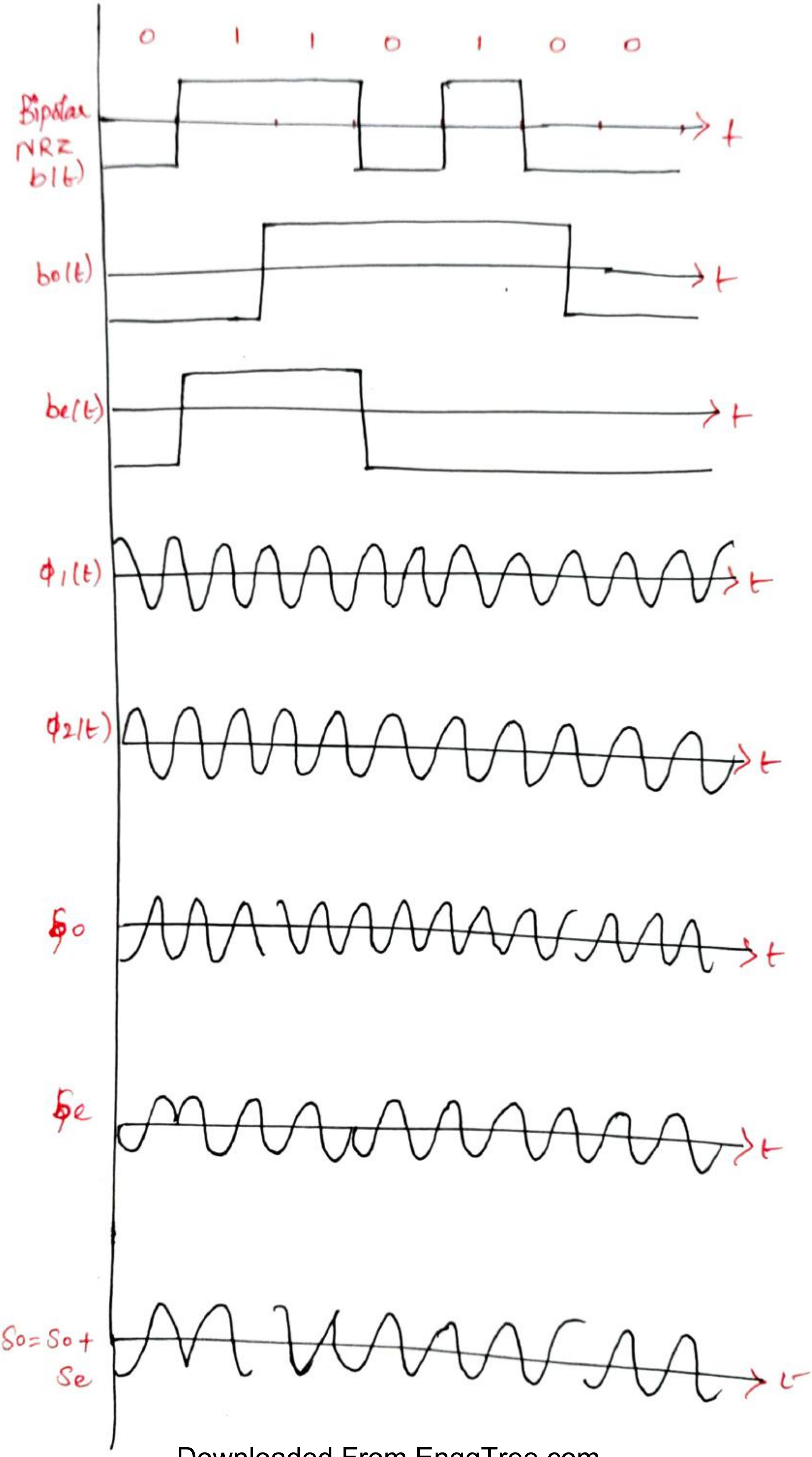


$S(t) = \sqrt{P_s}\cos 2\pi f_0 t - \sqrt{P_s}\sin 2\pi f_0 t$

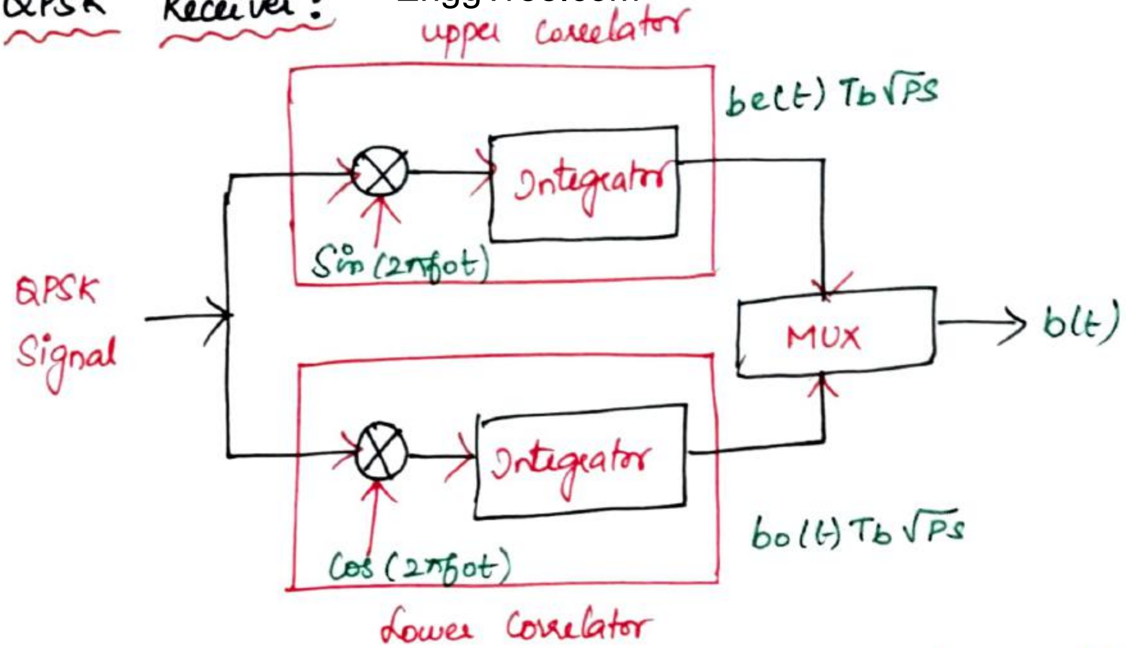
$S(t) = \sqrt{P_s}\cos 2\pi f_0 t + \sqrt{P_s}\sin 2\pi f_0 t$

$S(t) = -\sqrt{P_s}\cos 2\pi f_0 t + \sqrt{P_s}\sin 2\pi f_0 t$

QPSK waveform



QPSK Receiver :



→ Coherent carrier is to be recovered from the received signal s(t).

→ Received signal s(t) is first raised to its 4th power. Then it is passed through a bandpass filter centered around 4fo.

→ The output of the bandpass filter is a coherent carrier of frequency 4fo. This is divided by 4 and it gives two coherent quadrature carriers Cos (2πfo t) and sin (2πfot)

→ These coherent carrier are applied to two synchronous demodulators. These synchronous demodulators consist of multiplier and an integrator

→ upper integrator integrates over (2k-1) Tb to (2k+1) Tb and lower integrator integrates over 2kTb to (2k+2) Tb.

→ The output of two integrators are sampled at the offset of one bit period Tb. Odd and even sequences are combined by multiplexer.

→ upper integrator $b_e(t)\sqrt{P_s}\,T_b$

→ Lower integrator $b_o(t)\sqrt{P_s}\,T_b$

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t)$$

$$S(t) = \sqrt{E_b}\,b_o(t)\,\phi_1(t) + \sqrt{E_b}\,b_e(t)\,\phi_2(t)$$

$$d = 2\sqrt{E_b}$$

⇓

distance between signal points

Power Spectral density :

$$S_e(f) = P_s T_s\left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

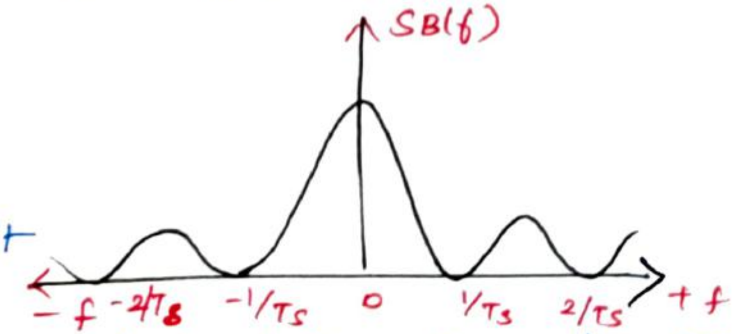
$$S_o(f) = P_s T_s\left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

$$S_B(f) = S_e(f) + S_o(f)$$

$$= 2P_s T_s\left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

Bandwidth :

$BW = f_b$



Advantages :

→ for the Same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK.

→ Because of reduced bandwidth, the information transmission rate of QPSK is higher.

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Quadrature Amplitude Shift Keying (QASK) / Quadrature Amplitude Modulation (QAM)

→ PSK signal has constant amplitude throughout. if amplitude of the signal is also varied, then the points will lie inside the circle also on the signal space diagram.

→ This further increases the noise immunity of the system. such system involves phase as well as amplitude shift keying. it is called Quadrature amplitude phase shift keying.

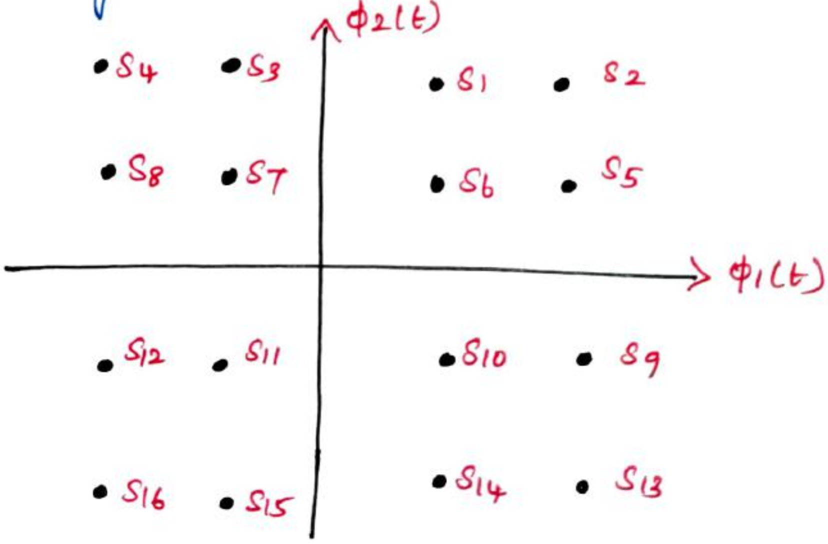
Geometrical Representation and Euclidean Distance of QASK / Signal space representation or signal Space constellation :

→ 16 QASK signals it covers distance from the neighbouring points is $d = 2a$.

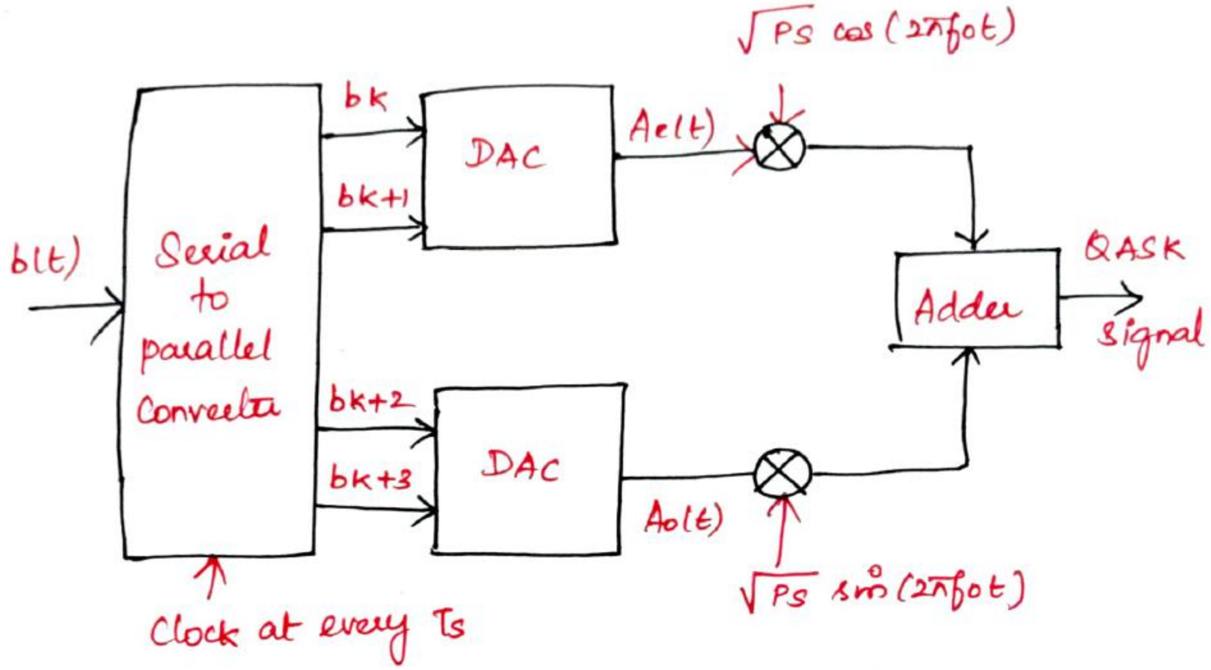
$$d_{QPSK} = 2\sqrt{E_b} = \sqrt{4E_b}$$

$$d_{16PSK} = \sqrt{0.6 E_b}$$

→ Thus the distance of 16-QASK is greater than 16 ary PSK where as it is less than QPSK.



Transmitter of QASK :



$$\rightarrow S(t) = k_1 a\, \phi_1(t) + k_2 a\, \phi_2(t)$$

$$\phi_1(t) = \sqrt{\frac{2}{Ts}} \cos(2\pi f_0 t)$$

$$\phi_2(t) = \sqrt{\frac{2}{Ts}} \cos(2\pi f_0 t)$$

$$Ps = \frac{Es}{Ts}$$

$$S(t) = k_1 \sqrt{0.2 Ps} \cos(2\pi f_0 t) + k_2 \sqrt{0.2 Ps} \sin(2\pi f_0 t)$$

→ Here k_1 and k_2 defined as the amplitude of the modulated signal.

→ Input bit stream is applied to a serial to parallel Converter.

→ four successive bits are applied to the digital to analog converters.

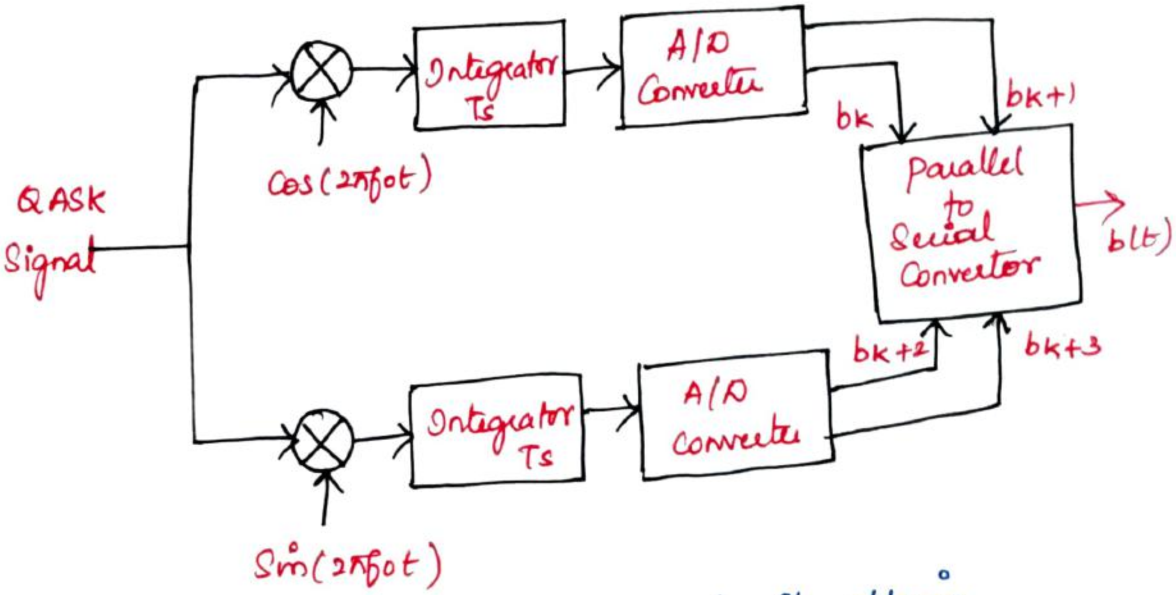
→ These bits are applied after every Ts Second.

→ bk and bk+1 are applied to upper digital to analog converter.

→ bk+2 and bk+3 are applied to lower digital to analog converter.

$$S(t) = A_e(t)\sqrt{P_s}\cos(2\pi f_o t) + A_o(t)\sqrt{P_s}\sin(2\pi f_o t)$$

Receiver of QASK :



→ The carrier recovery circuit obtains quadrature carriers from received QASK signal.

→ These carriers from received QASK signal are $\cos(2\pi f_o t)$ and $\sin(2\pi f_o t)$.

→ Integrators integrate the multiplied signals over one symbol period. The output of integrator at sampling period give $A_e(t)$ and $A_o(t)$.

→ Analog to Digital Converters gives the four bits bk, bk+1, bk+2 and bk+3.

→ The parallel to serial converter then generates the bit sequence b(t).

→ Inphase and quadrature coherent carriers are multiplied with QASK signal.

power Spectral Density :

$$S(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s} \right]^2$$

$$S(f) = \frac{P_s T_s}{2} \left[\frac{\sin \pi (f - f_0) T_s}{\pi (f - f_0) T_s} \right]^2 + \frac{P_s T_s}{2} \left[\frac{\sin \pi (f + f_0) T_s}{\pi (f + f_0) T_s} \right]^2$$

Bandwidth :

$$BW = 2 f_s$$

$$= \frac{2}{T_s}$$

→ Thus the Bandwidth and power Spectral density of QASK is similar to that of M-ary PSK.

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Differential phase shift keying (DPSK)/ Non-coherent version of BPSK :

→ Differential phase shift keying (DPSK) is differentially coherent modulation method.

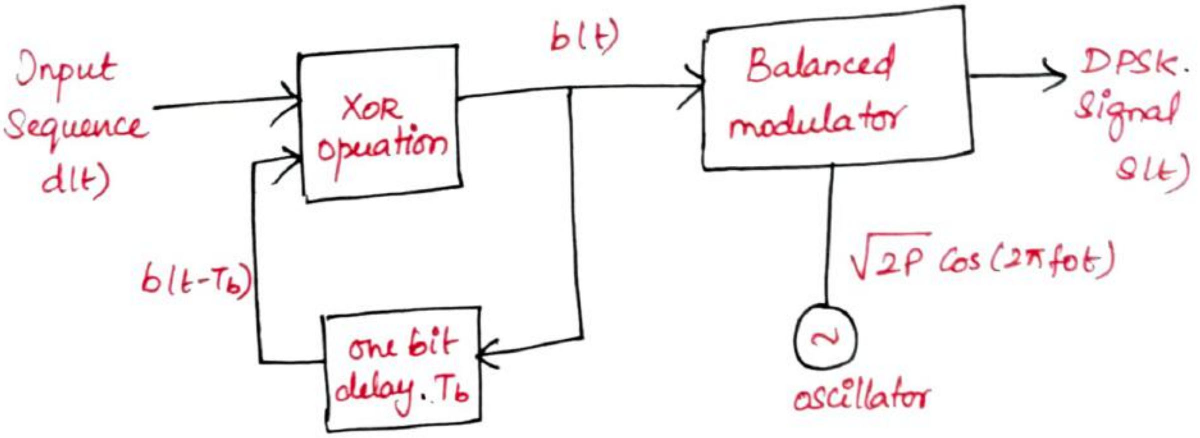
→ DPSK does not need a synchronous (coherent) carrier at the demodulator.

→ The input sequence of binary bits is modified such that the next bit depends upon the previous bit.

→ Therefore in the receiver the previous received bits are used to detect the present bit.

→ Non-Coherent detection of BPSK is not possible since message information lies in phases. Hence DPSK is also called non-coherent version of BPSK.

DPSK Transmitter :

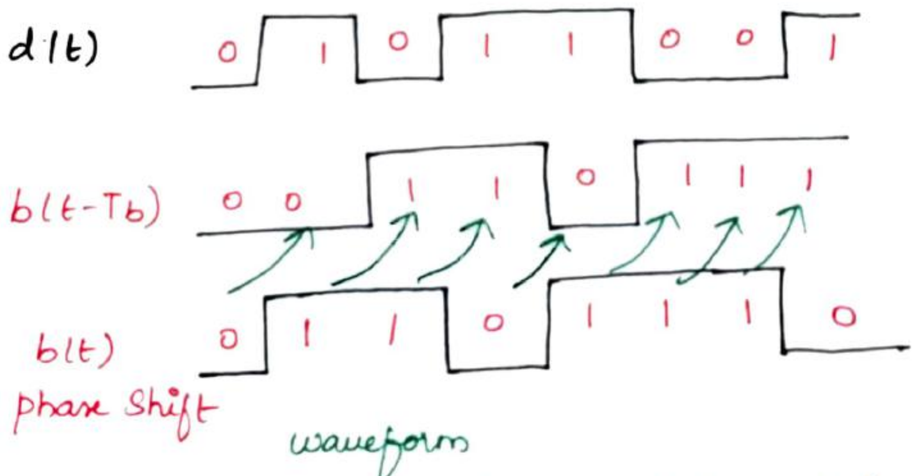


→ The differentially encoded signal b(t) then performs BPSK modulation of the carrier $\sqrt{2P}\cos(2\pi f_0 t)$

→ symbol duration $T = 2T_b$

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

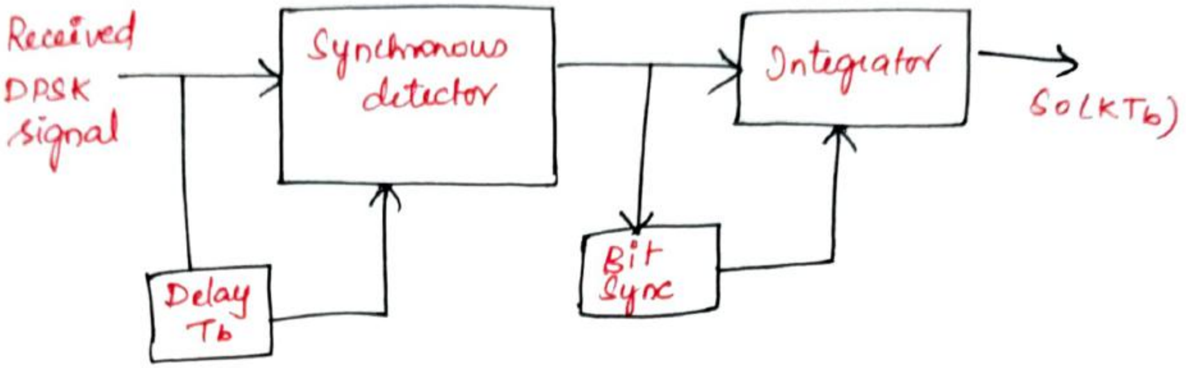
$$= \pm\sqrt{2P}\cos(2\pi f_0 t)$$



→ sequence b(t) is applied to a balanced modulator. The balanced modulator is also supplied with a carrier $\sqrt{2P}\cos(2\pi f_0 t)$.

→ BPSK phase of the carrier changes on both the symbol '1' and '0'.

DPSK Receiver :



→ phase shift in received signal, During the transmission, the DPSK signal undergoes some phase shift θ. Therefore the signal received at the input of the receiver.

→ Received signal $= b(t) \cdot \sqrt{2P} \cos(2\pi f_0 t + \theta)$

→ synchronous detector, This signal is multiplied with its delayed version by one bit.

$$\text{Multiplier output} = b(t) \cdot b(t-T_b)\ (2P)\ \cos(2\pi f_0 t+\theta) \cdot \cos(2\pi f_0 (t-T_b)+\theta)$$

→ f_0 is the carrier frequency and T_b is one bit period T_b contains integral number of cycles of f_0.

$$f_b = \frac{1}{T_b} \; ; \; f_0 = \frac{n}{T_b}$$

→ Integrator. The above signal is given to the integrator. In the kth bit interval,

$$S_o(kT_b) = b(kT_b)\ b[(k-1)T_b]\ PT_b$$

$b(t) \cdot b(t-T_b) = 1V$ then $d(t) = 0$

$b(t) \cdot b(t-T_b) = -1V$ then $d(t) = 1$

→ Decision device output

$$S_0 (KT_b) = \begin{cases} -PT_b, & \text{then } d(t)=1 \\ +PT_b, & \text{then } d(t)=0 \end{cases}$$

Bandwidth :

→ One symbol duration (T) is equivalent to two bits duration ($2T_b$).

$$BW = f_b \quad ; \quad T = 2T_b$$

→ Thus the minimum bandwidth in DPSK is equal to f_b.

Advantages :

→ DPSK does not need carrier at its receiver Hence the complicated circuitry for generation of local carrier is avoided.

→ The Bandwidth requirement of DPSK is reduced compared to that of BPSK.

Disadvantages :

→ The probability of error or bit error rate of DPSK is higher than that of BPSK.

→ Noise interference in DPSK is more.

→ Since DPSK uses two successive bits for its reception, error in the first bit creates error in the second bit. Error propagation in DPSK is more.

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Carrier Synchronisation (Carrier Recovery):

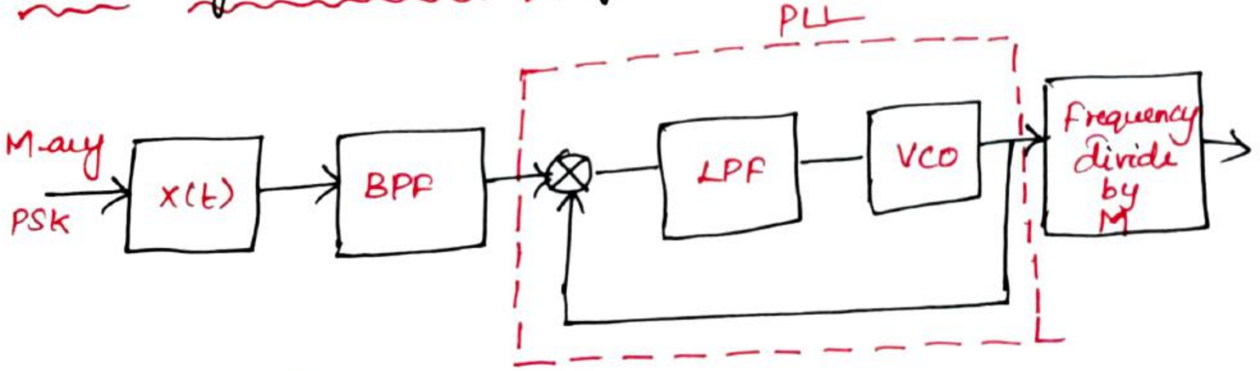
→ Carrier Synchronisation is required in coherent detection methods to generate a coherent reference at the receiver.

→ Modulated Carrier signal contains a discrete component at the carrier frequency.

→ phase lock loop can be used to track this component fc. The output frequency of phase locked loop is thus locked to the carrier frequency fc in the transmitted signal.

→ This output frequency of phase locked loop is used as a coherent reference signal for detection in the receiver.

Carrier synchronisation using M th power loop:



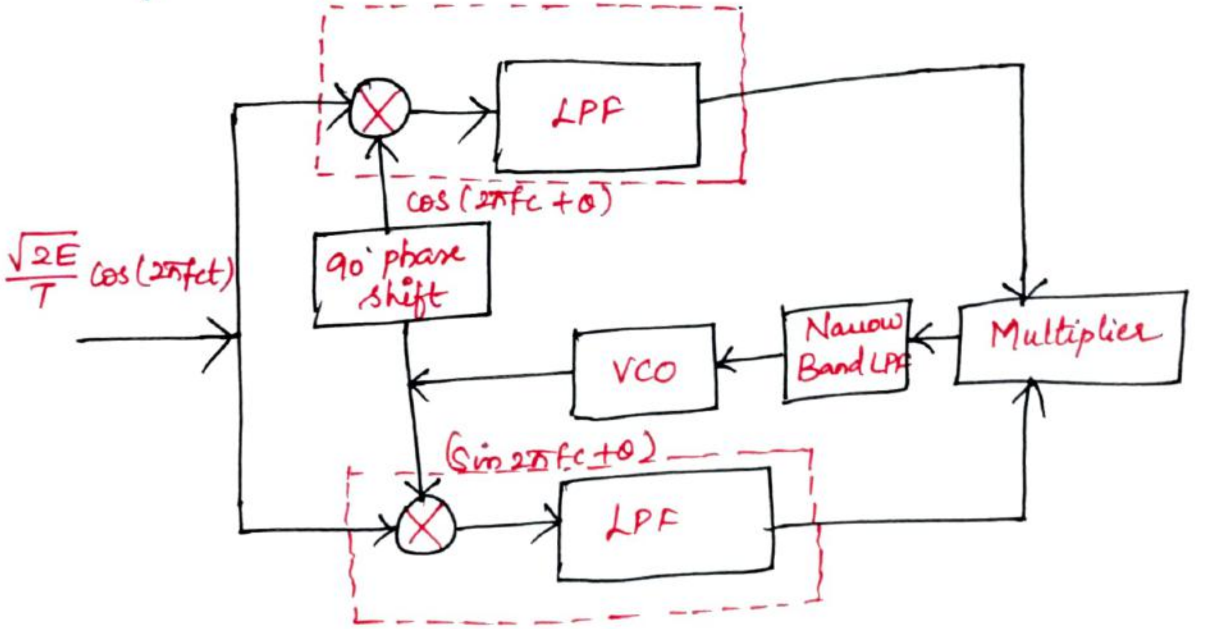
→ sign of the recovered carrier is always independent of sign of input signal carrier since it is squared. Therefore 180° error in the output.

Costas Loop for carrier synchronisation:

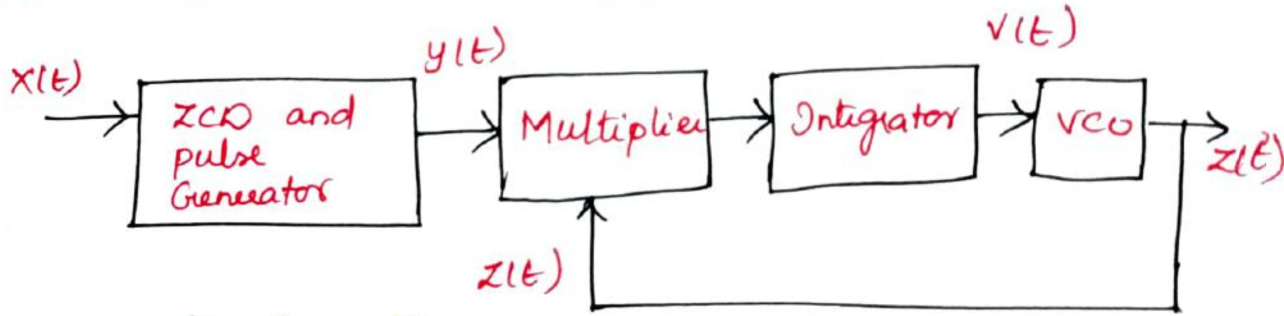
→ This is the alternative method for Carrier Synchronisation. This is used for binary phase shift keying.

→ There are two phase locked loops and Common VCO and separate phase comparators.

→ BPSK signal is supplied to both the phase comparators. The Low pass filters remove the double frequency terms generated in the phase comparators.



Closed loop bit synchronization :



→ when the baseband signal crosses zero, the output of the zero crossing detector is rectangular pulse of half bit duration.

→ Multiplication result is integrated and Low pass filtered to produce a control voltage v(t).

→ Average value of y(t) and z(t) is zero and Control Voltage remains constant.

— × —

UNIT 4 Two marks

1) Obtain the orthonormal basis functions for the signal.

$$S_1(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) ; & 0 \le t \le T_b \\ 0 ; & 0 \le t \le T_b \end{cases}$$

→ Sine and cosine functions are orthonormal to each other

$$S_2(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \sin(2\pi f_c t) ; & 0 \le t \le T_b \\ 0 & ; 0 \le t \le T_b \end{cases}$$

—— × ——

2) what is QPSK. write the expression for the signal set of QPSK.

→ Two successive bits in the data sequence are grouped together. This combination of two bits forms four distinct symbols. when the symbol is changed to next symbol the phase of the carrier is changed by 45° ($\pi/4$). phase shift will be $\pi/4$, $3\pi/4$, $5\pi/4$ or $7\pi/4$.

$$\rightarrow S(t) = b_o(t) \sqrt{P_s} \cos(2\pi f_o t) + b_e(t) \sqrt{P_s} \sin(2\pi f_o t)$$

—— × ——

3) what are the drawbacks of ASK?

→ peak transmitter power is not constant.

→ suitable only for simple transmission applications.

—— × ——

4) what is QAM?

→ The amplitude as well as well as phase of the quadrature carrier is modulated by input digital data. it results in QAM (Quadrature Amplitude Modulation)

—— × ——

5) Mention the advantages of PSK systems.

→ Reduced Bandwidth

→ peak transmitted power remains constant.

——— × ———

6) what are the advantages of QPSK over PSK?

→ Bandwidth of QPSK is reduced to half.

→ Amplitude variations in QPSK are reduced.

——— × ———

7) what are the drawbacks of binary PSK system?

→ it is difficult to detect +b(t) or -b(t) because of Squaring in the receiver.

→ problems of ISI and interchannel interference are present.

——— × ———

8) what is mean by memoryless modulation?

→ when the digital symbol modulates amplitude, phase or frequency of the carrier without any reference to previous symbol, it is called memoryless modulation. ex: ASK, FSK, PSK, QPSK.

——— × ———

9) Define QAM.

→ phase as well as amplitude of the Quadrature Carriers is modulated. Hence it is called Quadrature Amplitude modulation.

——— × ———

10) Difference between Coherent and Non-coherent transmission.

→ Coherent, the local carrier generated at the receiver is phase locked with the carrier at the transmitter.

→ Non Coherent, The receiver carrier need not be phase locked with transmitter carrier.

——— × ———

UNIT - 4 Big Questions

1) ASK
2) BFSK
3) BPSK
4) QPSK
5) QAM / QASK
6) DPSK
7) Carrier Recovery

UNIT-5 Demodulation Techniques

Elements of Detection Theory:

Concept of detection:

→ The modulator / transmitter sends one of the 'M' possible signals in the given time slot.

→ These signals are $s_1(t), s_2(t) \ldots s_M(t)$

→ when the signal travels along the channel, Additive white Gaussian Noise (AWGN) interferes and changes the signal's shape.

s_i → transmitted signal
x → received signal
w → noise vector

$$x = s_i + w \quad , \quad i = 1, 2, \ldots M$$

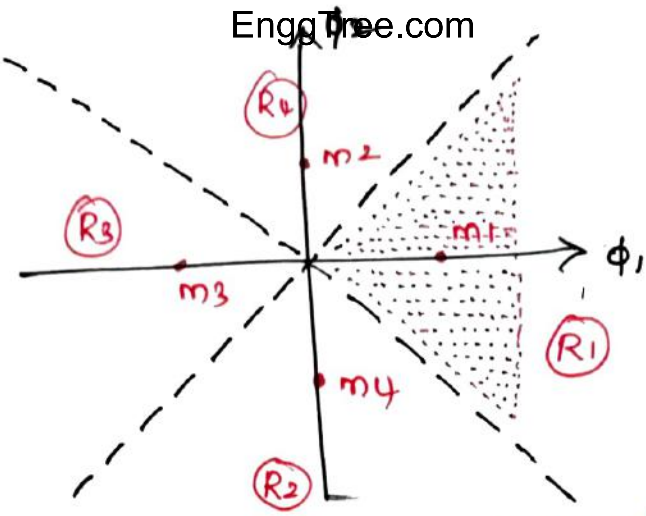
→ Based on the received observation vector 'x' the mapping is made to an estimate $\hat{m}$ of the transmitted symbol m_i in such a way that probability of error is minimum.

Maximum Likelihood Decoding:

→ observation vector be x. The decision is made as $\hat{m} = m_i$. The average probability of Symbol error in this decision is

$$P_e(m_i, x) = P(m_i \text{ not sent} / x) = 1 - P(m_i \text{ sent} / x)$$

Set $\hat{m} = m_i$, if $P(m_i \text{ sent} / x) \geq P(m_K \text{ sent} / x)$ for all $k \neq i$



Example of maximum likelihood decision $N=2$; $M=4$

→ There are four regions $R_1, R_2, R_3\ R_4$

→ messages m_1, m_2, m_3, m_4

→ vectors falls in region R_1 then message m_1 is selected.

Maximum Likelihood Detection process:

$P(x_1)$ = probability of receiving symbol x_1

$P(x_2)$ = " " " x_2

$f_X(v/x_1)$ = pdf of receiving voltage v when x_1 is sent

$f_X(v/x_2)$ = pdf of receiving voltage v when x_2 is sent.

λ = Decision threshold

$v > \lambda$, symbol x_1 is selected

$v < \lambda$, Symbol x_2 is selected

$$P_e = \int_{v<\lambda} P(x_1) f_X(v/x_1)\,dv + \int_{v>\lambda} P(x_2) f_X(v/x_2)\,dv$$

$$P(x_1) f_X(v/x_1) > P(x_2) f_X(v/x_2)$$

(or)

$$\frac{P(v/x_1)}{P(v/x_2)} > \frac{P(x_2)}{P(x_1)}$$

This equation called maximum likelihood detector.

Optimum filter (Receiver) Design:

→ The optimum filter is used to detect the baseband signals and unmodulated passband signals.

→ Transmitted signal sequence is 10011. pulse checked at the point T.

Requirements of signal receiver:

→ signal to noise ratio must be improved.

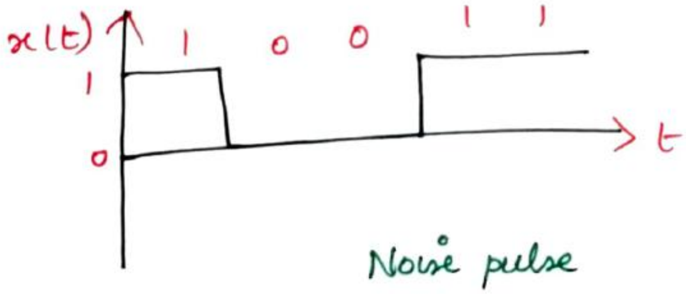
→ signal must be checked when signal to noise ratio is maximum.

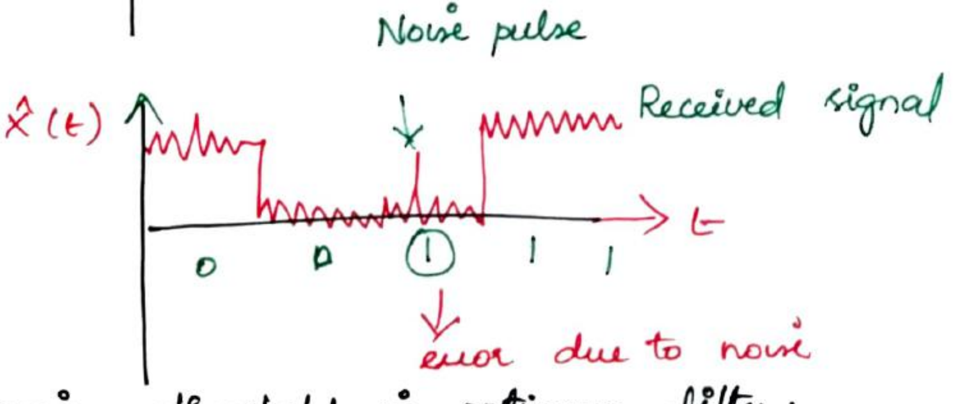
→ error probability should be minimum.

Matched filter:

→ it satisfies all the above requirements

→ it is called matched filter since its impulse response is matched to shape of input signal.



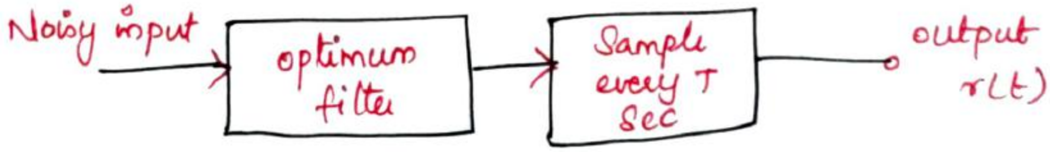


Decision threshold in optimum filter:

→ binary 1 ; $x_1(t) = +A$

→ binary 0 ; $x_2(t) = -A$

→ Thus the input signal $x(t)$ will be either $x_1(t)$ or $x_2(t)$ depending upon the polarity of the NRZ signal.



→ Input to the receiver $= x(t) + n(t)$

→ output from the receiver $= x_{01}(T) + n_0(T)$ or $x_{02}(T) + n_0(T)$

When noise is absent :

$$r(t) = x_{01}(T) \quad ; \quad x(t) = x_1(t)$$

$$r(t) = x_{02}(T) \quad ; \quad x(t) = x_2(t)$$

When noise is present :

$$\text{Decision boundary} = \frac{x_{01}(T) + x_{02}(T)}{2}$$

Error Conditions :

→ $x_{01}(T) > x_{02}(T)$

$$n_0(T) \geq \frac{x_{01}(T) + x_{02}(T)}{2} - x_{02}(T)$$

$$n_0(T) \geq \frac{x_{01}(T) - x_{02}(T)}{2}$$

→ $x_{02}(T) > x_{01}(T)$

$$n_0(T) \leq - \frac{x_{01}(T) + x_{02}(T)}{2} - x_{01}(T)$$

$$n_0(T) \leq \frac{x_{02}(T) - x_{01}(T)}{2}$$

Error probability:

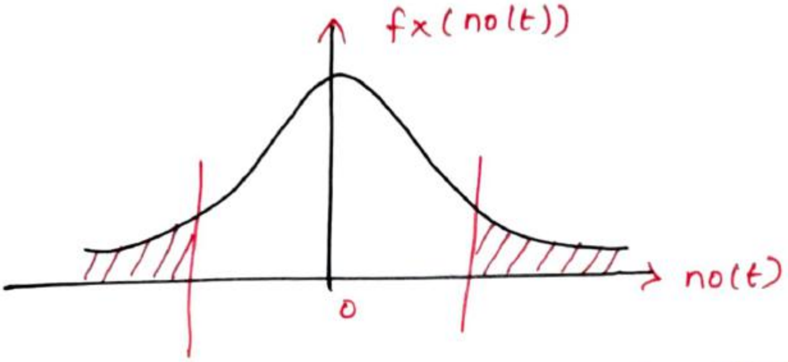
Gaussian Noise:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/2\sigma^2}$$

$f_X(x)$ is the Pdf of random Variable x

m → mean Value

σ → Standard deviation



probability that no(t) takes value in shaded area

$$P\left[n_o(t) \geq \frac{x_{01}(T) - x_{02}(T)}{2}\right] = P\left[n_o(t) \leq \frac{-x_{02}(T) - x_{0}}{2}\right.$$

Error probability

$$P_e = P\left[n_o(t) \geq \frac{x_{01}(T) - x_{02}(T)}{2}\right] = P\left[n_o(t) \leq \frac{-x_{02}(T) - x_{01}(T}{2}\right.$$

$$P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma}\right]$$

* * *

Matched filter:

→ The optimum filter becomes matched filter when white Gaussian noise is present.

Maximum signal to Noise power Ratio:

→ Standard deviation

$$\sigma = [\text{Mean square Value} - \text{Square of mean Value}]^{1/2}$$

Error probability:

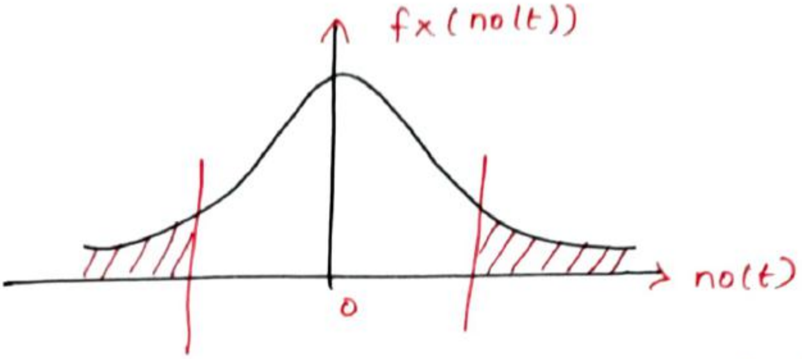
Gaussian Noise:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/2\sigma^2}$$

$f_X(x)$ is the Pdf of random Variable x

$m \rightarrow$ mean Value

$\sigma \rightarrow$ Standard deviation



probability that $n_0(t)$ takes Value in shaded area

$$P\left[n_0(t) \geq \frac{x_{01}(T) - x_{02}(T)}{2}\right] = P\left[n_0(t) \leq \frac{-x_{02}(T) - x_{01}}{2}\right.$$

Error probability

$$P_e = P\left[n_0(t) \geq \frac{x_{01}(T) - x_{02}(T)}{2}\right] = P\left[n_0(t) \leq \frac{-x_{02}(T) - x_{01}(T)}{2}\right.$$

$$P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma}\right]$$

——— X ———

Matched filter:

→ The optimum filter becomes matched filter when white Gaussian noise is present.

Maximum signal to Noise power Ratio:

→ Standard deviation

$\sigma = [$Mean square Value − Square of mean Value$]^{1/2}$

$$= [\overline{x}^2 - m_x^2]^{1/2}$$

→ mean square value

$$\overline{x}^2 = \sigma^2 = \overline{n_0^2(t)}$$

→ signal power, the erfc function is the monotonically decreasing function. Therefore Pe decreases as the ratio $\frac{x_{01}(T) - x_{02}(T)}{\sigma}$

$$\rho = \frac{x_0^2(T)}{\sigma^2}$$

$$x_0(T) = x_{01}(T) - x_{02}(T)$$

$$X_0(f) = H(f)\, X(f)$$

$$x_0(T) = \text{IFT}\{X_0(f)\}$$

$$= \int_{-\infty}^{\infty} X_0(f)\, e^{j2\pi ft}\, df$$

$$= \int_{-\infty}^{\infty} H(f)\, X(f)\, e^{j2\pi fT}\, df$$

Input and output power spectral densities of noise are related as,

$$S_{no}(f) = |H(f)|^2\, S_{ni}(f)$$

$$S_{ni}(f) = \frac{N_0}{2}$$

$$S_{no}(f) = |H(f)|^2 \cdot \frac{N_0}{2}$$

$$\rho = \frac{x_0^2(T)}{\sigma^2} = \frac{\left|\int_{-\infty}^{\infty} H(f)\, X(f)\, e^{j2\pi fT}\, df\right|^2}{\int_{-\infty}^{\infty} |H(f)|^2\, \frac{N_0}{2}\, df}$$

use of Schwarz's Inequality

$$Q_1(f) = \sqrt{\frac{N_0}{2}}\; H(f)$$

$$Q_2(f) = \frac{1}{\sqrt{\frac{N_0}{2}}}\; X(f)\, e^{j2\pi fT}$$

$$\rho = \frac{\left| \int_{-\infty}^{\infty} Q_1(f) \cdot Q_2(f)\, df \right|^2}{\int_{-\infty}^{\infty} |Q_1(f)|^2\, df}$$

Maximum Error probability of a Matched filter

$$\rho_{max} = \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]^2$$

parseval's power theorem states that,

$$\int_{-\infty}^{\infty} |X(f)|^2\, df = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{0}^{T} x^2(t)\, dt$$

$$P_e = \frac{1}{2}\, \text{erfc} \sqrt{\frac{E}{N_0}}$$

* * *

Impulse Response of a matched filter:

→ Impulse response can be obtained from the transfer function of the matched filter.

Transfer function of matched filter:

→ Transfer function of the matched filter should be maximize the signal to noise power ratio.

$$\rho \le \int_{-\infty}^{\infty} \frac{|X(f)|^2}{\frac{N_0}{2}}\, df$$

$$H(f) = K \cdot \frac{X^*(f)}{\frac{No}{2}} \cdot e^{-j2\pi f T}$$

Transfer function of matched filter

$$H(f) = \frac{2k}{No} x^*(f) \; e^{-j2\pi f t}$$

Impulse response:

→ Impulse response can be obtained by taking inverse fourier transform of the transfer function of matched filter.

$$H(f) = \frac{2k}{No} x(-f) \; e^{-j2\pi f t}$$

$$h(t) = IFT \{ H(f) \}$$

$$= IFT \left\{ \frac{2k}{No} x(-f) \cdot e^{-j2\pi f T} \right\}$$

$$FT \{ x(-T) \} = x(-f)$$

$$FT \{ x(T-t) \} = x(-f) \; e^{-j2\pi f T}$$

Impulse response of matched filter

$$h(t) = \frac{2k}{No} \{ x_1(T-t) - x_2(T-t) \}$$

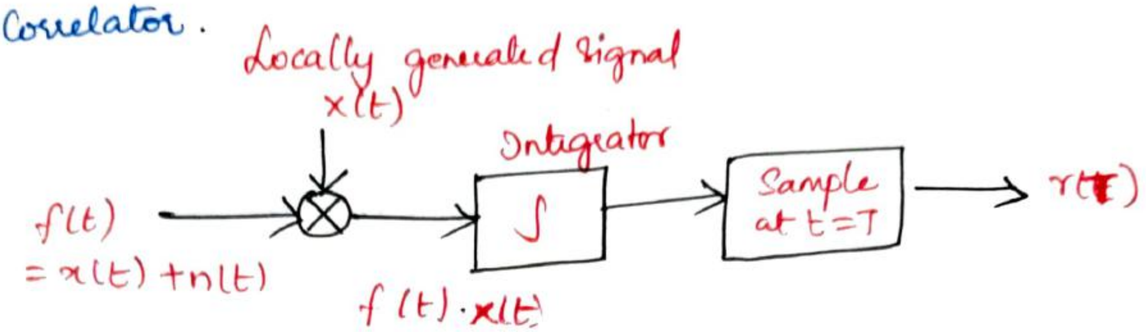
Properties of matched filter:

→ signal to noise ratio of the matched filter depends only upon the ratio the signal energy

→ The output Signal of a matched filter is proportional to a shifted version of the auto-correlation function of the input signal to which the filter is matched.

Correlation Receiver :

→ Different types of receiver is called Correlator.



→ The adjacent figure f(t) represents input noisy signal $f(t) = x(t) + n(t)$.

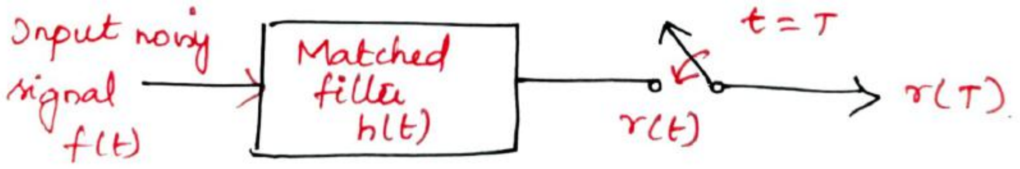
→ f(t) multiplied with x(t) and it is integrated by integrator.

→ Then based on the sampled value, decision is made.

$$r(t) = \int_0^T f(t) \cdot x(t)\, dt$$

$$t = T$$

$$r(T) = \int_0^T f(t) \cdot x(t)\, dt.$$



matched filter receiver

$$r(t) = f(t) * h(t) = \int_{-\infty}^{\infty} f(\tau) \cdot h(t-\tau)\, d\tau$$

$$h(t) = \frac{2k}{No}\, x(T-t)$$

$$h(t-\tau) = \frac{2k}{No}\, x(T-t+\tau)$$

$$r(t) = \int^{\infty} f(\tau)\, \frac{2k}{No}\, x(T-t+\tau)\, d\tau$$

$$r(T) = \frac{2k}{N_0} \int_0^T f(t)\, x(t)\, dt$$

* * *

Probability of Error for Coherently Detected BPSK:

→ In binary PSK (BPSK), the phase of the carrier is shifted by 180° for two symbols.

$$\text{binary } 1 \Rightarrow x_1(t) = \sqrt{2P}\cos(2\pi f_0 t)$$

$$\text{binary } 0 \Rightarrow x_2(t) - \sqrt{2P}\cos(2\pi f_0 t)$$

$$x_2(t) = -x_1(t)$$

$$P_e = \frac{1}{2}\,\text{erfc}\left\{\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}\right\}$$

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{2}{N_0}\int_0^T x^2(t)\, dt.$$

$$x(t) = 2x_1(t)$$

$$= \frac{8}{N_0}\int_0^T x_1^2(t)\, dt$$

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{8}{N_0}\cdot E$$

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]_{max} = \sqrt{\frac{8E}{N_0}}$$

$$P_e = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E}{N_0}}\;;\quad \boxed{P_e = Q\sqrt{\frac{2E}{N_0}}}$$

* * *

Error probability of binary FSK :

probability of Error for Coherently detected binary orthogonal FSK :

→ In the binary FSK transmission, two different Carrier frequencies are used to transmit two binary levels.

binary 1 ⇒ $x_1(t) = \sqrt{2P} \cos(2\pi f_0 + \Omega)t$

binary 0 ⇒ $x_2(t) = \sqrt{2P} \cos(2\pi f_0 - \Omega)t$

$$x_1(t) - x_2(t) = \sqrt{2P}\left\{\cos(2\pi f_0 + \Omega)t - \cos(2\pi f_0 - \Omega)t\right\}$$

$$[x_1(t) - x_2(t)]^2 = 2P\left[\cos(2\pi f_0 + \Omega)t - \cos(2\pi f_0 - \Omega)t\right]^2$$

$$= 2P\left[-2\sin 2\pi f_0 t \ \sin \Omega t\right]^2$$

$$2\pi f_0 = \omega_0$$

$$= 2P\left[4\sin^2 \omega_0 t \cdot \sin^2 \cancel{\omega_0}\cdot \Omega t\right]$$

$$= 2P\left\{(2\sin^2 \omega_0 t)(2\sin^2 \Omega t)\right\}$$

$$\{x_1(t) - x_2(t)\}^2 = 2P\left\{(1 - \cos 2\omega_0 t)(1 - \cos 2\Omega t)\right\}$$

$$= 2P\left\{1 - \cos 2\Omega t - \cos 2\omega_0 t + \cos 2\omega_0 t \cos 2\Omega t\right\}$$

$$\int_0^T [x_1(t) - x_2(t)]^2 dt = \int_0^T 2P\left\{1 - \cos 2\Omega t - \cos 2\omega_0 t + \frac{1}{2}\left[\cos 2(\omega_0 - \Omega)t + \cos 2(\omega_0 + \Omega)t\right\} dt\right.$$

$$\int_0^T [x_1(t) - x_2(t)]^2 \, dt = 2PT\left\{1 - \frac{\sin 2\Omega T}{2\Omega T}\right\}$$

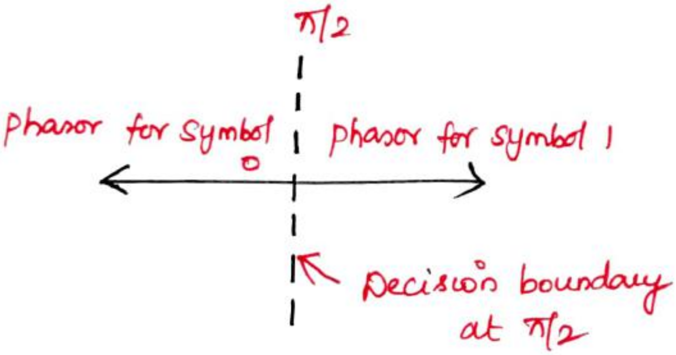
$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{0.6E}{N_0}}$$

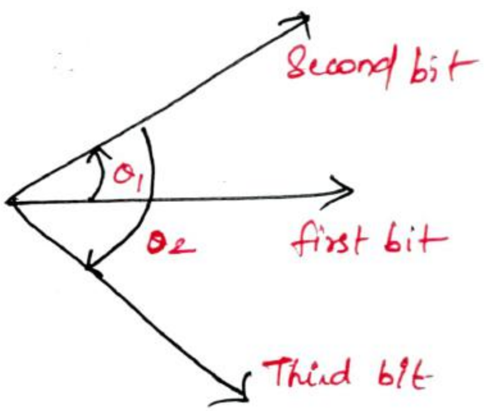
probability of error for Non-coherently detected binary orthogonal FSK :

$$P_e = \frac{1}{2} e^{-E/2N_0}$$

→ it Depends on signal energy.

probability of error for binary orthogonal DPSK :





→ Decision device 1

difference between two consecutive bits differs by $<\pi/2$

→ Decision device 0

difference between two consecutive bits differs by $>\pi/2$

$$E = 2E_b$$

$$P_e = \frac{1}{2} e^{-E_b/N_0}$$

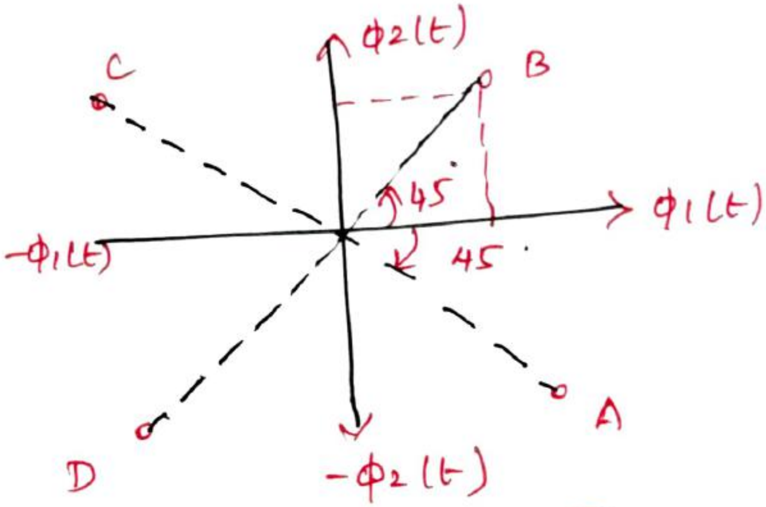
This is called average probability of error

probability of error for QPSK :

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_s \cos^2\theta}{N_0}}$$

→ Consider the receiver for QPSK signal

$$P_{e1} = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_s \cos^2\theta}{N_0}}$$



$$P_{e2} = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_s \cos^2\theta}{N_0}}$$

$$P_{e1} = P_{e2} = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_s \cos^2 45^\circ}{N_0}}$$

$$= \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_s}{2N_0}}$$

Error probability of QPSK $P_e = \mathrm{erfc}\sqrt{\frac{E_s}{2N_0}}$

$$P_e = 2Q\sqrt{\frac{2E_b}{N_0}} \quad ; \; E_s = 2E_b$$

— × —

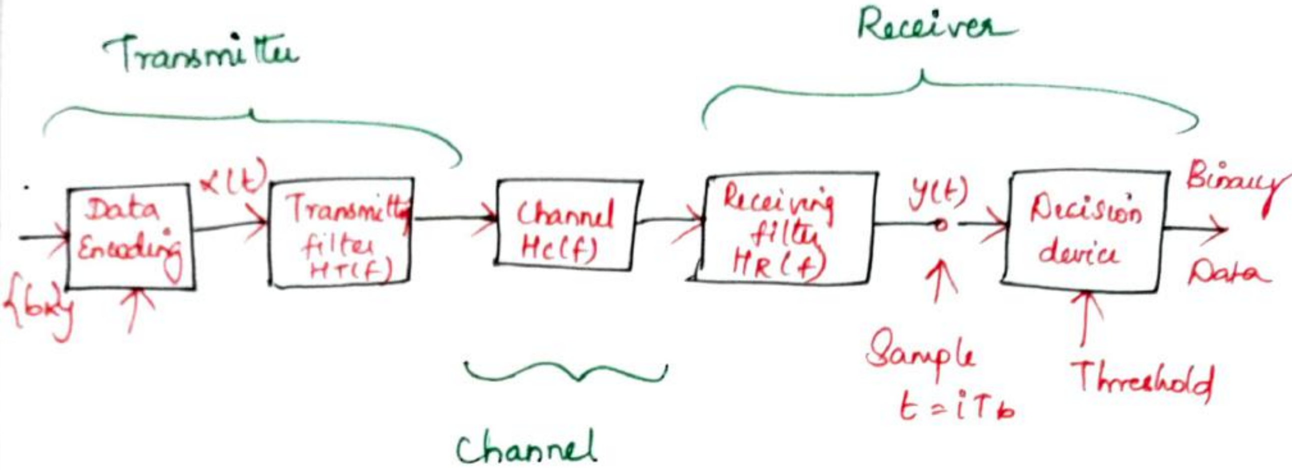
Intersymbol Interference (ISI):

Baseband Transmission of binary data:

→ As we already know that binary data can be transmitted in baseband or passband.

→ In baseband transmission, there is no modulation of high frequency carrier.

→ Two amplitude levels corresponding to binary 1 and 0.



→ These signal transmitted over the channel in baseband transmission.

$$x(t) = \sum_{k=-\infty}^{\infty} A_k \, g(t-kT_b)$$

$$A_k = \begin{cases} +a & \text{if} \quad b_k = 1 \\ -a & \text{if} \quad b_k = 0 \end{cases}$$

T_b → duration of each input binary bit

$g(t)$ → Shaping pulse.

→ $x(t)$ passed through transmitting filter $H_T(f)$.

→ The signal then passed to $H_c(f)$ receiving filter. Combined transfer function $H_R(f)$. output is $y(t)$

$y(t_i) > \lambda \Rightarrow$ symbol '1'

$y(t_i) \leq \lambda \Rightarrow$ symbol '0'

ISI:

→ The presence of outputs due to other bits interfere with the output of required bit. This effect is called ISI.

$$y(t_i) = \mu A_i$$

→ ISI can be reduced by proper design of pulse Spectrum $G(f)$, $H_R(f)$ and $H_c(f)$.

Intersymbol Interference (ISI) Problem :

$$y(t) = \mu \sum_{k=-\infty}^{\infty} A_k \, P(t - kT_b)$$

μ → Scaling factor

$P(t)$ → shape ~~of~~ different from $g(t)$

$$\mu A_k P(f) = H(f) A_k G(f)$$

⇓

fourier transform of $g(t)$ be $G(f)$. $P(t)$ be $P(f)$

$H(f)$ combined transfer function of transmitting filter,

$$H(f) = H_T(f) \cdot H_C(f) \cdot H_R(f)$$

$$y(t_i) = \mu A_i P(0) + \mu \sum_{k=-\infty}^{\infty} A_k P\left[(i-k)T_b\right]$$

$$i = 0, \pm 1, \pm 2 \ldots$$

——— × ———

Nyquist criterion for Distortionless Transmission:

Nyquist pulse shaping criterion:

Time domain criterion:

→ To Eliminate ISI second term of summation must be zero, $P(t)$ is

$$P[(i-k)T_b] = \begin{cases} 1 & \text{for } i = k \\ 0 & \text{for } i \neq k \end{cases}$$

→ $P(t)$ satisfies the above conditions

$$y(t_i) = \mu A_i$$

Frequency Domain:

$$P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f - n f_b)$$

→ P(t) are periodic with period f_b.

$$P_\delta(t) = \sum_{n=-\infty}^{\infty} P(nT_b)\, \delta(t - nT_b)$$

take fourier transform

$$P_\delta(f) = \int_{-\infty}^{\infty} P_\delta(t) \cdot e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} P(nT_b) \cdot \delta(t - nT_b)\right] \cdot e^{-j2\pi ft}\, dt.$$

$n = i - k$

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} P[(i-k)T_b] \cdot \delta[t - (i-k)T_b] \cdot e^{-j2\pi ft}\, dt.$$

$$P_\delta(f) = \begin{cases} \int_{-\infty}^{\infty} P(0)\, \delta(t) \cdot e^{-j2\pi ft}\, dt & ; \text{ for } i = k \\ \int_{-\infty}^{\infty} 0\ \delta(t) \cdot e^{-j2\pi ft}\, dt & ; \text{ for } i \neq k \end{cases}$$

$$P_\delta(f) = \int_{-\infty}^{\infty} P(0)\ \delta(t)\, e^{-j2\pi ft}\, dt$$

$P_\delta(f) = P(0)$; for $i = k$

$= 1$

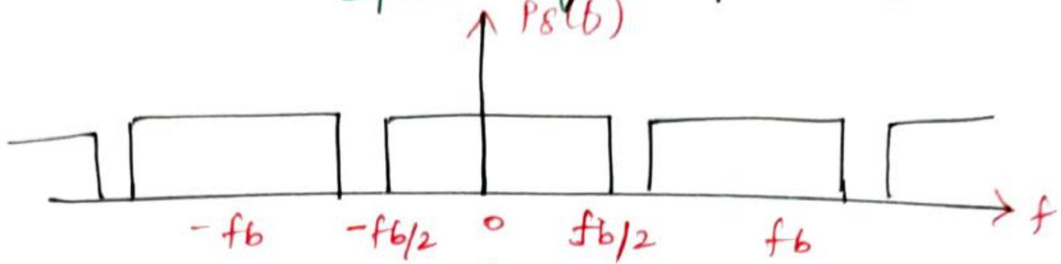
$$\sum_{n=-\infty}^{\infty} P(f - nf_b) = \frac{1}{f_b}$$

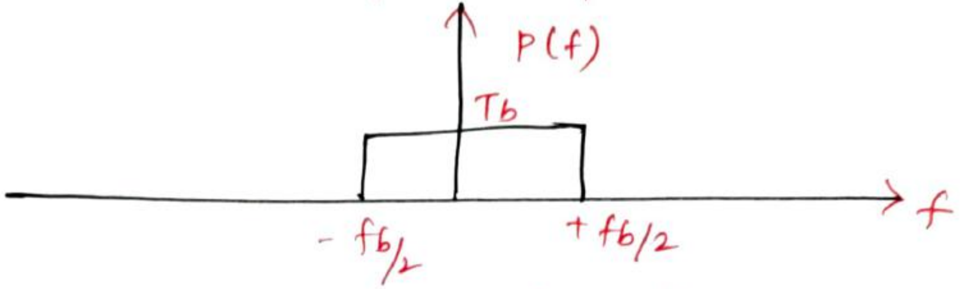
$1/f_b = T_b$

Ideal Nyquist channel:

$$P(f) = \frac{1}{f_b} \text{rect}\left(\frac{f}{f_b}\right)$$

Spectrum of sampled signal





Spectrum of P(t)

$$P(t) = \text{Sinc}(f_b t) \qquad B_0 = f_b/2$$

$$P(t) = \text{Sinc}(2B_0 t)$$

Nyquist Bandwidth:

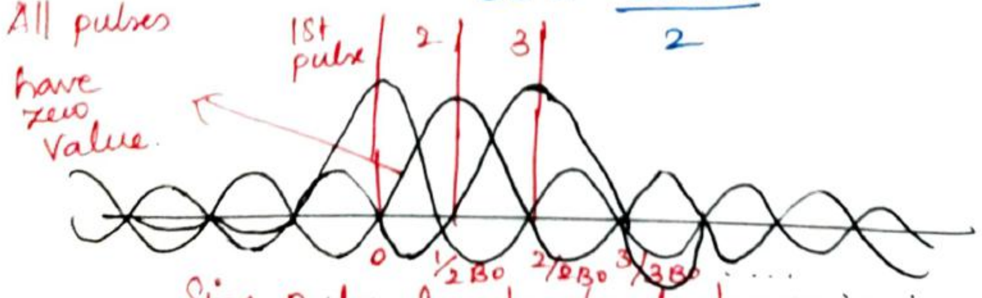
→ B_0 is called Nyquist Bandwidth.

Nyquist Bandwidth is the minimum transmission bandwidth for zero ISI.

$$\text{Bit period } T_b = \frac{1}{2B_0}$$

$$\text{Nyquist Bandwidth } B_0 = \frac{1}{2T_b}$$

$$B_0 = \frac{\text{Bitrate}}{2}$$



Sinc pulse for baseband transmission Zero ISI

→ Ideal sinc pulse physically not possible

$$P(t) = \text{sinc}\,(2B_0 t)$$

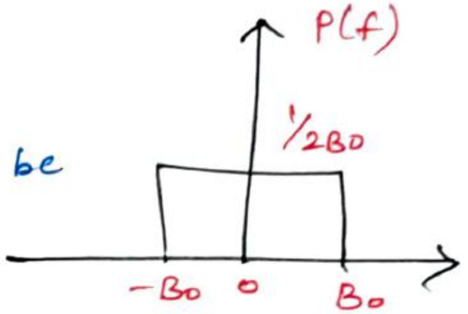
$$P(f) = \frac{1}{2B_0}\,\text{rect}\,\left(f/2B_0\right) \Rightarrow \text{fourier transform}$$

$$P(f) = \begin{cases} 1/2B_0 & \text{for } -B_0 < f \leq B_0 \\ 0 & \text{elsewhere} \end{cases}$$

merits :

→ it eliminates ISI

→ The method seems to be most easy.



Spectrum of sinc pulse

Limitations :

→ The transmission of exact sinc pulse is physically not possible.

Raised Cosine channel :

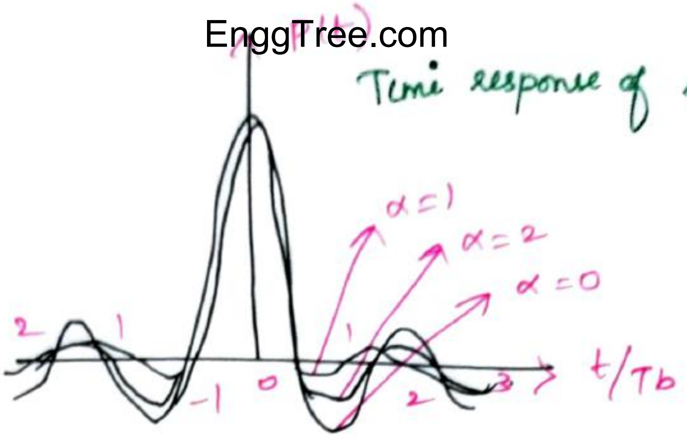
→ Raised cosine Spectrum, the frequency response p(f) decreases towards zero gradually

$$P(f) = \begin{cases} 1/2B_0 & \text{for } -f_1 < f < f_1 \\ 1/4B_0 \left\{ 1 + \cos\left[\frac{\pi(|f| - f_1)}{2B_0 - 2f_1}\right] \right. & \text{for } f_1 < |f| < 2B_0 - f_1 \\ 0 & \text{elsewhere.} \end{cases}$$

$$P(t) = \text{sinc}\,(2B_0 t)\,\frac{\cos\,(2\pi\alpha B_0 t)}{1 - 16\alpha^2 B_0^2 t^2}$$

⇒ Raised cosine spectrum

Time response of raised cosine Spectrum.



Bandwidth $= 2B_0 - f_1$; $\alpha = 1 - f_1/B_0$

$$f_1 = B_0 - B_0\alpha$$

$$B = 2B_0 - B_0 + B_0\alpha$$

$$B = B_0(1+\alpha)$$

for $\alpha = 0$; $B = B_0$

α increases Bandwidth is also increases

$\alpha = 1$; $B = 2B_0$

———— × ————

UNIT - 5 Two marks

1. what is Intersymbol Interference (ISI) in baseband PAM system?

→ presence of outputs due to other bits (symbols) interfere with the output of required bit is known as ISI.

$$y(t_i) = \mu A_i + \mu \sum_{k=-\infty}^{\infty} A_k P[(i-k)T_b]$$

μA_i is the contribution of i^{th} transmitted bit.

—— × ——

2. what is the condition for zero Inter symbol Interference?

→ zero ISI can be obtained if the transmitted pulse satisfies the following condition

Time domain : $P[[(i-k)T_b] = \begin{cases} 1 & \text{for } i=k \\ 0 & \text{for } i \neq k \end{cases}$

Frequency domain : $\sum_{n=-\infty}^{\infty} P(f-nf_b) = T_b$

—— × ——

3. what is an ideal Nyquist channel?

→ ideal Nyquist channel uses sinc pulse for transmission.

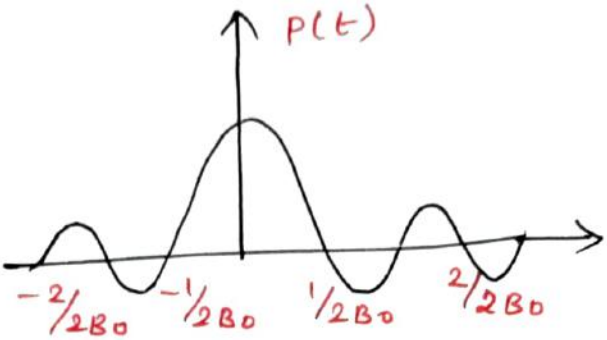
$$P(t) = \frac{\sin(2\pi B_0 t)}{2\pi B_0 t}$$

$$P(f) = \begin{cases} 1/2B_0 & \text{for } -B_0 < f \leq B_0 \\ 0 & \text{elsewhere} \end{cases}$$

—— × ——

4) Give the Nyquist criterion for zero ISI. plot the impulse response of an ideal Nyquist channel.

→ $$P(t) = \frac{Sin(2PB_ot)}{2PB_ot}$$



* * *

5) How does pulse shaping reduce inter symbol interference?

→ The Shape of pulse is selected such that at the instant of detection, the interference due to all other Symbols is zero.

→ The effect of ISI is totally eliminates if signal is Sampled at T_b, $2T_b$, $3T_b$....

* * *

6) ISI cannot be avoided. Justify the statement.

→ when the pulse P(t) is transmitted across the channel, the output y(t) is given by

$$y(t_i) = \mu A_i + M \sum_{k=-\infty}^{\infty} A_k P[(i-k)T_b]$$

$$i = 0, \pm 1, \pm 2 \ldots.$$

* * *

7) State Nyquist Second and third criteria to realize zero ISI.

→ Values at the pulse edge are distortionless. This means Values at pulse edges are Changed

$$P(t) = \begin{cases} 0.5 \text{ for } & -T/2 \le t \le T/2 \\ 0 \quad \text{for} & t = (2k-1)T/2 \end{cases}$$

$$P_r(f) = T \cos(f\, T/2) \text{ and } P_i(f) = 0$$

→ within each symbol period, the integration of signal is proportional to integration of the transmit signal.

$$p(\omega) = \begin{cases} \dfrac{\omega T/2}{\sin(\omega T/2)} & , |\omega| < \pi/T, \ |\omega| > \pi/T \end{cases}$$

$$P(t) = \frac{1}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{(\omega T/2)}{\sin(\omega T/2)} e^{j\omega t} dt$$

——— × ———

8. Mention the need of optimum transmitting and receiving filter in baseband data transmission.

→ optimum filter integrates the signal during the bit interval and checks the output at the time instant where signal to noise ratio is maximum.

→ Transfer function of the optimum filter is selected so as to maximize signal to noise ratio.

→ optimum filter minimize the probability of error.

——— × ———

9) which digital modulation technique gives better error probability?

→ Binary PSK gives reduced error probability compared to ASK and FSK.

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E}{N_0}}$$

——— × ———

④

10) write the expression for bit error rate for Coherent binary FSK.

→ The bit error rate of coherent binary FSK is given as,

$$Pe = \frac{1}{2} \text{erfc} \sqrt{\frac{0.6E}{N_0}}$$

11) what is the error probability of DPSK?

$$Pe = \frac{1}{2} e^{-E_b/N_0}$$

12) Compare the probability of error of PSK with FSK.

→ Error probability of PSK : $Pe = \frac{1}{2} \text{erfc} \sqrt{\frac{E}{N_0}}$

For same $\frac{E}{N_0}$, error probability is less.

→ Error probability of FSK : $Pe = \frac{1}{2} \text{erfc} \sqrt{\frac{0.6E}{N_0}}$

For same $\frac{E}{N_0}$, error probability is high

13) Give the signal space representation of QPSK. How is the performance of the system related to the distance between the symbols in the signal space?

$$Pe \leq \sum_{k=2}^{M} \frac{1}{2} \text{erfc} \sqrt{\frac{d_{k1}^2}{4N_0}}$$

Here 'M' is the number signal points

d_{k1} → Distance between S_1 and S_k in the signal space.

14) Define BER.

→ Bit error rate is the number of bits bits that go in error in specific number of transmitted bits.

UNIT -5 Big Questions

1) Elements of Detection theory
2) Matched filter / optimum filter design
3) Coherent Receiver / correlation Receiver
4) probability error of BPSK, FSK, DPSK, QPSK
5) ISI
6) Nyquist criterion for Distortion Less Transmission