UNIT 1.

AMPLITUDE MODULATION.

COMMUNICATION:

Communication is the process of establishing connection or link by which information is transferred from one point called as Source to the other point called destination.

Types of Communication Systems:

(i) Analog communication Systems.
(ii) Digital Communication Systems.

Analog Communication:

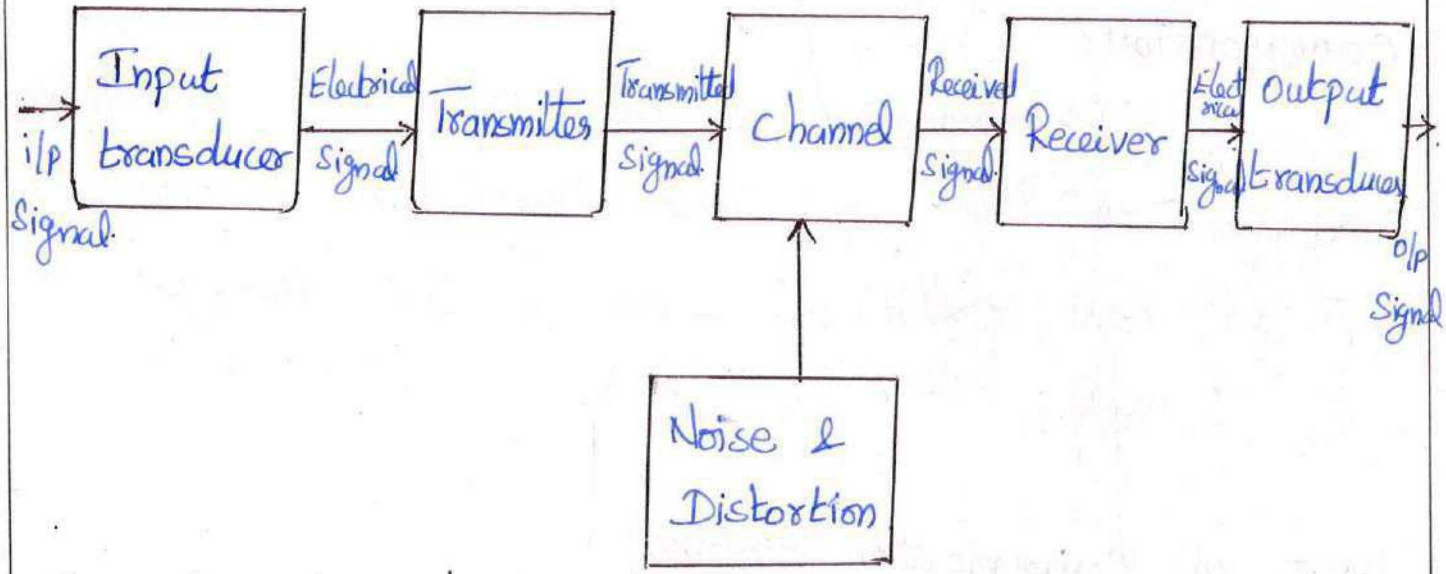
It is a system in which information signal is transmitted and received in analog form.

Information and carrier both are analog signals.

Digital Communication:

Digital pulses in the form of code words are transferred between two or more points in a communication system.

General communication System:



Input transducer / Information Source:

1. The message or information originates in the information source.
2. There can be various messages in the form of words, groups of words, code, symbols, sound signal etc.
3. Out of these the desired message is selected and conveyed <or> communicated.

Therefore the function of information source is to produce the required message that has to be transmitted.

Transmitter :

1). A transmitter converts the message signal into a suitable form for propagation over the communication medium.

2) It is achieved by modulating the carrier.

3) The output wave is called modulated signal.

Channel and noise :

1) channel is the medium through which the message travels from the transmitter to the receiver

2) channel can be of many forms like coaxial cable, microwave links, radio wave links or an optical fiber.

3) The signal gets distorted due to a noise introduced in the system.

4). Noise : It is an unwanted signal which tends to interfere with the required signal.

Receiver:

1). It reproduce the message signal in electrical form from the distorted received signal.

2). This is accomplished by a process known as demodulation or detection.

3) It is the reverse process of modulation.

Output transducer / Destination:

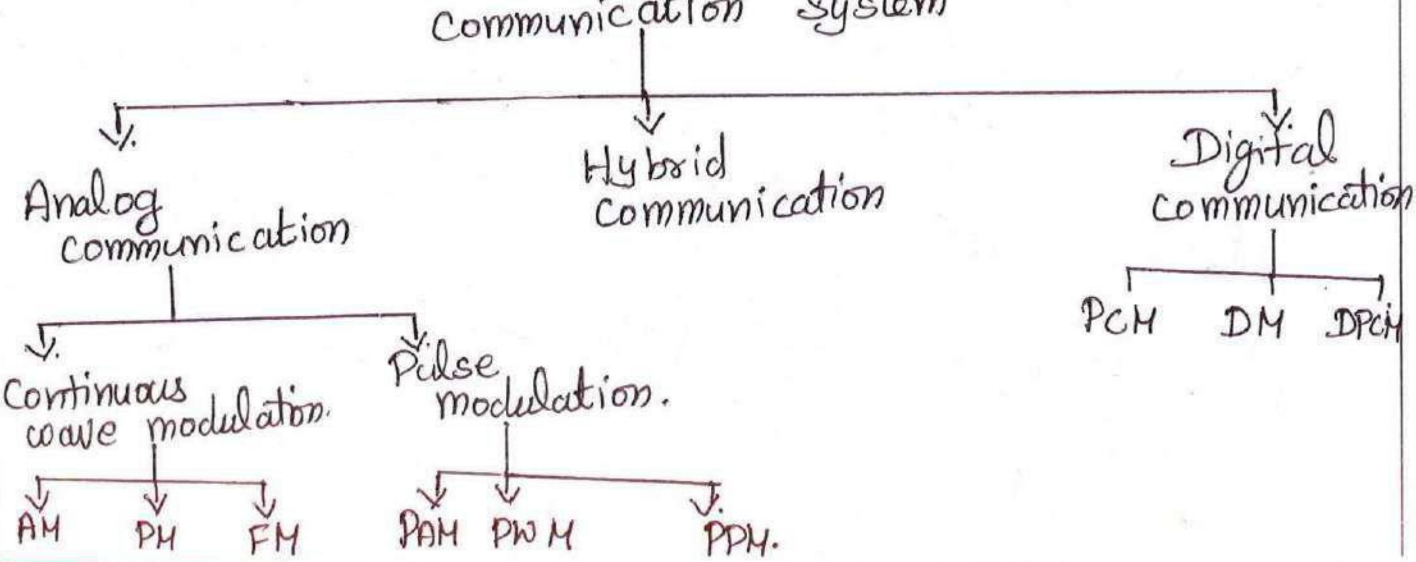
1) It is the final stage, used to convert an electrical message signal into its original form.

Types of channel:

(i) Wire <or> Line Communication.

(ii) Wireless <or> Radio communication.

Types of communication System:



Analog Communication System :

It is designed to transmit analog information using analog modulation schemes.

Digital Communication System :

It is designed to transmit digital information using digital modulation schemes.

Hybrid Communication System :

It is designed to used digital modulation schemes for transmitting sampled and quantized value of analog signals.

Modulation :

Modulation is process by which some characteristics of a carrier signal vary in accordance with the instantaneous value of a modulating signal or message signal.

Modulation – change.

Carrier signal <or> high frequency signal.

Base band signal <or> Information signal <or> Modulating signal <or> Low frequency signal.

Need for Modulation:

(i) Practical antenna height:

$$\text{wavelength } \lambda = \frac{\text{Velocity}}{\text{frequency}} = \frac{3 \times 10^8}{f\,(Hz)}\ (m).$$

For low frequency ranges, the length of transmitting antenna will be extremely large. Which is practically impossible to be implemented.

Hence the signal should be modulated.

(ii) Operating range:

Greater than frequency of the wave, greater the energy possessed by it. Audio of signal frequencies are small, so it cannot be transmitted over large distances if radiated directly into space.

(iii) Wire communication:

One feature of radio transmission is that it should be carried without wires. At audio frequencies, radiation is not practicable because the efficiency of radiation is poor.

(iv) Multiplexing :

If more than one Singal uses a Single channel then modulation may be used to translate different signals to different Spectral location, thus enabling the receiver to select the desired signal.

(v) To overcome equipment limitations :

If the frequency of the Signal to be Processed and frequency range of processing apparatus do not match, modulation can be used to accomplish frequency translation.

(vi) Modulation to reduce noise and interferences :

The effect of noise and interference cannot be completely eliminated in a communication System, it can be minimised by using certain types of modulation Schemes.

(vii) Increases the range of communication :

Modulation process increases the frequency of the Signal to be transmitted. Hence increases the range of Communication.

x————————x

Amplitude Modulation (AM):

It is the process by which amplitude of the carrier signal is varied in accordance with the instantaneous amplitude value of the modulating signal, but frequency and phase of the carrier remains constant.

Mathematical or Time domain representation of an AM:

Modulating Signal $V_m(t) = V_m \cos \omega_m t$.

Carrier Signal $V_c(t) = V_c \cos \omega_c t$.

V_c = Amplitude of the carrier signal

V_m = Amplitude of the modulating signal.

Amplitude of the carrier signal is changed after amplitude modulation.

$$V_{AM} = V_c + V_m(t).$$

Sub $V_m(t)$

$$V_{AM} = V_c + V_m \cos \omega_m t.$$

$$= V_c \left[1 + \frac{V_m}{V_c} \cos \omega_m t\right].$$

$$V_{AM} = V_c \left[1 + m_a \cos \omega_m t\right].$$

Modulation index of AM $= m_a = \frac{V_m}{V_c}$.

AM wave can be expressed as

$$V_{AM}(t) = V_{AM} \cos\omega_c t.$$

Sub V_{AM}

$$V_{AM}(t) = V_c (1 + m_a \cos\omega_m t) \cos\omega_c t.$$

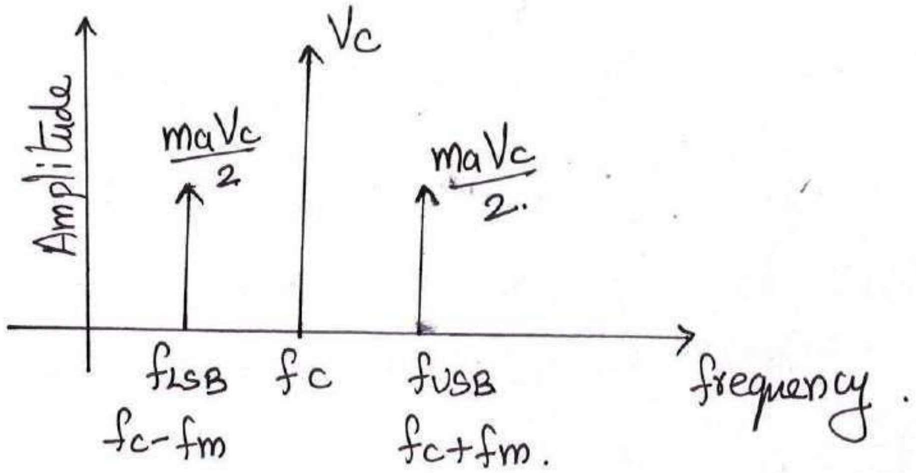
$$V_{AM}(t) = V_c \cos\omega_c t + m_a V_c \cos\omega_m t \cos\omega_c t.$$

$$\cos\omega_m t \cos\omega_c t = \frac{\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t}{2}.$$

$$V_{AM}(t) = V_c \cos\omega_c t + \frac{m_a V_c}{2} \left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t \right]$$

$$V_{AM}(t) = \underbrace{V_c \cos\omega_c t}_{\substack{\text{Carrier} \\ 1}} + \underbrace{\frac{m_a V_c}{2} \cos(\omega_c - \omega_m)t}_{\substack{\text{LSB} \\ 2}} + \underbrace{\frac{m_a V_c}{2} \cos(\omega_c + \omega_m)t}_{\substack{\text{USB.} \\ 3}}$$

1. Unmodulated Carrier Signal.
2. frequency $(f_c - f_m)$ and amplitude $\frac{m_a V_c}{2}$.
   Lower Side band.
3. frequency $(f_c + f_m)$ and amplitude $\frac{m_a V_c}{2}$.
   Upper Side band.



Band width of AM:

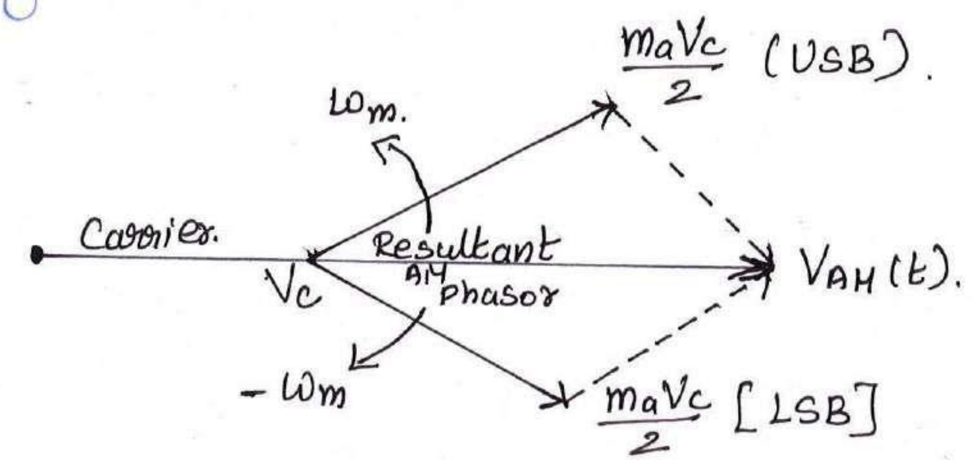
Bandwidth of AM Signal is given by Subtraction of the highest and lowest frequency

$$B = f_{USB} - f_{LSB}.$$

$$= (f_c + f_m) - (f_c - f_m).$$

$$B = 2f_m.$$

Bandwidth of AM Signal is twice the maximum frequency of modulating signal.



Phasor representation of AM.

Modulation Index :

It is the ratio of maximum amplitude of modulating signal to maximum amplitude of the carrier signal

$$m_a = \frac{V_m}{V_c} \quad . \quad V_c > V_m.$$

Modulation Index should be a number between 0 and 1 ($0 < m_a < 1$). $m_a = 1$ then $V_m = V_c$, this is called 100% modulation.

Percentage modulation:

When modulation index is expressed in percentage, then it is called percentage modulation

$$\% \text{ Modulation index} = m_a \times 100 = \frac{V_m}{V_c} \times 100.$$

Calculation :

$$V_{max} = V_c + V_m \quad \text{— ①}$$

$$V_{min} = V_c - V_m \quad \text{— ②}$$

① − ②.

$$V_{max} - V_{min} = \cancel{V_c} + V_m - \cancel{V_c} + V_m.$$

Modulation Index :

$$\boxed{V_m = \frac{V_{max} - V_{min}}{2}}$$

from ①.

$$V_c = V_{max} - V_m .$$

$$V_c = V_{max} - \left[\frac{V_{max} - V_{min}}{2}\right]$$

$$= V_{max} - \frac{V_{max}}{2} + \frac{V_{min}}{2} .$$

$$V_c = \frac{V_{max}}{2} + \frac{V_{min}}{2} .$$

$$\boxed{V_c = \frac{V_{max} + V_{min}}{2}}$$

$$m_a = \frac{V_m}{V_c} .$$

Sub V_m and V_c

$$m_a = \frac{(V_{max} - V_{min})/2}{(V_{max} + V_{min})/2}$$

$$\boxed{m_a = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}}$$

Modulation index for multiple modulating frequencies

$$m_T = \sqrt{m_1^2 + m_2^2 + \cdots}$$

m_T = Total resultant modulation index.

m_1, m_2 = Modulation indices due to individual modulating Components.

Power relations in AM wave :

Total Power in AM = P_T

P_T = Carrier power + Power in LSB + Power in USB.

$$P_T = P_C + P_{LSB} + P_{USB}.$$

$$P = \frac{V^2}{R}.$$

$$P_T = \frac{V_C^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}.$$

all the three voltages are in rms Values.

Carrier Power P_C :

$$P_C = \frac{V_{C(rms)}^2}{R}.$$

$$V_{C(rms)}^2 = \left(\frac{V_C}{\sqrt{2}}\right)^2 = \frac{V_C^2}{2}.$$

$$\boxed{P_c = \frac{V_c^2}{2R}}$$

Power in Sidebands:

$$P_{USB} = P_{LSB} = \frac{[V_{SB(rms)}]^2}{R}$$

$V_{SB} \rightarrow$ Voltage of Sidebands.

$$V_{SB} = \frac{m_a V_c}{2}$$

$$V_{SB(rms)} = \left(\frac{m_a V_c}{2\sqrt{2}}\right)^2 = \frac{m_a^2 V_c^2}{4 \times 2} = \frac{m_a^2 V_c^2}{8}$$

$$P_{USB} = P_{LSB} = \frac{m_a^2 V_c^2}{8R}$$

$$= \frac{m_a^2}{4}\left[\frac{V_c^2}{2R}\right]$$

$$P_{USB} = P_{LSB} = \frac{m_a^2}{4} P_c$$

$$P_T = P_c + P_{USB} + P_{LSB}$$

$$= \frac{V_c^2}{2R} + \frac{m_a^2 P_c}{4} + \frac{m_a^2 P_c}{4}$$

$$P_T = P_c + \frac{m_a^2 P_c}{4} + \frac{m_a^2 P_c}{4}$$

$$P_T = P_c\left[1 + \frac{m_a^2}{4} + \frac{m_a^2}{4}\right].$$

$$P_T = P_c\left[1 + \frac{m_a^2}{2}\right].$$

For $m_a = 1$, 100% modulation

$$P_T = P_c\left[1 + \frac{1}{2}\right].$$

$$P_T = P_c\,[1.5].$$

$$\boxed{P_T = 1.5\,P_c.}$$

Modulation index interms of P_T and P_c.

$$\frac{P_T}{P_c} = 1 + \frac{m_a^2}{2}.$$

$$\frac{m_a^2}{2} = \frac{P_T}{P_c} - 1.$$

$$m_a^2 = 2\left[\frac{P_T}{P_c} - 1\right].$$

$$\boxed{m_a = \sqrt{2\left(\frac{P_T}{P_c} - 1\right)}}$$

Transmission efficiency η :

It is the ratio power in side bands to the total transmitted power.

$$\eta\% = \frac{\text{Power in Side bands}}{\text{Total transmitted power}} \times 100.$$

$$\text{Power in Side bands} = P_{LSB} + P_{USB}.$$

$$\eta\% = \frac{P_{LSB} + P_{USB}}{P_T} \times 100.$$

$$P_{USB} = P_{LSB} = \frac{m_a^2 V_c^2}{8R}.$$

$$P_T = \frac{V_c^2}{2R}\left[1 + \frac{m_a^2}{2}\right].$$

$$\eta\% = \frac{\frac{m_a^2 V_c^2}{8R} + \frac{m_a^2 V_c^2}{8R}}{\frac{V_c^2}{2R}\left[1 + \frac{m_a^2}{2}\right]} \times 100.$$

$$= \frac{\frac{m_a^2}{4}\left(\frac{V_c^2}{2R}\right) + \frac{m_a^2}{4}\left(\frac{V_c^2}{2R}\right)}{\frac{V_c^2}{2R}\left(1 + \frac{m_a^2}{2}\right)}.$$

$$\frac{V_c^2}{2R} = P_c.$$

$$\eta\% = \frac{P_c \frac{m_a^2}{4} + P_c \frac{m_a^2}{4}}{P_c\left(1 + \frac{m_a^2}{2}\right)}.$$

$$\eta\% = \frac{\cancel{P_c}\left[\frac{m_a^2}{4}+\frac{m_a^2}{4}\right]}{\cancel{P_c}\left[1+\frac{m_a^2}{2}\right]}$$

$$\eta\% = \frac{m_a^2/2}{1+m_a^2/2}$$

$$= \frac{m_a^2/\cancel{2}}{(2+m_a^2)/\cancel{2}}$$

$$\boxed{\eta\% = \frac{m_a^2}{2+m_a^2} \times 100}$$

If $m_a = 1$

$$\eta\% = \frac{1}{3} \times 100$$

$$\boxed{\eta\% = 33.3\%}$$

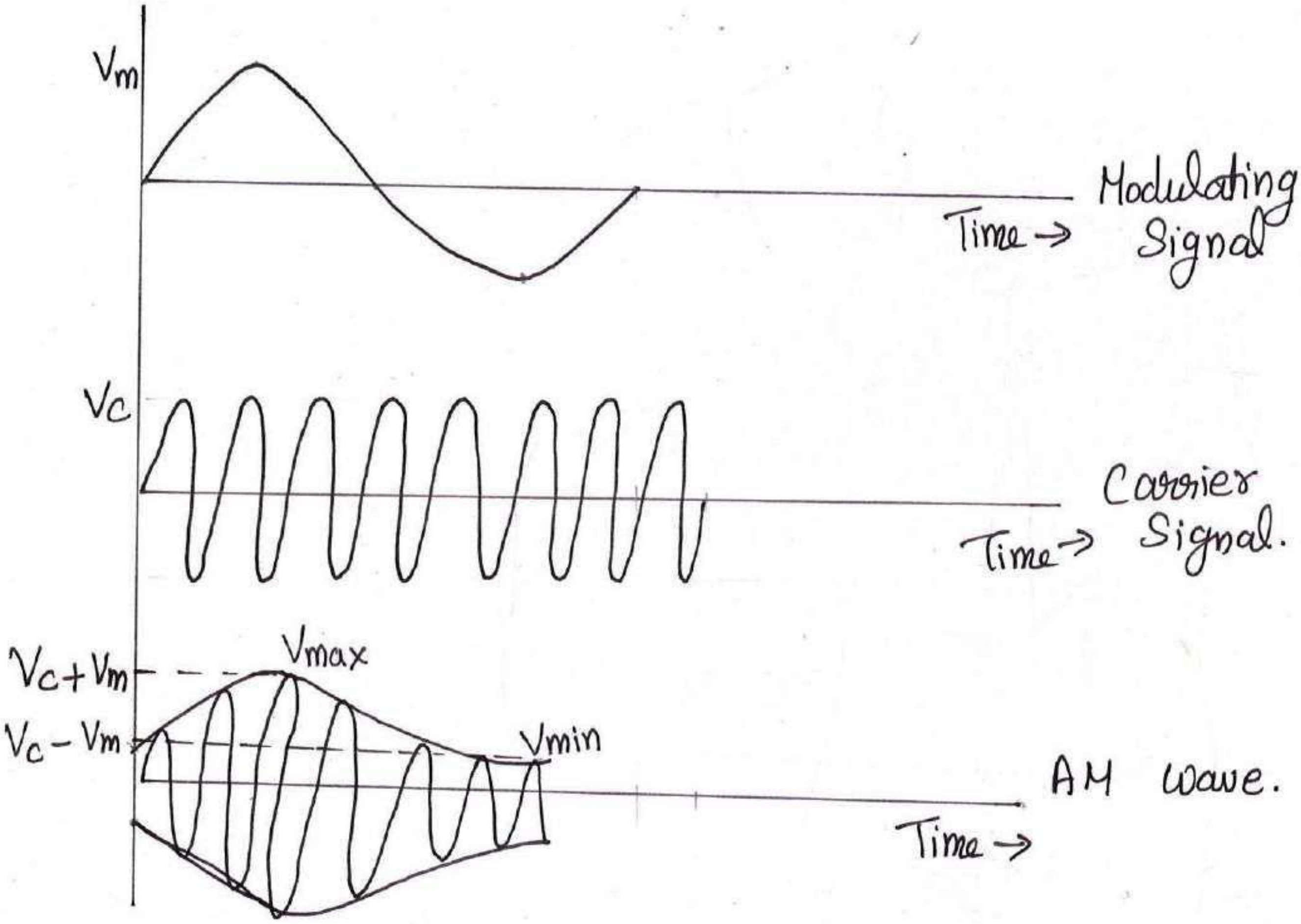
Only the transmitted power is used and remaining power is wasted by the carrier along with the side bands.

wasted Power $= 100 - 33.3$

$= 66.7$

66.7% of power is wasted.

AM waveform.



Generation of Amplitude Modulation :

It is divided into two types.

(i) Linear modulator <or> Large Signal modulator.

The devices are operated in the linear region of its transfer characteristics. The relation between the amplitude of the modulating signal and resulting depth of modulation is linear.

(ii) Non-linear modulator or Small signal modulator.

They make use of non-linear VI characteristics of the devices and are in general suited for use at low voltages.

Non-Linear Modulator:

A simple diode or transistor or FET can be used as a non-linear modulator by restricting the operation over the non-linear region of its characteristics.

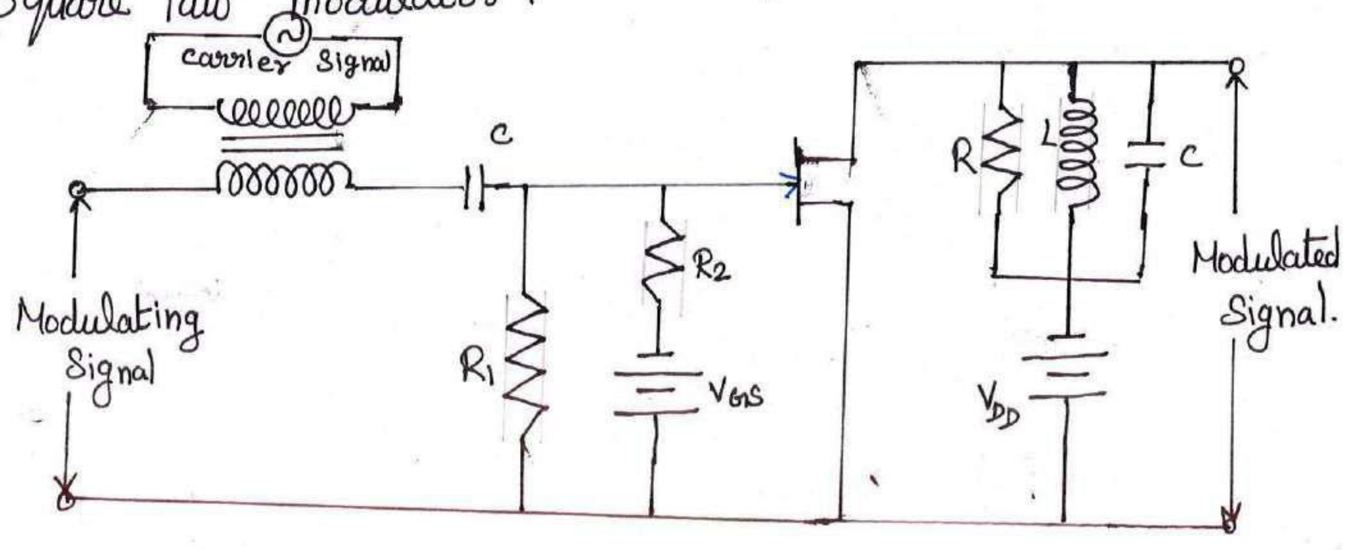
Methods:

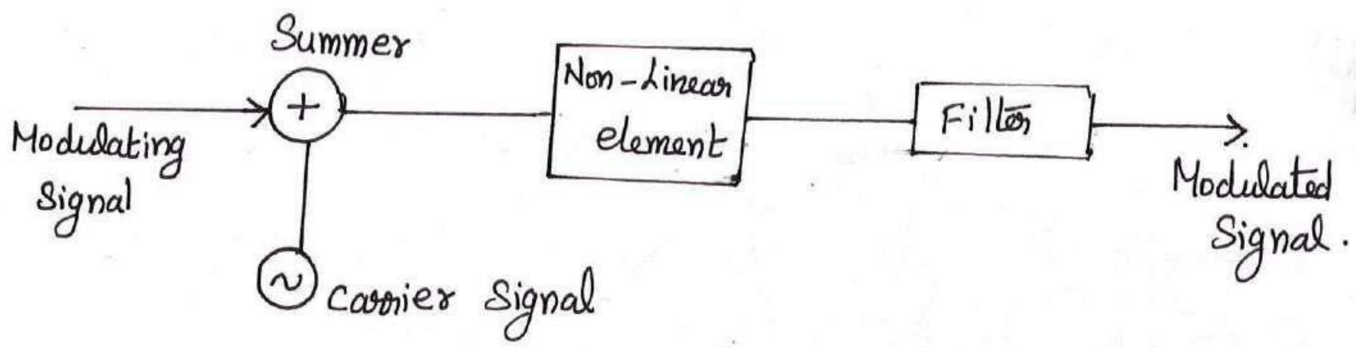
(i) Square law modulator.

(ii) Square law diode modulator.

(iii) Balanced modulator.

Square law modulator:





A Square law modulator requires to add up the carrier and modulating signal to obtain AM with carrier.

Features of Square law:

1. Summer - to add carrier and modulating signal
2. A non-linear (active) element.
3. Bandpass filter for extracting desired modulating products.

Construction:

1. This Square law detector uses FET.
2. FET is biased in a non-linear region of its transfer characteristics, to obtain the desired output.
3. Output tank circuit RLC is tuned to the carrier frequency to select the desired modulating components.

Analysis:

When FET is biased and operated in a restricted portion of its non-linear transfer characteristics, the resulting current i_0 will be given by

$$i_o = a_1 V_1 + a_2 V_1^2 + \cdots$$

$V_1 \rightarrow$ Input voltage applied to FET.

$$V_1 = A_m \sin\omega_m t + A_c \sin\omega_c t.$$

$$i_o(t) = a_1(A_m \sin\omega_m t + A_c \sin\omega_c t) + a_2(A_m \sin\omega_m t + A_c \sin\omega_c t)^2 + \cdots$$

Neglecting second and higher order terms.

$$i_o(t) = a_1 A_m \sin\omega_m t + a_1 A_c \sin\omega_c t + \cancel{a_2 A_m^2 \sin^2\omega_m t} + \cancel{a_2 A_c^2 \sin^2\omega_c t} + 2 A_m A_c a_2 \sin\omega_m t . \sin\omega_c t$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$i_o(t) = a_1 A_m \sin\omega_m t + a_1 A_c \sin\omega_c t + \frac{\cancel{2} A_m A_c a_2}{\cancel{2}}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

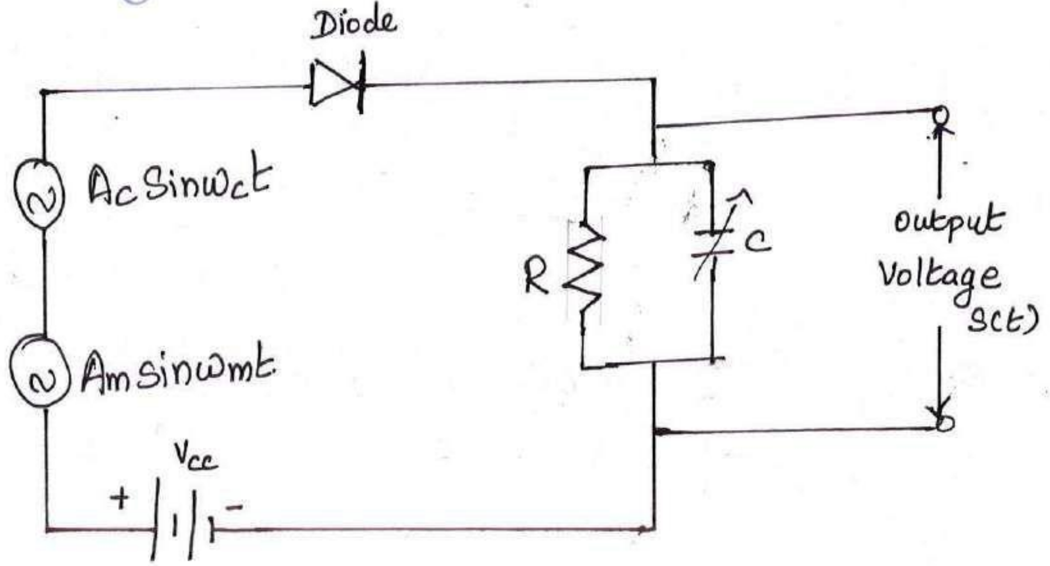
$$i_o(t) = a_1 A_m \sin\omega_m t + a_1 A_c \sin\omega_c t + a_2 A_m A_c \cos(\omega_c - \omega_m)t - a_2 A_m A_c \cos(\omega_c + \omega_m)t.$$

When BPF is tuned to carrier frequency, it allows only ω_c, $\omega_c - \omega_m$ and $\omega_c + \omega_m$.

$$\therefore i(t) = \underbrace{a_1 A_c \sin\omega_c t}_{\text{Carrier}} + \underbrace{a_2 A_m A_c \cos(\omega_c - \omega_m)t}_{\text{LSB}} - \underbrace{a_2 A_m A_c \cos(\omega_c + \omega_m)t}_{\text{USB}}.$$

Square law diode modulator :

This method is suited at low voltage levels because of the fact that current - voltage characteristic of a diode is highly non - linear particularly in low voltage region.



Construction:

The carrier and modulating Signals are applied across the diode. A D.C battery Vcc is Connected across the diode to get a fixed operating point on the V-I characteristics of diode.

Working:

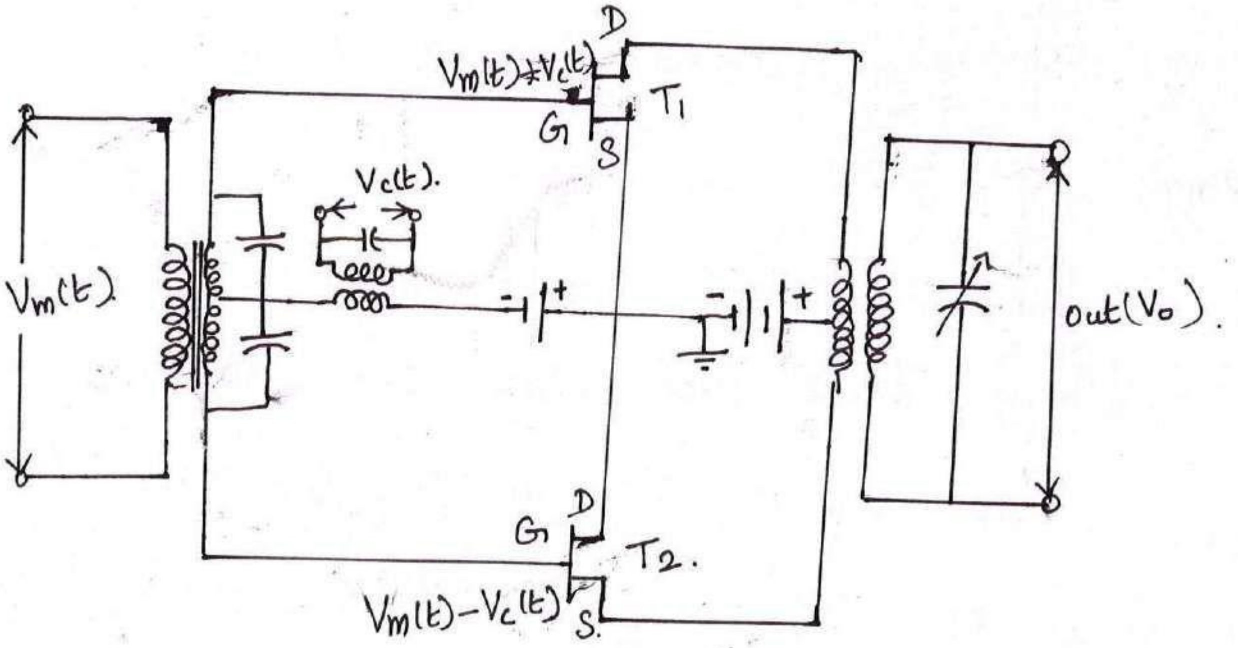
1) Working of this circuit may be explained by considering the fact when two different frequencies are passed through a non-linear device, the process of amplitude modulation takes place.

2). When carrier and modulating frequencies are applied at the input of diode, then different frequency terms appear at the output of diode.

3) These different frequency terms are applied across a tuned circuit which is tuned to the carrier frequency and has a narrow bandwidth just to pass two sidebands along with the carrier and reject other frequencies.

Hence at the output of tuned circuit, carrier and two side bands are obtained.

Balanced Modulator :



Construction:

1. Two non-linear devices are connected in the balanced mode, so as to supply the carrier wave. It is assumed that the two FETs are identical and the circuit is symmetrical.

2. The operation is confined in non-linear region of its transfer characteristics, the carrier voltage across the two windings of a centre tap transformer is equal and opposite in phase $V_c = -V_c'$.

Operation:

The input voltage to FET T_1 is given by

$$V_{GS} = C(t) + V_m(t).$$

$$C(t) = A_c \sin\omega_c t.$$

$$V_m(t) = A_m \sin\omega_m t.$$

Input voltage to FET T_2 is given by.

$$V'_{GS} = -C(t) + V_m(t).$$

$$= -A_c \sin\omega_c t + A_m \sin\omega_m t.$$

By using non-linearity relationship the drain Current can be written as

$$i_d = a_1 V_{GS} + a_2 V_{GS}^2.$$

$$i'_d = a_1 V'_{GS} + a_2 V'^2_{GS}.$$

$$i_d = a_1 [A_c \sin\omega_c t + A_m \sin\omega_m t] + a_2 [A_c \sin\omega_c t + A_m \sin\omega_m t]^2.$$

$$= a_1 [A_c \sin\omega_c t + A_m \sin\omega_m t] + a_2 A_c^2 \sin^2\omega_c t + a_2 A_m^2 \sin^2\omega_m t + 2a_2 A_c A_m \sin\omega_c t \sin\omega_m t.$$

Similarly

$$i_d' = a_1(-A_c \sin\omega_c t + A_m \sin\omega_m t) + a_2 A_c^2 \sin^2\omega_c t + a_2 A_m^2 \sin^2\omega_m t - 2a_2 A_m A_c \sin\omega_c t \sin\omega_m t.$$

Output AM Voltage V_o is given by

$$V_o = K(i_d - i_d').$$

i_d and i_d' flow in opposite direction.

K is a constant depending on impedance or other circuit parameters.

$$V_o = 2Ka_1 A_c \sin\omega_c t + 4Ka_2 A_m A_c \sin\omega_c t \sin\omega_m t$$

by sub i_d and i_d'.

$$V_o = 2KA_c a_1 \left[1 + \frac{2a_2 A_m}{a_1} \sin\omega_m t\right] \sin\omega_c t.$$

$$\frac{2a_2 A_m}{a_1} = m_a.$$

$$V_o = 2Ka_1 A_c [1 + m_a \sin\omega_m t] \sin\omega_c t.$$

Advantages of balanced modulator:

1. Undesirable harmonics are eliminated by BPF. in Non-linear circuit.
2. But in balanced modulator harmonics are balanced and filter is not required.

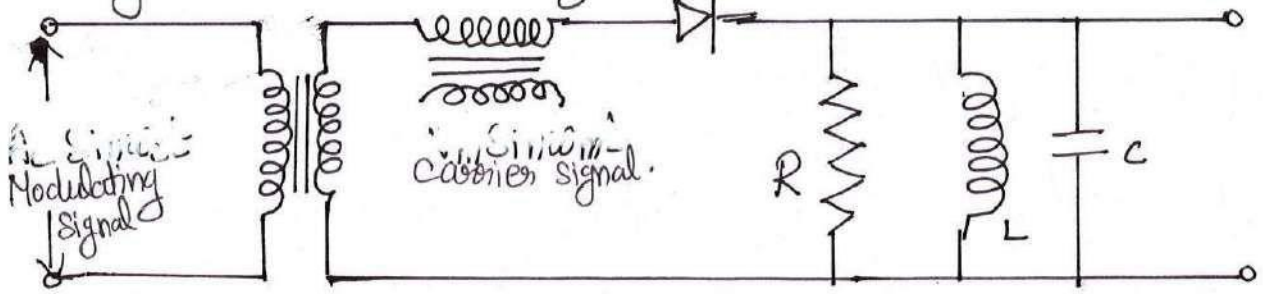
Linear Modulator.

Drawbacks of non-linear modulator:

1. Heavy filtering is required to remove unwanted terms presented in the output of the modulator.
2. The output power level is also very low.

To reduce these problems, linear modulators are used.

Switching modulator using diode:



Construction:

1) A BJT or diode serves as a switch driven at the carrier frequency.

2) RLC circuit acts as tank circuit and is tuned to resonate at fc.

3) The switching action causes the tank circuit to ring sinusoidally.

Operation:

1) The diode is forward biased for every positive half cycle of the carrier and behaves like a short circuited switch.

2) For negative half cycle of the carrier, the diode is reverse biased and behaves like an open switch. The signal does not reach the filter and no output is obtained.

3) Steady state output voltage in the absence of modulating voltage

$$V_o(t) = A_c \sin \omega_c t.$$

Adding message to the input through transformer gives an output of

$$S(t) = V_o(t) = A_c \sin \omega_c t + N A_m \sin \omega_m t \sin \omega_c t.$$

$N \rightarrow$ Turns ratio of transformer.

4). If V and N are correctly propotioned, the desired modulation can be accomplished without generation of unwanted components.

Comparison of Low level and High level modulation

S.No.	Low level Modulation	High level modulation
1.	Modulation done at low power level.	Modulation done at High power level.
2.	Depth of modulation is less than 100%	Depth of modulation is maximum.
3.	Modulation takes place prior to the output element of the final stage of the transmitter.	Modulation takes place at the last element of the final stage of the transmitter.
4.	Class B amplifier used to amplify the modulated signal.	It does not require any amplification after modulation.
5.	Efficiency and gain are Very low.	Efficiency and gain are Very high.
6.	Less modulating power is required to achieve high percentage modulation.	High modulating power is required to achieve high percent modulation.
7.	Used for wireless intercom, remote control, walkie-talkie etc	Used for transmit radio and television signals.

AM Demodulation :

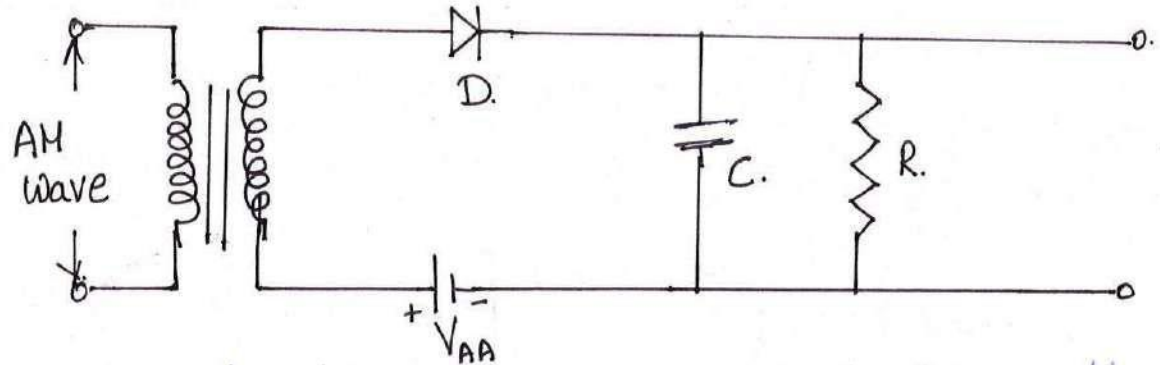
The process of recovering the original modulating signal from a modulated wave is termed as demodulation <or> detection.

Types of AM Detectors :

(i) Linear detectors
- Non-Coherent <or> envelope detection.
- Coherent <or> Synchronous detection

(ii) Non-linear detectors.

Envelope detection <or> diode detector.



A detector circuit output follows the envelope of the modulated signal which is used to reproduce the modulating signal is known as envelope detector.

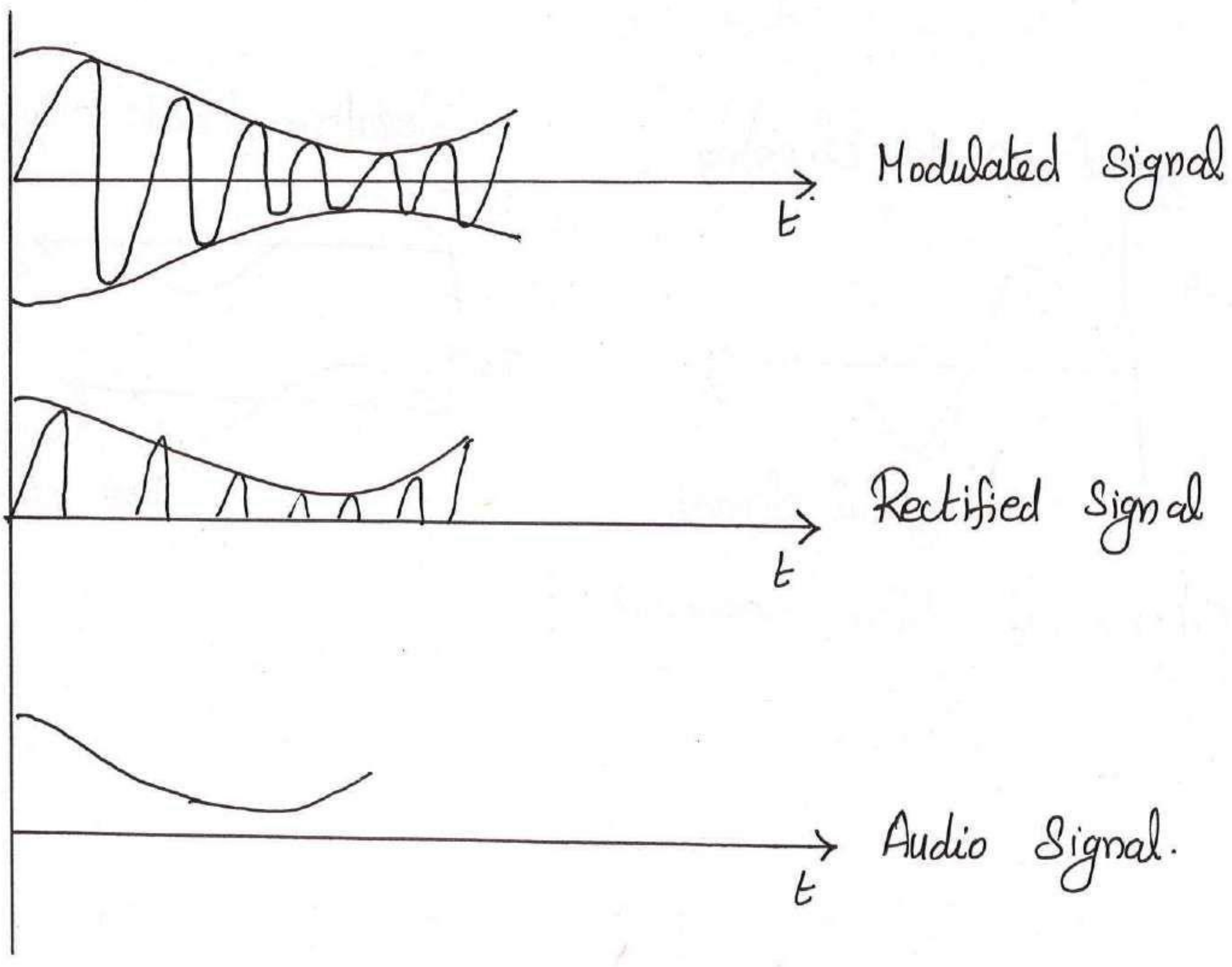
Principle of operation :

The modulated AM signal is applied to the primary winding of the transformer. The load impedance consist of a resistor R in shunt with a capacitor C.

During positive half cycle of AM, the diode conducts and capacitor 'C' charges to the peak Voltage of the carrier signal through R.

During negative half cycle of AM, the diode is reverse biased and no current flows. Hence the capacitor discharges.

Therefore only positive half of AM wave appears across the resistance.



Distortions :

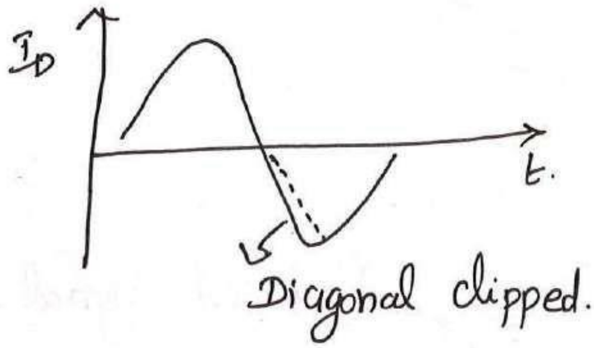
The spikes introduced by charging and discharging of the capacitor are called distortions.

It can be avoided by keeping the time constant Rc large, but that creates a problem called diagonal clipping.

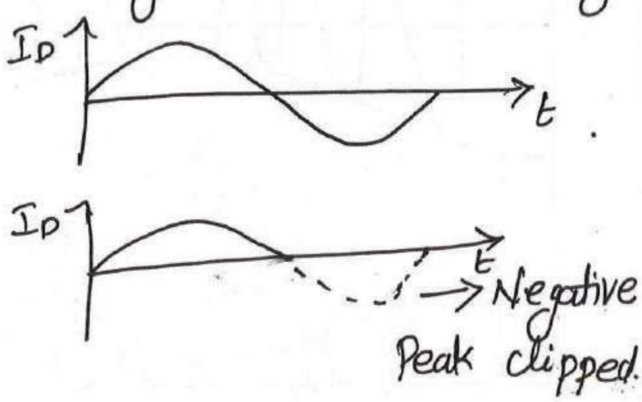
Types of distortions :

1. Diagonal Peak clipping.
2. Negative Peak clipping.

Diagonal peak clipping



Negative Peak clipping.



Choice of time constant R-C :

Optimum value of time constant should be chosen which will give a compromise between the two.

1. Spike or fluctuation in a detected envelope should be minimum.

2. Negative peaks of the detected envelope should not be missed even partially i.e. diagonal clipping.

Selecting the time Constant:

The envelope Voltage is $V_c = E_c (1+m_a \cos\omega_m t)$.

$$V_c = E_c + E_c m_a \cos\omega_m t.$$

$$\frac{dV_c}{dt} = -E_c m_a \sin\omega_m t. \quad \text{—} (1)$$

Discharging Capacitor Voltage is given as

$$V_c = E_c e^{-t/RC}.$$

$$\frac{dV_c}{dt} = \frac{-1}{RC} E_c e^{-t/RC}.$$

$$\frac{dV_c}{dt} = \frac{-V_c}{RC}.$$

Sub equation of V_c.

$$\frac{dV_c}{dt} = \frac{-E_c [1+m_a \cos\omega_m t]}{RC} \quad \text{—} (2).$$

To avoid diagonal clipping slope of discharge Curve given in (2) must be greater tha (1).

$$\frac{+E_c [1+m_a \cos\omega_m t]}{RC} \geq +E_c m_a \omega_m \sin\omega_m t.$$

$$\frac{1}{RC} \geqslant \frac{E_c\, m_a\, \omega_m \sin\omega_m t}{E_c\,[1+m_a\cos\omega_m t]}$$

$$\frac{1}{RC} \geqslant \frac{\omega_m\, m_a \sin\omega_m t}{1+m_a\cos\omega_m t}.$$

for maximum condition the derivative should be equated to zero.

$$\frac{d}{dt}\left[\frac{\omega_m\, m_a \sin\omega_m t}{1+m_a\cos\omega_m t}\right] = 0.$$

$$\frac{d}{dt}\left[\frac{u}{v}\right] = \frac{v\,du - u\,dv}{v^2}.$$

$$\frac{(1+m_a\cos\omega_m t)(\omega_m m_a\cos\omega_m t) - (\omega_m m_a\sin\omega_m t)(-m_a\sin\omega_m t)}{(1+m_a\cos\omega_m t)^2} = 0.$$

$$\frac{\omega_m m_a\cos\omega_m t + m_a^2\omega_m\cos^2\omega_m t + \omega_m m_a^2\sin^2\omega_m t}{1+m_a^2\cos^2\omega_m t + 2m_a\cos\omega_m t} = 0.$$

$$m_a\omega_m\cos\omega_m t + m_a^2\omega_m(\cos^2\omega_m t + \sin^2\omega_m t) = 0.$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$m_a\omega_m\cos\omega_m t + m_a^2\omega_m = 0.$$

$$m_a \omega_m \cos\omega_m t = -m_a^2 \omega_m.$$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

|||ly

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$$-m_a = \cos\omega_m t.$$

Squaring on both sides

$$\cos^2\omega_m t = m_a^2.$$

$$\therefore \sin\omega_m t = \sqrt{1-m_a^2}.$$

Sub $\sin\omega_m t$ in $\frac{1}{RC}$ equation

$$\frac{1}{RC} \geq \frac{\omega_m m_a \sin\omega_m t}{1 + m_a \cos\omega_m t}$$

$$\frac{1}{RC} \geq \omega_m \frac{m_a\sqrt{1-m_a^2}}{1 + m_a(-m_a)}.$$

$$\frac{1}{RC} \geq \omega_m \frac{m_a\sqrt{1-m_a^2}}{1 - m_a^2}.$$

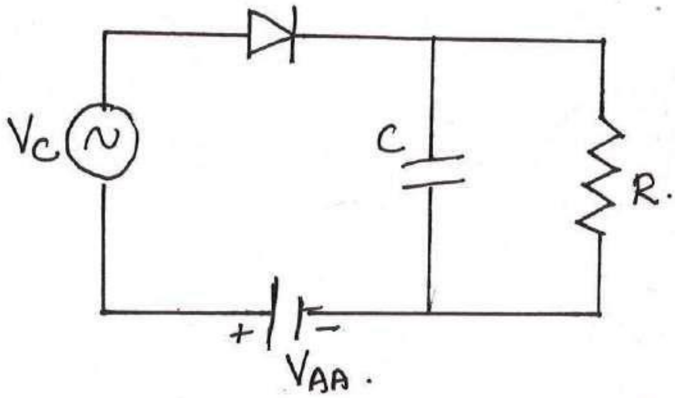
$$\frac{1}{RC} \geq \frac{\omega_m m_a}{\sqrt{1-m_a^2}}.$$

If $m_a << 1$ (Very small) $\sqrt{1-m_a^2}$ is neglected.

$$\boxed{\frac{1}{RC} \geq \omega_m m_a.}$$

This is the relation for obtaining optimum Value of time constant RC in terms of modulation index and modulating frequency.

Square-law detector:



It is used to demodulate or detect the modulated signal of small magnitude, so that the operating region may be restricted to the non-linear portion of the V-I characteristics of the device.

Construction:

Dc supply voltage V_{AA} is used to get the fixed operating point in the non-linear portion of the diode V-I characteristics.

Since the operation is limited to the non-linear region in the VI characteristics of the diode, the lower half-portion of the modulated waveform is compressed. This produces envelope distortion.

Operation:

The distorted output diode current is expressed by the non-linear V-I relationship

$$i = aV + bV^2.$$

$$V = \text{modulated voltage} = a(1+m_a \cos\omega_m t)\cos\omega_c t.$$

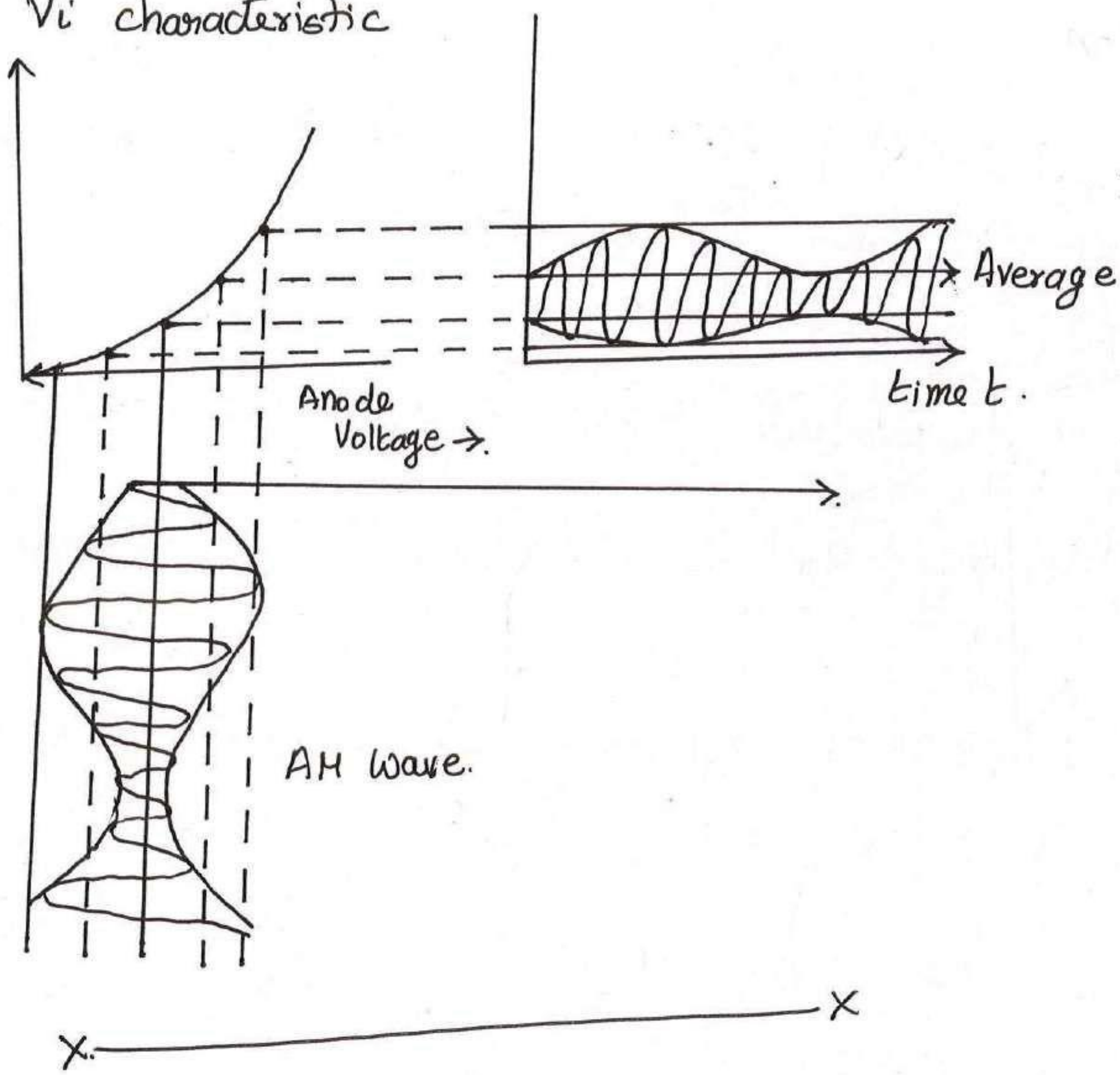
$$i = a\left[a(1+m_a\cos\omega_m t)\cos\omega_c t\right] + b\left[a(1+m_a\cos\omega_m t)\cos\omega_c t\right]^2.$$

The expression will have terms with frequencies $2\omega_c$, $2(\omega_c \pm \omega_m)$, ω_m, $2\omega_m$.

All these frequencies will be passed through a LPF which will allow frequencies less than or equal to the modulating frequency ω_m and reject other higher frequency components.

Thus the modulating signal with frequency ω_m is recovered from the input modulated signal.

Vi characteristic

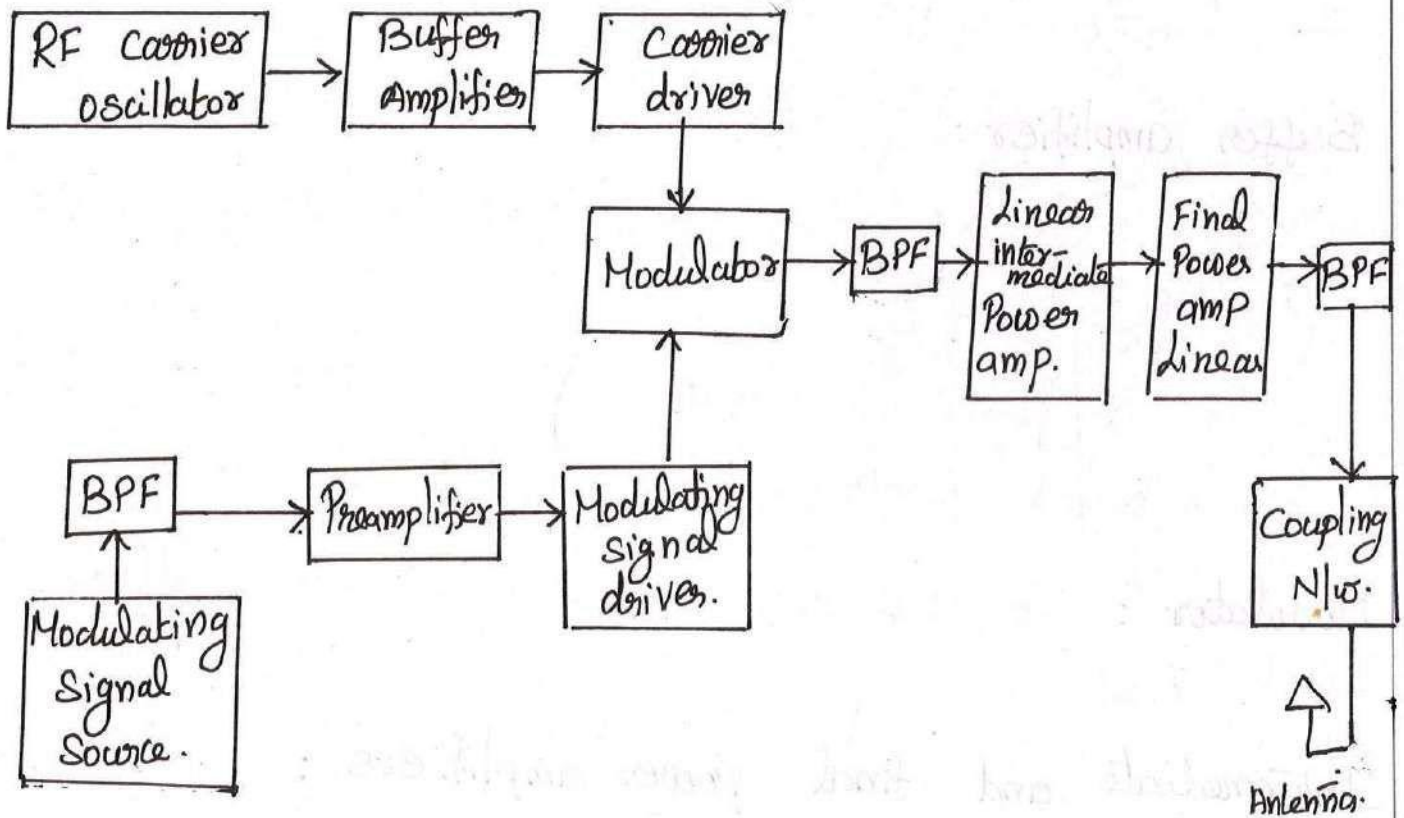


AM Transmitters :

Types:

1. Low level transmitters.
2. High level transmitters.

Low level transmitters:



Preamplifier:

1. It is a linear voltage amplifier with high input impedance.
2. It is used to raise source signal amplitude to a usable level with minimum non-linear distortion and as little thermal noise as possible.

Modulating signal driver (linear amplifier):

1) Amplifies the information signal to an adequate level to sufficiently drive the modulator.

RF Carrier oscillator:

1) It is used to generate the carrier signal.
2) Usually crystal controlled oscillators are used.

Buffer amplifier:

1) It is a low-gain, high-input impedance linear amplifier.
2) It is used to isolate the oscillator from the high-power amplifiers.

Modulator: It can use either emitter or collector modulation.

Intermediate and final power amplifiers:

These amplifiers are required with low-level transmitters to maintain symmetry in AM envelope.

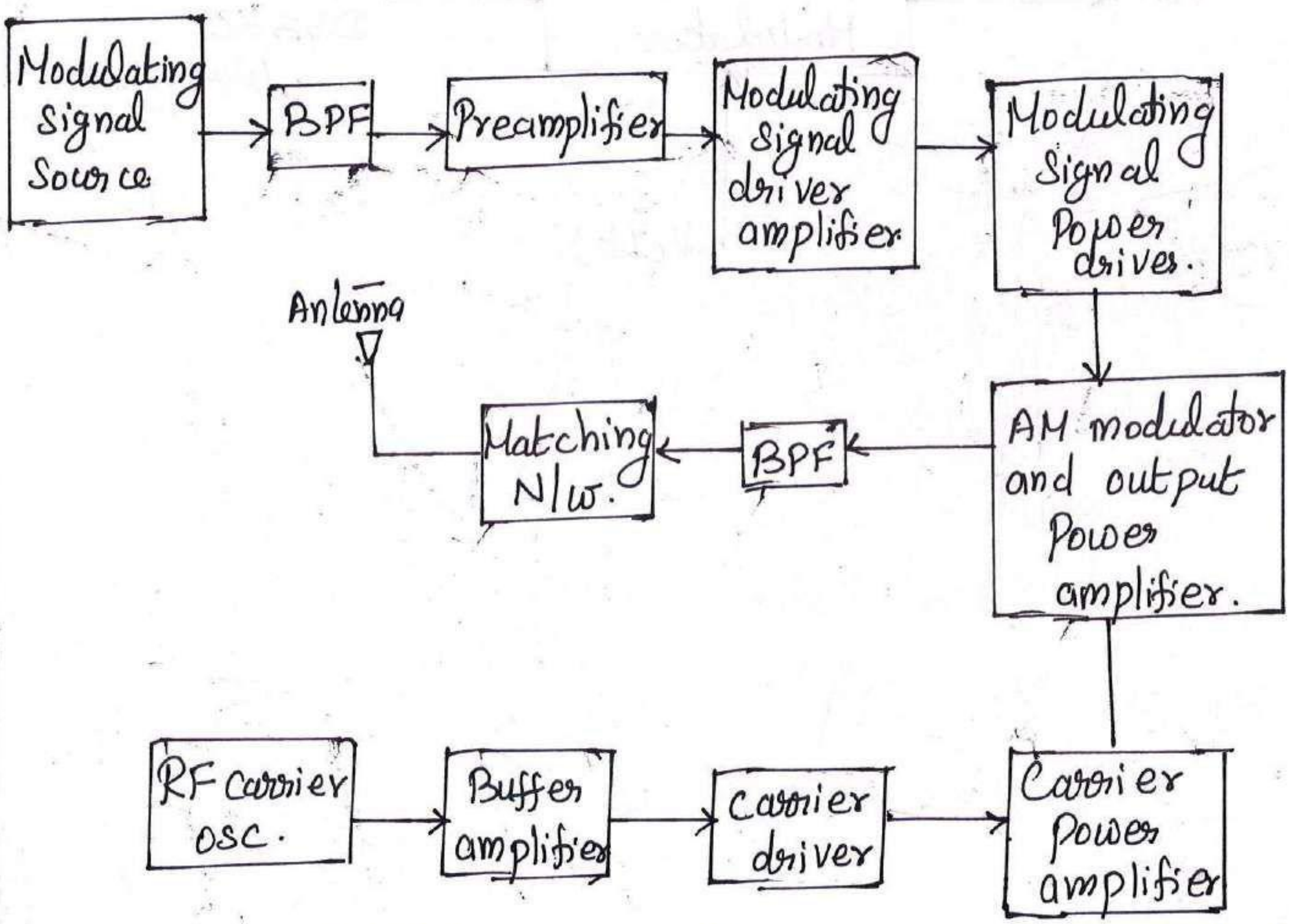
Coupling network:

It matches output impedance of the final amplifier to the transmission line / antenna

Applications.

1. It is used in low-power, low-capacity systems.
2. Wireless intercoms, remote control units, pagers and short-range walkie-talkie.

High level transmitter :

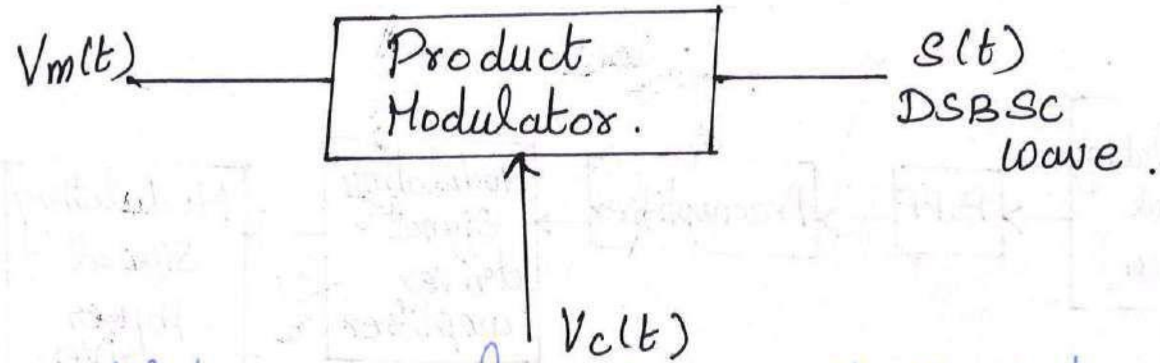


Modulating signal is processed similarly as in low-level transmitter except for the addition of power amplifier.

Double Side band Suppressed Carrier AM (DSBSC-AM)

In AM modulation, to save power, the carrier is suppressed because it does not contain any useful information, only lower and upper side band contains information. This is called DSB-SC-AM.

Representation of DSBSC Signal:



Modulating signal $V_m(t) = V_m \sin\omega_m t$.

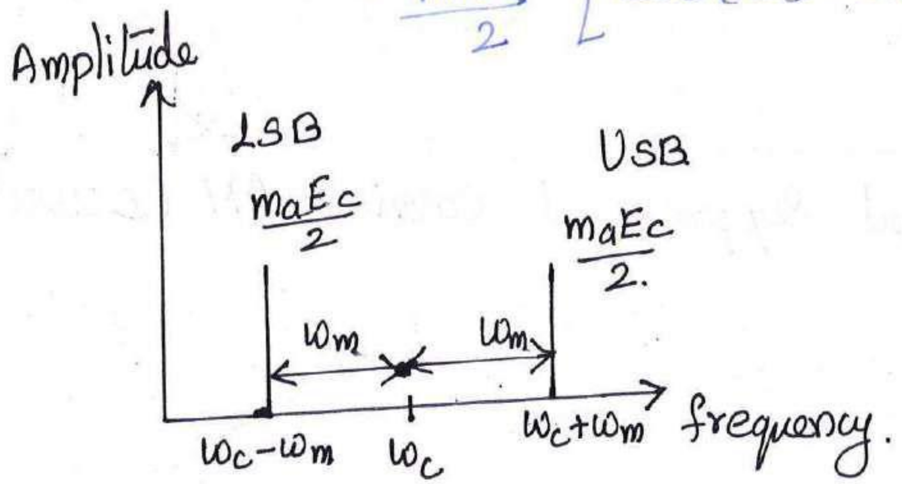
Carrier signal $V_c(t) = V_c \sin\omega_c t$.

When multiplying both the carrier and message signal, the resulting signal is DSB-SC AM signal.

$$S(t) = V_m(t) \cdot V_c(t).$$

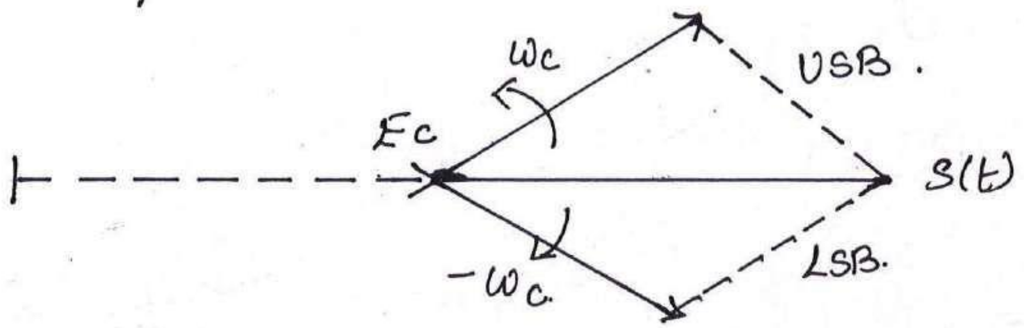
$$= V_m \sin\omega_m t \; V_c \sin\omega_c t.$$

$$= \frac{V_m V_c}{2} \left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t \right]$$



frequency spectrum of DSBSC-AM.

Phasor representation of DSBSC-AM.



Power calculation :

for AM:

$$P_t = P_c + P_{LSB} + P_{USB}.$$

$$P_t = P_c \left[1 + \frac{m_a^2}{2}\right].$$

for DSB-SC AM.

Since Carrier is Suppressed.

$$P_t' = P_{LSB} + P_{USB}.$$

$E = V$.

$$P_{LSB} = P_{USB} = \frac{m_a^2 E_c^2}{8R}$$

$$P_t' = \frac{m_a^2 E_c^2}{8R} + \frac{m_a^2 E_c^2}{8R}.$$

$$P_t' = \frac{m_a^2 E_c^2}{4R}.$$

$$P_t' = \frac{m_a^2}{2}\left[\frac{E_c^2}{2R}\right]$$

$$P_t' = \frac{m_a^2}{2} P_c.$$

Power Saving:

$$\text{Power Saving} = \frac{P_t - P_t'}{P_t}$$

$$= \frac{\left[1+\frac{m_a^2}{2}\right]P_c - \frac{1}{2}m_a^2 P_c}{\left[1+\frac{m_a^2}{2}\right]P_c}$$

$$= \frac{P_c + \cancel{\frac{1}{2}m_a^2 P_c} - \cancel{\frac{1}{2}m_a^2 P_c}}{\left[1+\frac{m_a^2}{2}\right]P_c}$$

$$= \frac{P_c}{\left(1+\frac{m_a^2}{2}\right)P_c} = \frac{1}{1+\frac{m_a^2}{2}} = \frac{2}{2+m_a^2}$$

$$\text{Power Saving} = \frac{2}{2+m_a^2} \times 100.$$

If $m_a = 1$.

Power Saving will be $\frac{2}{2+1} \times 100$.

$= 66.7\%$.

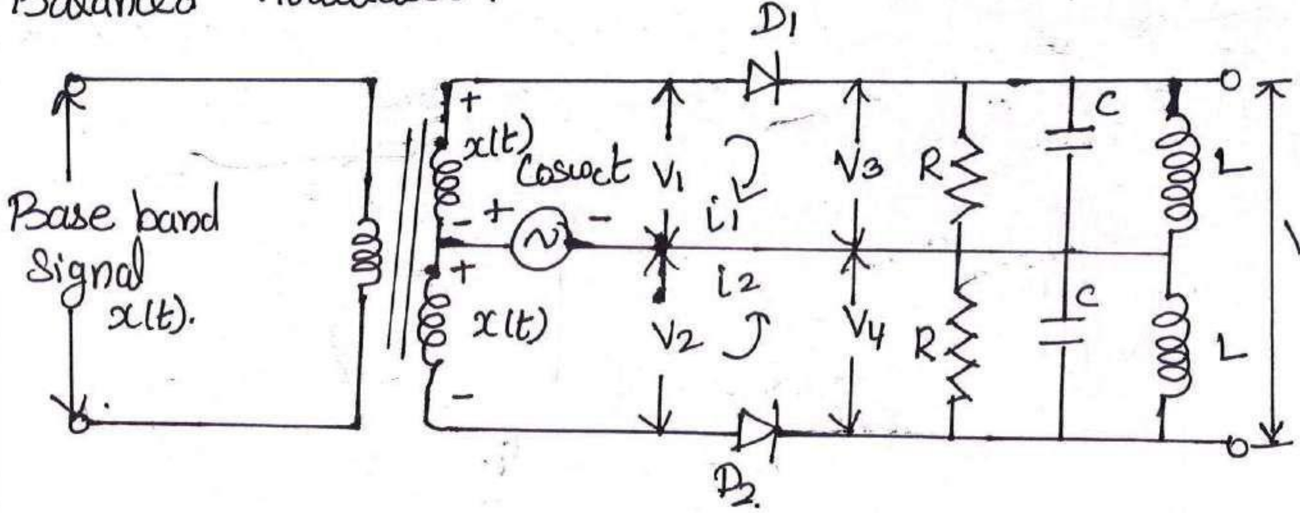
By Suppressing the carrier, power Saving is increased from 33.3 to 66.7%.

Generation of DSB-SC-AM:

(i) Balanced modulator.

(ii) Ring modulator.

Balanced Modulator:



Principle:

When two non-linear devices i.e diodes, transistors etc are connected in a balanced mode to suppress the carrier signals of each other, then only side bands will be left, thus generating DSB-SC signals.

Balanced modulator using diodes:

$$m(t) = E_m \cos \omega_m t.$$

It is applied to two diodes through a centre-tapped transformer.

Carrier signal : $E_c \cos \omega_c t$.

non linear V-I relationship.

$$i = aV + bV^2.$$

for two input voltages V_1 and V_2

$$V_1 = E_c \cos\omega_c t + E_m \cos\omega_m t$$

$$V_2 = E_c \cos\omega_c t - E_m \cos\omega_m t.$$

for diode D_1

$$i_1 = aV_1 + bV_1^2.$$

for diode D_2

$$i_2 = aV_2 + bV_2^2.$$

$$i_1 = a[E_c \cos\omega_c t + E_m \cos\omega_m t] + b[E_c \cos\omega_c t + E_m \cos\omega_m t]^2$$

$$= aE_c \cos\omega_c t + aE_m \cos\omega_m t + bE_c^2 \cos^2\omega_c t + bE_m^2 \cos^2\omega_m t + 2bE_c E_m \cos\omega_c t \cos\omega_m t$$

|||ly

$$i_2 = aE_c \cos\omega_c t - aE_m \cos\omega_m t + bE_c^2 \cos^2\omega_c t + bE_m^2 \cos^2\omega_m t - 2bE_m E_c \cos\omega_m t \cos\omega_c t.$$

Output V_0 will be

$$V_0 = K(i_1 - i_2).$$

$$V_0 = 2ka\,E_m \cos\omega_m t + 4Kb\,E_m E_c \cos\omega_m t \cos\omega_c t$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right].$$

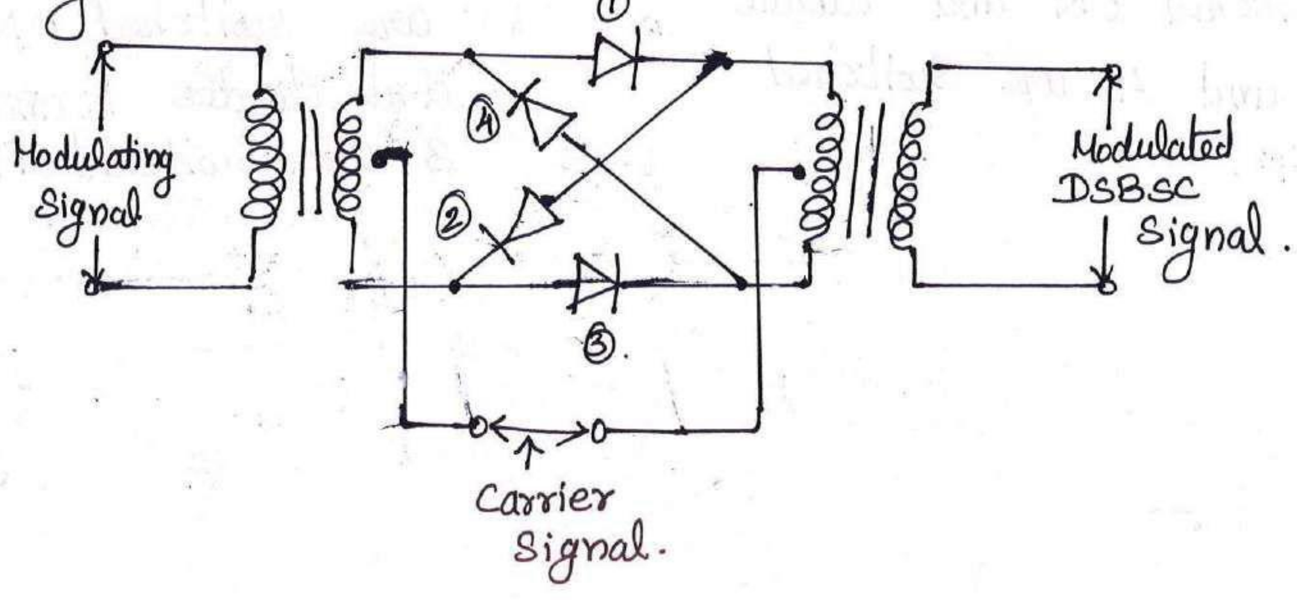
$$V_0 = 2ka\,E_m \cos\omega_m t + 2Kb\,E_m E_c \left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

The output of BPF centred around $\pm\omega_c$ is given by.

$$V_0 = 2Kb\,E_m E_c \left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right].$$

Hence DSBSC-AM is generated.

Ring modulator.

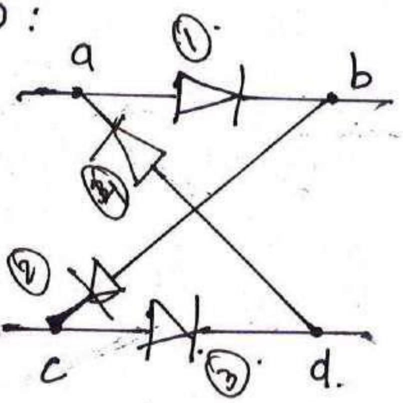


Construction:

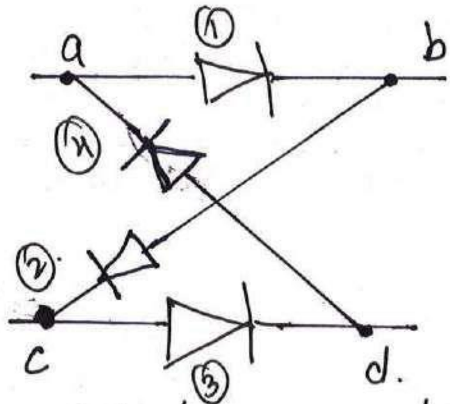
In a ring modulator, four diodes are connected in the form of a ring and all four diodes point in the same manner.

All four diodes in ring structure, are controlled by a square wave carrier signal c(t) of frequency f_c, applied through a centre-tapped transformer.

Operation:



Diodes 1 and 3 are switched ON and diodes 2 and 4 are switched OFF.



Diodes 2 and 4 are switched ON and diodes 1 and 3 are switched OFF.

When diodes are ideal and transformer are perfectly balanced, the two outer diodes (1 & 3) are switched on if the carrier signal is positive and the two inner diodes (2 & 4) are switched off and thus presenting very high impedance, the modulator multiplies the modulating signal $x(t)$ by $+1$.

If the carrier is negative the modulator multiplies the modulating signal by -1.

A ring modulator is also known as a double-balanced modulator, since it is balanced with respect to the base band signal or modulating signal as well as the square wave carrier signal.

Advantages:

1. Low power consumption.
2. Modulation system is simple.

Dis advantages:

1. Complex detection.

Application:

Analog TV systems to transmit colour information.

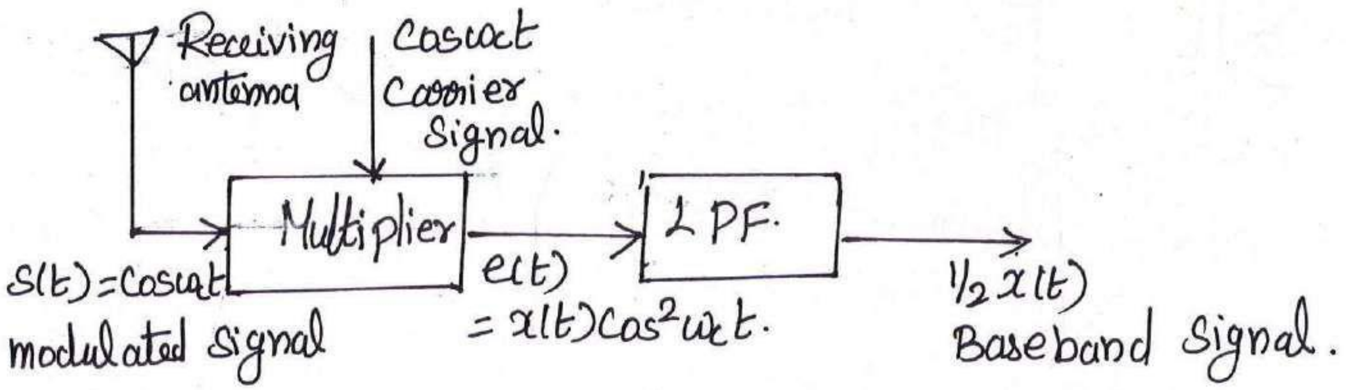
X ———————— X

Demodulation of DSB-SC Signals.

(i) Synchronous detection.

(2) Costa's receiver.

Synchronous detection:



Demodulation:

At the receiver end, the original modulating signal $x(t)$ is recovered from the modulated DSB-SC signal.

It can be achieved by retranslating the baseband or modulating signal from a higher spectrum, centered at $\pm\omega_c$ to the original spectrum.

This process of retranslation is called demodulation or detection.

Synchronous detection:

The base band signal $x(t)$ can be recovered from DSBSC wave $s(t)$ by

1. Multiply $s(t)$ with a locally generated sinusoidal signal (carrier) and then.

2) Low pass filtering the product.

3) If the local oscillator signal is exactly coherent <or> synchronized in both frequency and phase with the carrier wave $\cos\omega_c t$ used in transmitter to generate DSBSC.

The method is known as coherent detection or synchronous detection.

$$e(t) = x(t)\cos\omega_c t \cdot \cos\omega_c t$$

$$= x(t)\cos^2\omega_c t$$

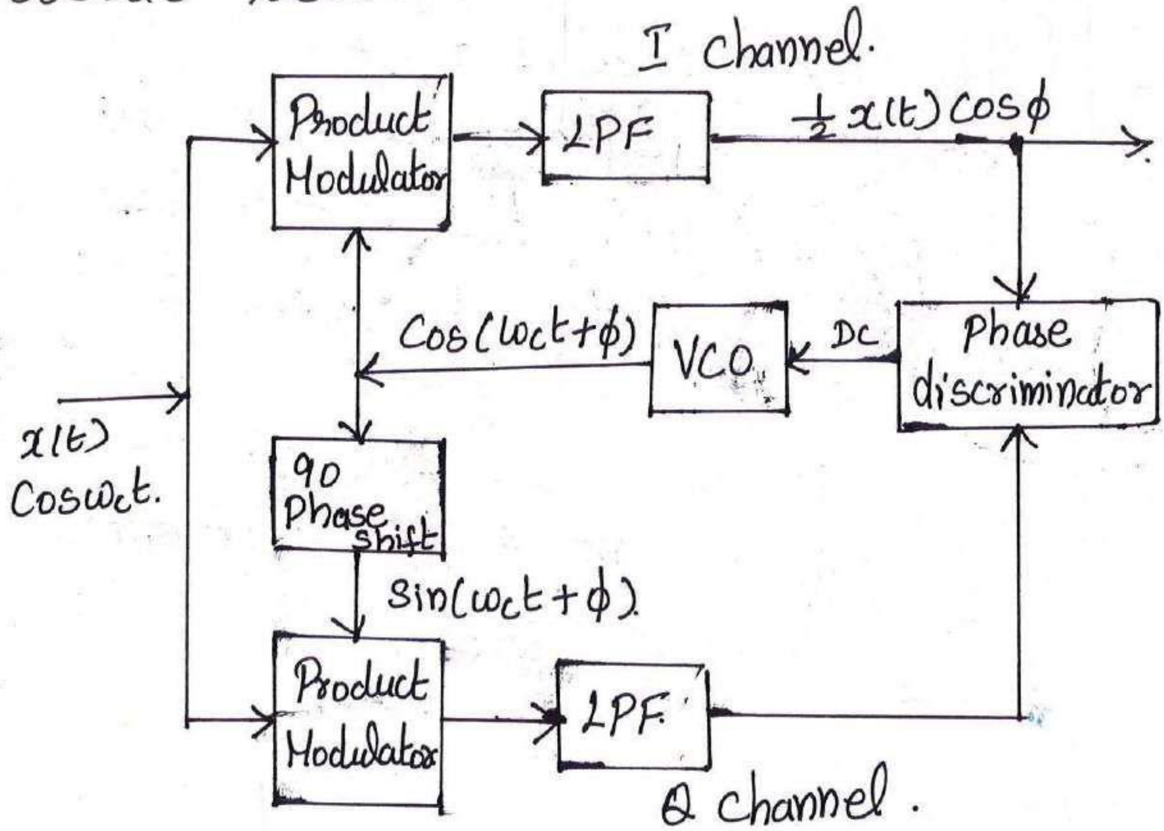
$$= \frac{x(t)}{2}\left[1 + \cos 2\omega_c t\right].$$

$\cos^2\theta = \frac{1}{2}\left[1 + \cos 2\theta\right]$

$$e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos 2\omega_c t.$$

$e(t)$ is then passed through a low pass filter centered at $\pm \omega_c$, hence the second term will be suppressed and the original modulating signal $\frac{1}{2}x(t)$ is obtained.

Costa's receiver:



Construction:

It has two synchronous detectors; one detector is used to be fed with a locally generated carrier which is inphase with the transmitted carrier signal. This detector is known as inphase coherent, I channel.

The other Synchronous detector employs a local carrier which is quadrature phase with transmitted carrier Signal, this is known as Q channel <or> Quadrature phase coherent detector.

Operation:

1. Let the local carrier Signal be Synchronized with the transmitted carrier. The output of the I channel is desired modulating Signal, but the output of Q channel is zero.

2. If the local oscillator frequency drifts slightly, I channel output is almost unchanged but the Q channel is not zero rather Some signal appears at its output.

Outputs of Q- channel:

1. It is proportional to ϕ.
2. Will have Same polarity as I- channel for one direction of phase shift in local oscillator, whereas the polarity will be opposite to I- channel to other direction of Phase Shift.

The Phase discriminator provides a DC control signal which may be used to control and correct the local oscillator phase error.

Local oscillator – VCO.

Limitations :

1. Costas receiver ceases phase control where there is no modulation.
2. The re-establishment is so rapid that distortion is not perceptible in Voice Communication.

X ———————— X

Hilbert Transform:

When phase angles of all the components of a given signal are shifted by 90°, the resulting function of time is called "Hilbert Transform".

Consider an LTI System with transfer function

$$H(f) = \begin{cases} -j, & f > 0 \\ 0, & f = 0 \\ j, & f < 0. \end{cases} \quad \text{and} \quad \mathrm{Sgn}(f) = \begin{cases} 1, & f > 0 \\ 0, & f = 0 \\ -1, & f < 0 \end{cases}$$

The function $H(f)$ can be expressed using Signum function

$$H(f) = -j\,sgn(f).$$

$$e^{-j\pi/2} = -j \quad ; \quad e^{j\pi/2} = j$$

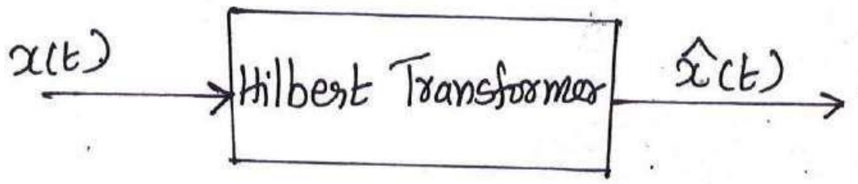
$$\therefore \quad H(f) = \begin{cases} 1 \cdot e^{-j\pi/2}, & f > 0. \\ 1 \cdot e^{j\pi/2}, & f < 0. \end{cases}$$

$$|H(f)| = 1 \quad \forall \text{ all } f.$$

$$\angle H(f) = \begin{cases} -\pi/2, & f > 0 \\ +\pi/2, & f < 0. \end{cases}$$

The device which satisfies this property is known as Hilbert transform.

When a signal is applied to Hilbert transformer, the amplitudes of all frequency components of the input signal remain unaffected. It produces a phase shift of $-90°$ for all positive frequencies while a phase shift of $+90°$ for all negative frequencies of the signal..



Properties of Hilbert transform:

1. The signal $x(t)$ and $\hat{x}(t)$ have the same energy density spectrum.
2. The signal $x(t)$ and $\hat{x}(t)$ have the same autocorrelation function.
3. The signal $x(t)$ and $\hat{x}(t)$ are mutually orthogonal

   i.e $$\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\, dt = 0.$$

4. $H[x(t)] = \hat{x}(t)$ then

   $H[\hat{x}(t)] = -x(t).$

Applications:

1. For generation of SSB signals.
2. For designing minimum phase type filters.
3. For representation of band pass signals.

Pre- envelope :

Consider a real valued signal $x(t)$. The pre-envelope $x_+(t)$ for positive frequencies of the signal $x(t)$ is defined as the complex valued function given by equation

$$x_+(t) = x(t) + j\,\hat{x}(t).$$

The pre-envelope is useful in treating band pass signals and systems.

$$X_+(f) = X(f) + j\,[-\operatorname{sgn}(f)\,X(f)].$$

$$X_+(f) = \begin{cases} 2X(f) & , f > 0 \\ X(0) & , f = 0 \\ 0 & , f < 0. \end{cases}$$

The pre-envelope $x(t)$ for negative frequencies

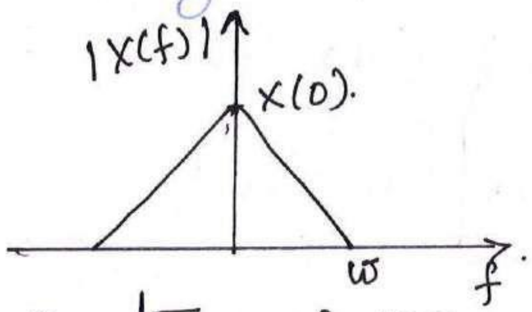
$$x_-(t) = x(t) - j\,\hat{x}(t).$$

The two pre-envelopes $x_+(t)$ and $x_-(t)$ are complex conjugate of each other

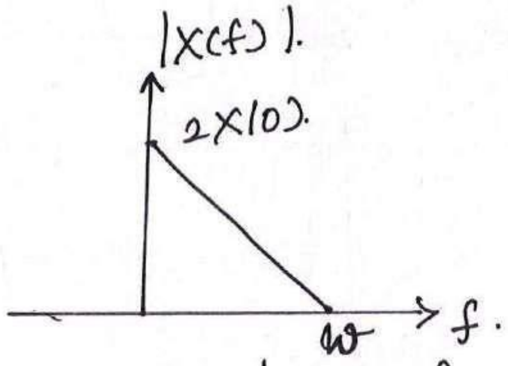
$$x_+(t) = x_-^*(t)$$

$$X(f) = \begin{cases} 0, & f > 0. \\ X(0), & f = 0. \\ 2X(f), & f < 0. \end{cases}$$

Thus the pre-envelopes $x_+(t)$ and $x_-(t)$ constitute a complementary pair of complex valued signals.



Spectrum of LPF $x(t)$



Spectrum of Pre-envelope.

Properties:

If $\hat{x}(t)$ is Hilbert transform of $x(t)$ then Hilbert transform of $\hat{x}(t)$ is

$$[-j \operatorname{sgn}(f)]^2 = -1.$$

Complex envelope:

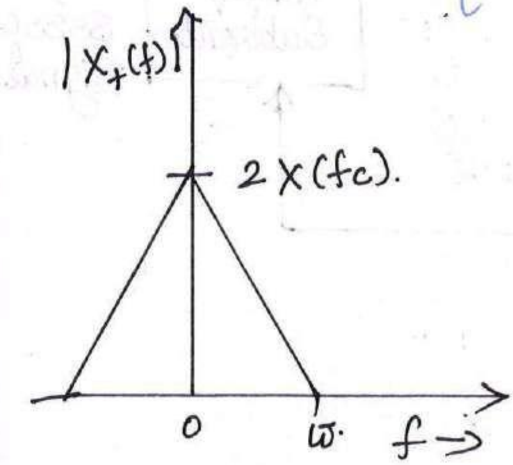
Complex envelope is defined as

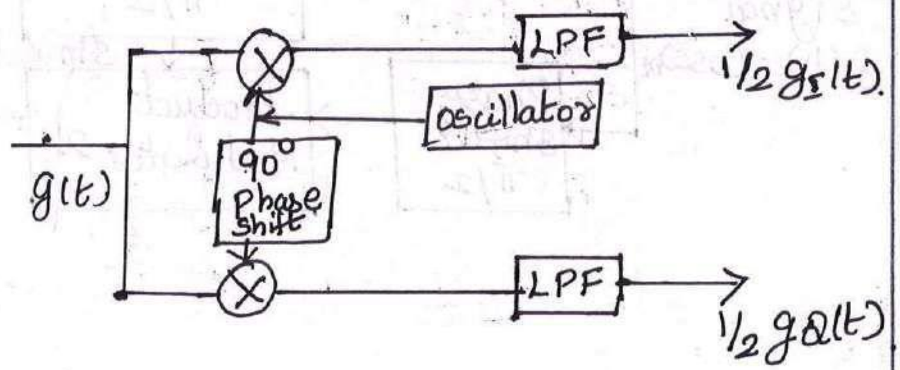
$$g_+(t) = \tilde{g}(t)\, e^{j2\pi f_c t}.$$

$\tilde{g}(t) \rightarrow$ complex envelope of a signal $g(t)$.

Complex envelope is the frequency shift version of analytic signal.

$$\tilde{G}(f) = \begin{cases} 2G(f-f_c) & \forall \; f > 0 \\ G(0) & \forall \; f = 0 \\ 0 & \forall \; f < 0 \end{cases}$$



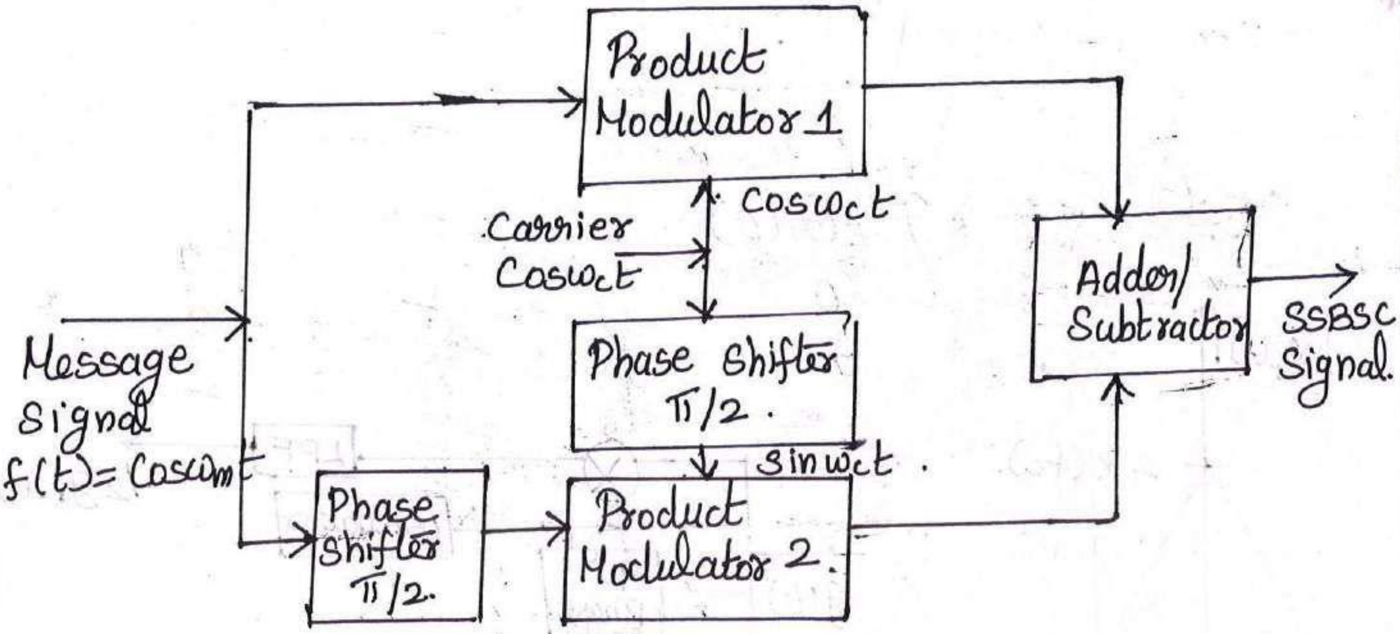


$$\hat{g}(t) = g_I(t) + j g_Q(t).$$

Single Sideband Suppressed Carrier AM (SSB-SC-AM):

In DSB-SC, both side bands carry the same information, which increases the bandwidth. Hence if the carrier signal and one of the sideband is suppressed. The system is called Single Sideband Suppressed Carrier system (SSB SC System).

Block diggram of SSB-SC AM.



Modulating Signal $x(t) = \cos\omega_m t$.

Carrier Signal $= \cos\omega_c t$.

The output of product modulator 1 will be

$$\cos\omega_m t \cos\omega_c t = \frac{1}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right].$$

The output of product modulator 2 will be

$$\sin\omega_m t \sin\omega_c t = \frac{1}{2}\left[\cos(\omega_c-\omega_m)t - \cos(\omega_c+\omega_m)t\right].$$

The above two product modulator outputs are added / subtracted to produce SSB-SC AM wave.

If the two outputs are added together.

adder $= \cos\omega_m t \cos\omega_c t + \sin\omega_m t \sin\omega_c t$

$= \cos(\omega_c - \omega_m)t$.

This is the expression for SSB-SC with lower side band.

If the two outputs are subtracted.

Subtractor $= \cos\omega_m t \cos\omega_c t - \sin\omega_m t \sin\omega_c t$

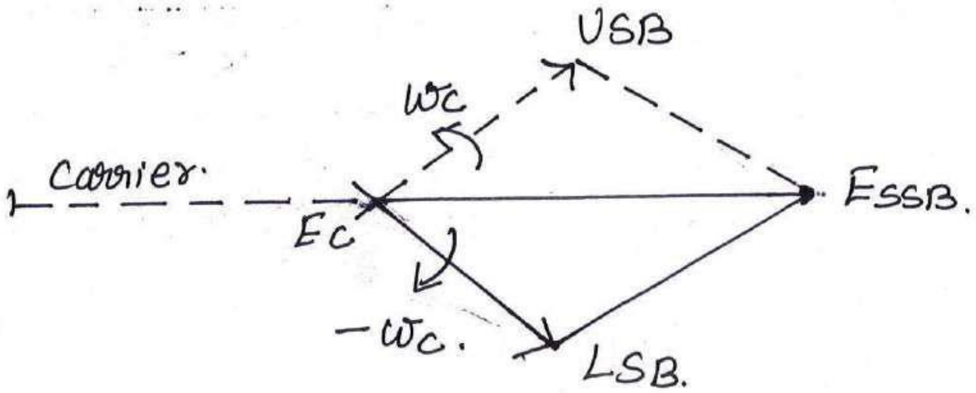
$= \cos(\omega_c + \omega_m)t$.

This is the expression for SSB-SC with upper side band.

Combining the two equations.

$$\phi_{SSB}(t) = \cos\omega_m t \cos\omega_c t \pm \sin\omega_m t \sin\omega_c t.$$

Phasor representation of SSB-SC wave.



Power Calculation in SSBSC-AM:

Total power transmitted in AM is

$$P_t = P_c\left[1 + \frac{m_a^2}{2}\right].$$

If carrier and one side band is suppressed, then total power in SSB-SC AM is

$$P_t'' = P_{LSB} = P_{USB} = \frac{m_a^2 E_c^2}{8R}.$$

$$P_t'' = \frac{1}{4} m_a^2 P_c.$$

$$\text{Power saving} = \frac{P_t - P_t''}{P_t}.$$

$$= \frac{\left[1+\frac{m_a^2}{2}\right]P_c - \left[\frac{m_a^2}{4}P_c\right]}{\left[1+\frac{m_a^2}{2}\right]P_c}.$$

$$= \frac{P_c\left[1+\frac{m_a^2}{2} - \frac{m_a^2}{4}\right]}{P_c\left[1+\frac{m_a^2}{2}\right]}.$$

$$\text{Power Saving} = \frac{1+m_a^2/4}{1+m_a^2/2}.$$

$$\text{Power Saving} = \frac{(4+m_a^2)/4}{(2+m_a^2)/2}$$

$$= \frac{4+m_a^2}{2(2+m_a^2)}$$

$$\boxed{\text{Power Saving} = \frac{4+m_a^2}{4+2m_a^2}}$$

If $m_a = 1$, then Power Saving in SSB SC AM is $\frac{5}{6} = 83.33\%$.

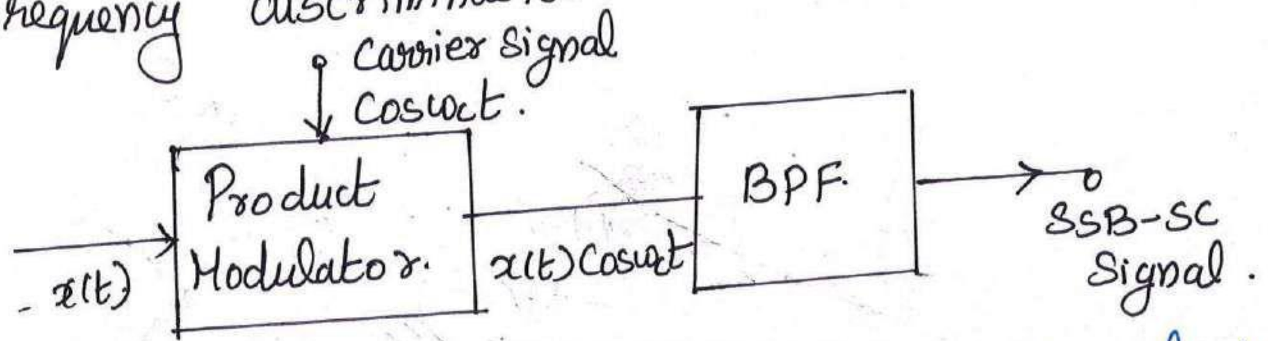
Generation of SSB - SC - AM.

(i) Frequency discrimination method <or> filter method.

(ii) Phase discrimination method <or> phase shift method.

(iii) Modified phase shift method <or> weaver's method.

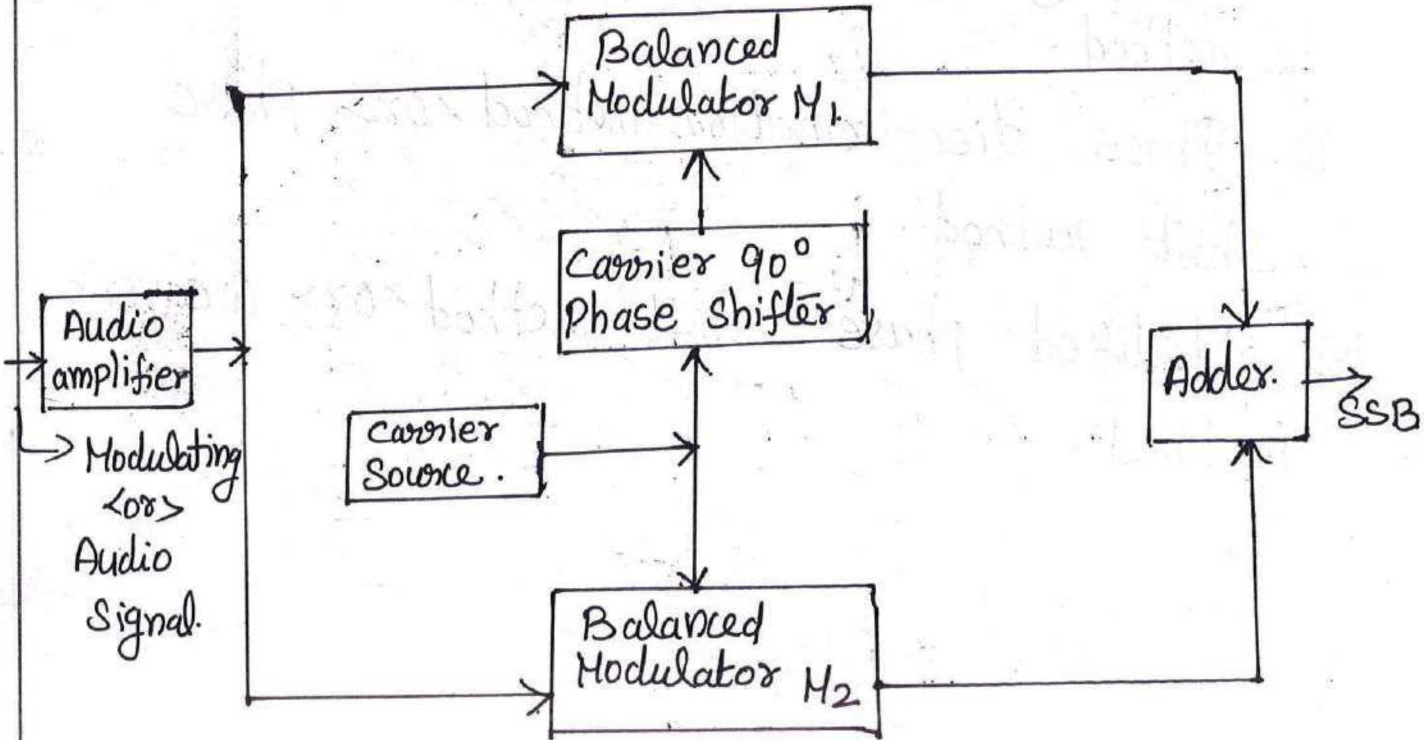
Frequency discrimination method :



In this method, DSBSC signal is first generated using an ordinary product modulator or balanced modulator.

Then, this DSBSC signal is passed through suitable BPF to obtain SSBSC signal.

Phase discrimination <or> Phase shift method :



Construction:

1) In this method two balanced modulators and two phase shifters are used.
2) One modulator accepts carrier with 90° phase shift from carrier oscillator and modulating signal directly.
3) Another modulator accepts modulating signal with phase shift of 90° and the carrier signal directly.

Operation:

1. Balanced modulator 1 accepts direct signal

$$V_m(t) = V_m \sin \omega_m t$$

and 90° phase shifted carrier signal

$$V_c(t) = V_c \sin(\omega_c t + 90).$$

2) Balanced modulator 2 accepts 90° phase shifted modulating signal

$$V_m(t) = V_m \sin(\omega_m t + 90^\circ)$$

and direct carrier

$$V_c(t) = V_c \sin \omega_c t.$$

Output V_1 of balanced modulator 1 is:

$$V_1 = V_m \sin\omega_m t \; V_c \sin(\omega_c t + 90).$$

$$= \frac{V_m V_c}{2}\Big[\cos[(\omega_c t + 90) - \omega_m t] - \cos[(\omega_c t + 90) + \omega_m t]\Big]$$

$$V_1 = \frac{V_m V_c}{2}\Big[\underbrace{\cos(\omega_c t - \omega_m t + 90)}_{LSB} - \underbrace{\cos(\omega_c t + \omega_m t + 90)}_{USB}\Big]$$

Output V_2 of balanced modulator 2 is

$$V_2 = V_m \sin(\omega_m t + 90)\, V_c \sin\omega_c t.$$

$$= \frac{V_m V_c}{2}\Big[(\cos(\omega_c t - (\omega_m t + 90)) - \cos(\omega_c t + (\omega_m t + 90))\Big].$$

$$V_2 = \frac{V_m V_c}{2.}\Big[\underbrace{\cos(\omega_c t - \omega_m t - 90)}_{LSB} - \underbrace{\cos(\omega_c t + \omega_m t + 90)}_{USB.}\Big]$$

Output of the sum will be

$$V_0 = V_1 + V_2.$$

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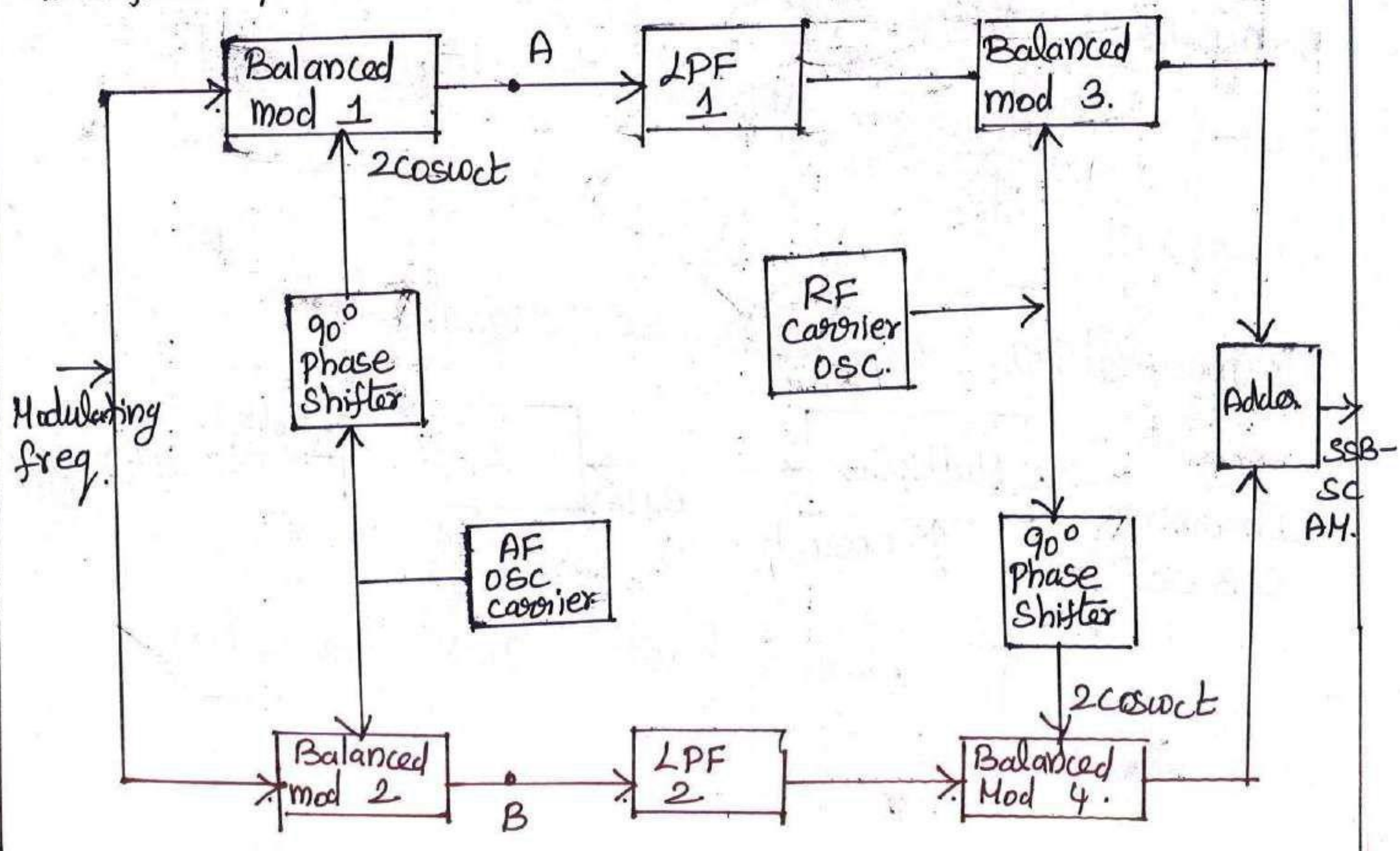
$$V_0 = V_1 + V_2.$$

$$= \frac{V_m V_c}{2}\left[\cos(\omega_c t - \omega_m t + 90) - \cos(\omega_c t + \omega_m t + 90)\right]$$

$$+ \frac{V_m V_c}{2}\left[\cos(\omega_c t - \omega_m t - 90) - \cos(\omega_c t + \omega_m t + 90)\right].$$

The LSB components have the phase shifted difference. Hence they can be cancelled. Therefore only upper sideband components are present and SSBSC is obtained.

Modified phase shift method <or> Weaver's method:



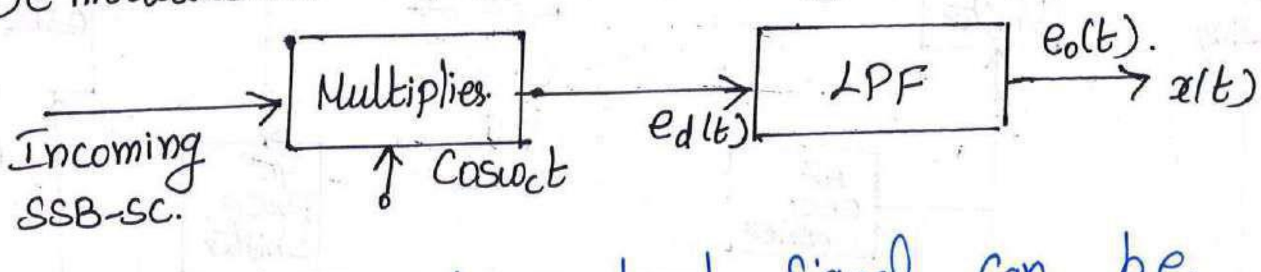
This method is used to generate SSB at any frequency and thus use low audio frequencies.

Construction:

The signal feeding to the balanced modulator 1 and 2 and its output signal at point AB is similar to that of phase shift method.

Instead of phase shifting the entire range of modulating frequencies, this method combines them with a fixed carrier and phase shift is applied to this fixed frequency only.

Demodulation of SSB-SC signals.



The base band signal can be recovered by synchronous detection.

Advantages of SSB:

1. Less bandwidth is required.
2. More power saving (83.33%).
3. Reduced noise interference.

Disadvantages of SSB:

1. The generation and reception of SSB signal is complicated.
2. SSB transmitter and receiver need to have an excellent frequency stability.

Applications of SSB:

1. When power saving and low bandwidth requirement are important.
2. Land and air mobile communication, navigation and amateur radio.

Vestigial side band modulation (VSB):

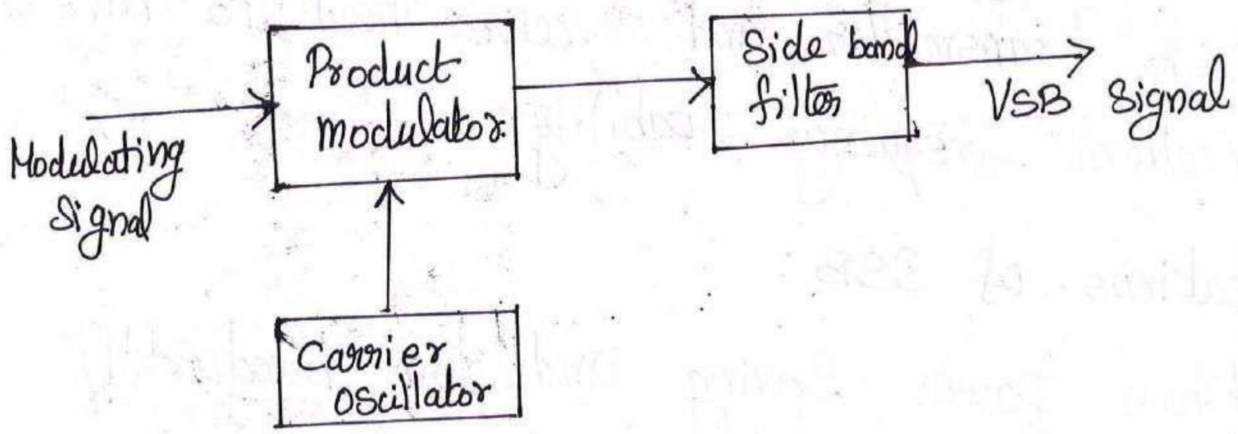
In VSB, the desired sideband is allowed to pass completely, and a small portion (trace or vestige) of the undesired.

Side band is also allowed.

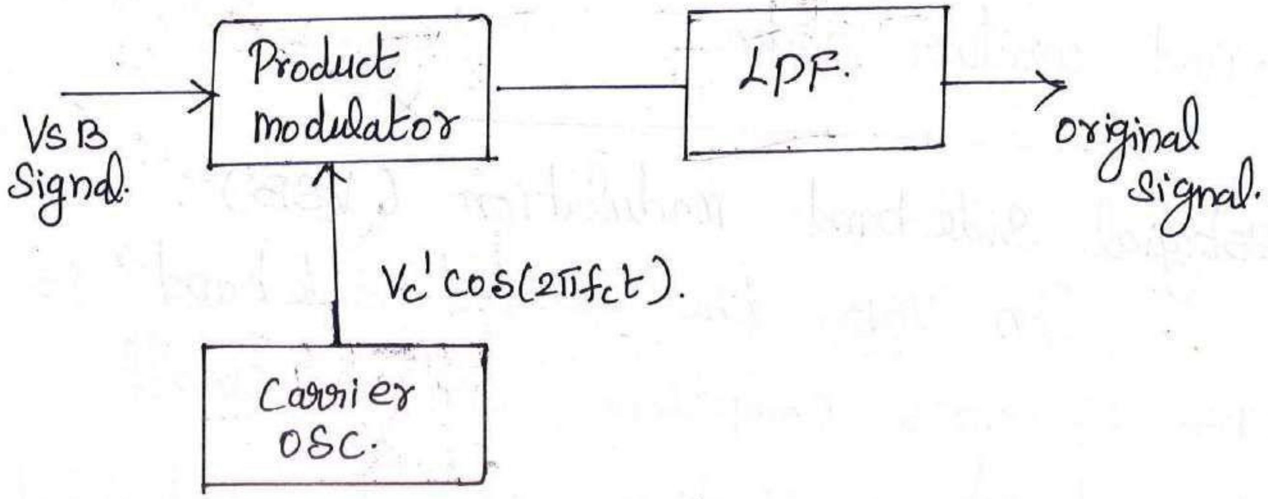
Reason:

SSB-SC signals are difficult to generate due to the difficulty in isolating the desired side band. This difficulty can be overcome by using VSB.

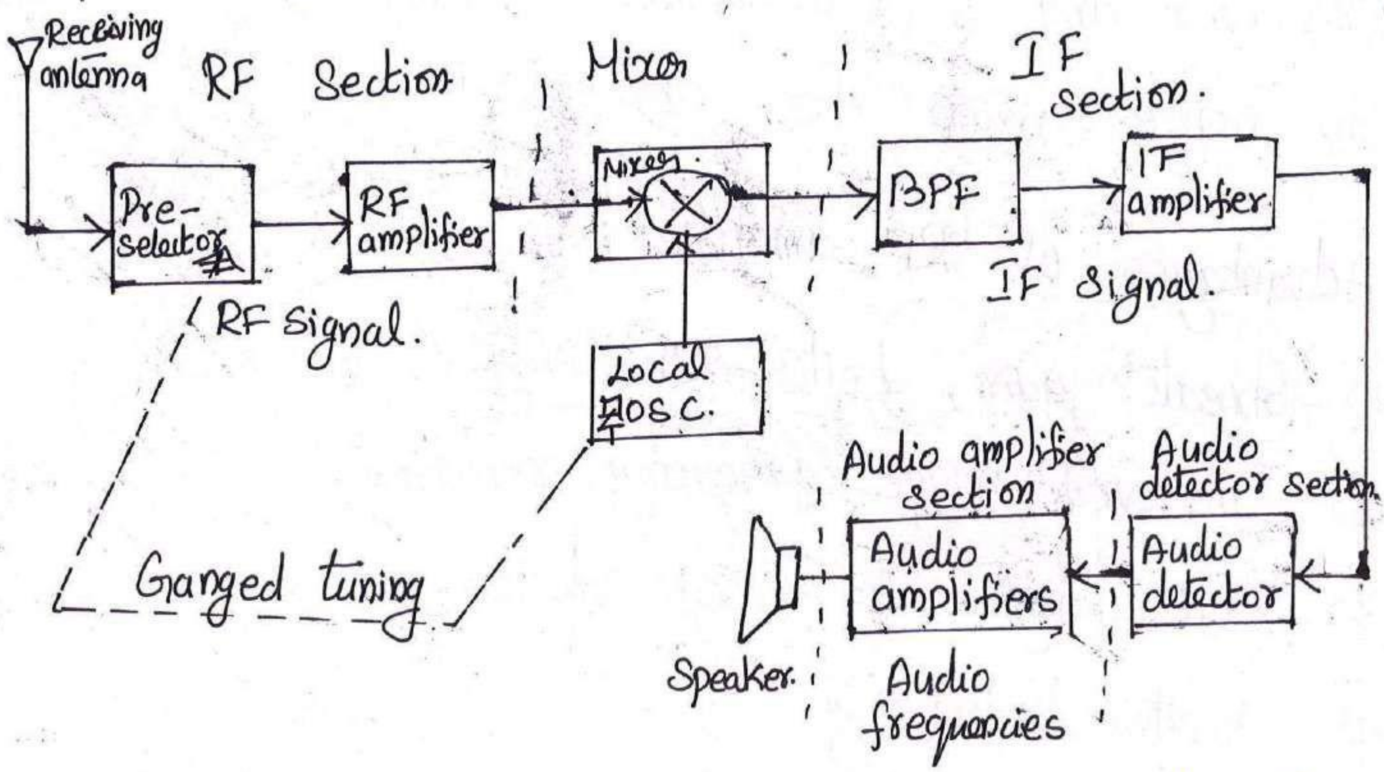
VSB modulator:



VSB Demodulator.



Superheterodyne receiver:



Heterodyne – Mix of two frequencies together in a non-linear device or to transmit one frequency to another using non-linear mixing.

Sections of a Superheterodyne receiver.

1. RF Section:

1) It consists of a pre-selector and an amplifier.

2) Pre-Selector is a broad-tuned BPF with an adjustable center frequency used to reject the unwanted radio frequency and to reduce the noise BW.

3. RF amplifier determines sensitivity of the receiver and a predominant factor in determining the noise figure for the receiver.

Advantages of RF amplifier:

1) Greater gain, better sensitivity.
2) Improved image frequency rejection.
3) Better Signal to noise ratio.
4) Better selectivity.

2) Mixer / Converter Section:

1) It converts the given RF frequency to IF by mixing with local oscillator.
2) The shape of the envelope and BW of the original information contained in the envelope remains unchanged although the carrier and sideband frequencies are changed from RF to IF.
3) The most common IF in AM is 455 KHz.

3) IF Section:

1) It consists of a series of IF amplifiers and BPF to achieve most of the receiver gain and selectivity.

2) The IF is always lower than the RF because it is easier and less expensive to construct high-gain.

3) IF amplifiers are also less likely to oscillate than their RF counterparts.

4) Detector Section:

1) To convert IF signals back to original source information.

2) Can be a diode or PLL.

5) Audio amplifier Section:

1) It consists of several cascaded audio amplifiers and one or more speakers.

2) Depending on the required audio output power, the number of amplifier stages are used.

unit - IV Noise Characterization

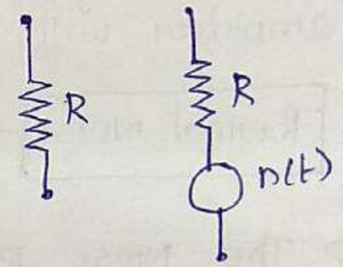
Noise sources - Noise figure - Noise temperature and noise bandwidth Noise in cascaded systems. Representation of narrow band noise, inphase and Quadrature, Envelope and phase - Noise performance analysis in AM & FM systems - Threshold effect, pre emphasis and deemphasis for FM.

① characterisation of noise sources:

→ Any conductive two terminal device is generally characterized as lossy and has some resistance say R ohms.

→ A resistor that is at a temperature T above absolute zero contains free electrons that exhibit random motion and thus result in a noise voltage across the terminals of the resistor. Such a noise voltage is called Thermal noise.

The o/p n(t) of the noise source is characterized as a sample function of a random process.



→ Based on quantum mechanics, Power spectral density of thermal noise is given as,

$$S_R(f) = \frac{2Rh|f|}{\left(e^{\frac{h|f|}{KT}} - 1\right)} \ (\text{volts})^2/Hz \quad \text{—— ①}$$

h → planks constant, K → Boltzmann's constant.

T → Temperature of the resistor in degrees kelvin

T = 273 + C, C → degrees centigrade.

$$e^{\frac{h|f|}{KT}} \approx 1 + \frac{h|f|}{KT} \quad \text{—— ②}$$

The power spectral density is approximated as,

$$S_R(f) = 2RKT \ (\text{volts})^2/Hz \quad \text{—— ③}$$

where $KT = \dfrac{h|f|}{\left(e^{\frac{h|f|}{KT}} - 1\right)}$

The power spectral density of the noise Voltage across the load resistor is,

$$S_n(f) = \frac{KT}{2} \text{ W/Hz} \quad \text{---} (4)$$

power spectral density of thermal noise is generally expressed as

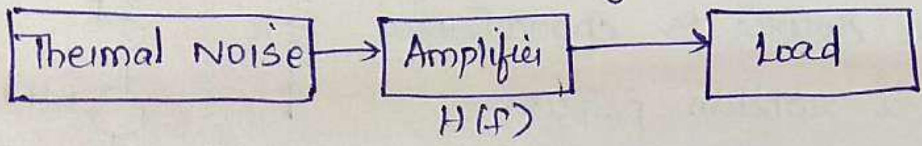
$$S_n(f) = \frac{N_0}{2} \text{ W/Hz} \quad \text{---} (5)$$

For eg. At room temperature $T_0 = 290^\circ K$, $N_0 = 4 \times 10^{-21}$ W/Hz

(2) Noise Figure and Noise temperature :

→ when we use amplifiers in communication systems to boost the level of a signal, we are also ~~amplify~~ amplifying the noise corrupting the signal.

→ since any amplifier has some finite passband, we model an amplifier with the frequency response characteristic $H(f)$.



→ The Noise power at the o/p of the network is,

$$P_{no} = \int_{-\infty}^{\infty} S_n(f) |H(f)|^2 df = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df \quad \text{---} (1)$$

The noise equivalent bandwidth of the filter is defined as,

$$B_{neq} = \frac{1}{2\mathcal{G}} \int_{-\infty}^{\infty} |H(f)|^2 df \quad \text{---} (2)$$

$\mathcal{G} = |H(f)|^2_{max}$ → max. available power gain of the amplifier.

o/p noise power from an ideal amplifier that introduces no additional noise may be expressed as,

$$P_{no} = \mathcal{G} N_0 B_{neq} \quad \text{---} (3)$$

Any practical amplifier introduces additional noise at its o/p due to internally generated noise.

Hence, the noise power at its o/p may be expressed as,

$$P_{no} = g N_0 B_{neq} + P_{ni} \qquad \boxed{N_0 = KT}$$

$$= g KT B_{neq} + P_{ni} \quad \text{—— (4)}$$

P_{ni} → power of the amplifier o/p due to internal noise,

$$P_{no} = g K B_{neq} \left(T + \frac{P_{ni}}{g K B_{neq}} \right) \quad \text{— (5)}$$

$$T_e = \frac{P_{ni}}{g K B_{neq}} \quad \text{—— (6)}$$

is the effective noise temperature of the two port network

Then, $P_{no} = g K B_{neq} (T + T_e)$ —— (7)

A signal source at the input to the amplifier with power P_{si} will produce an o/p with power.

$$P_{so} = g P_{si} \quad \text{—— (8)}$$

o/p SNR from two-port network is,

$$\left(\frac{S}{N}\right)_0 = \frac{P_{so}}{P_{no}} = \frac{g P_{si}}{g KT B_{neq} (1 + T_e/T)}$$

$$= \frac{P_{si}}{N_0 B_{neq} (1 + T_e/T)}$$

$$= \frac{1}{1 + T_e/T} (S/N)_i \quad \text{—— (9)}$$

$(S/N)_i$ → input SNR to the two port network.

→ SNR at the o/p of the amplifier is degraded (reduced) by the factor $(1 + T_e/T)$.

T_e → measure of noisiness of the amplifier.

$(1 + T_e/T_0)$ → noise figure of the amplifier.

→ The noise figure of a two port network is defined as the ratio of the o/p noise power P_{no} to the o/p noise power of an ideal two port network.

$F = (1 + T_e/T_0)$ —— (10)

is the noise figure of the amplifier.

$$\left(\frac{S}{N}\right)_0 = \frac{1}{F}\left(S/N\right)_i \quad \text{—— (11)}$$

Taking logarithm of both sides of Eqn. (11),

$$10 \log (S/N)_0 = -10 \log F + 10 \log (S/N)_i \quad \text{—— (12)}$$

$10 \log F \rightarrow$ loss in SNR due to the additional noise introduced by the amplifier.

→ The noise figure for many low-noise amplifiers is below 3dB.

→ conventional integrated circuits amplifiers have noise figures of 6dB to 7dB.

over all noise figure of a cascade of K amplifiers with gains g_k and corresponding noise figure F_k, $1 \le k \le K$ is,

$$F = F_1 + \frac{F_2 - 1}{g_1} + \frac{F_3 - 1}{g_1 g_2} + \cdots + \frac{F_K - 1}{g_1 g_2 \cdots g_{K-1}} \quad \text{—— (13)}$$

This expression is known as Fries formula.

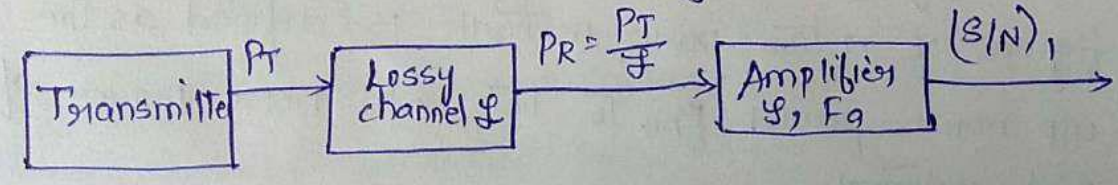
→ We observe that the dominant term is F_1, which is the noise figure of the first amplifier stage.

→ ∴ The front end of a receiver should have a low noise figure and high gain.

→ Remaining terms in the sum will be negligible.

(3) Noise in cascaded systems:

→ Analog repeaters are basically amplifiers that are generally used in telephone wireline channels and microwave line of sight radio channels to boost the signal level.



Input signal power at the input to the repeater is,

$$P_R = P_T / \mathcal{L} \quad \text{——} \ (1)$$

The o/p power from the repeater is,

$$P_o = g P_R = g P_T / \mathcal{L} \quad \text{——} \ (2)$$

We may select the amplifier gain g to offset the transmission loss. $g = \mathcal{L}$ and $P_o = P_T$

(SNR) at the o/p of the repeater is,

$$\left(\frac{S}{N}\right)_1 = \frac{1}{F_a}\left(\frac{S}{N}\right)_1 \quad \text{——} \ (3)$$

$$= \frac{1}{F_a}\left(\frac{P_R}{N_0 B_{neq}}\right) = \frac{1}{F_a}\left(\frac{P_T}{N_0 \mathcal{L} B_{neq}}\right) \quad \text{——} \ (4)$$

$$\left(\frac{S}{N}\right)_1 = \frac{1}{F_a \mathcal{L}}\left(\frac{P_T}{N_0 B_{neq}}\right) \quad \text{——} \ (5)$$

Based on this result eqn. (5), we may view the lossy transmission medium followed by the amplifier as a cascade of two networks: one with <u>noise figure $\mathcal{L}$</u> and the other with a <u>noise figure F_a</u>.

For cascade connection, the overall noise figure is,

$$F = \mathcal{L} + \frac{F_a - 1}{g_a} \quad \text{——} \ (6) \qquad \mathcal{L} + g(f_a - 1)$$

1+

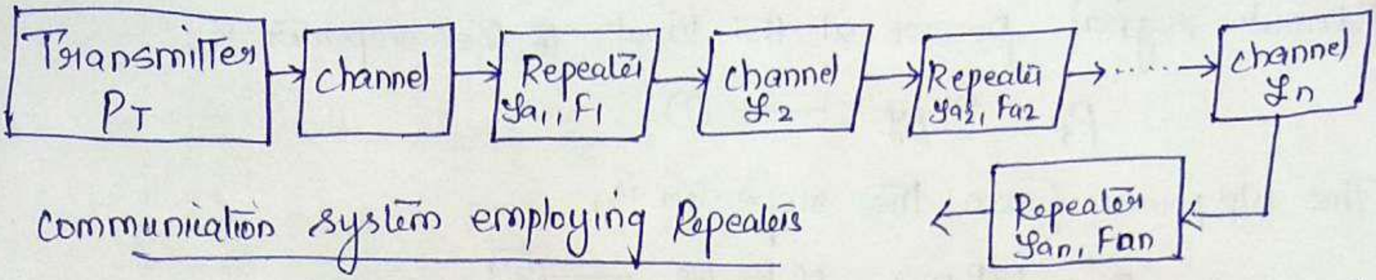
$g_a = \frac{1}{\mathcal{L}}$, then,

$$F = \mathcal{L} + \frac{F_a - 1}{1/\mathcal{L}} = \cancel{\mathcal{L} F_a} \ \cancel{(4)}$$

$$\frac{\mathcal{L}(1/\mathcal{L}) + \cancel{\mathcal{L}} F_a - 1}{1/\mathcal{L}} = \frac{\cancel{1} + F_a - \cancel{1}}{1/\mathcal{L}}$$

$$= \frac{F_a}{1/\mathcal{L}} = \mathcal{L} F_a \quad \text{——} \ (7)$$

→ Hence, the cascade of the lossy transmission medium and the amplifier is equivalent to a single network with noise figure $\mathcal{L} F_a$.



Communication system employing Repeaters

$F_i = \mathcal{L} F_{ai}$ is the noise figure of the ith section, the overall noise figure for the k sections is,

$$F = \mathcal{L}_1 F_{a1} + \frac{\mathcal{L}_2 F_{a2} - 1}{G_{a1}/\mathcal{L}_1} + \frac{\mathcal{L}_3 F_{a3} - 1}{(G_{a2}/\mathcal{L}_2)(G_{a1}/\mathcal{L}_1)} \cdots$$

$$+ \frac{\mathcal{L}_K F_{aK} - 1}{(G_{a1}/\mathcal{L}_1)(G_{a2}/\mathcal{L}_2)\cdots(G_{aK}/\mathcal{L}_K)} \quad \text{--- (8)}$$

∴ Signal to noise ratio at the o/p of the repeater at the receiver is,

$$\left(\frac{S}{N}\right)_0 = \frac{1}{F}(S/N)_i$$

$$= \frac{1}{F}\left(\frac{P_T}{N_0 B_{neq}}\right) \quad \text{--- (9)}$$

If k segments are identical, ie., $\mathcal{L}_i = \mathcal{L}$, for all i and $F_{ai} = F_a$ for all i values.

$G_{ai} = \mathcal{L}_i$ for all i values.

The overall noise figure becomes,

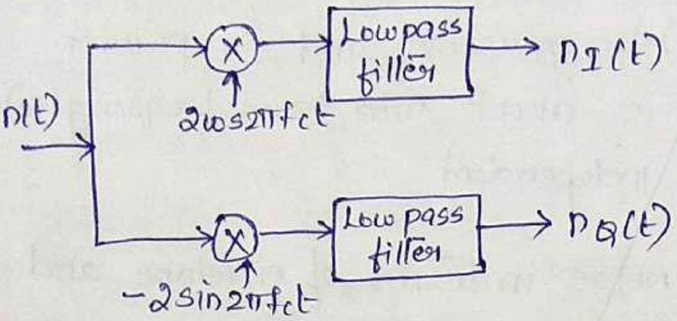
$$F = K\mathcal{L}F_a - (K-1) \approx K\mathcal{L}F_a \quad \text{--- (10)}$$

$$\left(\frac{S}{N}\right)_0 \approx \frac{1}{K\mathcal{L}F_a}\left(\frac{P_T}{N_0 B_{neq}}\right) \quad \text{--- (11)}.$$

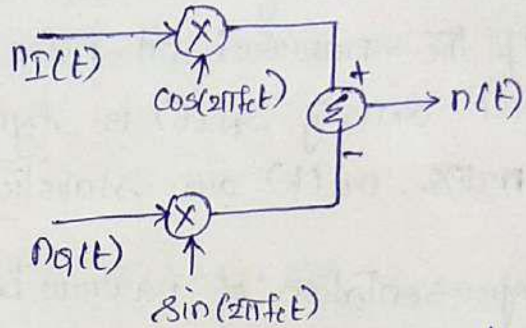
∴ overall noise figure for the cascade of the k identical segments is simply k times the noise figure of one segment.

④ Representation of narrow band noise in terms of in phase and Quadrature components.

In phase components:



Extraction of inphase & Quadrature components



Generation of narrowband ~~noise~~ process.

$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$ —— ①

$n_I(t) \rightarrow$ inphase component of $n(t)$

$n_Q(t) \rightarrow$ quadrature component of $n(t)$

$n_I(t)$ & $n_Q(t)$ are low pass signals.

The inphase and Quadrature components of a narrowband noise have important properties that are summarised:

(i) The inphase component $n_I(t)$ and quadrature component $n_Q(t)$ of narrow band noise $n(t)$ have zero mean.

(ii) If $n(t)$ is gaussian, then $n_I(t)$ and $n_Q(t)$ are jointly gaussian

(iii) If $n(t)$ is stationary, then $n_I(t)$ and $n_Q(t)$ are jointly stationary.

(iv) Both $n_I(t)$ & $n_Q(t)$ have same power spectral density.

$$S_{NI}(f) = S_{NQ}(f) = \begin{cases} S_N(f-f_c) + S_N(f+f_c) & , -B \le f \le B \\ 0 & , \text{otherwise} \end{cases} \quad \text{—②}$$

(v) The inphase component $n_I(t)$ and quadrature component $n_Q(t)$ have same variance as $n(t)$.

(vi) The cross spectral density of $n_I(t)$ & $n_Q(t)$ is purely imaginary.

$$S_{NINQ}(f) = -S_{NQNI}(f)$$

$$= \begin{cases} j[S_N(f+f_c) - S_N(f-f_c)] & , -B \le f \le B \\ 0 & , \text{otherwise} \end{cases} \quad \text{—③}$$

vii) If the narrowband noise $n(t)$ is gaussian and it's power spectral density $S_N(t)$ is symmetric about mid band frequency (f_c), then $n_I(t)$ & $n_Q(t)$ are statistically independent.

⑤ Representation of narrow band noise in terms of envelope and phase components.

Representation of noise $n(t)$ in terms of envelope & phase is,

$$n(t) = r(t)\cos[2\pi f_c t + \psi(t)] \quad \text{—①}$$

where,
$$r(t) = [n_I^2(t) + n_Q^2(t)]^{1/2} \quad \text{—②}$$

$$\psi(t) = \tan^{-1}\left[\frac{n_Q(t)}{n_I(t)}\right] \quad \text{—③}$$

$r(t) \to$ envelope of $n(t)$

$\psi(t) \to$ phase of $n(t)$.

→ Both $r(t)$ & $\psi(t)$ are sample functions of low pass random process.

→ The time interval between two successive peaks of envelope $r(t)$ is approximately $1/B$, where $2B \to$ Bandwidth of $n(t)$.

→ N_I and N_Q are independent gaussian random variables of zero mean and variance σ^2.

Joint probability density function is,

$$f_{NI,NQ}(n_I, n_Q) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{n_I^2 + n_Q^2}{2\sigma^2}\right) \quad \text{—④}$$

Probability of N_I lies between n_I and $n_I + dn_I$ and N_Q lies between n_Q and $n_Q + dn_Q$ is given by.

$$f_{NI,NQ}(n_I, n_Q)\, dn_I\, dn_Q = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{n_I^2 + n_Q^2}{2\sigma^2}\right) \cdot dn_I\, dn_Q \quad \text{—⑤}$$

$$n_I = r\cos\psi \longrightarrow (6)$$

$$n_Q = r\sin\psi \longrightarrow (7)$$

$$dn_I\, dn_Q = r\,dr\, d\psi \longrightarrow (8)$$

→ R & ψ denote the Random variables obtained by observing the random process represented by envelope $r(t)$ and phase $\psi(t)$.

probability of random variable is,

Substitute Eqn. (6), (7), (8) in (5),

$$= \frac{1}{2\pi\sigma^2} \exp\left(-\left(\frac{r^2\cos^2\psi + r^2\sin^2\psi}{2\sigma^2}\right)\right) r\,dr\,d\psi$$

$$\boxed{\cos^2\psi + \sin^2\psi = 1}$$

$$= \frac{r}{2\pi\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) dr\, d\psi \longrightarrow (9)$$

Joint probability density function of R & ψ is,

$$f_{R\psi}(r,\psi) = \frac{r}{2\pi\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right)$$

prob. density function is independent of angle ψ.

→ Random Variable ψ is uniformly distributed inside the range 0 to 2π,

$$f_\psi(\psi) = \begin{cases} \frac{1}{2\pi}, & 0 \le \psi \le 2\pi \\ 0, & \text{elsewhere} \end{cases} \longrightarrow (10)$$

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), & r \geq 0 \\ 0, & \text{elsewhere} \end{cases} \longrightarrow (11).$$

$$\nu = \frac{r}{\sigma} \rightarrow (12)$$

$$f_\nu = \sigma f_R(r) - (13)$$

Rayleigh distribution of Eqn. (11),

$$f_V(\nu) = \begin{cases} \nu \exp\left(-\nu^2/2\right), & \nu \geq 0 \\ 0, & \text{elsewhere} \end{cases} \longrightarrow (12)$$

Rayleigh distribution is zero for negative values of ν, because envelope can assume only non-negative values

⑥ Threshold effect in Angle modulation :

→ The noise analysis of angle demodulation schemes is based on the assumption that SNR at the demodulator input is high.

→ This high SNR is a simplifying assumption that is usually made in the analysis of non-linear modulation systems.

→ The signal and noise processes at the o/p of the demodulator are completely mixed in a single process by a complicated non linear functional.

→ At low signal to noise ratios, signal and noise components are so intermingled that we cannot recognize the signal from the noise, ∴ Threshold effect is present.

→ There is a specific (SNR) at the I/P of demodulator (Threshold SNR) below which signal mutilation occurs.

→ The existence of threshold effect places an upper limit on the trade off between bandwidth and power in FM system. This limit is a practical limit in the value of the modulation index β_f. At threshold relation between,

$$\frac{P_R}{N_0 W} = \left(\frac{S}{N}\right)_b \quad \text{--- (i)}$$

and β_f holds in FM system:

$$\left(\frac{S}{N}\right)_{b,th} = 20(\beta+1) \quad \text{--- ①}$$

$P_R \rightarrow$ Received power.

We can calculate the max. allowed β to make sure that the system works above threshold.

$B_c \rightarrow$ Bandwidth allocation

using carson's rule,

$$B_c = 2(\beta+1)W \quad \text{--- ②}$$

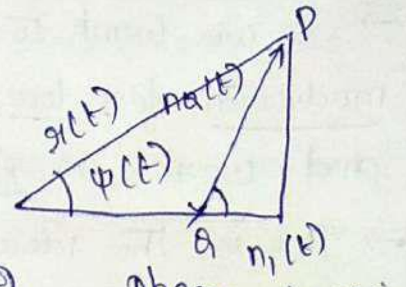
→ There are two factors that limit the value of the modulation index β.

(i) Limitation of channel bandwidth.

(ii) Limitation on received power which limits the value of β.

$$\frac{PM}{(\max|m(t)|)^2} = 1/2 \quad \text{--- (3)}$$

$$(S/N)_o = 3/2\, \beta^2 (S/N)_b \quad \text{--- (4)}$$



Phasor diagram of $x(t) = [A_c + n_I(t)] \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$.

for $\beta = 5$, $(S/N)_{b,th} = 120 \sim 20.8\,dB$ --- (5)

for $\beta = 2$, $(S/N)_{b,th} = 60 \approx 17.8\,dB$ --- (6)

if $(S/N)_b = 20dB$, we cannot use $\beta = 5$, system will operate below threshold.

if $(S/N)_b = 17.8\,dB$, at the o/p of the receiver.

→ If we want to employ the maximum available B.W., we must choose the largest possible β that guarantees that the system will operate above threshold.

This is the value of β that satisfies,

$$(S/N)_{b,th} = 20(\beta+1) \quad \text{--- (7)}$$

$$(S/N)_o = 60\beta^2(\beta+1)\, P_{Mn} \quad \text{--- (8)}$$

relates a desired o/p SNR to the highest possible β that achieves SNR.

7) pre emphasis and deemphasis filtering:

→ The modulator filter, which emphasizes high frequencies, is called pre emphasis filter, the demodulator filter, which is the inverse of the modulator filter, is called deemphasis filter.

→ The noise power spectral density at the o/p of the demodulator in PM is flat, for FM, noise power spectrum has a parabolic shape.

→ This means that FM performs better in low frequency components of the message signal and PM performs better in high frequency components.

→ ∴ we want to design a system that performs frequency modulation for low-frequency components of the message signal and works as phase modulation for high-frequency components.

→ This is the idea behind preemphasis and deemphasis filtering techniques.

→ The objective in this technique is to design a system which behaves like an ordinary frequency-modulation-demodulation pair in the low frequency band of the message signal and like a phase modulation demodulation pair in high frequency band of message signal.

→ Demodulation side → low freq. have simple FM demodulation.

→ ↳ high freq. have " PM demodulation.

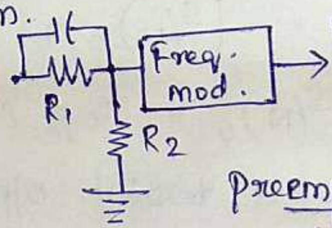
∴ The demodulation needs a filter that has constant gain at low frequencies and behaves as integrator at high frequencies.

→ Inorder to compensate attenuation of high freq. components, we can amplify at the Tx before modulation.

→ We need high pass filter at the Tx.

" " low pass filter at the Rx.

Net effect → Flat frequency response.



Preemphasis circuit

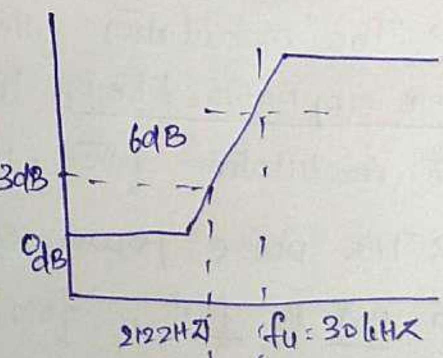
Frequency response of the receiver (deemphasis) is,

$$H_d(f) = \frac{1}{1+j\frac{f}{f_0}} \quad \text{—} \ (1)$$

$$f_0 = \frac{1}{2\pi \times 75 \times 10^{-6}} \approx 2100 = 3dB \quad \text{—} \ (2)$$

$$S_{nPD}(f) = S_{no}(f)\,|H_d(f)|^2 \quad \text{—} \ (3)$$

$$S_{no}(f) = \frac{N_0}{A_c^2} f^2 \quad \text{—} \ (4)$$



preemphasis curve.

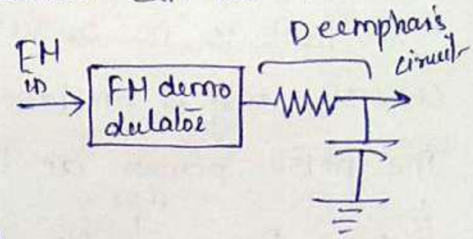
substitute Eqn. (1) & (4) in (3),

$$S_{nPD}(f) = \frac{N_0}{A_c^2} f^2 \frac{1}{1+\frac{f^2}{f_0^2}} \quad \text{—} \ (5)$$

Eqn. 5 is the noise component after the deemphasis filter has a power spectral density.

The noise power at the o/p of the demodulator can be obtained as,

$$P_{nPD} = \int_{-W}^{+W} S_{nPD}(f)\,df \quad \text{--- (6)}$$

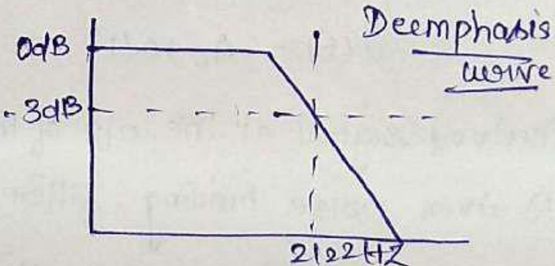


$$= \frac{N_0}{A_c^2} \int_{-W}^{+W} \frac{f^2}{1+f^2/f_0^2}\,df \quad \text{--- (7)}$$

$$= \frac{2N_0 f_0^3}{A_c^2}\left[\frac{W}{f_0} - \arctan\frac{W}{f_0}\right] \quad \text{--- (8)}$$

Since the demodulated msg signal power in this case is equal to simple FM system with no pre emphasis & deemphasis filtering, the ratio of o/p SNR's in these two cases is inversely proportional to noise power ratios ie.,

$$\frac{\left(\frac{S}{N}\right)_{oPD}}{(S/N)_o} = \frac{P_{no}}{P_{nPD}} \quad \text{--- (9)}$$



$$= \frac{\dfrac{2N_0W^3}{3A_c^2}}{\dfrac{2N_0 f_0^3}{A_c^2}\left[\dfrac{W}{f_0} - \arctan\dfrac{W}{f_0}\right]} \quad \text{--- (10)}$$

$$= \frac{1}{3}\,\frac{(W/f_0)^3}{\left[\dfrac{W}{f_0} - \arctan\dfrac{W}{f_0}\right]} \quad \text{--- (11)}$$

Eqn. 11 is the improvement obtained by employing preemphasis and deemphasis filtering.

→ The only filter that has an effect on the received noise is the receiver filter, which shapes the power spectral density of the noise within the message bandwidth.

(8) Noise performance analysis in AM and FM systems:

(i) Noise Effect on a baseband system.

→ There is no carrier demodulation to be performed. The receiver consists only of an ideal low pass filter with the bandwidth W. The noise power at the o/p of the receiver, for a white noise i/p is,

$$P_{no} = \int_{-W}^{+W} \frac{N_0}{2} \, df \quad \text{—①}$$

P_R → Received power

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} \quad \text{—②}$$

(ii) Effect of Noise on DSB-SC AM:

In DSB-SC AM, the transmitted signal is,

$$u(t) = A_c \, m(t) \cos(2\pi f_c t) \quad \text{—①}$$

Received signal at the o/p of the Receiver noise limiting filter } = I/p signal + filtered noise.

$$r(t) = u(t) + n(t) \quad \text{—②}$$

$$u(t) = A_c m(t) \cos(2\pi f_c t) \quad \text{—③}$$

$$n(t) = n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \quad \text{—④}$$

substitute Eqn. ④ & ③ in ②,

$$r(t) = A_c m(t) \cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \quad \text{—⑤}$$

multiply r(t) with $\cos(2\pi f_c t + \phi)$ yields in Eqn. ⑤,

$$r(t) \cos(2\pi f_c t + \phi) = \left[A_c m(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) + n(t) \cos(2\pi f_c t + \phi) \right] \quad \text{—⑥}$$

using the formula for cosA . cosB:

$$= \frac{1}{2} A_c m(t) \cos(\phi) + \frac{1}{2} \left[n_c(t) \cos(\phi) + n_s(t) \sin\phi \right] \quad \text{—⑦}$$

→ The effect of phase difference between the received carrier and a locally generated carrier at the receiver is a drop equal to $\cos^2(\phi)$ in the received signal power.

→ If a phase locked loop is employed, then $\phi = 0$, and the demodulator is called coherent (or) synchronous demodulator.

without the loss of generality, we assume $\phi = 0$, in eqn. (7),

$$y(t) = \frac{1}{2} A_c m(t) \cos(0) + 1/2 [n_c(t) \cos(0) + n_s(t) \sin(0)] \quad \text{— (8)}$$

$\cos(0) = 1$

$\sin(0) = 0$

$$y(t) = 1/2 \, A_c m(t) + 1/2 \, n_c(t) \quad \text{— (9)}$$

$$= 1/2 [A_c m(t) + n_c(t)] \quad \text{— (10)}$$

The message signal power is,

$$P_0 = \frac{A_c^2}{4} P_M \quad \text{— (11)}$$

P_M → power content of the message signal.

$$P_{n0} = \frac{1}{4} P_{n_c}$$

$$= 1/4 \, P_n \quad \text{— (12)}$$

power spectral density of $n(t)$ is given by,

$$S_n(f) = \begin{cases} N_0/2, & |f - f_c| < W \\ 0, & \text{otherwise} \end{cases} \quad \text{— (13)}$$

The noise power is,

$$P_n = \int_{-\infty}^{\infty} S_n(f)\, df$$

$$= N_0/2 \times 4W$$

$$\boxed{P_n = 2N_0 W} \quad \text{— (14)}$$

$$(S/N)_o = \frac{P_0}{P_{n0}} \quad \text{— (15)}$$

Substitute Eqn. (14) in (12),

$$P_{no} = \frac{1}{4} P_n$$

$$= \frac{1}{4} \times 2WN_0 = \frac{WN_0}{2} \quad \text{--- (16)}$$

Substitute Eqn. (6) & (11) in Eqn. (15),

$$= \frac{\frac{A_c^2}{4} P_H}{\frac{1}{2} WN_0} = \frac{A_c^2}{\cancel{4}_2} P_H \times \frac{\cancel{2}}{WN_0}$$

$$\left(S/N\right)_o = \frac{A_c^2 P_H}{2WN_0} \quad \text{--- (17)}$$

$$\left(S/N\right)_{ODSB} = \frac{P_R}{N_0 W} \quad \text{--- (18)} \qquad \left[\because P_R = \frac{A_c^2 P_H}{2}\right]$$

∴ In DSBSC AM, the o/p SNR is the same as the SNR for a baseband system.

Noise :

- Noise
  - External Noise
    - Atmospheric noise
    - Galactic noise
    - Manmade noise
  - Internal Noise
    - Shot Noise
    - Thermal noise
    - White noise

Noise can be defined as an unwanted signals that ends to disturb the transmission and Processing of signals in communication system. The sources of noise may be external to the system or internal to the system.

Shot Noise :

Shot Noise arises in electronic devices such as diodes and transistors because of the nature of the current flow in these devices. Thus the total current flowing through the photodetector may be modeled as an infinite sum of current pulses, as given by

$$x(t) = \sum_{k=-\infty}^{\infty} h(t - \tau_k)$$

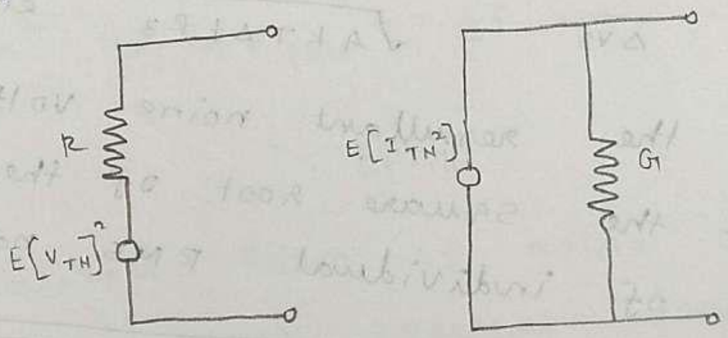
Where, $h(t - \tau_k)$ is the current pulse generate at τ_k.

Thermal Noise :

Thermal noise is the name given to the electrical noise arising from the random motion of electrons in a conductor. The mean-square value of the thermal noise voltage V_{TN} appearing across the terminals of a resistor, measured in a bandwidth of Δf Hertz.

$$E\left[V_{TN}^2\right] = 4KTR\Delta f \text{ Volts}^2$$

Hence, RMS value of voltage across the resistor $R = 2\sqrt{KTR\Delta f}$ volts.



Models of a noisy resistor, Thevenin equivalant circuit, Norton equivalant circuit

The central limit theorem indicates that thermal noise is Gaussian distributed with zero mean.

$$\boxed{V_{TN} = \sqrt{4KTBR}} \qquad \boxed{B = \Delta f}$$

Addition of Noise due to several source in series :

Let us consider several thermal noise sources (i.e) resistor R_1, R_2, R_3 etc in series Producing noise voltage $\overline{\Delta V_1}$, $\overline{\Delta V_2}$ etc. We know that the RMS noise voltage Produced by resistor R is given as

$$\overline{\Delta V_n} = \sqrt{4KTR\Delta f}$$

$$\overline{\Delta V_1} = \sqrt{4KT\Delta f R_1}$$

$$\overline{\Delta V_2} = \sqrt{4KT\Delta f R_2}$$

$$\overline{\Delta V_3} = \sqrt{4KT\Delta f R_3} \quad \text{etc,....}$$

Then the resultant noise voltage $\overline{\Delta V_{nr}}$ is given by the square root of the sum of the squares of individual RMS noise voltages.

$$\overline{\Delta V_{nr}} = \sqrt{\overline{\Delta V_1^2} + \overline{\Delta V_2^2} + \overline{\Delta V_3^2} + \cdots}$$

$$\overline{\Delta V_{nr}} = \sqrt{4KT\Delta f (R_1 + R_2 + R_3 + \cdots)}$$

R_S is the equivalant series resistance of the individual resistances and is given as

$$R_S = R_1 + R_2 + R_3 + \cdots$$

$$\boxed{\overline{\Delta V_{nr}} = \sqrt{4KT\Delta f (R_S)}}$$

Addition of Noise due to several sources in

Parallel :

With resistors in parallel it is best to work in terms of conductance. Thus let G_{pair} represent the parallel combination.

Where, $G_{pair} = G_1 + G_2 + G_3 + \cdots$

$$\overline{\Delta I_n^2} = 4 G_{pair} KT \Delta f$$

$$= 4 (G_1 + G_2 + G_3 + \cdots) KT \Delta f$$

$$= \overline{\Delta I_1^2} + \overline{\Delta I_2^2} + \overline{\Delta I_3^2} + \cdots$$

$$\overline{\Delta I_n^2} = 4 R_{pair} KT \Delta f$$

White Noise :

An idealized form of noise called white noise, the power spectral density of which is independent of the operating frequency. The adjective white is used in the sense that white light contains equal amount of all frequencies within the visible band of electromagnatic radiation.

$$S_w(f) = \frac{N_0}{2}$$

The parameter N_0 is usually referred to the input stage of the receiver of a communication system.

It may be expressed as,

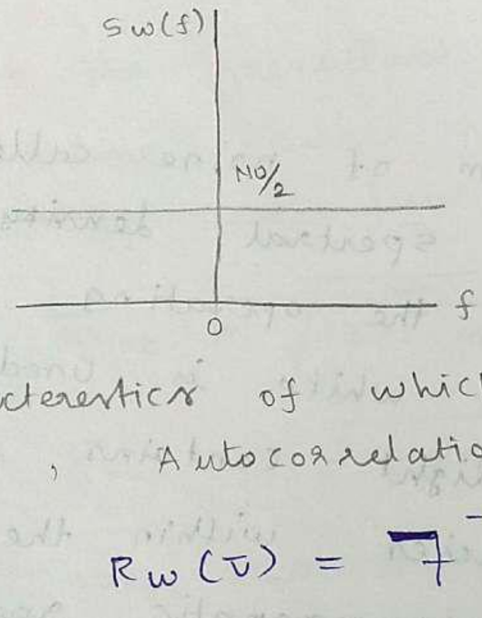
$$N_0 = kT_e$$

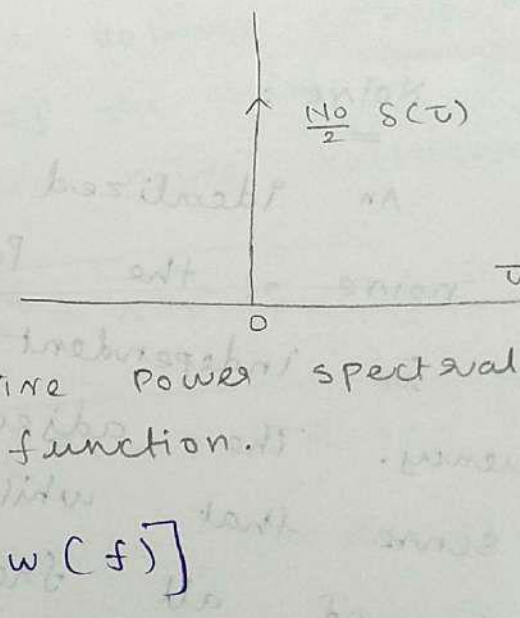
$k \Rightarrow$ Boltzmann's constant

$T_e \Rightarrow$ The equivalant noise temparature of the receiver.

The important feature of the equivalant noise temperature is that it depends only on the parameters of the system.

Since the autocorrelation function is the inverse fourier transform of the Power spectral density, it follows that for white noise.





characterestics of which noise power spectral density, Autocorrelation function.

$$R_w(\tau) = \mathcal{F}^{-1}[S_w(f)]$$

$$= \mathcal{F}^{-1}\left[\frac{N_0}{2}\right]$$

$$= \frac{N_0}{2}\mathcal{F}^{-1}[1]$$

$$= \frac{N_0}{2}\,\delta(\tau)$$

The autocorrelation function of white noise consist of a delta function weighted by the factor $\frac{N_0}{2}$ and $\tau = 0$.

$$\frac{T_e}{T} = \frac{T_1}{T} + \frac{T_2}{G_1 T} + \frac{T_3}{G_1 G_2 T} + \cdots$$

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots$$

Where,

$T_1, T_2 \Rightarrow$ equivalant noise temparature for individual network

$G_1, G_2, G_3 \Rightarrow$ available power gain.

Externel Noise:

It is a noise that is generated outside the device or a circuit. The three primary sources of externel noise are atmosphere, extraterrestrial and man-made.

Atmosphoric Noise:

It is naturally occuring electrical disturbances that originate within Earth's atmosphere. It is also known as static electricity that as lightning, sputtering, cracking... magnitude of energy α frequency. frequency above 30 MHz or 50, atmospheric noise is relatively significant.

Extraterrestrial Noise:

Extra terrestrial noise consists of electrical system that originate from Earth's atmosphere and is therefore sometimes called deep space noise. It originate from milky way, other galaxies and sun.

It is of two types.

solar Noise :

It is generated directly from sum heat. There are 2 parts to solar noise. a quiet condition and sporadic disturbance caused by sun spot activity and solar flare

cosmic noise :

They are continiously distributed throughout the galaxies cosmic noise is after called as black body and is distributed fairly throught the sky.

Man made noise :

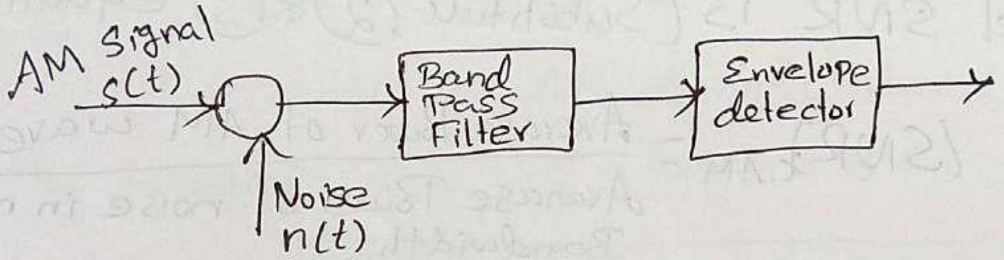
man made noise is simply noise that is produced by mankind the predominant sources of manmade noise are spark producing mechanisms such as commutators in electric motors, automobile ignition systems, ac power generating and switching equipment and flurosence lights.

(Effect)
Noise Performance in AM System:

- → Conventional AM
- → DSB-SC-AM
- → SSB-SC-AM

Derive the figure of merit for AM Receiver:

→ I would like to perform noise analysis for an AM receiver using an envelope detector.



→ Need to determine the channel signal to noise ratio, and then the output signal to noise ratio.

We know that the Amplitude modulated (AM) wave is

$$s(t) = V_c[1 + K_a m(t)] \cos(2\pi f_c t) \rightarrow (1)$$

$$= V_c \cos 2\pi f_c t + V_c K_a m(t) \cos 2\pi f_c t$$

Aveage Power of AM is
→ need to take RMS (Root mean square)

$$P_S = \left(\frac{V_c}{\sqrt{2}}\right)^2 + \left(\frac{V_c K_a m(t)}{\sqrt{2}}\right)^2$$

$$= \frac{V_c^2}{2} + \frac{V_c^2 K_a^2 P_M}{2}$$

$$\boxed{P_S = \frac{V_c^2(1 + K_a^2 P_M)}{2}} \rightarrow (2)$$

Average Power of noise in the message bandwidth is.

$$P_{nc} = \int_{-\cancel{2}B}^{\cancel{2}B} \frac{N_0}{2}\, df$$

$$= \frac{N_0}{2}\left[B-(-B)\right] = \frac{N_0}{\cancel{2}}\cancel{4}B$$

$$\boxed{P_{nc} = \cancel{2}N_0 B} \longrightarrow 3$$

Channel SNR is (Substitute 2 & 3 equations)

$$(SNR)_{C,AM} = \frac{\text{Average Power of AM wave}}{\text{Average Power of noise in message Bandwidth.}}$$

$$\boxed{(SNR)_{C,AM} = \frac{V_c^2(1+k_a^2 P_M)}{2 N_0 B}} \longrightarrow 4$$

$P_M \rightarrow$ Power of the message signal $= \frac{V_m^2}{2}$

$B \rightarrow$ Bandwidth

Assume the band Pass noise is mixed with AM wave in the channel shown in the above figure.

This Combination is applied at the input of AM demodulator.

Hence, the input of AM demodulator is

$$V(t) = S(t) + n(t) \longrightarrow 5$$

$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q \sin 2\pi f_c t$$

$$V(t) = V_c[1 + K_a m(t)] \cos(2\pi f_c t) + n_I(t) \cos(2\pi f_c t)$$
$$- n_Q(t) \sin(2\pi f_c t)$$

$$V(t) = [V_c + V_c K_a m(t) + n_I(t)] \cos(2\pi f_c t)$$
$$- n_Q(t) \sin 2\pi f_c t$$

$n_I(t) \rightarrow$ In Phase component of noise

$n_Q(t) \rightarrow$ Quardrature phase components of noise

The output of AM demodulator is nothing but the envelope of the above signal.

$$d(t) = \sqrt{[V_c + V_c K_a m(t) + n_I(t)]^2 + n_Q(t)^2}$$

$$d(t) = V_c + V_c K_a m(t) + n_I(t)$$

Average ~~signa~~ Power of the demodulated signal is.

$$P_m = \left(\frac{V_c K_a m(t)}{\sqrt{2}}\right)^2 = \frac{V_c^2 K_a^2 P}{2} \rightarrow (6)$$

Average power of noise at the output is.

$$P_n = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} P_{no}$$

$$P_{no} = \int_{-2B}^{2B} \frac{N_0}{2} df = \frac{N_0}{2}[+2B - (-2B)] = \frac{N_0}{\cancel{2}} \times \cancel{4}^{2} B$$

$$P_{no} = 2 N_0 B$$

$$P_n = \frac{\cancel{2} N_0 B}{\cancel{2}}$$

$$\boxed{P_n = N_0 B} \rightarrow (7)$$

Output SNR is (Substitute 6 & 7 equations)

$$(SNR)_{o,AM} = \frac{\text{Average Power of demodulated signal}}{\text{Average Power of noise at Output}}$$

$$\boxed{(SNR)_{o,AM} = \frac{V_c^2 K_a^2 P}{2 B N_0}} \longrightarrow 8$$

Substitute 4th & 8th equation in Figure of merit of AM receiver.

$$F = \frac{V_c^2 K_a^2 P}{2 N_0 B} \Big/ \frac{V_c^2[1+K_a^2 P]}{2 N_0 B}$$

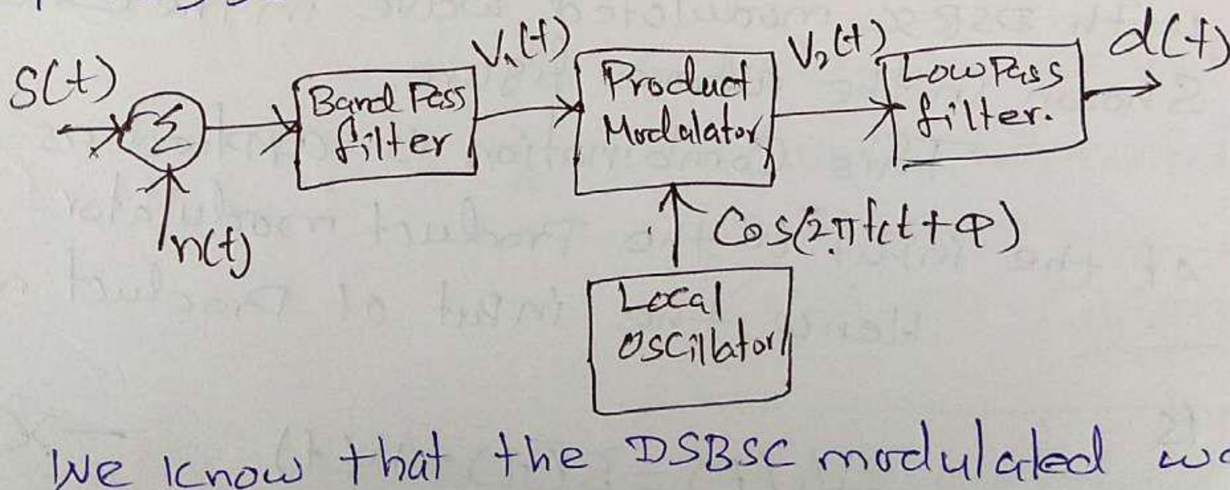
$$= \frac{\cancel{V_c^2} K_a^2 P}{2 \cancel{N_0 B}} \times \frac{\cancel{2 N_0 B}}{\cancel{V_c^2}[1+K_a^2 P]}$$

$$\boxed{F = \frac{K_a P^2}{1+K_a^2 P}}$$

Therefore the Figure of Merit of AM receiver is less than one.

Derive the Figure of Merit for DSB-SC AM receiver (or) Noise Performance in DSB-SC AM receiver (Effect)

→ Consider the following receiver model of DSB-SC system to analyse noise.



We know that the DSBSC modulated wave is

$$s(t) = V_c m(t) \cos 2\pi f_c t \longrightarrow ①$$

Average power of DSB-SC-AM modulated wave is

$$P_s = \left(\frac{V_c m(t)}{\sqrt{2}}\right)^2 = \frac{V_c^2 P}{2} \longrightarrow ②$$

Average Power of DSB-SC modulated wave is

$$P_{nc} = \int_{-B}^{B} \frac{N_0}{2}\, df$$

$$= \frac{N_0}{2}\left[B - (-B)\right] = \frac{N_0}{\cancel{2}}\, \cancel{2}B$$

$$\boxed{P_{nc} = N_0 B} \longrightarrow (3)$$

Substitute 2nd and 3rd equation in Channel SNR.

$$(SNR)_{C\text{-}DSBSC} = \frac{\text{Average Power of DSBSC modulated wave}}{\text{Average Power of noise in message bandwidth}}$$

$$\boxed{(SNR)_{C\text{-}DSBSC} = \frac{V_c^2 P}{2 N_0 B}} \longrightarrow (4)$$

Assumbe the band pass noise is mixed with DSB-SC modulated wave in the channel as shown in the above figure.

This Combination is applied as one of the input to the product modulator.

Hence the input of product modulator is.

$$V_1(t) = s(t) + n(t) \longrightarrow (5)$$

$$V_1(t) = V_c m(t) \cos 2\pi f_c t + n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \longrightarrow (6)$$

Consider ~~envelope d~~

Coherent or synchronous detector.

→ Local oscillator generates the carrier signal $c(t) = \cos(2\pi f_c t + \phi)$ → (7)

→ This signal is applied as another input to the product modulator.

→ Therefore, the product modulator produces an output, which is the product of $v_1(t)$ and $c(t)$

$$V_2(t) = V_1(t)\, c(t) \quad \rightarrow (8)$$

Substitute 6^{th} and 7^{th} equation in 8^{th} equation

$$V_2(t) = \left[V_c m(t) \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t\right]\left[\cos(2\pi f_c t + \phi)\right]$$

$$= V_c m(t) \cos(2\pi f_c t + \phi) * \cos(2\pi f_c t)$$

$$+ n_I(t) \cos(2\pi f_c t + \phi) * \cos(2\pi f_c t)$$

$$- n_Q(t) \cos(2\pi f_c t + \phi) * \sin 2\pi f_c t$$

$$\boxed{\cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}}$$

$$= \left[V_c m(t) + n_I(t)\right] \cos(2\pi f_c t + \phi) \cos 2\pi f_c t$$

$$- n_Q(t) \cos(2\pi f_c t + \phi) \sin 2\pi f_c t$$

$$= \left[\frac{V_c m(t) + n_I(t)}{2}\right]\left[\cos(4\pi f_c t + \phi) + \cos\phi\right]$$

$$- \frac{n_Q(t)}{2}\left[\sin(4\pi f_c t + \phi) - \sin\phi\right]$$

$$V_2(t) = \left[\frac{V_c m(t) + n_I(t)}{2}\right] \cos(4\pi f_c t + \varphi)$$

$$+ \left[\frac{V_c m(t) + n_I(t)}{2}\right] \cos\varphi$$

$$- n_Q(t) \sin(2\pi f_c(t) + \varphi) + n_Q(t) \sin\varphi \rightarrow 9$$

This signal is applied to Low Pass filter

Low Pass filter rejects the double frequency Components and Passes only the low Pass Components.

$$d(t) = \frac{V_c m(t) + n_I(t)}{2} \cos\varphi + n_Q(t) \sin\varphi$$

Apply $\varphi = 0$

$$\boxed{d(t) = \frac{V_c m(t)}{2} + \frac{n_I(t)}{2}} \rightarrow 10$$

Avarage Power of demodulated signal

$$= \left(\frac{V_c m(t)/\cancel{2}}{2\sqrt{2}}\right)^2 + \frac{\cancel{(1/\sqrt{2})^2 n_I(t)}}{2}$$

$$\boxed{P_S = \frac{V_c^2 P}{8}} \rightarrow 11$$

Average noise Power:-

$$P_{no} = \left(\frac{1/2}{\sqrt{2}\,2}\right)^2 n_I(t) = \frac{1}{8} P_{no}$$

$$P_n = \int_{-2B}^{2B} \frac{N_0}{2}\, df = \frac{N_0}{2}\left[2B-(-2B)\right]$$

$$P_n = \frac{N_0}{2} \times 4B = 2N_0B$$

$$= \frac{1}{\cancel{8}_{4}} \times \cancel{2}N_0B$$

$$\boxed{P_{no} = \frac{N_0B}{4}} \rightarrow 12$$

Substitute 11[th] & 12[th] equations in Output SNR

$$(SNR)_{o\,DSBSC} = \frac{\text{Average Power of demodulation signal}}{\text{Average Power of noise at Output}}$$

$$= \frac{V_c^2P}{8} \Big/ \frac{N_0B}{4}$$

$$= \frac{V_c^2P}{\cancel{8}_{2}} \times \frac{\cancel{4}}{N_0B}$$

$$\boxed{(SNR)_{o\,DSB\text{-}SC} = \frac{V_c^2P}{2N_0B}} \rightarrow 13$$

Substitute 4th & 13th equation in figure of Merit of DSB-SC

$$F = \frac{(SNR)_{o\,DSB\text{-}SC}}{(SNR)_{C\,DSB\text{-}SC}}$$

$$= \frac{V_c^2 P}{2 N_0 B} \Big/ \frac{V_c^2 P}{2 N_0 B}$$

$$= \frac{\cancel{V_c^2 P}}{\cancel{2 N_0 B}} \times \frac{\cancel{2 N_0 B}}{\cancel{V_c^2 P}}$$

$$\boxed{F = 1}$$

Therefore, the Figure of merit of DSB-SC receiver is 1.

Noise Performance in SSBSC AM using coherent detection.

Let the SSBSC signal is written as,

$$s(t) = A_c\, m(t) \cos 2\pi f_c t + A_c\, m(t) \sin 2\pi f_c t$$

the noise be $n(t)$

$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

The signal Power at the channel is

$$P_{SC} = \frac{A_c^2 P_m}{2} + \frac{A_c^2 P_m}{2} = A_c^2 P_m$$

The Noise Power at the channel is,

$$P_{NC} = P_n = \int_{-W}^{W} S_N(f)\, df$$

$$= \int_{-W}^{W} \frac{N_0}{2}\, df = \frac{N_0}{2}(2W)$$

$$= N_0 W$$

The signal to noise ratio at the channel is

$$\left(\frac{S}{N}\right)_c = \frac{P_{SC}}{P_{NC}}$$

$$= \frac{A_c^2 P_m}{N_0 W}$$

The output of the BPF is,

$$x(t) = s(t) + n(t)$$

$$= \left[A_c\, m(t) \cos 2\pi f_c t + A_c\, m(t) \sin 2\pi f_c t\right] + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

$$= [A_c m(t) + n_I(t)] \cos 2\pi f_c t + [A_c m(t) - n_Q(t)] \sin 2\pi f_c t$$

The output of the product modulator is

$$v(t) = x(t) \cos 2\pi f_c t$$

$$= [(A_c m(t) + n_I(t)) \cos 2\pi f_c t + (A_c m(t) - n_Q(t)) \sin 2\pi f_c t] \cos 2\pi f_c t$$

$$= [A_c m(t) + n_I(t)] \left(\frac{1 + \cos 4\pi f_c t}{2}\right) + [A_c m(t) - n_Q(t)] \left(\frac{\sin 4\pi f_c t}{2}\right)$$

The output of the LPF is

$$y(t) = \frac{A_c m(t) + n_I(t)}{2}$$

The signal power at the output is

$$P_{SO} = \frac{A_c^2}{4} P_m$$

The noise power at the output is

$$P_{no} = \frac{1}{2} P_n$$

$$P_n = \int_{-W/2}^{W/2} S_N(f)\, df$$

$$= \int_{-W/2}^{W/2} \frac{N_0}{2}\, df$$

$$= \frac{N_0}{2} [W] = \frac{N_0 W}{2}$$

$$P_{no} = \frac{N_0 W}{4}$$

$$\left(\frac{S}{N}\right)_0 = \frac{P_{SO}}{P_{no}}$$

$$\left(\frac{S}{N}\right)_0 = \frac{A_c^2 P_m / 4}{N_0 W / 4}$$

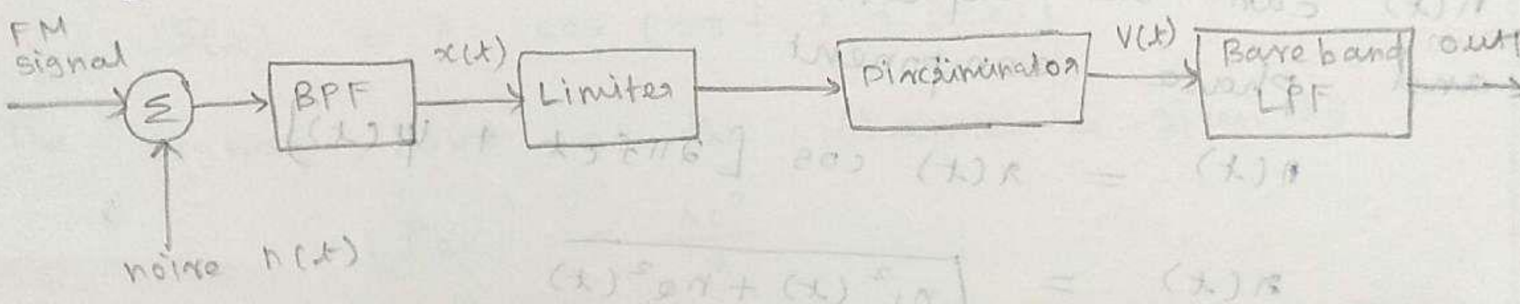
$$= \frac{A_c^2 P_m}{N_0 W}$$

$$\text{Figure of merit} = \frac{\left(\frac{S}{N}\right)_0}{\left(\frac{S}{N}\right)_c}$$

$$= \frac{A_c^2 P_m / N_0 W}{A_c^2 P_m / N_0 W}$$

$$= 1$$

Noise in FM receiver:



The output of the limiter is then fed to the discriminator which consists of two components.

1). A slope network or differentiator with a purely imaginary frequency response that varies linearly with frequency.

2) An envelope detector that recovers the amplitude variation and thus reproducing the message signal.

The output of the discriminator is then fed to the post detection filter or base-band low pass filter which is used to remove the

the out of band components of the noise of the discriminator output and thereby keeps the effect of output noise to a minimum.

Consider a narrow band noise n(t) which consist of inphase component and quadrature component.

$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

n(t) can be expressed in terms of envelope and phase component

$$n(t) = r(t) \cos [2\pi f_c t + \psi(t)]$$

$$r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$$

and phase $\psi(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right)$

The noise power at the channel is given by,

$$P_{NC} = P_n$$

$$P_n = \int_{-W}^{W} S_N(f)\, df$$

$$S_N(f) = \frac{N_0}{2}$$

$$= \int_{-W}^{W} \frac{N_0}{2}\, df$$

$$= \frac{N_0}{2} [f]_{-W}^{W} = \frac{N_0}{2} [2W]$$

$$P_{NC} = N_0 W$$

The incoming signal $s(t)$ is defined by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

Where, A_c is the carrier amplitude
f_c is the carrier frequency
k_f is the frequency sensitivity
$m(t)$ is a message signal

$$\phi(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

The $s(t)$ can be expressed in simple form

$$s(t) = A_c \cos\left(2\pi f_c t + \phi(t)\right]$$

The signal power at the channel is given by

$$P_{SC} = \frac{A_c^2}{2}$$

The signal to noise ratio of the channel is given by,

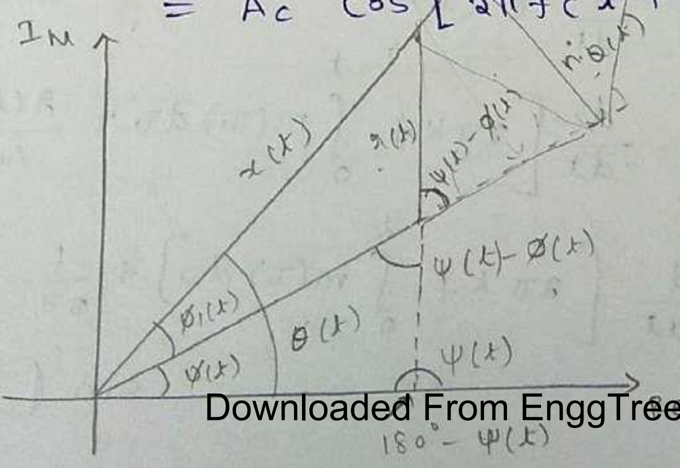
$$\left(\frac{S}{N}\right)_C = \frac{P_{SC}}{P_{NC}}$$

$$\left(\frac{S}{N}\right)_C = \frac{A_c^2}{2N_0W}$$

The noisy signal at the bandpass filter output is,

$$x(t) = s(t) + n(t)$$

$$= A_c \cos\left[2\pi f_c t + \phi(t)\right] + r(t) \cos\left[2\pi f_c t + \psi(t)\right]$$



$$\sin(\psi(t) - \phi(t)) = \frac{n_Q(t)}{r(t)}$$

$$n_Q(t) = r(t)\sin(\psi(t) - \phi(t)).$$

$$\cos(\psi(t) - \phi(t)) = \frac{n_I(t)}{r(t)}$$

$$n_I(t) = r(t)\cos(\psi(t) - \phi(t))$$

$$x(t) = A_c + n_I(t) = A_c + r(t)\cos(\psi(t) - \phi(t))$$

$$\phi_1(t) = \tan^{-1}\left(\frac{n_Q(t)}{A_c + n_I(t)}\right)$$

$$\phi_1(t) = \tan^{-1}\left[\frac{r(t)\sin[\psi(t) - \phi(t)]}{A_c + r(t)\cos(\psi(t) - \phi(t))}\right]$$

From the phase diagram,

$$\theta(t) = \phi(t) + \phi_1(t)$$

$$= \phi(t) + \tan^{-1}\left[\frac{r(t)\sin(\psi(t) - \phi(t))}{A_c + r(t)\cos[\psi(t) - \phi(t)]}\right]$$

$$\theta(t) \simeq \phi(t) + \frac{r(t)}{A_c}\sin[\psi(t) - \phi(t)]$$

$$\theta(t) \simeq 2\pi k_f \int_0^t m(\tau)\,d\tau + \frac{r(t)}{A_c}\sin[\psi(t) - \phi(t)]$$

The output of the discriminator is,

$$v(t) = \frac{1}{2\pi}\theta'(t)$$

$$= \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f \int_0^t m(\tau)\,d\tau + \frac{r(t)}{A_c}\sin(\psi(t) - \phi(t))\right]$$

$$\simeq \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f \int_0^t m(\tau)\,d\tau\right] + \frac{1}{2\pi}\frac{d}{dt}\left[\frac{r(t)}{A_c}\sin[\psi(t) - \phi(t)]\right]$$

$$\simeq \frac{2\pi k_f}{2\pi} \frac{d}{dt}\left[\int_0^t m(\tau)\,d\tau\right] + \frac{1}{2\pi A_c}\frac{d}{dt}\left[r(t)\sin[\psi(t)-\phi(t)\right.$$

$$\simeq k_f\, m(t) + n_d(t)$$

Where the noise term $n_d(t)$ is defined by,

$$n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}\left[r(t)\sin\left[\psi(t)-\phi(t)\right]\right]$$

$$n_d(t) \simeq \frac{1}{2\pi A_c}\frac{d}{dt}\left[r(t)\sin\psi(t)\right]$$

The quadrature component $n_Q(t)$ of the filtered noise is

$$n_Q(t) = r(t)\sin\psi(t)$$

$$\therefore n_d(t) \simeq \frac{1}{2\pi A_c}\frac{d}{dt}\left[r(t)\sin\psi(t)\right]$$

$$\simeq \frac{1}{2\pi A_c} j2\pi f\left[r(t)\sin\psi(t)\right]$$

$$= \frac{jf}{A_c}\left[r(t)\sin\psi(t)\right]$$

Thus the additive noise $n_d(t)$ appearing at the discriminator output is determined effectively by the carrier amplitude A_c and the quadrature component $n_Q(t)$ of the narrowband noise $n(t)$.

The signal appearing at the output of the low pass filter is same as the discriminator output $V(t)$.

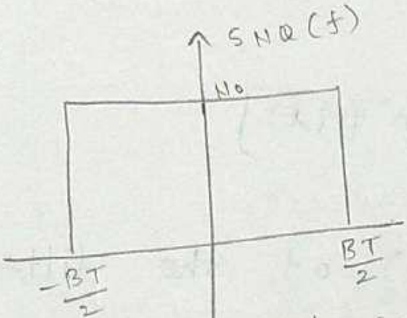
The signal Power at the output is given by

$$P_{so} = k_f^2\, P_m$$

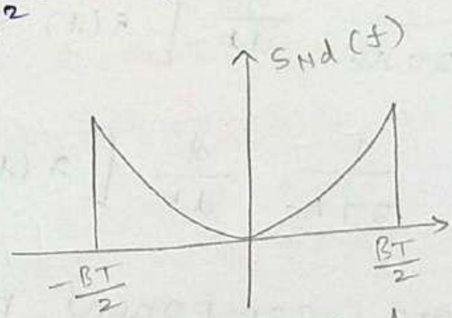
The Power Spectral density of $n_d(t)$ is given by

$$S_{Nd}(f) = \frac{|jf|^2}{|A_c|^2} S_{NQ}(f)$$

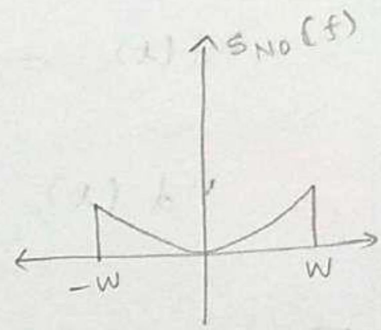
$$= \frac{f^2}{A_c^2} S_{NQ}(f)$$



Power spectral density of quadrature component of narrow band noise



Power spectral density of noise $n_d(t)$ at the discriminator output



Power spectral density of noise $n_o(t)$ at the receiver output

$$S_{NQ}(f) = \begin{cases} N_o & ; -\frac{BT}{2} \le f \le \frac{BT}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

$$S_{Nd}(f) = \begin{cases} \frac{N_o f^2}{A_c^2} & ; -\frac{BT}{2} \le f \le \frac{BT}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

The outband components of noise $n_d(t)$ will be removed.

Therefore, the Power Spectral density $S_{No}(f)$ appearing at the receiver output is defined by

$$S_{No}(f) = \begin{cases} \frac{N_o f^2}{A_c^2} & ; -W \le f \le W \\ 0 & ; \text{otherwise} \end{cases}$$

The avarage noise Power at the o/p is defined by

$$P_{NO} = \int_{-W}^{+W} S_{No}(f)\, df$$

Capture Effect :-

The inherent ability of an FM system to minimize the effects of unwanted signals also applies to interference produced by another frequency modulated signal whose frequency content is close to the carrier frequency of the desired FM wave.

However, interference suppression in an FM receiver works well only when the interference is weaker than the desired FM input.

When the interference is the stronger one of the two, the receiver locks onto the stronger signal and thereby suppresses the desired FM input.

When they are of nearly equal strength, the receiver fluctuates back and forth between them.

This phenomenon is known as the capture effect.