ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT I – FUNDAMENTAL CONCEPTS IN DESIGN

1.1 INTRODUCTION TO DESIGN PROCESS

Introduction

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Classifications of Machine Design

The machine design may be classified as follows:

1. *Adaptive design.* In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. *Development design.* This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. *New design.* This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

(*a*) *Rational design.* This type of design depends upon mathematical formulae of principle of mechanics.

(*b*) *Empirical design.* This type of design depends upon empirical formulae based on the practice and past experience.

(*c*) ***Industrial design.*** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(*d*) ***Optimum design.*** It is the best design for the given objective function under the specified constraints. It may be achieved by minimising the undesirable effects.

(*e*) ***System design.*** It is the design of any complex mechanical system like a motor car.

(*f*) ***Element design.*** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(*g*) ***Computer aided design.*** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimisation of a design.

General Considerations in Machine Design

Following are the general considerations in designing a machine component:

1. ***Type of load and stresses caused by the load.*** The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed later.

2. ***Motion of the parts or kinematics of the machine.*** The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required.

The motion of the parts may be:

(*a*) Rectilinear motion which includes unidirectional and reciprocating motions.

(*b*) Curvilinear motion which includes rotary, oscillatory and simple harmonic.

(*c*) Constant velocity.

(*d*) Constant or variable acceleration.

3. ***Selection of materials.*** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed later.

4. ***Form and size of the parts.*** The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. ***Frictional resistance and lubrication.*** There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. ***Convenient and economical features.*** In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. ***Use of standard parts.*** The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. ***Safety of operation.*** Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. ***Workshop facilities.*** A design engineer should be familiar with the limitations of this employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. ***Number of machines to be manufactured.*** The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. *Cost of construction.* The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.

12. *Assembling.* Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

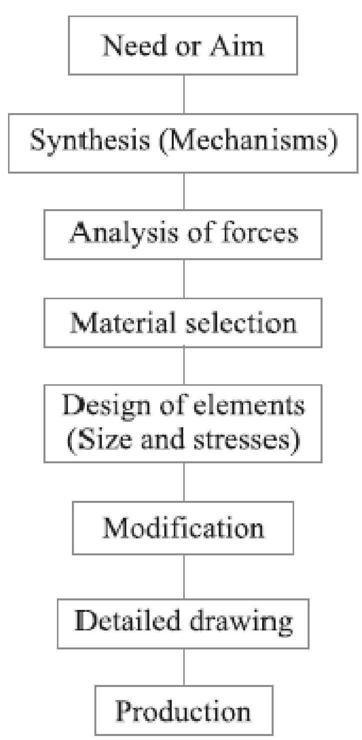

Fig.1. General Machine Design Procedure

1. ***Recognition of need.*** First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. ***Synthesis*** **(*Mechanisms*).** Select the possible mechanism or group of mechanisms which will give the desired motion.

3. ***Analysis of forces.*** Find the forces acting on each member of the machine and the energy transmitted by each member.

4. ***Material selection.*** Select the material best suited for each member of the machine.

5. ***Design of elements*** **(*Size and Stresses*).** Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. ***Modification.*** Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. ***Detailed drawing.*** Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. ***Production.*** The component, as per the drawing, is manufactured in the workshop.
The flow chart for the general procedure in machine design is shown in Fig.

Note: When there are number of components in the market having the same qualities of efficiency, durability and cost, then the customer will naturally attract towards the most appealing product. The aesthetic and ergonomics are very important features which gives grace and lustre to product and dominates the market.

Engineering materials and their properties

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

Classification of Engineering Materials

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.

2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

(a) Ferrous metals and (*b*) Non-ferrous metals.

The ******ferrous metals*** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The ***non-ferrous*** metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,

2. Suitability of the materials for the working conditions in service, and

3. The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. Strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2. Stiffness. It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. Elasticity. It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

7. Malleability. It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. Toughness. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers and turbines.

12. Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue**. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

(a) Brinell hardness test,

(b) Rockwell hardness test,

(c) Vickers hardness (also called Diamond Pyramid) test, and

(d) Shore scleroscope.

Steel

It is an alloy of iron and carbon, with carbon content up to a maximum of 1.5%. The carbon occurs in the form of iron carbide, because of its ability to increase the hardness and strength of the steel. Other elements *e.g.* silicon, sulphur, phosphorus and manganese are also present to greater or lesser amount to impart certain desired properties to it. Most of the steel produced now-a-days is ***plain carbon steel*** or simply ***carbon steel*.** A carbon steel is defined as a steel which has its properties mainly due to its carbon content and does not contain more than 0.5% of silicon and 1.5% of manganese.

The plain carbon steels varying from 0.06% carbon to 1.5% carbon are divided into the following types depending upon the carbon content.

1. Dead mild steel — up to 0.15% carbon

2. Low carbon or mild steel — 0.15% to 0.45% carbon

3. Medium carbon steel — 0.45% to 0.8% carbon

4. High carbon steel — 0.8% to 1.5% carbon

According to Indian standard *[IS: 1762 (Part-I)–1974], a new system of designating the steel is recommended. According to this standard, steels are designated on the following two basis: (*a*) On the basis of mechanical properties, and (*b*) On the basis of chemical composition. We shall now discuss, in detail, the designation of steel on the above two basis, in the following pages.

Steels Designated on the Basis of Mechanical Properties

These steels are carbon and low alloy steels where the main criterion in the selection and inspection of steel is the tensile strength or yield stress. According to Indian standard IS: 1570 (Part–I)- 1978 (Reaffirmed 1993), these steels are designated by a symbol 'Fe' or 'Fe E' depending on whether the steel has been specified on the basis of minimum tensile strength or yield strength, followed by the figure indicating the minimum tensile strength or yield stress in N/mm2. For example 'Fe 290' means a steel having minimum tensile strength of 290 N/mm2 and 'Fe E 220' means a steel having yield strength of 220 N/mm2.

Steels Designated on the Basis of Chemical Composition

According to Indian standard, IS : 1570 (Part II/Sec I)-1979 (Reaffirmed 1991), the carbon steels are designated in the following order :

(*a*) Figure indicating 100 times the average percentage of carbon content,

(*b*) Letter 'C', and

(*c*) Figure indicating 10 times the average percentage of manganese content. The figure after multiplying shall be rounded off to the nearest integer.

For example 20C8 means a carbon steel containing 0.15 to 0.25 per cent (0.2 per cent on average) carbon and 0.60 to 0.90 per cent (0.75 per cent rounded off to 0.8 per cent on an average) manganese.

Effect of Impurities on Steel

The following are the effects of impurities like silicon, sulphur, manganese and phosphorus on steel.

1. *Silicon.* The amount of silicon in the finished steel usually ranges from 0.05 to 0.30%. Silicon is added in low carbon steels to prevent them from becoming porous. It removes the gases and oxides, prevent blow holes and thereby makes the steel tougher and harder.

2. *Sulphur.* It occurs in steel either as iron sulphide or manganese sulphide. Iron sulphide because of its low melting point produces red shortness, whereas manganese sulphide does not affect so much. Therefore, manganese sulphide is less objectionable in steel than iron sulphide.

3. *Manganese*. It serves as a valuable deoxidising and purifying agent in steel. Manganese also combines with sulphur and thereby decreases the harmful effect of this element remaining in the steel. When used in ordinary low carbon steels, manganese makes the metal ductile and of good bending qualities. In high speed steels, it is used to toughen the metal and to increase its critical temperature.

4. *Phosphorus.* It makes the steel brittle. It also produces cold shortness in steel. In low carbon steels, it raises the yield point and improves the resistance to atmospheric corrosion. The sum of carbon and phosphorus usually does not exceed 0.25%.

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UNIT I – FUNDAMENTAL CONCEPTS IN DESIGN

1.2 MANUFACTURING CONSIDERATIONS IN MACHINE DESIGN

Manufacturing Processes

The knowledge of manufacturing processes is of great importance for a design engineer. The following are the various manufacturing processes used in Mechanical Engineering.

1. *Primary shaping processes*. The processes used for the preliminary shaping of the machine component are known as primary shaping processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, powder metal forming, squeezing, etc.

2. *Machining processes*. The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, grinding, hobbing, etc.

3. *Surface finishing processes*. The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, super finishing, sheradizing, etc.

4. *Joining processes*. The processes used for joining machine components are known as joining processes. The common operations used for this process are welding, riveting, soldering, brazing, screw fastening, pressing, sintering, etc.

5. *Processes effecting change in properties*. These processes are used to impart certain specific properties to the machine components so as to make them suitable for particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot peening.

Other considerations in Machine design

1. Workshop facilities.
2. Number of machines to be manufactured
3. Cost of construction

4. Assembling

Interchangeability

The term interchangeability is normally employed for the mass production of identical items within the prescribed limits of sizes. A little consideration will show that in order to maintain the sizes of the part within a close degree of accuracy, a lot of time is required. But even then there will be small variations. If the variations are within certain limits, all parts of equivalent size will be equally fit for operating in machines and mechanisms. Therefore, certain variations are recognized and allowed in the sizes of the mating parts to give the required fitting. This facilitates to select at random from a large number of parts for an assembly and results in a considerable saving in the cost of production.

In order to control the size of finished part, with due allowance for error, for interchangeable parts is called ***limit system*.** It may be noted that when an assembly is made of two parts, the part which enters into the other, is known as ***enveloped surface*** (or **shaft** for cylindrical part) and the other in which one enters is called ***enveloping surface*** (or **hole** for cylindrical part). The term ***shaft*** refers not only to the diameter of a circular shaft, but it is also used to designate any external dimension of a part. The term ***hole*** refers not only to the diameter of a circular hole, but it is also used to designate any internal dimension of a part.

Important Terms used in Limit System

The following terms used in limit system (or interchangeable system) are important from the subject point of view:

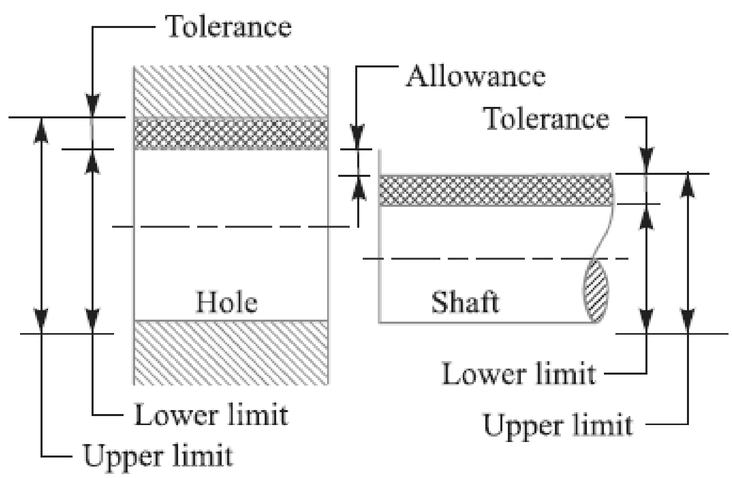


Fig. Limits of sizes.

1. Nominal size. It is the size of a part specified in the drawing as a matter of convenience.

2. Basic size. It is the size of a part to which all limits of variation (i.e. tolerances) are applied to arrive at final dimensioning of the mating parts. The nominal or basic size of a part is often the same.

3. Actual size. It is the actual measured dimension of the part. The difference between the basic size and the actual size should not exceed a certain limit; otherwise it will interfere with the interchangeability of the mating parts.

4. Limits of sizes. There are two extreme permissible sizes for a dimension of the part as shown in Fig. The largest permissible size for a dimension of the part is called **upper** or **high** or **maximum limit,** whereas the smallest size of the part is known as **lower** or **minimum limit.**

5. Allowance. It is the difference between the basic dimensions of the mating parts. The allowance may be **positive** or **negative**. When the shaft size is less than the hole size, then the allowance is **positive** and when the shaft size is greater than the hole size, then the allowance is **negative.**

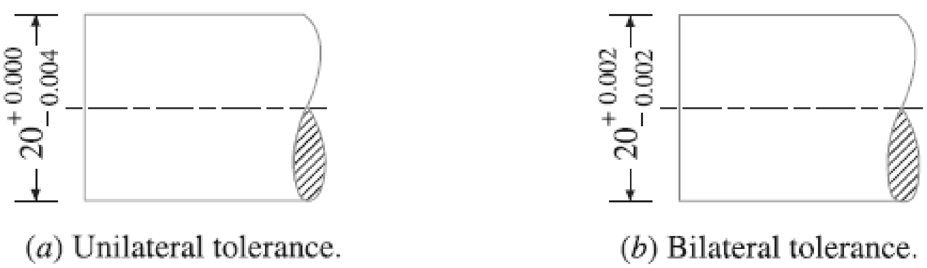


Fig. Method of assigning Tolerances

6. Tolerance zone. It is the zone between the maximum and minimum limit size.

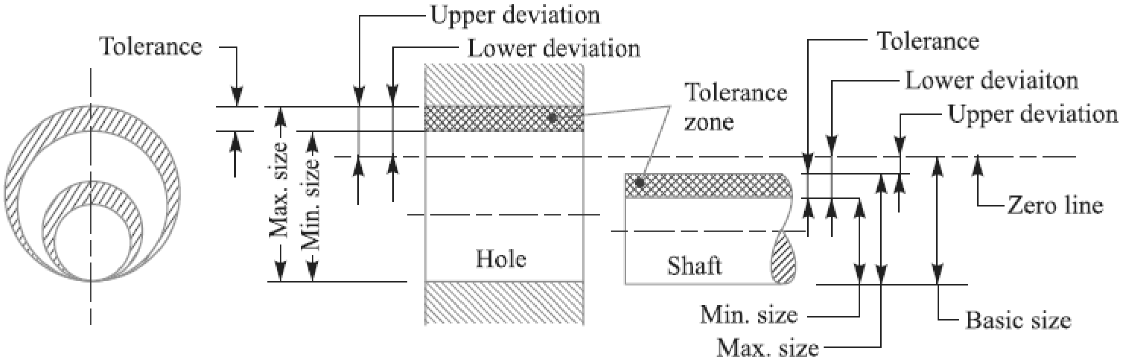


Fig. Tolerance Zone

7. Zero line. It is a straight line corresponding to the basic size. The deviations are measured from this line. The positive and negative deviations are shown above and below the zero line respectively.

8. Upper deviation. It is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol ES (Ecart Superior) and of a shaft, it is represented by es.

9. Lower deviation. It is the algebraic difference between the minimum size and the basic size. The lower deviation of a hole is represented by a symbol EI (Ecart Inferior) and of a shaft, it is represented by ei.

10. Actual deviation. It is the algebraic difference between an actual corresponding basic size.

11. Mean deviation. It is the arithmetical mean between the upper and lower deviations.

12. Fundamental deviation. It is one of the two deviations which are conventionally chosen to define the position of the tolerance zone in relation to zero line, as shown in Fig.

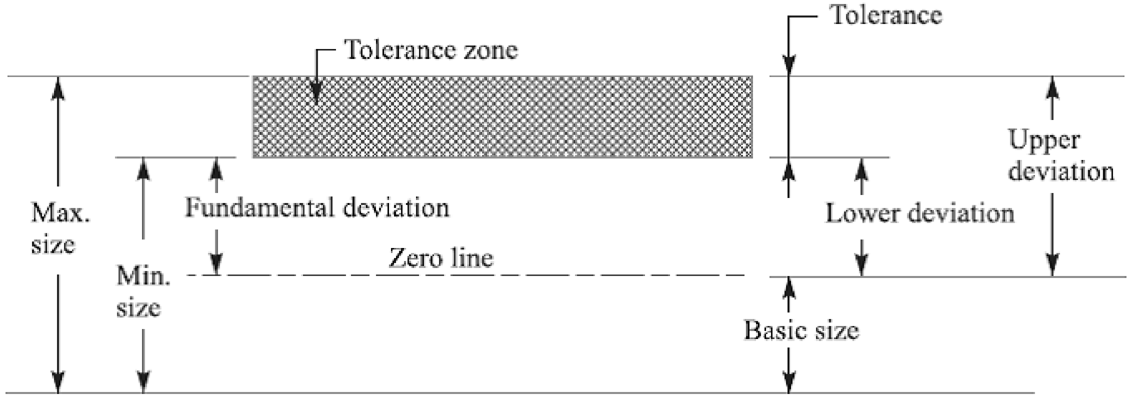


Fig. Fundamental deviation.

Fits : The degree of tightness or looseness between the two mating parts is known as a *fit* of the parts. The nature of fit is characterized by the presence and size of clearance and interference. The ***clearance*** is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assembly as shown in Fig. 3.5 (*a*). In other words, the clearance is the difference between the sizes of the hole and the shaft before assembly. The difference must be ***positive.***

The ***clearance*** is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assembly as shown in Fig. (*a*). In other words, the clearance is the difference between the sizes of the hole and the shaft before assembly. The difference must be ***positive.***

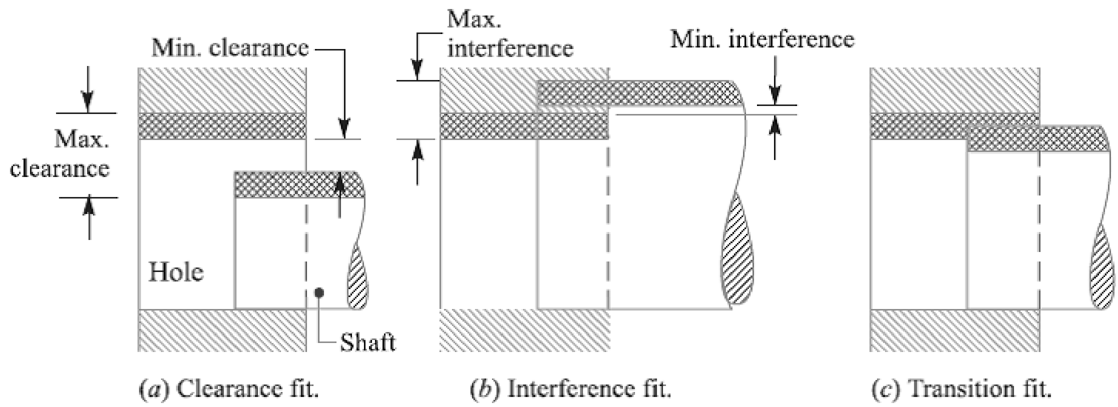


Fig. Types of fits.

The ***interference*** is the amount by which the actual size of a shaft is larger than the actual finished size of the mating hole in an assembly as shown in Fig. (*b*). In other words, the

interference is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference must be ***negative.***

Types of Fits

According to Indian standards, the fits are classified into the following three groups:

1. *Clearance fit.* In this type of fit, the size limits for mating parts are so selected that clearance between them always occur, as shown in Fig. (*a*). It may be noted that in a clearance fit, the tolerance zone of the hole is entirely above the tolerance zone of the shaft. In a clearance fit, the difference between the minimum size of the hole and the maximum size of the shaft is known as ***minimum clearance*** whereas the difference between the maximum size of the hole and minimum size of the shaft is called ***maximum clearance*** as shown in Fig. (*a*). The clearance fits may be slide fit, easy sliding fit, running fit, slack running fit and loose running fit.

2. *Interference fit.* In this type of fit, the size limits for the mating parts are so selected that interference between them always occur, as shown in Fig. (*b*). It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft. In an interference fit, the difference between the maximum size of the hole and the minimum size of the shaft is known as ***minimum interference,*** whereas the difference between the minimum size of the hole and the maximum size of the shaft is called ***maximum interference,*** as shown in Fig. (*b*). The interference fits may be shrink fit, heavy drive fit and light drive fit.

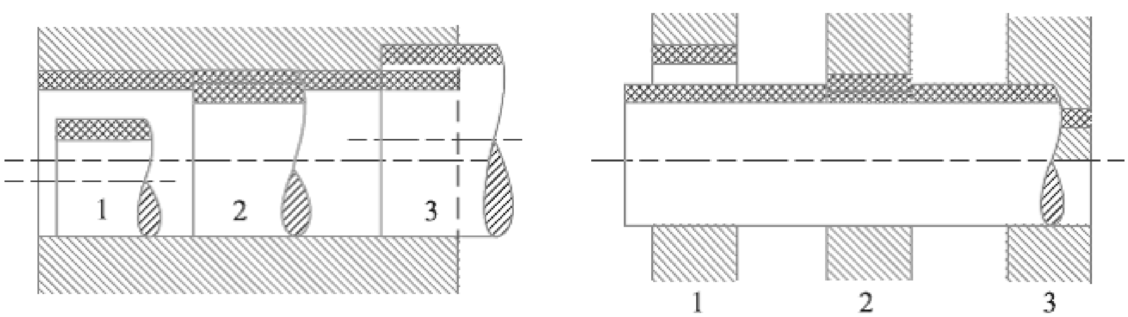
3. *Transition fit.* In this type of fit, the size limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts, as shown in Fig. (*c*). It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap. The transition fits may be force fit, tight fit and push fit.

Basis of Limit System

The following are two bases of limit system:

1. *Hole basis system*. When the hole is kept as a constant member (*i.e.* when the lower deviation of the hole is zero) and different fits are obtained by varying the shaft size, as shown in Fig. (*a*), then the limit system is said to be on a hole basis.

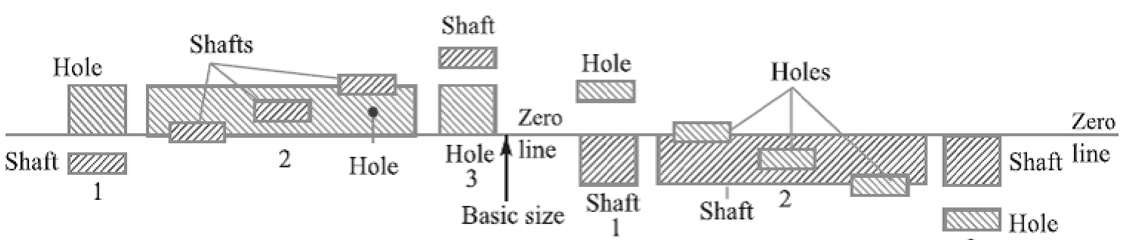
2. *Shaft basis system*. When the shaft is kept as a constant member (*i.e.* when the upper deviation of the shaft is zero) and different fits are obtained by varying the hole size, as shown in Fig.(*b*), Then the limit system is said to be on a shaft basis.



1. Clearance fit. 2. Transition fit. 3. Interference fit.

(*a*) Hole basis system. (*b*) Shaft basis system.

Fig. Bases of Limit System.



1. Clearance fit. 2. Transition fit. 3. Interference fit.

(*a*) Hole basis system. (*b*) Shaft basis system.

Fig. Bases of Limit System

The hole basis and shaft basis system may also be shown as in Fig. with respect to the zero line. It may be noted that from the manufacturing point of view, a hole basis system is always preferred. This is because the holes are usually produced and finished by standard tooling like drill, reamers, etc., whose size is not adjustable easily. On the other hand, the size of the shaft (which is to go into the hole) can be easily adjusted and is obtained by turning or grinding operations.

Problem-1:

The dimensions of the mating parts, according to basic hole system, are given as follows:
Hole : 25.00 mm Shaft : 24.97 mm
25.02 mm 24.95 mm
Find the hole tolerance, shaft tolerance and allowance.

Solution. Given : Lower limit of hole = 25 mm ; Upper limit of hole = 25.02 mm ; Upper limit of shaft = 24.97 mm ; Lower limit of shaft = 24.95 mm

Hole tolerance

We know that hole tolerance

= Upper limit of hole – Lower limit of hole

= 25.02 – 25 = 0.02 mm **Ans.**

Shaft tolerance

We know that shaft tolerance

= Upper limit of shaft – Lower limit of shaft

= 24.97 – 24.95 = 0.02 mm **Ans.**

Allowance

We know that allowance

= Lower limit of hole – Upper limit of shaft

= 25.00 – 24.97 = 0.03 mm **Ans.**

Problem-2:

Calculate the tolerances, fundamental deviations and limits of sizes for the shaft designated as 40 H8 / f7.

Solution. Given: Shaft designation = 40 $H8$ / $f7$

The shaft designation 40 $H8$ / $f7$ means that the basic size is 40 mm and the tolerance grade for the hole is 8 (*i.e.* $IT\,8$) and for the shaft is 7 (*i.e.* $IT\,7$).

Tolerances

Since 40 mm lies in the diameter steps of 30 to 50 mm, therefore the geometric mean diameter,

$$D = \sqrt{30 \times 50} = 38.73 \text{ mm}$$

We know that standard tolerance unit,

$$i = 0.45\sqrt[3]{D} + 0.001\,D$$

$$= 0.45\sqrt[3]{38.73} + 0.001 \times 38.73$$

$$= 0.45 \times 3.38 + 0.03873 = 1.559\,73 \text{ or } 1.56 \text{ microns}$$

$$= 1.56 \times 0.001 = 0.001\,56 \text{ mm} \qquad \text{...}(\because 1 \text{ micron} = 0.001 \text{ mm})$$

From Table 3.2, we find that standard tolerance for the hole of grade 8 ($IT\,8$)

$$= 25\,i = 25 \times 0.001\,56 = 0.039 \text{ mm } \textbf{Ans.}$$

and standard tolerance for the shaft of grade 7 ($IT\,7$)

$$= 16\,i = 16 \times 0.001\,56 = 0.025 \text{ mm } \textbf{Ans.}$$

Fundamental deviation

We know that fundamental deviation (lower deviation) for hole H,

$$EI = 0$$

From Table 3.7, we find that fundamental deviation (upper deviation) for shaft f,

$$es = -5.5\,(D)^{0.41}$$
$$= -5.5\,(38.73)^{0.41} = -24.63 \text{ or } -25 \text{ microns}$$
$$= -25 \times 0.001 = -0.025 \text{ mm } \textbf{Ans.}$$

$\therefore$ Fundamental deviation (lower deviation) for shaft f,

$$ei = es - IT = -0.025 - 0.025 = -0.050 \text{ mm } \textbf{Ans.}$$

The –ve sign indicates that fundamental deviation lies below the zero line.

Limits of sizes

We know that lower limit for hole

= Basic size = 40 mm **Ans.**

Upper limit for hole = Lower limit for hole + Tolerance for hole

= 40 + 0.039 = 40.039 mm **Ans.**

Upper limit for shaft = Lower limit for hole or Basic size – Fundamental deviation (upper deviation) ...($\because$ Shaft f lies below the zero line)

= 40 – 0.025 = 39.975 mm **Ans.**

and lower limit for shaft = Upper limit for shaft – Tolerance for shaft

= 39.975 – 0.025 = 39.95 mm **Ans.**

Problem-3:

A journal of nominal or basic size of 75 mm runs in a bearing with close running fit. Find the limits of shaft and bearing. What is the maximum and minimum clearance?

Solution. Given: Nominal or basic size = 75 mm

From Table 3.5, we find that the close running fit is represented by $H\,8/g\,7$, *i.e.* a shaft g 7 should be used with H 8 hole.

Since 75 mm lies in the diameter steps of 50 to 80 mm, therefore the geometric mean diameter,

$$D = \sqrt{50 \times 80} = 63 \text{ mm}$$

We know that standard tolerance unit,

$$i = 0.45\,\sqrt[3]{D} + 0.001\,D = 0.45\,\sqrt[3]{63} + 0.001 \times 63$$
$$= 1.79 + 0.063 = 1.853 \text{ micron}$$
$$= 1.853 \times 0.001 = 0.001\,853 \text{ mm}$$

$\therefore$ Standard tolerance for hole 'H' of grade 8 (IT 8)

$$= 25\,i = 25 \times 0.001\,853 = 0.046 \text{ mm}$$

and standard tolerance for shaft 'g' of grade 7 (IT 7)

$$= 16\,i = 16 \times 0.001\,853 = 0.03 \text{ mm}$$

From Table 3.7, we find that upper deviation for shaft g,

$$es = -2.5\,(D)^{0.34} = -2.5\,(63)^{0.34} = -10 \text{ micron}$$
$$= -10 \times 0.001 = -0.01 \text{ mm}$$

∴ Lower deviation for shaft *g*,

$$ei = es - IT = -0.01 - 0.03 = -0.04 \text{ mm}$$

We know that lower limit for hole

= Basic size = 75 mm

Upper limit for hole = Lower limit for hole + Tolerance for hole

= 75 + 0.046 = 75.046 mm

Upper limit for shaft = Lower limit for hole – Upper deviation for shaft

...(∵ Shaft *g* lies below zero line)

= 75 – 0.01 = 74.99 mm

and lower limit for shaft = Upper limit for shaft – Tolerance for shaft

= 74.99 – 0.03 = 74.96 mm

We know that maximum clearance

= Upper limit for hole – Lower limit for shaft

= 75.046 – 74.96 = 0.086 mm **Ans.**

and minimum clearance = Lower limit for hole – Upper limit for shaft

= 75 – 74.99 = 0.01 mm **Ans.**

ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT I – FUNDAMENTAL CONCEPTS IN DESIGN

1.4 Impact Stress

Impact Stress

Sometimes, machine members are subjected to the load with impact. The stress produced in the member due to the falling load is known as ***impact stress.*** Consider a bar carrying a load W at a height h and falling on the collar provided at the lower end, as shown in Fig.

Let A = Cross-sectional area of the bar,

E = Young's modulus of the material of the bar,

l = Length of the bar,

δl = Deformation of the bar,

P = Force at which the deflection δl is produced,

σ_i = Stress induced in the bar due to the application of impact load, and

h = Height through which the load falls.

We know that energy gained by the system in the form of strain energy

$$= \frac{1}{2} \times P \times \delta l$$

And potential energy lost by the weight

$$= W(h + \delta l)$$

Since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\frac{1}{2} \times P \times \delta l = W\ (h + \delta l)$$

$$\frac{1}{2}\, \sigma_i \times A \times \frac{\sigma_i \times l}{E} = W\left(h + \frac{\sigma_i \times l}{E}\right) \qquad \ldots\left[\because P = \sigma_i \times A, \text{ and } \delta l = \frac{\sigma_i \times l}{E}\right]$$

$$\therefore \qquad \frac{Al}{2E}(\sigma_i)^2 - \frac{Wl}{E}(\sigma_i) - Wh = 0$$

From this quadratic equation, we find that

$$\sigma_i = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2\,h\,A\,E}{W\,l}}\right) \qquad \ldots \text{[Taking +ve sign for maximum value]}$$

When $h = 0$, then $\sigma_i = 2W/A$. This means that the stress in the bar when the load in applied suddenly is double of the stress induced due to gradually applied load.

Problem:

An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm2 in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take E = 200 kN/mm^2.

Solution. Given : $h = 10$ mm ; $l = 3$ m $= 3000$ mm ; $A = 600\ \text{mm}^2$; $\delta l = 2$ mm ; $E = 200\ \text{kN/mm}^2 = 200 \times 10^3\ \text{N/mm}^2$

Stress in the bar

Let σ = Stress in the bar.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma . l}{\delta l}$$

$$\therefore \quad \sigma = \frac{E . \delta l}{l} = \frac{200 \times 10^3 \times 2}{3000} = \frac{400}{3} = 133.3\ \text{N/mm}^2 \quad \text{Ans.}$$

Value of the unknown weight

Let W = Value of the unknown weight.

We know that
$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2h\,A\,E}{Wl}}\right]$$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}}\right]$$

$$\frac{400 \times 600}{3\,W} = 1 + \sqrt{1 + \frac{800\ 000}{W}}$$

$$\frac{80\ 000}{W} - 1 = \sqrt{1 + \frac{800\ 000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160000}{W} = 1 + \frac{800000}{W}$$

$$\frac{6400 \times 10^2}{W} - 16 = 80 \quad \text{or} \quad \frac{6400 \times 10^2}{W} = 96$$

$$\therefore \quad W = 6400 \times 10^2 / 96 = 6666.7\ \text{N} \quad \text{Ans.}$$

Resilience

When a body is loaded within elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as ***strain energy*****.** The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as ***resilience*** and the maximum energy which can be stored in a body up to the elastic limit is called ***proof resilience****.* The proof resilience per unit volume of a material is known as

modulus of resilience. It is an important property of a material and gives capacity of the material to bear impact or shocks. Mathematically, strain energy stored in a body due to tensile or compressive load or resilience,

$$U = \frac{\sigma^2 \times V}{2E}$$

And Modulus of resilience

$$= \frac{\sigma^2}{2E}$$

Where σ = Tensile or compressive stress,

V = Volume of the body, and

E = Young's modulus of the material of the body.

When a body is subjected to a shear load, then modulus of resilience (shear)

$$= \frac{\tau^2}{2C}$$

Where τ = Shear stress, and

C = Modulus of rigidity.

When the body is subjected to torsion, then modulus of resilience

$$= \frac{\tau^2}{4C}$$

Problem:

A wrought iron bar 50 mm in diameter and 2.5 m long transmits shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take E = 200 GN/m^2.

Solution. Given : $d = 50$ mm ; $l = 2.5$ m $= 2500$ mm ; $U = 100$ N-m $= 100 \times 10^3$ N-mm ; $E = 200$ GN/m^2 $= 200 \times 10^3$ N/mm^2

Maximum instantaneous stress

Let σ = Maximum instantaneous stress.

We know that volume of the bar,

$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4}(50)^2 \times 2500 = 4.9 \times 10^6 \text{ mm}^3$$

We also know that shock or strain energy stored in the body (U),

$$100 \times 10^3 = \frac{\sigma^2 \times V}{2E} = \frac{\sigma^2 \times 4.9 \times 10^6}{2 \times 200 \times 10^3} = 12.25\,\sigma^2$$

$$\therefore \quad \sigma^2 = 100 \times 10^3 / 12.25 = 8163 \quad \text{or} \quad \sigma = 90.3 \text{ N/mm}^2 \text{ Ans.}$$

Elongation produced

Let δl = Elongation produced.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\delta l / l}$$

$$\therefore \quad \delta l = \frac{\sigma \times l}{E} = \frac{90.3 \times 2500}{200 \times 10^3} = 1.13 \text{ mm} \quad \textbf{Ans.}$$

Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion.*** The stress set up by torsion is known as ***torsional shear stress.*** It is zero at the centroidal axis and maximum at the outer surface. Consider a shaft fixed at one end and subjected to a torque (*T*) at the other end as shown in Fig. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. We have discussed above that the torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \quad \text{-------- (i)}$$

Where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

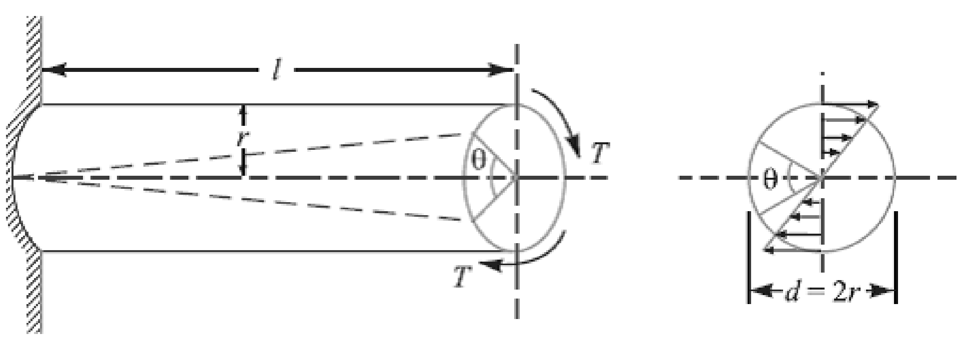
T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length *l*.



The above equation **is** known as ***torsion equation.*** It is based on the following assumptions:

1. The material of the shaft is uniform throughout.

2. The twist along the length of the shaft is uniform.

3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.

4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.

5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Note: 1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance *x* from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

2. From equation (*i*), we know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \text{or} \qquad T = \tau \times \frac{J}{r}$$

For a solid shaft of diameter (*d*), the polar moment of inertia,

$$J - I_{XX} + I_{YY} - \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$

Therefore,

$$T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$

In case of a hollow shaft with external diameter (d_o) and internal diameter (d_i), the polar moment of inertia,

$$J - \frac{\pi}{32} \left[(d_o)^4 \quad (d_i)^4\right] \text{ and } r - \frac{d_o}{2}$$

$$T = \tau \times \frac{\pi}{32} \left[(d_o)^4 - (d_4)^4\right] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

$$= \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4) \qquad \ldots \left(\text{Substituting, } k - \frac{d_i}{d_o}\right)$$

3. The expression ($C \times J$) is called ***torsional rigidity*** of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2 \pi N . T}{60} = T . \omega \qquad \ldots \left(\because \omega = \frac{2 \pi N}{60}\right)$$

Where T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Problem:

A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 160$ r.p.m ; $T_{max} = 1.25\, T_{mean}$; $\tau = 70$ MPa $= 70$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\,\pi\, N\, .\, T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\, T_{mean}$$

$$\therefore \quad T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \quad \text{or} \quad d = 81.5 \text{ mm } \textbf{Ans.}$$

Bending Stress

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses. Consider a straight beam subjected to a bending moment M as shown in Fig.

The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is perfectly homogeneous (*i.e.* of the same material throughout) and isotropic (*i.e.* of equal elastic properties in all directions).

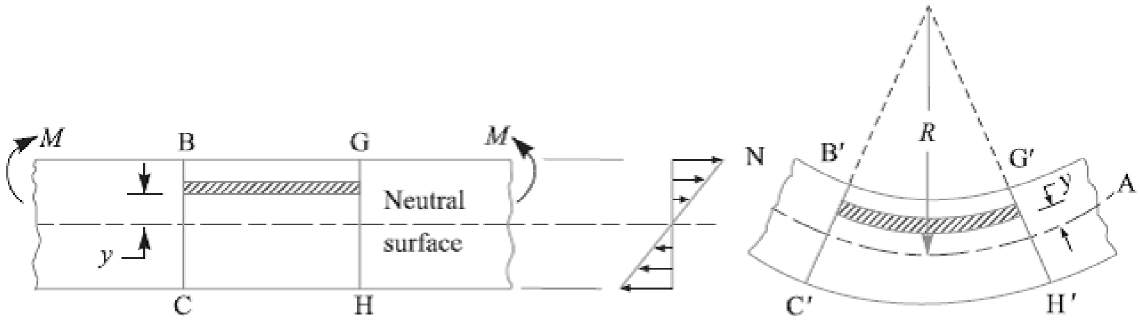
2. The material of the beam obeys Hooke's law.

3. The transverse sections (*i.e.* *BC* or *GH*) which were plane before bending remain plane after bending also.

4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.

5. The Young's modulus (E) is the same in tension and compression.

6. The loads are applied in the plane of bending.



A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called ***neutral surface*.** The intersection of the neutral surface with any normal cross-section of the beam is known *as* ***neutral axis*.** The stress distribution of a beam is shown in Fig. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y, *i.e.* the distance of the point from the neutral axis.

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

The ratio I/y is known as ***section modulus*** and is denoted by Z.

Notes: 1. the neutral axis of a section always passes through its centroid.

2. In case of symmetrical sections such as circular, square or rectangular, the neutral axis passes through its geometrical centre and the distance of extreme fibre from the neutral axis is $y = d / 2$, where d is the diameter in case of circular section or depth in case of square or rectangular section.

3. In case of unsymmetrical sections such as L-section or T-section, the neutral axis does not pass through its geometrical centre. In such cases, first of all the centroid of the section is calculated and then the distance of the extreme fibres for both lower and upper side of the section is obtained. Out of these two values, the bigger value is used in bending equation.

Problem:

A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.

Solution. Given: $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa $= 40$ N/mm^2 ; $h = 2b$

The beam is shown in Fig. 5.7.

Let b = Width of the beam in mm, and

h = Depth of the beam in mm.

$\therefore$ Section modulus,

$$Z = \frac{b \cdot h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3}\ \text{mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

$\therefore$ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

Problem:

A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.

Solution. Given : $P = 10$ kW $= 10 \times 10^3$ W ; $N = 400$ r.p.m ; $D = 1.2$ m $= 1200$ mm or $R = 600$ mm ; $\sigma_b = 15$ MPa $= 15$ N/mm^2

Let T = Torque transmitted by the pulley.

We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\pi N.T}{60} = \frac{2\pi \times 400 \times T}{60} = 42\,T$$

$$\therefore \quad T = 10 \times 10^3/42 = 238 \text{ N-m} = 238 \times 10^3 \text{ N-mm}$$

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\,520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm $= 2 \times 2b = 4b$ ...(Given)

$\therefore$ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4}(2b)^2 \times b = \pi\, b^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59\,520}{\pi\, b^3} = \frac{18\,943}{b^3}$$

or $b^3 = 18\,943/15 = 1263$ or $b = 10.8$ mm

$\therefore$ Minor axis, $2b = 2 \times 10.8 = 21.6$ mm **Ans.**

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2$ mm **Ans.**

Principal Stresses and Principal Planes

In the previous chapter, we have discussed about the direct tensile and compressive stress as well as simple shear. Also we have always referred the stress in a plane which is at right angles to the line of action of the force. But it has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as ***principal planes*** and the direct stresses along these planes are known as ***principal stresses.*** The planes on which the maximum shear stress act are known as planes of maximum shear.

Determination of Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (*i.e.* direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

Consider a rectangular body *ABCD* of uniform cross-sectional area and unit thickness subjected to normal stresses σ_1 and σ_2 as shown in Fig. (*a*). In addition to these normal stresses, a shear stress τ also acts. It has been shown in books on '***Strength of Materials***' that the normal stress across any oblique section such as *EF* inclined at an angle θ with the direction of σ_2, as shown in Fig. (*a*), is given by

$$\sigma_t = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta \qquad \text{...}(i)$$

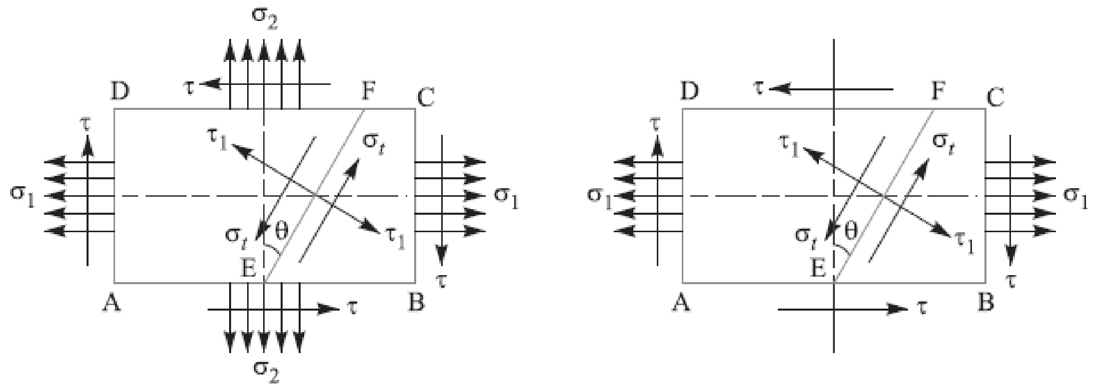
And tangential stress (*i.e.* shear stress) across the section *EF*,

$$\tau_1 = \frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta \qquad \text{...}(ii)$$

Since the planes of maximum and minimum normal stress (*i.e.* principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating $\tau_1 = 0$ in the above equation **(*ii*),** *i.e.*

$$\frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta = 0$$

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} \qquad \text{...}(iii)$$



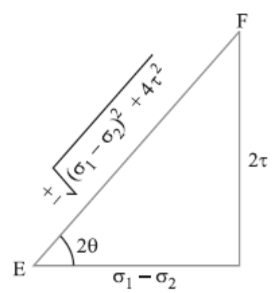
(*a*) Direct stress in two mutually prependicular planes accompanied by a simple shear stress.

(*b*) Direct stress in one plane accompanied by a simple shear stress.

Fig. Principal stresses for a member subjected to bi-axial stress

We know that there are two principal planes at right angles to each other. Let θ_1 and θ_2 be the inclinations of these planes with the normal cross-section. From the following Fig., we find that

$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$



$$\therefore \qquad \sin 2\theta_1 = + \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\text{and} \qquad \sin 2\theta_2 = - \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\text{Also} \qquad \cos 2\theta = \pm \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\therefore \qquad \cos 2\theta_1 = + \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\text{and} \qquad \cos 2\theta_2 = - \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

The maximum and minimum principal stresses may now be obtained by substituting the values of sin 2θ and cos 2θ in equation ***(i)***.

So, Maximum principal (or normal) stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2} \qquad ...(iv)$$

And minimum principal (or nor al) stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2} \qquad ...(v)$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by ***one-half the algebraic difference between the principal stresses,*** *i.e.*

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2} \qquad ...(vi)$$

Notes: 1. when a member is subjected to direct stress in one plane accompanied by a simple shear stress, then the principal stresses are obtained by substituting $\sigma_2 = 0$ in equation **(*iv*), (*v*)** and **(*vi*).**

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

2. In the above expression of σt2, the value of $\frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$ is more than $\sigma_1/2$ Therefore the nature of σ_{t2} will be opposite to that of σ_{t1}, *i.e.* if σ_{t1} is tensile then σ_{t2} will be compressive and *vice-versa*.

<u>Application of Principal Stresses in Designing Machine Members</u>

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained. The results obtained in the previous article may be written as follows:

1. Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

2. Maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

3. Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

Where σ_t = Tensile stress due to direct load and bending,

σ_c = Compressive stress, and

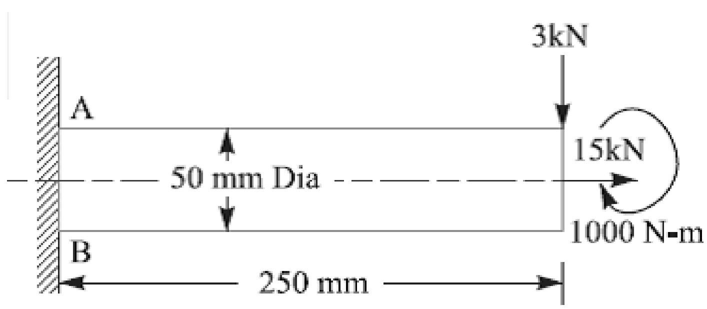
τ = Shear stress due to torsion.

Notes: 1. When $\tau = 0$ as in the case of thin cylindrical shell subjected in internal fluid pressure, then $\sigma_{tmax} = \sigma_t$

2. When the shaft is subjected to an axial load (*P*) in addition to bending and twisting moments as in the propeller shafts of ship and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). This will give the resultant tensile stress or compressive stress (σ_t or σ_c) depending upon the type of axial load (*i.e.* pull or push).

Problem:

A shaft, as shown in Fig., is subjected to a bending load of 3 kN, pure torque of 1000 N-m and an axial pulling force of 15 kN. Calculate the stresses at A and B.



Solution. Given : $W = 3$ kN $= 3000$ N ; $T = 1000$ N-m $= 1 \times 10^6$ N-mm ; $P = 15$ kN $= 15 \times 10^3$ N ; $d = 50$ mm; $x = 250$ mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4} (50)^2 = 1964 \text{ mm}^2$$

$\therefore$ Tensile stress due to axial pulling at points A and B,

$$\sigma_o = \frac{P}{A} = \frac{15 \times 10^3}{1964} = 7.64 \text{ N/mm}^2 = 7.64 \text{ MPa}$$

Bending moment at points A and B,

$$M = W.x = 3000 \times 250 = 750 \times 10^3 \text{ N-mm}$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} (50)^3$$

$$= 12.27 \times 10^3 \text{ mm}^3$$

$\therefore$ Bending stress at points A and B,

$$\sigma_b = \frac{M}{Z} = \frac{750 \times 10^3}{12.27 \times 10^3}$$

$$= 61.1 \text{ N/mm}^2 = 61.1 \text{ MPa}$$

This bending stress is tensile at point A and compressive at point B.

$\therefore$ Resultant tensile stress at point A,

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64$$

$$= 68.74 \text{ MPa}$$

and resultant compressive stress at point B,

$$\sigma_B = \sigma_b - \sigma_o = 61.1 - 7.64 = 53.46 \text{ MPa}$$

We know that the shear stress at points A and B due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\, d^3} = \frac{16 \times 1 \times 10^6}{\pi\,(50)^3} = 40.74 \text{ N/mm}^2 = 40.74 \text{ MPa} \qquad \ldots\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Stresses at point A

We know that maximum principal (or normal) stress at point *A*,

$$\sigma_{A(max)} = \frac{\sigma_A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right]$$

$$= \frac{68.74}{2} + \frac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 34.37 + 53.3 = 87.67 \text{ MPa (tensile) } \textbf{Ans.}$$

Minimum principal (or normal) stress at point *A*,

$$\sigma_{A(min)} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = 34.37 - 53.3 = -18.93 \text{ MPa}$$

$$= 18.93 \text{ MPa (compressive) } \textbf{Ans.}$$

and maximum shear stress at point *A*,

$$\tau_{A(max)} = \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 53.3 \text{ MPa } \textbf{Ans.}$$

Stresses at point B

We know that maximum principal (or normal) stress at point *B*,

$$\sigma_{B(max)} = \frac{\sigma_B}{2} + \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right]$$

$$= \frac{53.46}{2} + \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

$$= 26.73 + 48.73 = 75.46 \text{ MPa (compressive) } \textbf{Ans.}$$

$$= 48.73 \text{ MPa } \textbf{Ans.}$$

ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT I – FUNDAMENTAL CONCEPTS IN DESIGN

1.3 STRESS

Stress

When some external system of forces or loads acts on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as ***unit stress*** or simply a ***stress***. It is denoted by a Greek letter sigma (σ). Mathematically,

$$\text{Stress, } \sigma = P/A$$

Where P = Force or load acting on a body, and

A = Cross-sectional area of the body.

In S.I. units, the stress is usually expressed in Pascal (Pa) such that 1 Pa = 1 N/m^2. In actual practice, we use bigger units of stress *i.e*. megapascal (MPa) and gigapascal (GPa), such that

$$1 \text{ MPa} = 1 \times 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$$

And $$1 \text{ GPa} = 1 \times 10^9 \text{ N/m}^2 = 1 \text{ kN/mm}^2$$

Strain

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as ***unit strain*** or simply a ***strain.*** It is denoted by a Greek letter epsilon (ε). Mathematically, Strain, $\varepsilon = \delta l / l$ or $\delta l = \varepsilon.$

Where δl = Change in length of the body, and

l= Original length of the body.

Tensile Stress and Strain

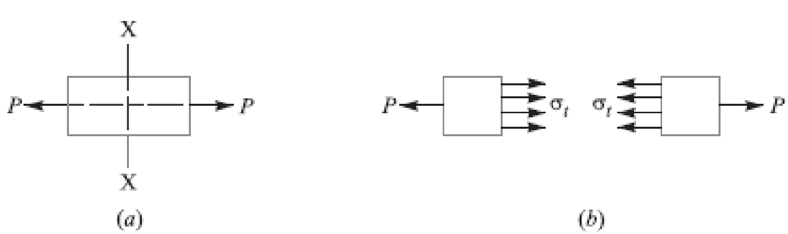


Fig. Tensile stress and strain

When a body is subjected to two equal and opposite axial pulls P (also called tensile load) as shown in Fig. (*a*), then the stress induced at any section of the body is known as ***tensile stress*** as shown in Fig. (*b*). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as ***tensile strain.***

Let P = Axial tensile force acting on the body,

A = Cross-sectional area of the body,

l = Original length, and

δl = Increase in length.

Then Tensile stress, $\sigma_t = P/A$

and tensile strain, $\varepsilon_t = \delta l / l$

Young's Modulus or Modulus of Elasticity

Hooke's law* states that when a material is loaded within elastic limit, the stress is directly proportional to strain, *i.e.*

$$\sigma \propto \varepsilon \qquad \text{or} \qquad \sigma = E.\varepsilon$$

$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

where E is a constant of proportionality known as ***Young's modulus*** or ***modulus of elasticity.*** In S.I. units, it is usually expressed in GPa *i.e.* GN/m^2 or kN/mm^2. It may be noted that Hooke's law holds good for tension as well as compression.

The following table shows the values of modulus of elasticity or Young's modulus (E) for the materials commonly used in engineering practice.

Values of E for the commonly used engineering materials.

Material	*Modulus of elasticity (E) in GPai.e. GN/m^2 for kN/mm^2*
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminium	60 to 80
Timber	10

Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called ***shear stress.***

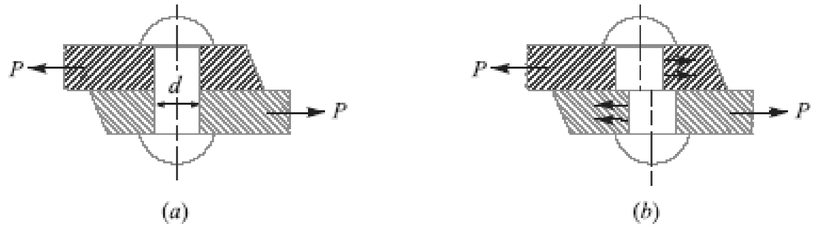


Fig. Single shearing of a riveted joint.

The corresponding strain is known as ***shear strain*** and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau (τ) and phi (φ) respectively. Mathematically, Shear stress, τ = Tangential force * Resisting area.

Consider a body consisting of two plates connected by a rivet as shown in Fig. (*a*). In this case, the tangential force *P* tends to shear off the rivet at one cross-section as shown in Fig. (*b*). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be in ***single shear.*** In such a case, the area resisting the shear off the rivet,

$$A = \frac{\pi}{4} \times d^2$$

Now let us consider two plates connected by the two cover plates as shown in Fig. *(a)*. In this case, the tangential force *P* ten s to shear off the rivet at two cross-sections as shown in Fig.

(b). It may be noted that when the tangential force is resisted by two cross-sections of the rivet (or when the shearing takes place at Two cross-sections of the rivet), then the rivets are said to be in *double shear.* In su h a case, the area resisting the shear off the rivet,

$$A = 2 \times \frac{\pi}{\ } \times d^2$$

and shear stress on the rivet cross-section.

$$\tau = \frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2} = \frac{2P}{\pi d^2}$$

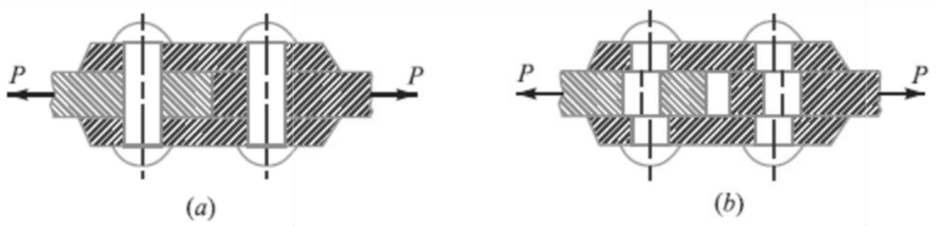


Fig. Double shearing of a riveted joint.

Notes:

1. All lap joints and single cover butt joints are in single shear, while the butt joints with double cover plates are in double shear.

2. In case of shear, the area involved is parallel to the external force applied.

3. When the holes are to be punched or drilled in the metal plates, then the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter '*d*' is to be punched in a metal plate of thickness '*t*', then the area to be sheared,

$$A = \pi d \times t$$

And the maximum shear resistance of the tool or the force required to punch a hole,

$$P = A \times \tau_u = \pi\, d \times t \times \tau_u$$

Where σ_u= Ultimate shear strength of the material of the plate.

Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$$\tau \propto \phi \quad \text{or} \quad \tau = C \,.\, \phi \quad \text{or} \quad \tau / \phi = C$$

Where τ= Shear stress,

φ= Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by *N* or *G*.

The following table shows the values of modulus of rigidity (*C*) for the materials in every day use:

Values of C for the commonly

sed materials s

Material	*Modulus of rigidity (C) in GPa i.e. GN/m² or kNmm²*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

Linear and Lateral Strain

Consider a circular bar of diameter *d* and length *l*, subjected to a tensile force *P* as shown in Fig. (*a*).

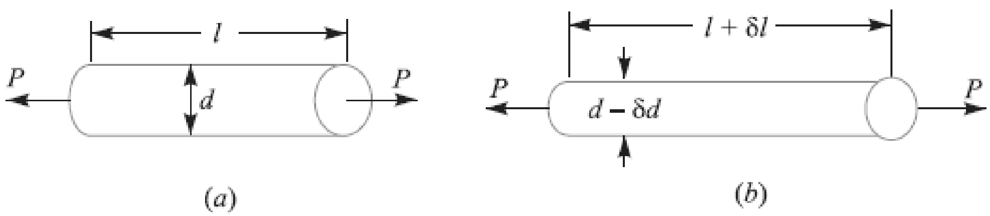


Fig. Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount δl and the diameter decreases by an amount δd, as shown in Fig. (*b*). similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as ***linear strain*** and an opposite kind of strain in every direction, at right angles to it, is known as ***lateral strain.***

4.18 Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{LateralStrain}}{\text{LinearStrain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by $1/m$ or μ.

Following are the values of Poisson's ratio for some of the materials commonly used in engineering practice.

Values of Poisson’s ratio for commonly used materials

S.No.	*Material*	*Poisson 's ratio* (*1/m or* μ)
1	**Steel** Cast	0.25 **to** 0.33
2	iron Copper	0.23 **to** 0.27
3	Brass	0.31 to 0.34
4	Aluminium	0.32 **to** 0.42
5	Concrete	0.32 to 0.36
6	Rubber	0.08 to 0.18
7		0.45 to 0.50

Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as ***volumetric strain.*** Mathematically, volumetric strain,

$$\varepsilon_v = \delta V / V$$

Where δV = Change in volume, and V = Original volume

Notes : 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right);\ \text{where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

where ε_x, ε_y and ε_z are the strains in the directions *x*-axis, *y*-axis and *z*-axis respectively.

Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as ***bulk modulus.*** It is usually denoted by *K*. Mathematically, bulk modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V / V}$$

Relation Between Bulk Modulus and Young's Modulus

The bulk modulus (*K*) and Young's modulus (*E*) are related by the following relation,

$$K = \frac{m.E}{3\,(m-2)} = \frac{E}{3\,(1-2\mu)}$$

Relation between Young's Modulus and Modulus of Rigidity

The Young's modulus (*E*) and modulus of rigidity (*G*) are related by the following relation,

$$G = \frac{m.E}{2\,(m+1)} = \frac{E}{2\,(1+\mu)}$$

Factor of Safety

It is defined, in general, as the **ratio of the maximum stress to the working stress.** Mathematically,

Factor of safety = Maximum stress/ Working or design stress

In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

Factor of safety = Yield point stress/ Working or design stress

In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

Factor of safety = Ultimate stress/ Working or design stress

This relation may also be used for ductile materials.
The above relations for factor of safety are for static loading.

Problem:

A steel bar 2.4 m long and 30 mm square is elongated by a load of 500 kN. If poisson's ratio is 0.25, find the increase in volume. Take E = 0.2×10^6 N/mm^2.

Solution. Given : $l = 2.4$ m = 2400 mm ; $A = 30 \times 30 = 900$ mm^2 ; $P = 500$ kN = 500×10^3 N ; $1/m = 0.25$; $E = 0.2 \times 10^6$ N/mm^2

Let δV = Increase in volume.

We know that volume of the rod,

$$V = \text{Area} \times \text{length} = 900 \times 2400 = 2160 \times 10^3 \text{ mm}^3$$

and Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{P/A}{\varepsilon}$$

$$\therefore \quad \varepsilon = \frac{P}{A.E} = \frac{500 \times 10^3}{900 \times 0.2 \times 10^6} = 2.8 \times 10^{-3}$$

We know that volumetric strain,

$$\frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right) = 2.8 \times 10^{-3}\ (1 - 2 \times 0.25) = 1.4 \times 10^3$$

$$\therefore \quad \delta V = V \times 1.4 \times 10^{-3} = 2160 \times 10^3 \times 1.4 \times 10^{-3} = 3024 \text{ mm}^3 \text{ Ans.}$$

Unit II

SHAFTS AND COUPLINGS

2.1 SHAFTS

2.1.1 Introduction

A shaft is a rotating machine element which is used to transmit power from one place to another. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

An ***axle,*** though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

A ***spindle*** is a short shaft that imparts motion either to a cutting tool (*e.g*. drill press spindles) or to a work piece (*e.g*. lathe spindles).

2.1.2 Material Used for Shafts

The material used for shafts should have the following properties :
1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.
The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.
The mechanical properties of these grades of carbon steel are given in the following table.

Table 2.1. Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

2.1.3 Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses. The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

2.1.4 Types of Shafts

The following two types of shafts are important from the subject point of view

1. *Transmission shafts*. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. *Machine shafts*. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

2.1.5 Stresses in Shafts

The following stresses are induced in the shafts :

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).

2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

2.1.6 Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(***a***) 112 MPa for shafts without allowance for keyways.

(***b***) 84 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible tensile stress (σ_t) may be taken as 60 per cent of the elastic limit in tension (σ_{el}), but not more than 36 per cent of the ultimate tensile strength (σ_u). In other words, the permissible tensile stress,

$$\sigma_t = 0.6\ \sigma_{el} \text{ or } 0.36\ \sigma_u, \text{ whichever is less.}$$

The maximum permissible shear stress may be taken as

(***a***) 56 MPa for shafts without allowance for key ways.

(***b***) 42 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible shear stress (τ) may be taken as 30 per cent of the elastic limit in tension (σ_{el}) but not more than 18 per cent of the ultimate tensile strength (σ_u). In other words, the permissible shear stress,

$$\tau = 0.3\ \sigma_{el} \text{ or } 0.18\ \sigma_u, \text{ whichever is less.}$$

2.1.7 Design of Shafts

The shafts may be designed on the basis of

1. Strength, and **2.** Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(***a***) Shafts subjected to twisting moment or torque only,

(***b***) Shafts subjected to bending moment only,

(***c***) Shafts subjected to combined twisting and bending moments, and

(***d***) Shafts subjected to axial loads in addition to combined torsional and bending loads.

We shall now discuss the above cases, in detail, in the following pages.

2.1.8 Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion

$$\frac{T}{J} = \frac{\tau}{r} \qquad \text{.....(i)}$$

Where T = Twisting moment (or torque) acting upon the shaft,
J = Polar moment of inertia of the shaft about the axis of rotation,
τ = Torsional shear stress, and
r = Distance from neutral axis to the outer most fibre
= d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i*) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation ***(i)***, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad ...(iii)$$

Let k = Ratio of inside diameter and outside diameter of the shaft
$= d_i / d_o$

Now the equation (*iii*) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad ...(iv)$$

From the equations ***(iii)*** or ***(iv)***, the outside and inside diameter of a hollow shaft may be determined. It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft. When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :
We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and
N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T1 - T2)\, R$$

where $T1$ and $T2$ = Tensions in the tight side and slack side of the belt respectively, and
R = Radius of the pulley.

Example 2.1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Given Data :
N = 200 r.p.m.
P = 20 kW = 20 × 10^3 W
τ= 42 MPa = 42 N/mm^2

To Find:
diameter of the shaft

Solution:
Let d = Diameter of the shaft.
We know that torque transmitted by the shaft,

$$T = \frac{P\ X\ 60}{2\pi N}$$

$$= \frac{20\ X\ 10^3\ X\ 60}{2\ \pi N}$$

= 955 N-m = 955 × 10^3 N-mm

We also know that torque transmitted by the shaft (T),

$$955\ X\ 10^3 = \frac{\pi}{16}\ X\ \tau\ X\ d^3$$

$$= \frac{\pi}{16}\ X\ 42\ X\ d^3$$

$$= 8.25\ d^3$$

d^3= 955 × 10^3 / 8.25 = 115 733 or d = 48.7 say 50 mm **Ans.**

Example 2.2. *Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.*

Given Data :
$P = 20$ kW $= 20 \times 10^3$ W
$N = 200$ r.p.m.
$\tau_u = 360$ MPa $= 360$ N/mm^2
F.S. = 8 ; $k = di / do = 0.5$

To Find
inside and outside diameter

Solution

We know that the allowable shear stress

$$\tau = \frac{\tau_u}{F.S} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft
Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \; X \; 60}{2 \; \pi \; N} = \frac{20 \; X \; 10^3 \; X \; 60}{2 \; \pi \; N} = 955 \text{ N-m} = 955 \text{ x } 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \text{ X } 10^3 = \frac{\pi}{16} \; X \; \tau \; X \; d^3 = \frac{\pi}{16} \; X \; \tau \; X \; d^3 = 8.84 \; d^3$$

$d^3 = 955 \times 10^3 / 8.84 = 108\,032$ or $d = 47.6$ say 50 mm **Ans.**

Diameter of hollow shaft
Let di = Inside diameter, and
do = Outside diameter.
We know that the torque transmitted by the hollow shaft (T),

$$955 \text{ x } 10^3 = \frac{\pi}{16} \; X \; \tau \; (d_o)^3 \; (1 - k^4)$$

$$= \frac{\pi}{16} \; X \; 45 \; (d_o)^3 \; (1 - (0.5)^4)$$
$$= 8.3 \; (do)^3$$

$(do)^3 = 955 \times 10^3 / 8.3 = 115\,060$ or $do = 48.6$ say 50 mm **Ans.**
and $di = 0.5 \; do = 0.5 \times 50 = 25$ mm **Ans.**

2.1.9 Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{...(i)}$$

where M = Bending moment,
I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,
σ_b = Bending stress, and
y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation ***(i)***, we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.
We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \quad \text{...(where } k = d_i / d_o)$$

and

$$y = d_o / 2$$

Again substituting these values in equation ***(i)***, we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \, (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (do) may be obtained.

Example 2.3. *A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.*

Given Data:
$W = 50$ kN $= 50 \times 10^3$ N
$L = 100$ mm ;
$x = 1.4$ m
$\sigma_b = 100$ MPa $= 100$ N/mm^2

To Find: *diameter of the axle between the wheels*

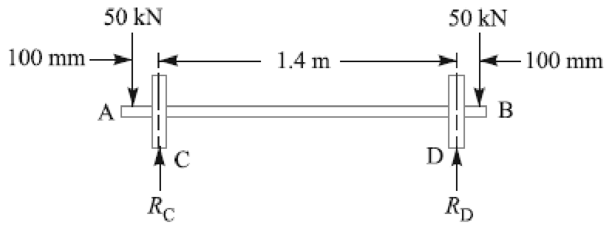


Fig. 1.1.

The axle with wheels is shown in Fig. 1.1.
A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

$$M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6 \text{ N-mm}$$

Let d = Diameter of the axle.

We know that the maximum bending moment (M)

$$5 \text{ X } 10^6 = \frac{\pi}{32} \text{ X } \sigma_b \text{ X } d^3 = \frac{\pi}{32} \text{ X } 100 \text{ X } d^3 = 9.32 d^3$$

$d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6$ or $d = 79.8$

say 80mm **Ans.**

2.1.10 Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σ_b from Art. 14.9 and Art. 14.10, we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \quad ...(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by *Te*. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τmax) equal to the allowable shear stress (τ) for the material, the equation ***(i)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad ...(iii)$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or

$$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad ...(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad ...(v)$$

Example 2.4. *A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted? The pressure angle of the gear may be taken as 20°.*

Given Data :
P = 7.5 kW = 7500 W
N = 300 r.p.m.
D = 150 mm = 0.15 m
L = 200 mm = 0.2 m
τ = 45 MPa = 45 N/mm^2
α = 20°

To Find:

1. *diameter of the shaft*
2. *sketch how the gear will be mounted on the shaft*

Solution:
Fig. 2.2 shows a shaft with a gear mounted on the bearings.

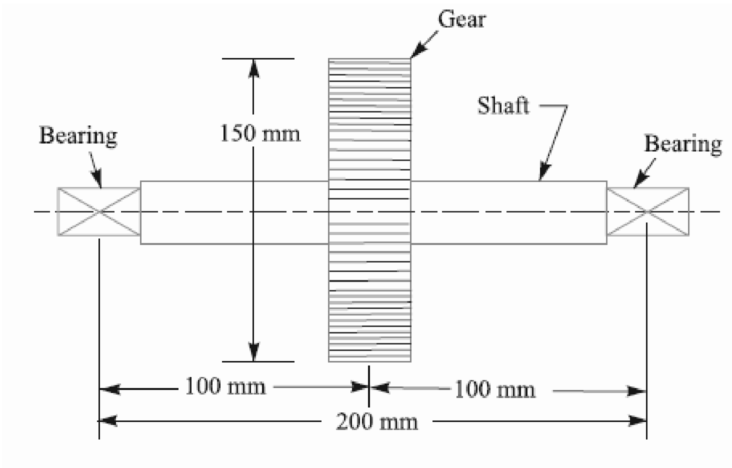


Fig. 2.2

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

∴ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos\alpha} = \frac{3182.7}{\cos 20^\circ} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$

$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

∴ $d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3$ or $d = 32$ say 35 mm **Ans.**

Example 2.5. *A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.*

Given Data:

D = 1.5 m or R = 0.75 m;
T_1 = 5.4 kN = 5400 N
T_2 = 1.8 kN = 1800 N
L = 400 mm
τ = 42 MPa = 42 N/mm^2

To Find:

diameter of the shaft

Solution .

A line shaft with a pulley is shown in Fig 2.3.

We know that torque transmitted by the shaft,

$T = (T_1 - T_2) R = (5400 - 1800)0.75 = 2700 \text{ N-m} = 2700 \times 10^3 \text{ N-mm}$

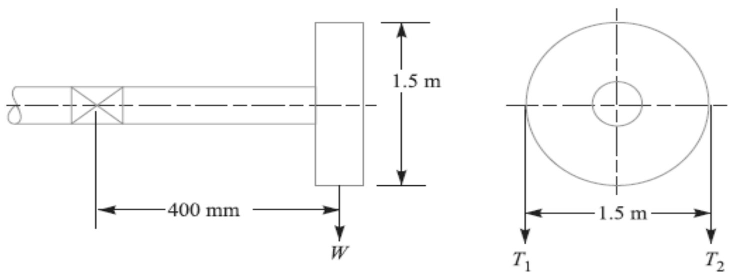


Fig. 2.3

Neglecting the weight of shaft, total vertical load acting on the pulley,

$$W = T_1 + T_2 = 5400 + 1800 = 7200 \text{ N}$$

$\therefore$ Bending moment, $M = W \times L = 7200 \times 400 = 2880 \times 10^3$ N-mm

Let d = Diameter of the shaft in mm.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$
$$= 3950 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3950 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$d^3 = 3950 \times 10^3/8.25 = 479 \times 10^3$ or d = 78 say 80 mm **Ans.**

Example 2.6. *A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley*

Given Data :

AB = 1 m
D_C = 600 mm or R_C = 300 mm = 0.3 m
AC = 300 mm = 0.3 m
T_1 = 2.25 kN = 2250 N
D_D = 400 mm or R_D = 200 mm = 0.2 m
BD = 200 mm = 0.2 m
θ= 180° = π rad
μ = 0.24
σ_b = 63 MPa = 63 N/mm²
τ = 42 MPa = 42 N/mm²

To Find :

suitable diameter for a solid shaft

Solution.

The space diagram of the shaft is shown in Fig. 2.4 (*a*).
Let T_1 = Tension in the tight side of the belt on pulley *C* = 2250 N...(Given)

T_2 = Tension in the slack side of the belt on pulley *C*.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127 \qquad \text{...(Taking antilog of 0.3278)}$$

and

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

∴ Vertical load acting on the shaft at *C*,
$W_C = T_1 + T_2$ = 2250 + 1058 = 3308 N
and vertical load on the shaft at *D*
= 0
The vertical load diagram is shown in Fig. 2.4 (*c*).
We know that torque acting on the pulley *C*,

$T = (T_1 – T_2) R_C$ = (2250 – 1058) 0.3 = 357.6 N-m
The torque diagram is shown in Fig. 2.4 (*b*).

Let T_3 = Tension in the tight side of the belt on pulley *D*, and
T_4 = Tension in the slack side of the belt on pulley *D*.

Since the torque on both the pulleys (*i.e. C* and *D*) is same, therefore

$$(T_3 - T_4) R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \qquad \text{...}(i)$$

We know that

$$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4 \qquad \text{...}(ii)$$

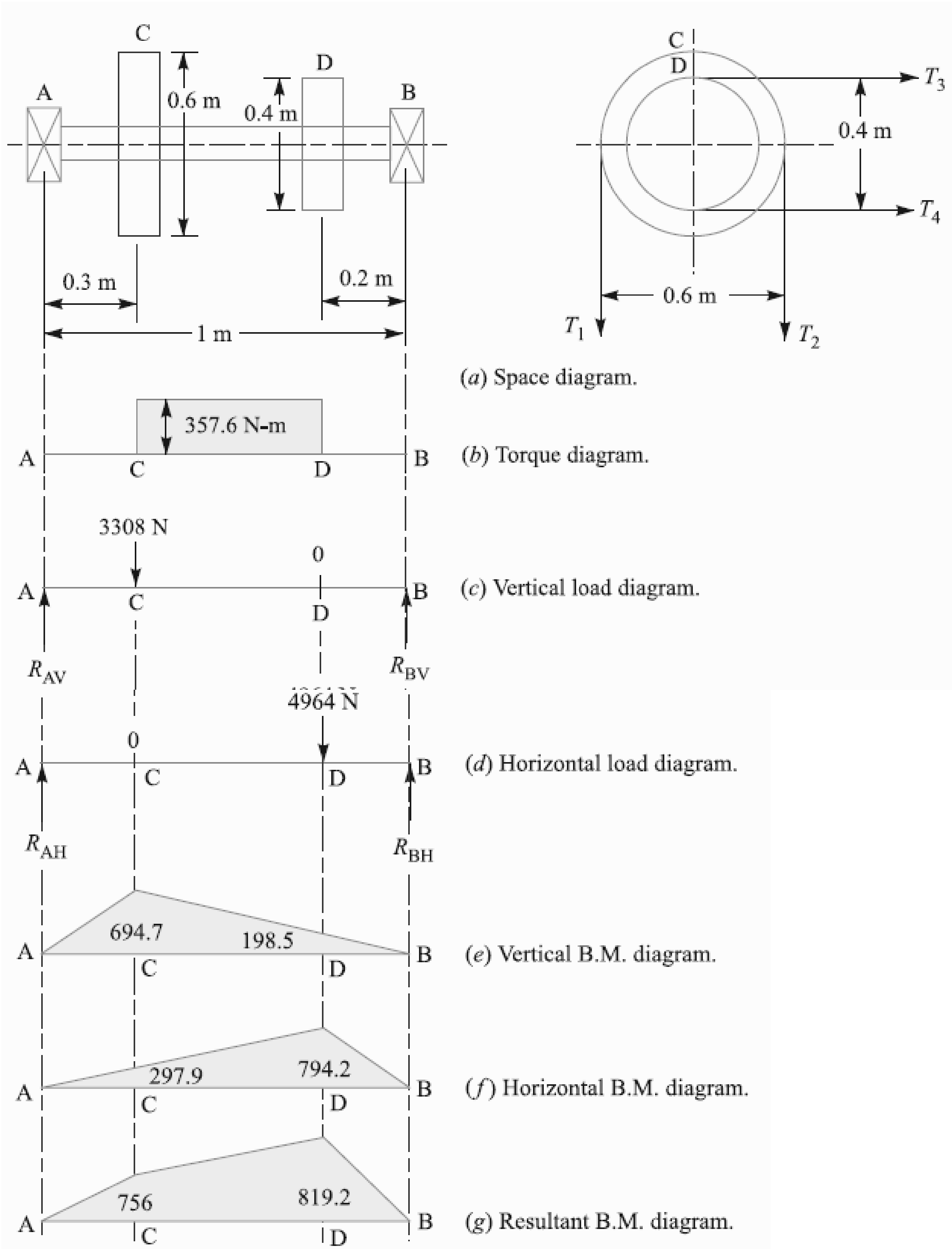

C
D
A
B
0.6 m
0.4 m
0.3 m
0.2 m
1 m
T3
T4
T1
T2
0.4 m
0.6 m
(a) Space diagram.
357.6 N-m
(b) Torque diagram.
3308 N
0
(c) Vertical load diagram.
RAV
RBV
4964 N
0
(d) Horizontal load diagram.
RAH
RBH
694.7
198.5
(e) Vertical B.M. diagram.
297.9
794.2
(f) Horizontal B.M. diagram.
756
819.2
(g) Resultant B.M. diagram.

Fig. 2.4

From equations (*i*) and (*ii*), we find that

$T3 = 3376$ N, and $T4 = 1588$ N

∴ Horizontal load acting on the shaft at *D*,

$WD = T3 + T4 = 3376 + 1588 = 4964$ N

and horizontal load on the shaft at $C = 0$

The horizontal load diagram is shown in Fig. 2.4 (*d*).

Now let us find the maximum bending moment for vertical and horizontal loading First of all, considering the vertical loading at *C*. Let *R*AV and *R*BV be the reactions at the bearings *A* and *B* respectively.

We know that

$RAV + RBV = 3308$ N

Taking moments about *A*,

$RBV \times 1 = 3308 \times 0.3$ or $RBV = 992.4$ N

and $RAV = 3308 - 992.4 = 2315.6$ N

We know that B.M. at *A* and *B*,

$MAV = MBV = 0$

B.M. at *C*, $MCV = RAV \times 0.3 = 2315.6 \times 0.3 = 694.7$ N-m

B.M. at *D*, $MDV = RBV \times 0.2 = 992.4 \times 0.2 = 198.5$ N-m

The bending moment diagram for vertical loading in shown in Fig. 2.4 (*e*).

Now considering horizontal loading at *D*. Let *R*AH and *R*BH be the reactions at the bearings *A* and *B* respectively.

We know that

$RAH + RBH = 4964$ N

Taking moments about *A*,

$RBH \times 1 = 4964 \times 0.8$ or $RBH = 3971$ N

and $RAH = 4964 - 3971 = 993$ N

We know that B.M. at *A* and *B*,

$MAH = MBH = 0$

B.M. at *C*, $MCH = RAH \times 0.3 = 993 \times 0.3 = 297.9$ N-m

B.M. at *D*, $MDH = RBH \times 0.2 = 3971 \times 0.2 = 794.2$ N-m

The bending moment diagram for horizontal loading is shown in Fig. 2.4 (*f*).

Resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 2.4 (*g*).

We see that bending moment is maximum at *D*.

∴ Maximum bending moment,

$M = M_D = 819.2$ N-m

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$
$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3 \text{ or } d = 47.6 \text{ mm}$$

Again we know that equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}(M + T_e)$$
$$= \tfrac{1}{2}(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (Me),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\, d^3$$

$$\therefore \quad d^3 = 856.6 \times 10^3/6.2 = 138.2 \times 10^3 \text{ or } d = 51.7 \text{ mm}$$

Taking larger of the two values,
we have

d = 51.7 say 55 mm **Ans.**

Example 2.7. *A shaft is supported on bearings A and B, 800 mm between centres. A 20° straight tooth spur gear having 600 mm pitch diameter, is located 200 mm to the right of the left hand bearing A, and a 700 mm diameter pulley is mounted 250 mm towards the left of bearing B. The gear is driven by a pinion with a downward tangential force while the pulley drives a horizontal belt having 180° angle of wrap. The pulley also serves as a flywheel and weighs 2000 N. The maximum belt tension is 3000 N and the tension ratio is 3 : 1. Determine the maximum bending moment and the necessary shaft diameter if the allowable shear stress of the material is 40 MPa.*

Given Data :

AB = 800 mm

α_C = 20°

D_C = 600 mm or R_C = 300 mm

AC = 200 mm ;

D_D = 700 mm or R_D = 350 mm

DB = 250 mm

θ = 180° = π rad

W = 2000 N

T_1 = 3000 N

T_1/T_2 = 3

τ = 40 MPa = 40 N/mm^2

To Find:

1. *maximum bending moment*
2. *shaft diameter*

Solution.

The space diagram of the shaft is shown in Fig. 2.5 (*a*).

We know that the torque acting on the shaft at *D*,

$$T = (T_1 - T_2)\,R_D = T_1\left(1 - \frac{T_2}{T_1}\right) R_D$$

$$= 3000\left(1 - \frac{1}{3}\right) 350 = 700 \times 10^3 \text{ N-mm} \qquad \ldots(\because\ T_1/T_2 = 3)$$

The torque diagram is shown in Fig. 2.5 (*b*).

Assuming that the torque at *D* is equal to the torque at *C*, therefore the tangential force acting on the gear *C*,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear *C*,

$$W_C = \frac{F_{tc}}{\cos \alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. 2.6.

Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at *C*,

$$W_{CV} = W_C \cos 20°$$

$$= 2483 \times 0.9397 = 2333 \text{ N}$$

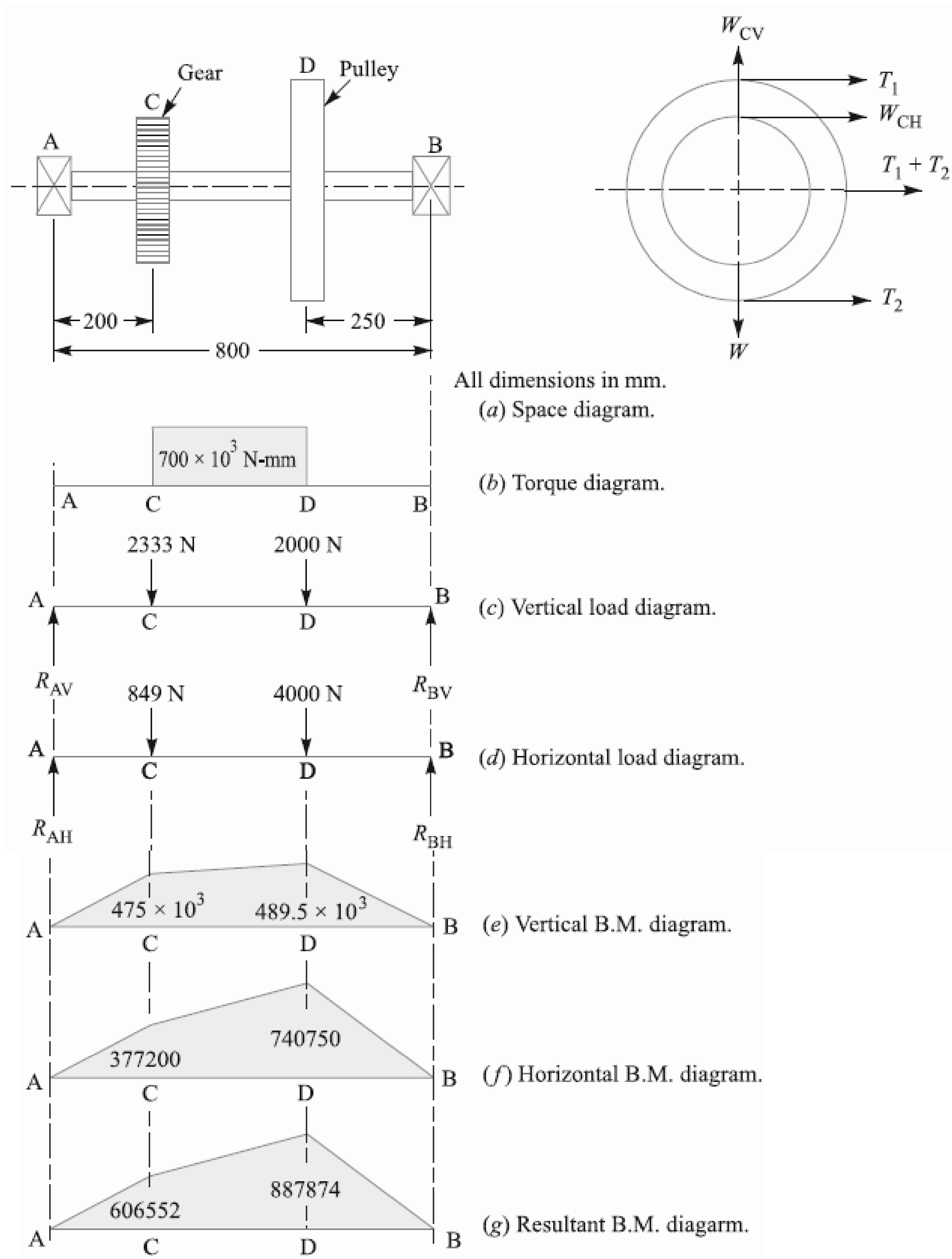
and horizontal component of W_C *i.e*. the horizontal load acting on the shaft at *C*,

$$W_{CH} = W_C \sin 20°$$

$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and $T_1 = 3000$ N, therefore

$$T2 = T1 / 3 = 3000 / 3 = 1000 \text{ N}$$



Gear
C
D
Pulley
A
B
200
250
800
W_{CV}
T_1
W_{CH}
$T_1 + T_2$
T_2
W
All dimensions in mm.
(a) Space diagram.
700×10^3 N-mm
A C D B
(b) Torque diagram.
2333 N
2000 N
A C D B
(c) Vertical load diagram.
R_{AV}
R_{BV}
849 N
4000 N
A C D B
(d) Horizontal load diagram.
R_{AH}
R_{BH}
475×10^3
489.5×10^3
A C D B
(e) Vertical B.M. diagram.
377200
740750
A C D B
(f) Horizontal B.M. diagram.
606552
887874
A C D B
(g) Resultant B.M. diagarm.

Fig. 2.5

∴ Horizontal load acting on the shaft at D,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at D,

$$W_{DV} = W = 2000 \text{ N}$$

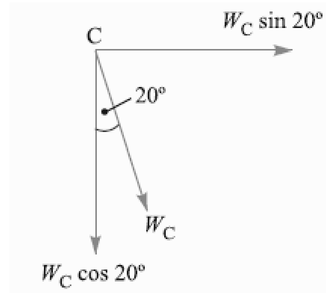


Fig. 2.6

The vertical and horizontal load diagram at C and D is shown in Fig. 2.5 (c) and (d) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all considering the vertical loading at C and D. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2333 + 2000 = 4333 \text{ N}$$

Taking moments about A, we get

$$R_{BV} \times 800 = 2000\ (800 – 250) + 2333 \times 200$$

$$= 1\ 566\ 600$$

$$\therefore R_{BV} = 1\ 566\ 600 / 800 = 1958 \text{ N}$$

And $RAV = 4333 – 1958 = 2375$ N

We know that B.M. at A and B,

$$MAV = MBV = 0$$

B.M. at C, $MCV = RAV \times 200 = 2375 \times 200 = 475 \times 10^3$ N-mm

B.M. at D, $MDV = RBV \times 250 = 1958 \times 250 = 489.5 \times 10^3$ N-mm

The bending moment diagram for vertical loading is shown in Fig. 2.5 (e).

Now consider the horizontal loading at C and D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 4849 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 800 = 4000\ (800 – 250) + 849 \times 200 = 2\ 369\ 800$$

$$\therefore \quad R_{BH} = 2\ 369\ 800 / 800 = 2963 \text{ N}$$

and $R_{AH} = 4849 – 2963 = 1886$ N

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $M_{CH} = R_{AH} \times 200 = 1886 \times 200 = 377\ 200$ N-mm

B.M. at *D*, $M_{DH} = R_{BH} \times 250 = 2963 \times 250 = 740\ 750$ N-mm

The bending moment diagram for horizontal loading is shown in Fig. 2.5 (*f*).

We know that resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(475 \times 10^3)^2 + (377\ 200)^2}$$
$$= 606\ 552 \text{ N-mm}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(489.5 \times 10^3)^2 + (740\ 750)^2}$$
$$= 887\ 874 \text{ N-mm}$$

Maximum bending moment

The resultant B.M. diagram is shown in Fig. 2.5 (*g*). We see that the bending moment is maximum at *D*, therefore

Maximum B.M., $M = M$D = 887 874 N-mm **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\ 874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

$$\therefore \quad d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3 \quad \text{or} \quad d = 52.4 \text{ say } 55 \text{ mm Ans.}$$

2.1.11 Shafts Subjected to Fluctuating Loads

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (*T*) and bending moment (*M*). Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where K_m = Combined shock and fatigue factor for bending, and

K_t = Combined shock and fatigue factor for torsion.

The following table shows the recommended values for K_m and K_t.

Table 2.2. Recommended values for *Km* and *Kt*.

Nature of load	K_m	K_t
1. Stationary shafts		
(*a*) Gradually applied load	1.0	1.0
(*b*) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating shafts		
(*a*) Gradually applied or steady load	1.5	1.0
(*b*) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(*c*) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Example 2.8. *Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200 N and is located at 300 mm from the centre of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 r.p.m. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. The shock and fatigue factors for bending and twisting are 1.5 and 2.0 respectively.The allowable shear stress in the shaft may be taken as 35 MPa.*

Given Data :
W = 200 N
L = 300 mm
D = 200 mm or R = 100 mm
P = 1 kW = 1000 W
N = 120 r.p.m.
θ = 180° = π rad
μ = 0.3
K_m = 1.5
K_t = 2
τ= 35 MPa = 35 N/mm^2

To Find:

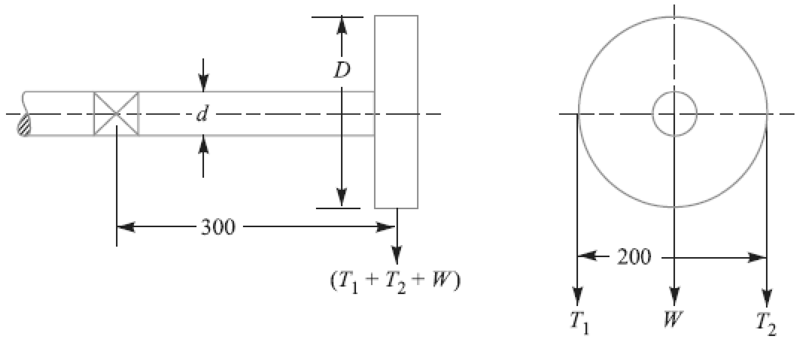
Design a shaft

Solution:

The shaft with pulley is shown in Fig. 14.9.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{1000 \times 60}{2\pi \times 120} = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Fig. 2.7

Let $T1$ and $T2$ = Tensions in the tight side and slack side of the belt respectively in newtons.

∴ Torque transmitted (T),

$$79.6 \times 103 = (T1 - T2)\, R = (T1 - T2)\, 100$$

$$T1 - T2 = 79.6 \times 103 / 100 = 796 \text{ N} \qquad \text{...........}\textbf{(\textit{i})}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\,\pi = 0.9426$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.9426}{2.3} = 0.4098 \text{ or } \frac{T_1}{T_2} = 2.57 \qquad ...(ii)$$

...(Taking antilog of 0.4098)

From equations **(*i*)** and **(*ii*)**, we get,

$$T1 = 1303 \text{ N, and } T2 = 507 \text{ N}$$

We know that the total vertical load acting on the pulley,

$$W_T = T1 + T2 + W = 1303 + 507 + 200 = 2010 \text{ N}$$

∴ Bending moment acting on the shaft,

$$M = W_T \times L = 2010 \times 300 = 603 \times 103 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

$$= \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$918 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 35 \times d^3 = 6.87\, d^3$$

$$\therefore \quad d^3 = 918 \times 10^3 / 6.87 = 133.6 \times 10^3 \text{ or } d = 51.1 \text{ say } 55 \text{ mm Ans.}$$

Example 2.9. *A horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250 mm and 400 mm respectively from the centre line of the left and right bearings. The pitch diameter of the gear C is 600 mm and that of gear D is 200 mm. The distance between the centre line of the bearings is 2400 mm. The shaft transmits 20 kW at 120 r.p.m. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure F_{tC} of the gear C and F_{tD} of the gear D act vertically downwards.*

Find the diameter of the shaft, if the working stress is 100 MPa in tension and 56 MPa in shear. The gears C and D weighs 950 N and 350 N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively.

Given Data :

AC = 250 mm
BD = 400 mm
D_C = 600 mm or R_C = 300 mm
D_D = 200 mm or R_D = 100 mm
AB = 2400 mm
P = 20 kW = 20×10^3 W
N = 120 r.p.m
σ_t = 100 MPa = 100 N/mm^2
τ = 56 MPa = 56 N/mm^2
W_C = 950 N
W_D = 350 N
K_m = 1.5
K_t = 1.2

To Find:
diameter of the shaft

Solution:

The shaft supported in bearings and carrying gears is shown in Fig. 2.8.

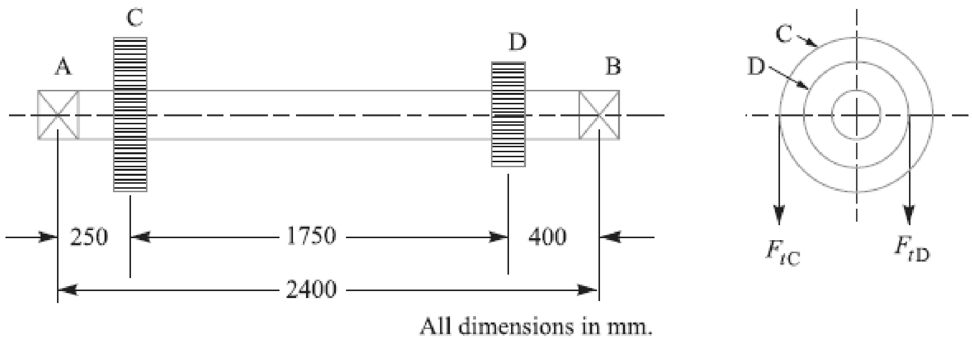


Fig. 2.8

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 120} = 1590 \text{ N-m} = 1590 \times 10^3 \text{ N-mm}$$

Since the torque acting at gears *C* and *D* is same as that of the shaft, therefore the tangential force acting at gear *C*,

$$F_{tC} = \frac{T}{R_C} = \frac{1590 \times 10^3}{300} = 5300 \text{ N}$$

and total load acting downwards on the shaft at *C*

$= F_{tC} + W_C = 5300 + 950 = 6250$ N

Similarly tangential force acting at gear *D*,

$$F_{tD} = \frac{T}{R_D} = \frac{1590 \times 10^3}{100} = 15\,900 \text{ N}$$

and total load acting downwards on the shaft at *D*

$= F_{tD} + W_D = 15\,900 + 350 = 16\,250$ N

Now assuming the shaft as a simply supported beam as shown in Fig. 2.9, the maximum bending moment may be obtained as discussed below :

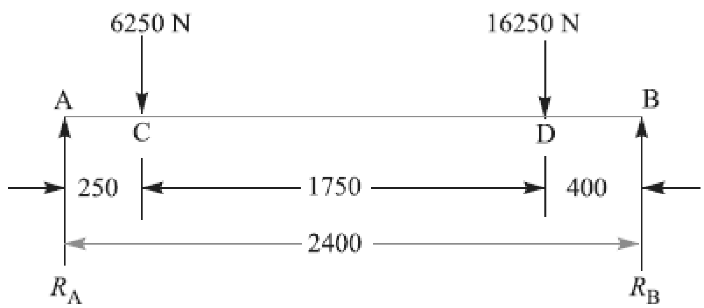


Fig. 2.9

Let R_A and R_B = Reactions at *A* and *B* respectively.

∴ $R_A + R_B$ = Total load acting downwards at *C* and *D*

$= 6250 + 16\,250 = 22\,500$ N

Now taking moments about *A*,

$R_B \times 2400 = 16\,250 \times 2000 + 6250 \times 250 = 34\,062.5 \times 103$

∴ $R_B = 34\,062.5 \times 103 / 2400 = 14\,190$ N

and $R_A = 22\,500 - 14\,190 = 8310$ N

A little consideration will show that the maximum bending moment will be either at *C* or *D*. We know that bending moment at *C*,

$M_C = R_A \times 250 = 8310 \times 250 = 2077.5 \times 103$ N-mm

Bending moment at *D*,

$M_D = R_B \times 400 = 14\,190 \times 400 = 5676 \times 103$ N-mm

∴ Maximum bending moment transmitted by the shaft,

$M = M_D = 5676 \times 103$ N-mm

Let *d* = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 5676 \times 10^3)^2 + (1.2 \times 1590 \times 10^3)^2}$$

$$= 8725 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment (T_e),

$$8725 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 56 \times d^3 = 11\ d^3$$

$$\therefore \quad d^3 = 8725 \times 10^3 / 11 = 793 \times 10^3 \text{ or } d = 92.5 \text{ mm}$$

Again we know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}(K_m \times M + T_e)$$

$$= \tfrac{1}{2}\left[1.5 \times 5676 \times 10^3 + 8725 \times 10^3\right] = 8620 \times 10^3 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$8620 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\ d^3 \quad \text{...(Taking } \sigma_b = \sigma_t)$$

$$\therefore \quad d^3 = 8620 \times 10^3 / 9.82 = 878 \times 10^3 \text{ or } d = 95.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 95.7 \text{ say } 100 \text{ mm } \textbf{Ans.}$$

2.1.12 Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \text{ or } \sigma_b = \frac{M \cdot y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32M}{\pi d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4F}{\pi d^2} \quad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \quad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi (d_o)^2 (1 - k^2)} \quad \text{... } (\because k = d_i/d_o)$$

$\therefore$ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32M}{\pi d^3} + \frac{4F}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F \times d}{8}\right) \quad \text{...}(i)$$

$$= \frac{32 M_1}{\pi d^3} \quad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32M}{\pi (d_o)^3 (1 - k^4)} + \frac{4F}{\pi (d_o)^2 (1 - k^2)}$$

$$= \frac{32}{\pi (d_o)^3 (1 - k^4)}\left[M + \frac{F\, d_o (1 + k^2)}{8}\right] = \frac{32 M_1}{\pi (d_o)^3 (1 - k^4)}$$

$$...\left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F\,d_o\,(1+k^2)}{8}\right]$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

$\therefore$ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4F}{\pi d^2} \quad \text{...(For round solid shaft)}$$

$$= \frac{\alpha \times 4F}{\pi (d_o)^2\,(1-k^2)} \quad \text{...(For hollow shaft)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation :

$$\text{Column factor,} \quad \alpha = \frac{1}{1 - 0.0044\,(L/K)}$$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation :

$$\text{**Column factor,} \quad \alpha = \frac{\sigma_y\,(L/K)^2}{C\pi^2 E}$$

where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C =1, for hinged ends,

= 2.25, for fixed ends,

=1.6, for ends that are partly restrained as in bearings.

Example 2.10. *A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN. Determine :*

1. The maximum shear stress developed in the shaft, and

2. The angular twist between the bearings.

Given Data:

d_o = 0.5 m
d_i = 0.3 m
P = 5600 kW = 5600 × 10^3 W
L = 6 m
N = 150 r.p.m.

$F = 500$ kN $= 500 \times 10^3$ N
$W = 70$ kN $= 70 \times 10^3$ N

To Find:

maximum shear stress

angular twist between the bearings.

Solution:

Maximum shear stress developed in the shaft

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356\ 460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\ 500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2]\,[(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

$\therefore$ Slenderness ratio,

$$L / K = 6 / 0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \quad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $\quad k = d_i / d_o = 0.3 / 0.5 = 0.6$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\,(1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\,500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\,(1 + 0.6^2)}{8}\right]^2 + (1 \times 356\,460)^2}$$

$$= \sqrt{(78\,750 + 51\,850)^2 + (356\,460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) = \frac{\pi}{16} \times \tau\,(0.5)^3\,[1 - (0.6)^4] = 0.02\,\tau$$

$$\therefore \quad \tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19 \text{ MPa } \textbf{Ans.}$$

2. *Angular twist between the bearings*

Let θ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}\,[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}\,[(0.5)^4 - (0.3)^4] = 0.005\,34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\,460 \times 6}{84 \times 10^9 \times 0.00\,534} = 0.0048 \text{ rad}$$

... (Taking G = 84 GPa = 84×10^9 N/m^2)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \textbf{ Ans.}$$

2.1.13 Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. *Torsional rigidity.* The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft. The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G\,.\,\theta}{L} \quad \text{or} \quad \theta = \frac{T\,.\,L}{J\,.\,G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$= \frac{\pi}{32} \times d^4$ ...(For solid shaft)

$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$ ...(For hollow shaft)

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. *Lateral rigidity*. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam,

Example 2.11. *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Given Data :

P = 4 kW = 4000 W

N = 800 r.p.m.

$\theta = 0.25° = 0.25 \text{ X } \frac{\pi}{180} = 0.0044$ rad

L = 1 m = 1000 mm

$G = 84 \text{ GPa} = 84 \times 10^9 \text{ N/m}^2 = 84 \times 10^3 \text{ N/mm}^2$

To Find:

1. *diameter of the spindle*
2. *shear stress induced in the spindle*

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\,740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\,740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\,167$$

$\therefore$ $d^4 = 129\,167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\,740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$\therefore$ $\tau = 47\,740 / 8420 = 5.67 \text{ N/mm}^2 = 5.67$ MPa **Ans.**

2.2 KEYS

2.2.1 Introduction

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

2.2.2 Types of Keys

The following types of keys are important from the subject point of view :
1. Sunk keys, **2.** Saddle keys, **3.** Tangent keys, **4.** Round keys, and **5.** Splines.

2.2.3 Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1. ***Rectangular sunk key*.** A rectangular sunk key is shown in Fig. 2.10. The usual proportions of this key are :

Width of key, $w = d / 4$
and thickness of key, $t = 2w / 3 = d / 6$
where d = Diameter of the shaft or diameter of the hole in the hub.
The key has taper 1 in 100 on the top side only.

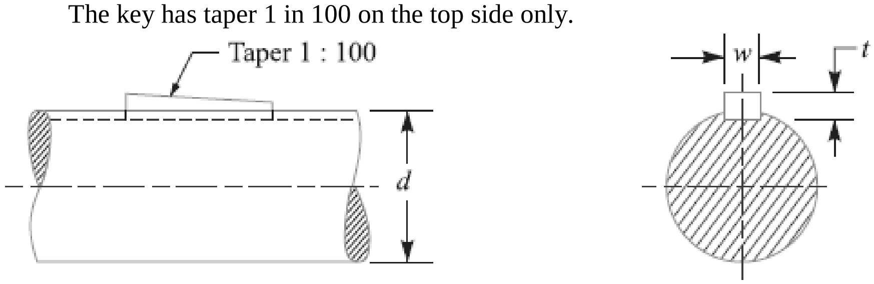


Fig. 2.10. Rectangular sunk key.

2. ***Square sunk key*.** The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.*

$$w = t = d / 4$$

3. ***Parallel sunk key*.** The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. ***Gib-head key*.** It is a rectangular sunk key with a head at one end known as ***gib head*.** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. 2.11 (*a*) and its use in shown in Fig. 2.11 (*b*).

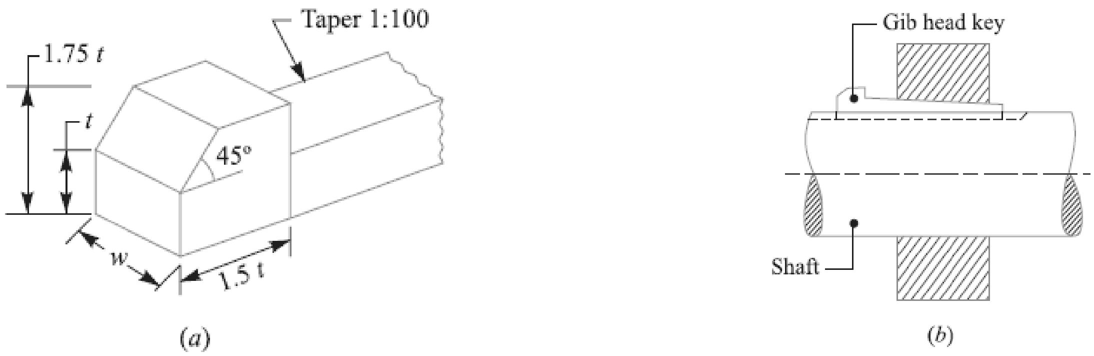


Fig. 2.11. Gib-head key.

The usual proportions of the gib head key are :
Width, $w = d / 4$;
and thickness at large end, $t = 2w / 3 = d / 6$

5. *Feather key*. A key attached to one member of a pair and which permits relative axial movement is known as ***feather key*.** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

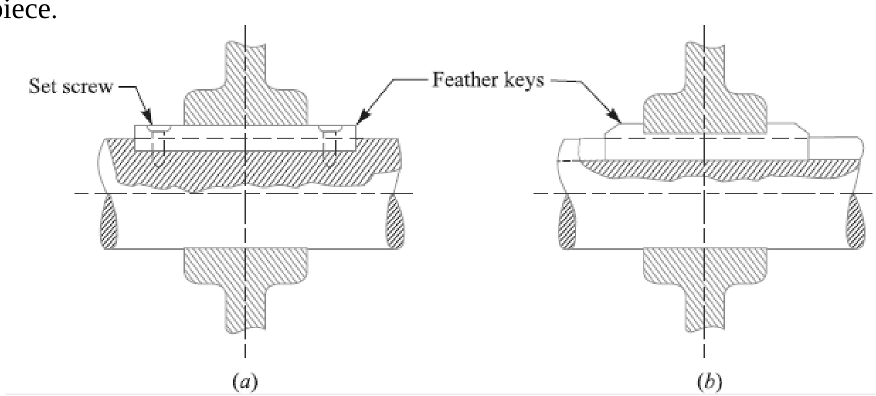


Fig. 2.12. Feather key.

The feather key may be screwed to the shaft as shown in Fig. 2.12 (*a*) or it may have double gib heads as shown in Fig. 2.12 (*b*). The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

The following table shows the proportions of standard parallel, tapered and gib head keys, according to IS : 2292 and 2293-1974 (Reaffirmed 1992).

Table 2.2. Proportions of standard parallel, tapered and gib head keys.

Shaft diameter (mm) upto and including	*Key cross-section*		*Shaft diameter (mm) upto and including*	*Key cross-section*	
	Width (mm)	*Thickness (mm)*		*Width (mm)*	*Thickness (mm)*
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

6. *Woodruff key*. The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. 2.13. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

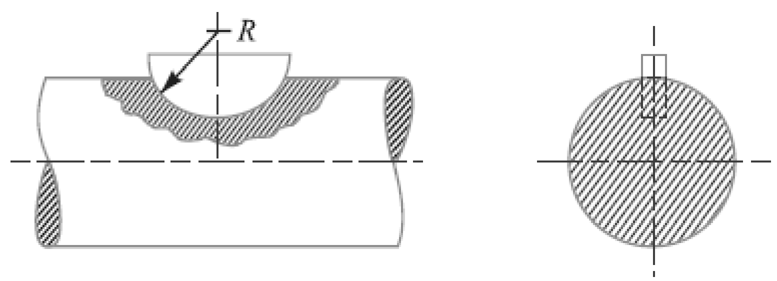


Fig. 2.13. Woodruff key

The main advantages of a woodruff key are as follows :

1. It accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.

The disadvantages are :

1. The depth of the keyway weakens the shaft.
2. It cannot be used as a feather.

2.2.4 Saddle keys

The saddle keys are of the following two types :
1. Flat saddle key, and **2.** Hollow saddle key.

A***flat saddle key*** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. 2.14. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

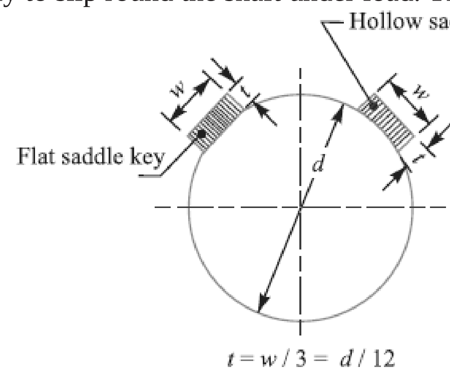


Fig. 2.14. Saddle key

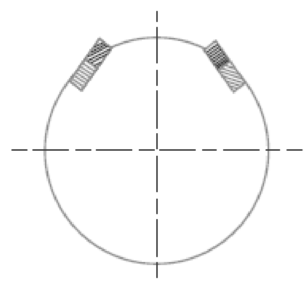


Fig. 2.15. Tangent key.

A ***hollow saddle key*** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

2.2.5 Tangent Keys

The tangent keys are fitted in pair at right angles as shown in Fig. 2.15. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

2.2.6 Round Keys

The round keys, as shown in Fig. 2.16 (*a*), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

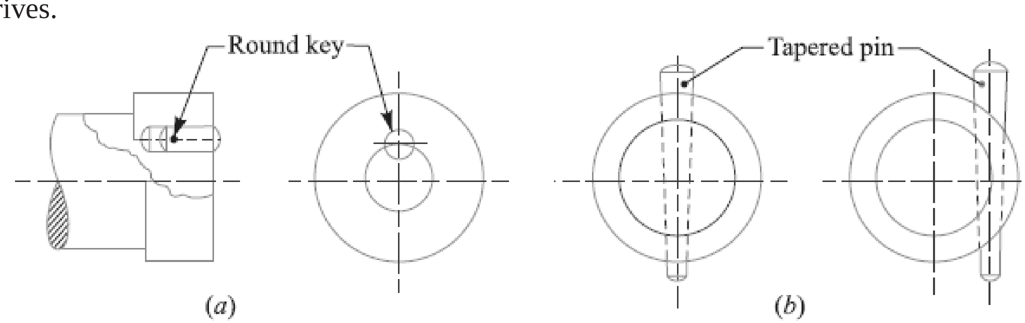
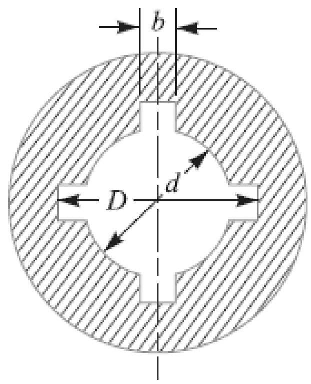


Fig. 2.16. Round keys.

Sometimes the tapered pin, as shown in Fig. 2.16 (*b*), is held in place by the friction between the pin and the reamed tapered holes.

2.2.7 Splines

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as ***splined shafts*** as shown in Fig. 2.17. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway. The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.



$D = 1.25\ d$ and $b = 0.25\ D$

Fig. 2.17. Splines.

2.2.8 Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces (*F*1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (*F*) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub
.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig. 2.18.

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

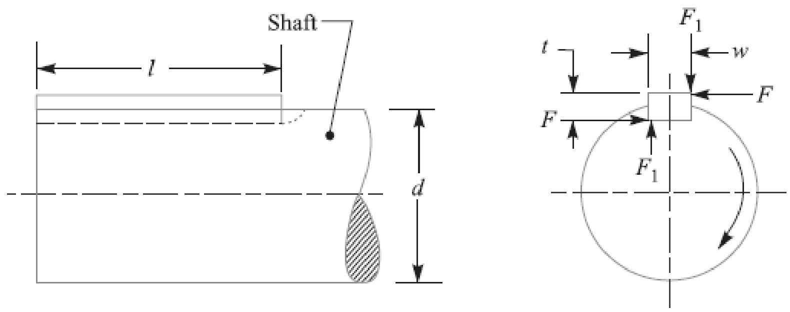


Fig. 2.18. Forces acting on a sunk key.

2.2.9 Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig. 2.18.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

τ and σ_C = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \quad \text{...}(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{...}(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{...[Equating equations } (i) \text{ and } (ii)\text{]}$$

or
$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \quad \text{...}(iii)$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress. Therefore from equation ***(iii)***, we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{...}(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \qquad \text{...}(v)$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations (*iv*) and (*v*), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \qquad \text{... (Taking } w = d/4) \quad \text{...}(vi)$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$$\therefore \quad l = 1.571\, d \qquad \text{... [From equation } (vi)]$$

Example 2.12. *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.*

Given :
d = 50 mm
τ= 42 MPa = 42 N/mm^2
σ_c = 70 MPa = 70 N/mm^2

To Find:
Design the rectangular key

Solution:
The rectangular key is designed as discussed below:

From Table 2.2, we find that for a shaft of 50 mm diameter,

Width of key, w = 16 mm **Ans.**
and thickness of key, t = 10 mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

2.3 SHAFT COUPLING

2.3.1 Introduction

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following :

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.
5. It should have no projecting parts.

Requirements of a Good Shaft Coupling

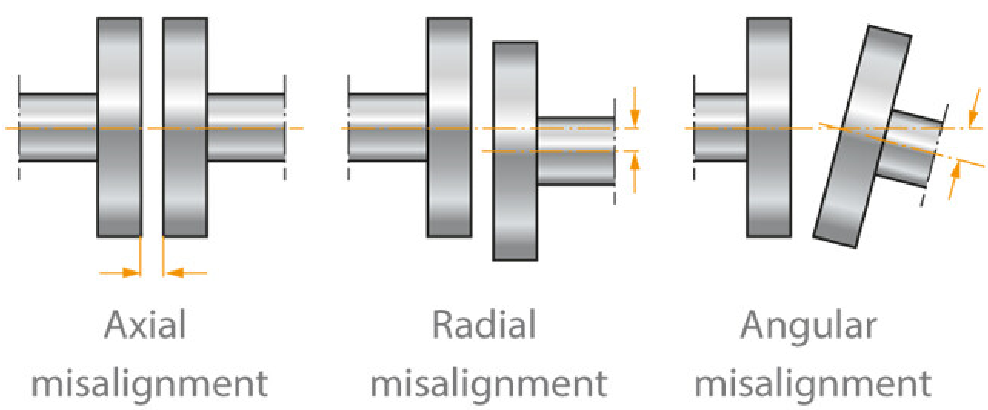
A good shaft coupling should have the following requirements :

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. It should have no projecting parts.

2.3.2 Types of Shaft Couplings

Shaft couplings are divided into two main groups as follows

1. *Rigid coupling*. It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view :
(*a*) Sleeve or muff coupling.



(*b*) Clamp or split-muff or compression coupling, and
(*c*) Flange coupling.
2. ***Flexible coupling.*** It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view :

(*a*) Bushed pin type coupling,
(*b*) Universal coupling, and
(*c*) Oldham coupling.

We shall now discuss the above types of couplings, in detail,

2.3.4 Sleeve or Muff-coupling

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig. 2.19. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. The usual proportions of a cast iron sleeve coupling are as follows :

Outer diameter of the sleeve, $D = 2d + 13$ mm
and length of the sleeve, $L = 3.5\ d$

where d is the diameter of the shaft.
In designing a sleeve or muff-coupling, the following procedure may be adopted.

1. ***Design for sleeve***
The sleeve is designed by considering it as a hollow shaft

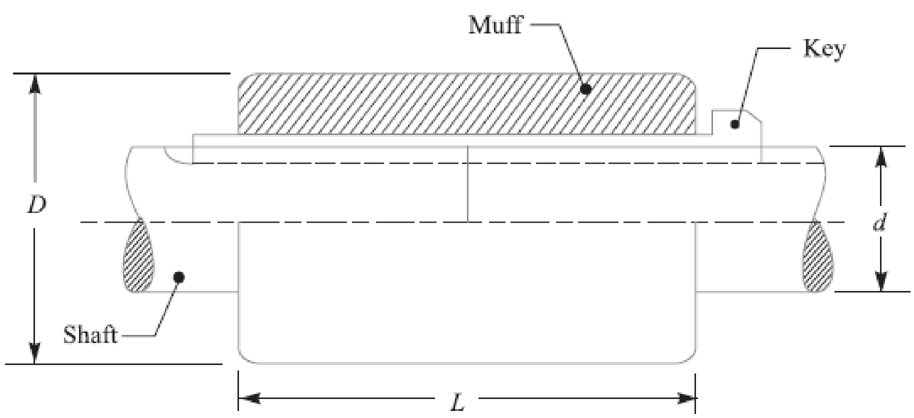


Fig. 2.19. Sleeve or muff coupling.

Let T = Torque to be transmitted by the coupling, and
τ_c = Permissible shear stress for the material of the sleeve which is cast rion.
The safe value of shear stress for cast iron may be taken as 14 MPa.
We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \times D^3\ (1 - k^4) \qquad \ldots (\because\ k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. ***Design for key***
The key for the coupling may be designed in the similar way as discussed in Art. 2.2.9. The width and thickness of the coupling key is obtained from the proportions. The length of the coupling key is atleast equal to the length of the sleeve (*i.e.* 3.5 d). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\ d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{... (Considering crushing of the key)}$$

Example 2.13. *Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.*

Given Data:

P = 40 kW = 40 × 10^3 W
N = 350 r.p.m.
τ_s = 40 MPa = 40 N/mm^2
σ_{cs} = 80 MPa = 80 N/mm^2
τ_c = 15 MPa = 15 N/mm^2

To Find:

The muff coupling is shown in Fig. 2.19. It is designed as discussed below :

1.*Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\ \pi N} = \frac{40 \times 10^3 \times 60}{2\ \pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

$\therefore \quad d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3$ or d = 52 say 55 mm **Ans.**

2. *Design for sleeve*

We know that outer diameter of the muff,

D= 2d + 13 mm = 2 × 55 + 13 = 123 say 125 mm **Ans.**

and length of the muff,
L = 3.5 d = 3.5 × 55 = 192.5 say 195 mm **Ans.**

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (*T*),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125} \right]$$

$$= 370 \times 10^3 \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm2, therefore the design of muff is safe.

3. *Design for key*
From Table 2.2, we find that for a shaft of 55 mm diameter,
Width of key, w = 18 mm **Ans.**
Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

∴ Thickness of key, $t = w$ = 18 mm **Ans.**
We know that length of key in each shaft,
$l = L / 2 = 195 / 2 = 97.5$ mm **Ans.**
Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (*T*),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (*T*),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

2.3.5 Clamp or Compression Coupling

It is also known as ***split muff coupling.*** In this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig. 2.20. The halves of the muff are made of cast iron. The shaft ends are made to abutt each other and a single key is fitted directly in the keyways of both the shafts. One-half of the muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings. This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling. The usual proportions of the muff for the clamp or compression coupling are :
Diameter of the muff or sleeve, $D = 2d + 13$ mm
Length of the muff or sleeve, $L = 3.5\ d$
where d = Diameter of the shaft

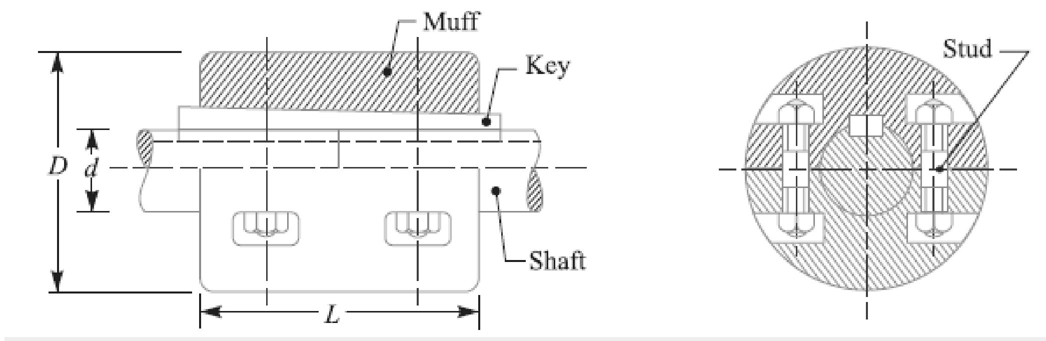


Fig. 2.20. Clamp or compression coupling.

In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1. *Design of muff and key*

The muff and key are designed in the similar way as discussed in muff coupling (Art. 13.14).

2. *Design of clamping bolts*

Let T = Torque transmited by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,

μ= Coefficient of friction between the muff and shaft, and

L = Length of muff.

We know that the force exerted by each bolt

$$= \mu \times \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n$$

and the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu\,(d_b)^2 \sigma_t \times n \times d$$

$$= \frac{\pi}{4}(d_b)^2 \sigma_t$$

∴ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

∴ Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2}\pi d \times L$$

From this relation, the root diameter of the bolt (*db*) may be evaluated.

Example 2.14. *Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.*
Given :

$P = 30 \text{ kW} = 30 \times 10^3 \text{ W}$
$N = 100$ r.p.m.
$\tau = 40 \text{ MPa} = 40 \text{ N/mm}^2$
$n = 6$
$\sigma_t = 70 \text{ MPa} = 70 \text{ N/mm}^2$
$\mu = 0.3$

1. *Design for shaft*
Let d = Diameter of shaft.
We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3 \text{ or } d = 71.4 \text{ say } 75 \text{ mm } \textbf{Ans.}$$

2. *Design for muff*
We know that diameter of muff,
$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163$ say 165 mm **Ans.**
and total length of the muff,
$L = 3.5\, d = 3.5 \times 75 = 262.5$ mm **Ans.**

3. *Design for key*
The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :
Width of key, $w = 22$ mm **Ans.**
Thickness of key, $t = 14$ mm **Ans.**
and length of key = Total length of muff = 262.5 mm **Ans.**

4. *Design for bolts*
Let db = Root or core diameter of bolt.
We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\, (d_b)^2\ 70 \times 6 \times 75 = 5830(d_b)^2$$

$$\therefore \quad (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

From Table **11.1,** we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

2.3.6 Flange Coupling

A flange coupling usually applies to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flange has a projected portion and the other flange has a corresponding recess.

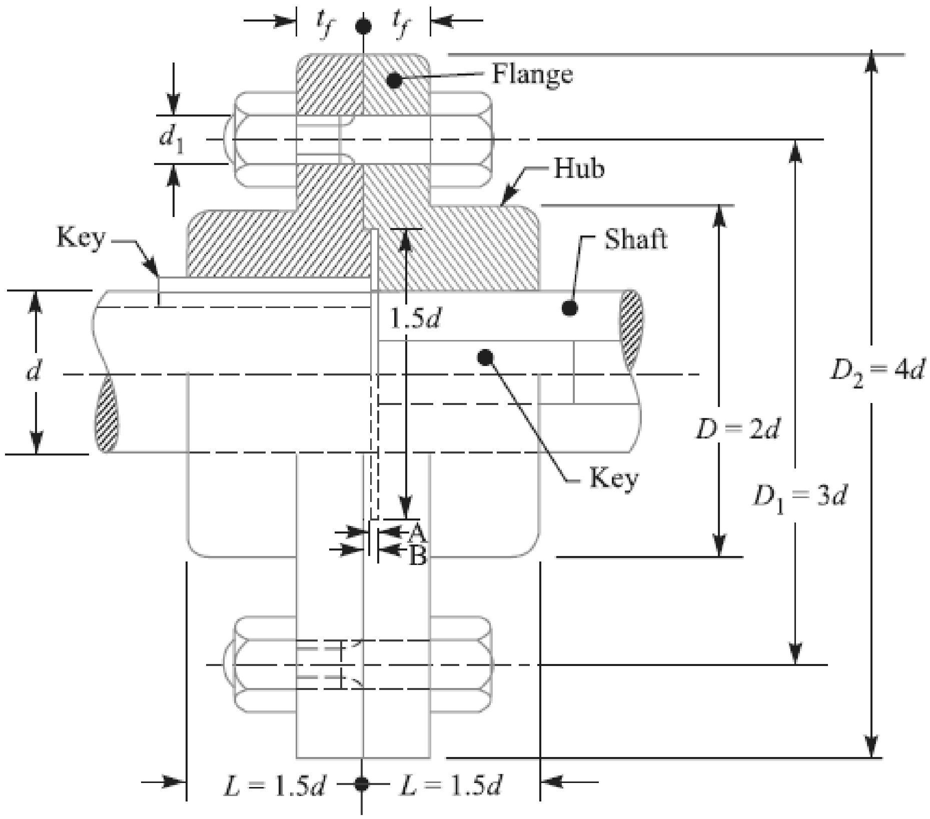


Fig. 2.21. Unprotected type flange coupling.

This helps to bring the shafts into line and to maintain alignment. The two flanges are coupled together by means of bolts and nuts. The flange coupling is adopted to heavy loads and hence it is used on large shafting. The flange couplings are of the following three types :

1. *Unprotected type flange coupling*. In an unprotected type flange coupling, as shown in Fig. 2.21, each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts. Generally, three, four or six bolts are used. The keys are staggered at right angle along the circumference of the shafts in order to divide the weakening effect caused by keyways. The usual proportions for an unprotected type cast iron flange couplings, as shown in

Fig. 2.21, are as follows :

If *d* is the diameter of the shaft or inner diameter of the hub, then Outside diameter of hub,

$D = 2d$

Length of hub, $L = 1.5d$

Pitch circle diameter of bolts,

$D1 = 3d$

Outside diameter of flange,

$D2 = D1 + (D1 - D) = 2D1 - D = 4d$

Thickness of flange, $tf = 0.5d$

Number of bolts = 3, for *d* upto 40 mm

= 4, for *d* upto 100 mm

= 6, for *d* upto 180 mm

2. *Protected type flange coupling*. In a protected type flange coupling, as shown in Fig. 2.22, the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.

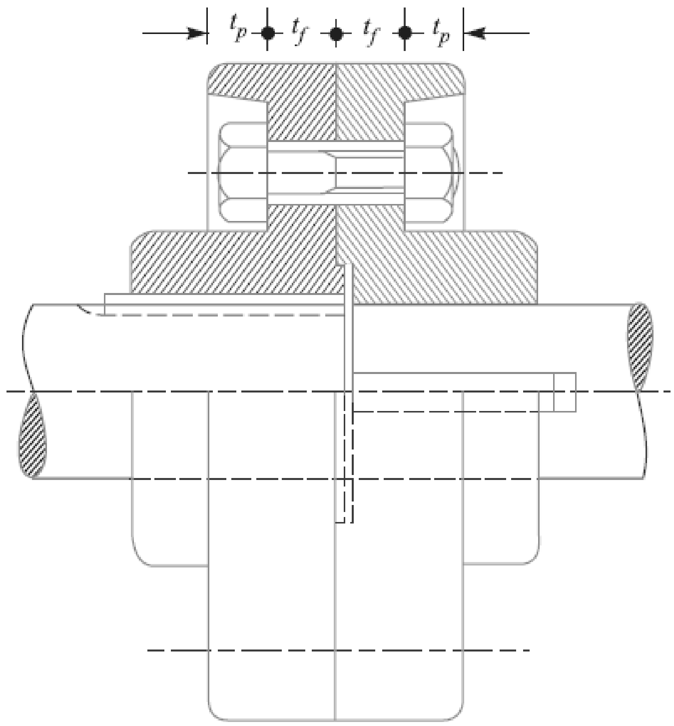


Fig. 2.22. Protective type flange coupling.

3. ***Marine type flange coupling*.** In a marine type flange coupling, the flanges are forged integral with the shafts as shown in Fig. 2.23. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft.
The number of bolts may be choosen from the following table.

Table 2.3. Number of bolts for marine type flange coupling.
[According to IS : 3653 – 1966 (Reaffirmed 1990)]

Shaft diameter (mm)	35 to 55	56 to 150	151 to 230	231 to 390	Above 390
No. of bolts	4	6	8	10	12

The other proportions for the marine type flange coupling are taken as follows :
Thickness of flange = d / 3
Taper of bolt = 1 in 20 to 1 in 40
Pitch circle diameter of bolts, $D1 = 1.6\ d$
Outside diameter of flange, $D2 = 2.2\ d$

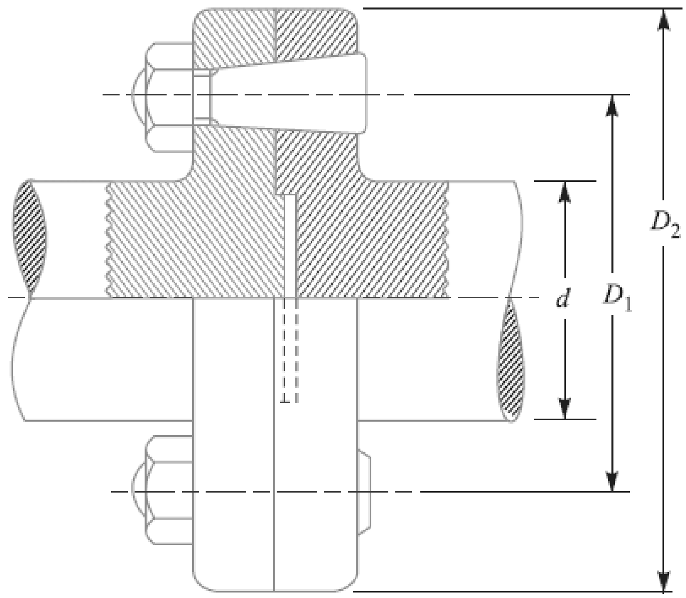


Fig. 2.23. Marine type flange coupling.

2.3.7 Design of Flange Coupling

Consider a flange coupling as shown in Fig. 2.22 and Fig. 2.23.

Let d = Diameter of shaft or inner diameter of hub,
D = Outer diameter of hub,
$d1$ = Nominal or outside diameter of bolt,
$D1$ = Diameter of bolt circle,
n = Number of bolts,
t_f = Thickness of flange,
τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively
τ_c = Allowable shear stress for the flange material *i.e.* cast iron,
σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

The flange coupling is designed as discussed below :

1. *Design for hub*

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked. The length of hub (L) is taken as 1.5 d.

2. *Design for key*

The key is designed with usual proportions and then checked for shearing and crushing stresses.The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.**3.**

3. *Design for flange*

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the troque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

$$\text{Load on each bolt} = \frac{\pi}{4} (d_1)^2 \tau_b$$

∴ Total load on all the bolts

$$= \frac{\pi}{4} (d_1)^2 \tau_b \times n$$

and torque transmitted,

$$T = \frac{\pi}{4} (d_1)^2 \tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts

$$= n \times d_1 \times t_f$$

and crushing strength of all the bolts

$$= (n \times d_1 \times t_f) \sigma_{cb}$$

∴ Torque,

$$T = (n \times d_1 \times t_f \times \sigma_{cb}) \frac{D_1}{2}$$

From this equation, the induced crushing stress in the bolts may be checked.

Example 2.15. *Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :*
Shear stress for shaft, bolt and key material = 40 MPa
Crushing stress for bolt and key = 80 MPa
Shear stress for cast iron = 8 MPa
Draw a neat sketch of the coupling.

Given : $P = 15$ kW $= 15 \times 10^3$ W
$N = 900$ r.p.m.
Service factor = 1.35
$\tau_s = \tau_b = \tau_k = 40$ MPa = 40 N/mm^2
$\sigma_{cb} = \sigma_{ck} = 80$ MPa = 80 N/mm^2
$\tau_c = 8$ MPa = 8 N/mm^2
The protective type flange coupling is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2 \pi N} = \frac{15 \times 10^3 \times 60}{2 \pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

$$T_{max} = 1.35 \times 159.13 = 215 \text{ N-m} = 215 \times 10^3 \text{ N-mm}$$

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the hub,

$$D = 2d = 2 \times 35 = 70 \text{ mm } \textbf{Ans.}$$

and length of hub, $L = 1.5\, d = 1.5 \times 35 = 52.5$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

$$\therefore \quad \tau_c = 215 \times 10^3 / 63\,147 = 3.4 \text{ N/mm}^2 = 3.4 \text{ MPa}$$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From Table 2.2, we find that for a shaft of 35 mm diameter,

Width of key, w = 12 mm **Ans.**

and thickness of key, $t = w$ = 12 mm **Ans.**

The length of key (l) is taken equal to the length of hub.

∴ $l = L$ = 52.5 mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (*Tmax*),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\, \tau_k$$

$$\therefore \quad \tau_k = 215 \times 10^3 / 11\,025 = 19.5 \text{ N/mm}^2 = 19.5 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\, \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 215 \times 10^3 / 5512.5 = 39 \text{ N/mm}^2 = 39 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of flange (*tf*) is taken as 0.5 d.

∴ $t_f = 0.5\, d = 0.5 \times 35 = 17.5$ mm **Ans.**

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (*Tmax*),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\,713\, \tau_c$$

$$\therefore \quad \tau_c = 215 \times 10^3 / 134\,713 = 1.6 \text{ N/mm}^2 = 1.6 \text{ MPa}$$

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. *Design for bolts*

Let *d*1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

$n = 3$

and pitch circle diameter of bolts,

$D1 = 3d = 3 \times 35 = 105$ mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (*Tmax*),

$$215 \times 10^3 = \frac{\pi}{4}(d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$$(d_1)^2 = 215 \times 10^3/4950 = 43.43 \text{ or } d_1 = 6.6 \text{ mm}$$

Assuming coarse threads, the nearest standard size of bolt is M 8. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$D2 = 4\ d = 4 \times 35 = 140$ mm **Ans.**

Thickness of the protective circumferential flange,

$tp = 0.25\ d = 0.25 \times 35 = 8.75$ say 10 mm **Ans.**

Example 2.16. *Design a rigid flange coupling to transmit a torque of 250 N-m between two coaxial shafts. The shaft is made of alloy steel, flanges out of cast iron and bolts out of steel. Four bolts are used to couple the flanges. The shafts are keyed to the flange hub. The permissible stresses are given below:*

Shear stress on shaft =100 MPa
Bearing or crushing stress on shaft =250 MPa
Shear stress on keys =100 MPa
Bearing stress on keys =250 MPa
Shearing stress on cast iron =200 MPa
Shear stress on bolts =100 MPa

After designing the various elements, make a neat sketch of the assembly indicating the important dimensions. The stresses developed in the various members may be checked if thumb rules are used for fixing the dimensions.

Given :

$T = 250$ N-m $= 250 \times 10^3$ N-mm
$n = 4$; $\tau_s = 100$ MPa $= 100$ N/mm^2
$\sigma_{cs} = 250$ MPa $= 250$ N/mm^2
$\tau_k = 100$ MPa $= 100$ N/mm^2
$\sigma_{ck} = 250$ MPa $= 250$ N/mm^2
$\tau_c = 200$ MPa $= 200$ N/mm^2
$\tau_b = 100$ MPa $= 100$ N/mm^2

To Find : *Design a rigid flange coupling*

Design for hub

First of all, let us find the diameter of the shaft (*d*). We know that the torque transmitted by the shaft (*T*),

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 100 \times d^3 = 19.64\ d^3$$

∴ $d^3 = 250 \times 10^3 / 19.64 = 12\ 729$ or $d = 23.35$ say 25 mm **Ans.**

We know that the outer diameter of the hub,

$D = 2\ d = 2 \times 25 = 50$ mm

and length of hub, $L = 1.5\ d = 1.5 \times 25 = 37.5$ mm

Let us now check the induced shear stress in the hub by considering it as a hollow shaft. We know that the torque transmitted (T),

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(50)^4 - (25)^4}{50}\right] = 23\ 013\ \tau_c$$

$$\therefore \quad \tau_c = 250 \times 10^3/23\ 013 = 10.86 \text{ N/mm}^2 = 10.86 \text{ MPa}$$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than 200 MPa, therefore the design for hub is safe.

2. *Design for key*

From Table 2.2, we find that the proportions of key for a 25 mm diameter shaft are :

Width of key, $w = 10$ mm **Ans.**

and thickness of key, $t = 8$ mm **Ans.**

The length of key (l) is taken equal to the length of hub,

∴ $l = L = 37.5$ mm **Ans.**

Let us now check the induced shear and crushing stresses in the key. Considering the key in shearing. We know that the torque transmitted (T),

$$250 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 37.5 \times 10 \times \tau_k \times \frac{25}{2} = 4688\ \tau_k$$

$$\therefore \quad \tau_k = 250 \times 10^3 / 4688 = 53.3 \text{ N/mm}^2 = 53.3 \text{ MPa}$$

Considering the key in crushing. We know that the torque transmitted (T),

$$250 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 37.5 \times \frac{8}{2} \times \sigma_{ck} \times \frac{25}{2} = 1875\ \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 250 \times 10^3 / 1875 = 133.3 \text{ N/mm}^2 = 133.3 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the given stresses, therefore the design of key is safe.

3. *Design for flange*

The thickness of the flange (tf) is taken as 0.5 d.

∴ $t_f = 0.5\ d = 0.5 \times 25 = 12.5$ mm **Ans.**

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear. We know that the torque transmitted (T),

$$250 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (50)^2}{2} \times \tau_c \times 12.5 = 49\ 094\ \tau_c$$

$$\tau_c = 250 \times 10^3 / 49\ 094 = 5.1 \text{ N/mm}^2 = 5.1 \text{ MPa}$$

Since the induced shear stress in the flange of cast iron is less than 200 MPa, therefore design of flange is safe.

4. *Design for bolts*

Let $d1$ = Nominal diameter of bolts.

We know that the pitch circle diameter of bolts,

∴ $D1 = 3\ d = 3 \times 25 = 75$ mm **Ans.**

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$250 \times 10^3 = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2 100 \times 4 \times \frac{75}{2} = 11\,780\,(d_1)^2$$

∴ $(d1)^2 = 250 \times 10^3 / 11\,780 = 21.22$ or $d1 = 4.6$ mm

Assuming coarse threads, the nearest standard size of the bolt is M 6. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$D2 = 4\,d = 4 \times 25 = 100$ mm **Ans.**

Thickness of the protective circumferential flange,

$t_p = 0.25\,d = 0.25 \times 25 = 6.25$ mm **Ans.**

2.3.8 Flexible Coupling

flexible coupling is used to join the abutting ends of shaftswhen they are not in exact alignment. In the case of a direct coupled drive from a prime mover to an electric generator, we should have four bearings at a comparatively close distance. In such a case and in many others, as in a direct electric drive from an electric motor to a machine tool, a flexible coupling is used so as to permit an axial misalignemnt of the shaft without undue absorption of the power which the shaft are transmitting. Following are the different types of flexible couplings :

1. Bushed pin flexible coupling, **2.** Oldham's coupling, and **3.** Universal coupling.

2.3.9 Bushed-pin Flexible Coupling

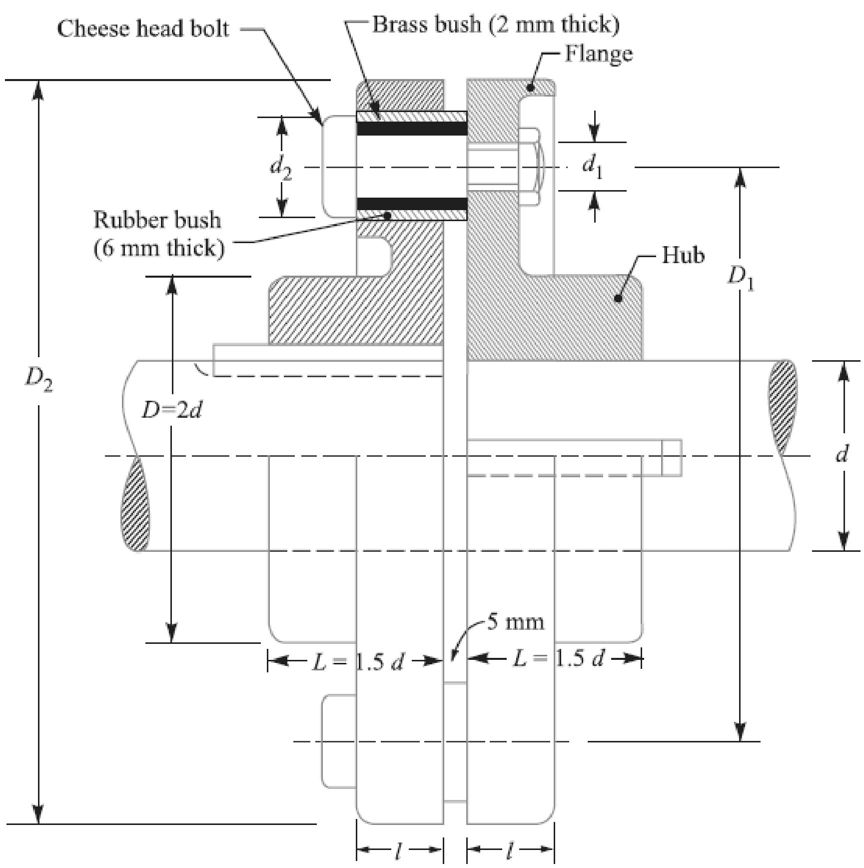


Fig. 2.24. Bushed-pin flexible coupling.

A bushed-pin flexible coupling, as shown in Fig. 2.24, is a modification of the rigid type of flange coupling. The coupling bolts are known as pins. The rubber or leather bushes are used over the pins. The two halves of the coupling are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of the coupling. There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.

In designing the bushed-pin flexible coupling, the proportions of the rigid type flange coupling are modified. The main modification is to reduce the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm2. In order to keep the low bearing pressure, the pitch circle diameter and the pin size is increased.

Let l = Length of bush in the flange,

$d2$ = Diameter of bush,

p_b = Bearing pressure on the bush or pin,

n = Number of pins, and

$D1$ = Diameter of pitch circle of the pins.

We know that bearing load acting on each pin,

$W = p_b \times d_2 \times l$

∴ Total bearing load on the bush or pins

$= W \times n = p_b \times d_2 \times l \times n$

and the torque transmitted by the coupling,

$$T = W \times n \left(\frac{D_1}{2}\right) = p_b \times d_2 \times l \times n \left(\frac{D_1}{2}\right)$$

The threaded portion of the pin in the right hand flange should be a tapping fit in the coupling hole to avoid bending stresses.

The threaded length of the pin should be as small as possible so that the direct shear stress can be taken by the unthreaded neck.

Direct shear stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2}$$

Since the pin and the rubber or leather bush is not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin as shown in Fig. 2.25. The bush portion of the pin acts as a cantilever beam of length l. Assuming a uniform distribution of the load W along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5 \text{ mm}\right)$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{W\left(\frac{l}{2} + 5 \text{ mm}\right)}{\frac{\pi}{32}(d_1)^3}$$

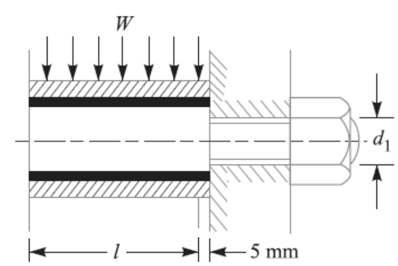


Fig. 2.25.

Since the pin is subjected to bending and shear stresses, therefore the design must be checked either for the maximum principal stress or maximum shear stress by the following relations :

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

and the maximum shear stress on the pin

$$= \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

The value of maximum principal stress varies from 28 to 42 MPa.

Example 2.17. *Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 r.p.m. The overall torque is 20 percent more than mean torque.*
The material properties are as follows :
(*a*) *The allowable shear and crushing stress for shaft and key material is 40 MPa and 80 MPa respectively.*
(*b*) *The allowable shear stress for cast iron is 15 MPa.*
(*c*) *The allowable bearing pressure for rubber bush is 0.8 N/mm*^{2}.
(*d*) *The material of the pin is same as that of shaft and key.*
Draw neat sketch of the coupling.

Given : P = 32 kW = 32 × 10^3 W
N = 960 r.p.m.
$Tmax$ = 1.2 $Tmean$
$\tau_s = \tau_k$ = 40 MPa= 40 N/mm2
$\sigma_{cs} = \sigma_{ck}$ = 80 MPa = 80 N/mm2
τ_c = 15 MPa = 15 N/mm2
P_b = 0.8 N/mm2

The bushed-pin flexible coupling is designed as discussed below :
1. *Design for pins and rubber bush*
First of all, let us find the diameter of the shaft (d). We know that the mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \text{ N-m}$$

and the maximum or overall torque transmitted,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

We also know that the maximum torque transmitted by the shaft (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 382 \times 10^3 / 7.86 = 48.6 \times 10^3 \text{ or } d = 36.5 \text{ say } 40 \text{ mm}$$

We have discussed in rigid type of flange coupling that the number of bolts for 40 mm diameter shaft are 3. In the flexible coupling, we shall use the number of pins (n) as 6.

$$\therefore \quad \text{Diameter of pins, } d_1 = \frac{0.5\, d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm}$$

In order to allow for the bending stress induced due to the compressibility of the rubber bush, the diameter of the pin (*d*1) may be taken as 20 mm. **Ans.**

The length of the pin of least diameter *i.e.* *d*1 = 20 mm is threaded and secured in the right hand coupling half by a standard nut and washer. The enlarged portion of the pin which is in the left hand coupling half is made of 24 mm diameter. On the enlarged portion, a brass bush of thickness 2 mm is pressed. A brass bush carries a rubber bush. Assume the thickness of rubber bush as 6 mm.

∴ Overall diameter of rubber bush,

$d2 = 24 + 2 \times 2 + 2 \times 6 = 40$ mm **Ans.**

and diameter of the pitch circle of the pins,

$D1 = 2\,d + d2 + 2 \times 6 = 2 \times 40 + 40 + 12 = 132$ mm **Ans.**

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$W = p_b \times d2 \times l = 0.8 \times 40 \times l = 32\,l$ N

and the maximum torque transmitted by the coupling (*Tmax*),

$$382 \times 10^3 = W \times n \times \frac{D_1}{2} = 32\,l \times 6 \times \frac{132}{2} = 12\,672\,l$$

$$\therefore \quad l = 382 \times 10^3 / 12\,672 = 30.1 \text{ say } 32 \text{ mm}$$

$$\text{and} \quad W = 32\,l = 32 \times 32 = 1024 \text{ N}$$

∴ Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2} = \frac{1024}{\frac{\pi}{4}(20)^2} = 3.26 \text{ N/mm}^2$$

Since the pin and the rubber bush are not rigidly held in the left hand flange, therefore the tangential load (*W*) at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load (*W*) along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right) = 1024\left(\frac{32}{2} + 5\right) = 21\,504 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3 = 785.5 \text{ mm}^3$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{21\,504}{785.5} = 27.4 \text{ N/mm}^2$$

∴ Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[27.4 + \sqrt{(27.4)^2 + 4\,(3.26)^2}\right]$$

$$= 13.7 + 14.1 = 27.8 \text{ N/mm}^2$$

and maximum shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(27.4)^2 + 4\,(3.26)^2}\right] = 14.1 \text{ N/mm}^2$$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. *Design for hub*

We know that the outer diameter of the hub,

$D = 2\,d = 2 \times 40 = 80$ mm

and length of hub, $L = 1.5\,d = 1.5 \times 40 = 60$ mm

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted ($Tmax$),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(80)^4 - (40)^4}{80}\right] = 94.26 \times 10^3\,\tau_c$$

$\tau_c = 382 \times 10^3 / 94.26 \times 10^3 = 4.05$ N/mm^2 = 4.05 MPa

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\,\tau_k$), therefore a square key may be used. From Table 2.2, we find that for a shaft of 40 mm diameter,

Width of key, $w = 14$ mm **Ans.**

and thickness of key, $t = w = 14$ mm **Ans.**

The length of key (L) is taken equal to the length of hub, *i.e.*

$L = 1.5\,d = 1.5 \times 40 = 60$ mm

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted ($Tmax$),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 60 \times 14 \times \tau_k \times \frac{40}{2} = 16\,800\,\tau_k$$

$$\therefore \quad \tau_k = 382 \times 10^3/16\,800 = 22.74 \text{ N/mm}^2 = 22.74 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2} = 8400\,\sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 382 \times 10^3/8400 = 45.48 \text{ N/mm}^2 = 45.48 \text{ MPa}$$

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. *Design for flange*

The thickness of flange (t_f) is taken as 0.5 d.

$\therefore \quad t_f = 0.5\,d = 0.5 \times 40 = 20$ mm

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted ($Tmax$),

$$382 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (80)^2}{2} \times \tau_c \times 20 = 201 \times 10^3\,\tau_c$$

$$\therefore \quad \tau_c = 382 \times 10^3 / 201 \times 10^3 = 1.9 \text{ N/mm}^2 = 1.9 \text{ MPa}$$

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.

2.3.10 Oldham Coupling

It is used to join two shafts which have lateral mis-alignment. It consists of two flanges *A* and *B* with slots and a central floating part *E* with two tongues *T*1 and *T*2 at right angles as shown in Fig. 2.26. The central floating part is held by means of a pin passing through the flanges and the floating part. The tongue *T*1 fits into the slot of flange *A* and allows for 'to and fro' relative motion of the shafts, while the tongue *T*2 fits into the slot of the flange *B* and allows for vertical relative motion of the parts. The resultant of these two components of motion will accommodate lateral misalignment of the shaft as they rotate.

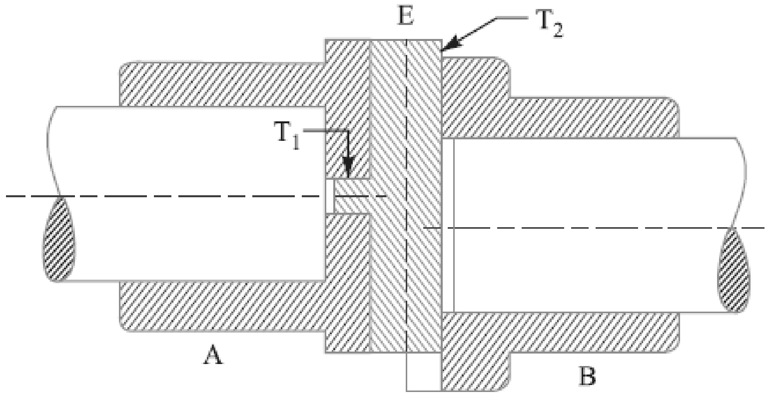


Fig. 2.26. Oldham coupling.

2.3.11 Universal (or Hooke's) Coupling

A universal or Hooke's coupling is used to connect two shafts whose axes intersect at a small angle. The inclination of the two shafts may be constant, but in actual practice, it varies when the motion is transmitted from one shaft to another. The main application of the universal or Hooke's coupling is found in the transmission from the gear box to the differential or back axle of the automobiles. In such a case, we use two Hooke's coupling, one at each end of the propeller shaft, connecting the gear box at one end and the differential on the other end. A Hooke's coupling is also used for transmission of power to different spindles of multiple drilling machine. It is used as a knee joint in milling machines. In designing a universal coupling, the shaft diameter and the pin diameter is obtained as discussed below. The other dimensions of the coupling are fixed by proportions as shown in Fig.

Let d = Diameter of shaft,

dp = Diameter of pin, and

$\tau$and τ_1 = Allowable shear stress for the material of the shaft and pin respectively.

We know that torque transmitted by the shafts,

$$T = \frac{\pi}{16} \times \tau \times d^3$$

From this relation, the diameter of shafts may be determined.

Since the pin is in double shear, therefore the torque transmitted.

$$T = 2 \times \frac{\pi}{4} (d_p)^2 \tau_1 \times d$$

From this relation, the diameter of pin may be determined.

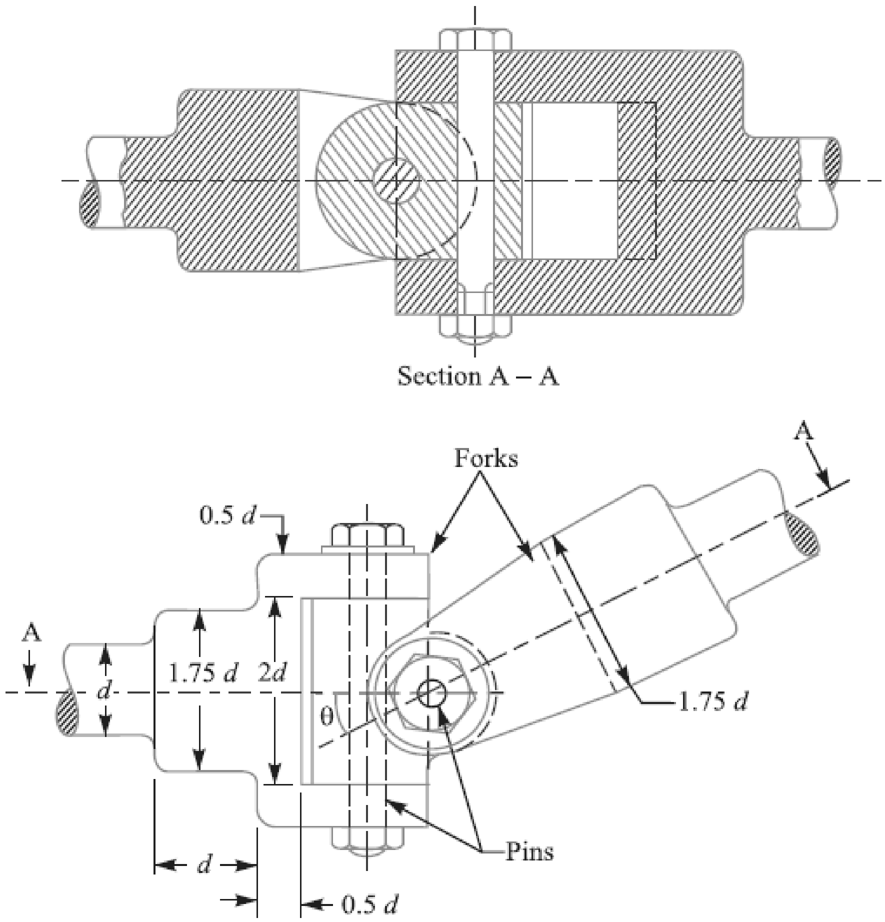


Fig. 2.27. Universal (or Hooke's) coupling.

EXERCISES

1. A shaft running at 400 r.p.m. transmits 10 kW. Assuming allowable shear stress in shaft as 40 MPa, find the diameter of the shaft.**[Ans. 35 mm]**

2. A hollow steel shaft transmits 600 kW at 500 r.p.m. The maximum shear stress is 62.4 MPa. Find the outside and inside diameter of the shaft, if the outer diameter is twice of inside diameter, assuming that the maximum torque is 20% greater than the mean torque.**[Ans. 100 mm ; 50 mm]**

3. A hollow shaft for a rotary compressor is to be designed to transmit a maximum torque of 4750 N-m. The shear stress in the shaft is limited to 50 MPa. Determine the inside and outside diameters of the shaft, if the ratio of the inside to the outside diameter is 0.4. **[Ans. 35 mm ; 90 mm]**

4. A motor car shaft consists of a steel tube 30 mm internal diameter and 4 mm thick. The engine develops 10 kW at 2000 r.p.m. Find the maximum shear stress in the tube when the power is transmitted through a 4 : 1 gearing. **[Ans. 30 MPa]**

5. A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static loads consisting of a bending moment of 10 kN-m and a torsional moment of 30 kN-m. Determine the diameter of the shaft using two different theories of failure and assuming a factor of safety of 2.**[Ans. 100 mm]**

6. A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The allowable shear stress for the material of the shaft is 42 MPa. If the shaft carries a central load of 900 N and is simply supported between bearing 3 metre apart, determine the diameter of the shaft. The maximum tensile or compressive stress is not to exceed 56 MPa. **[Ans. 50 mm]**

7. Two 400 mm diameter pulleys are keyed to a simply supported shaft 500 mm apart. Each pulley is 100 mm from its support and has horizontal belts, tension ratio being 2.5. If the shear stress is to be limited to 80 MPa while transmitting 45 kW at 900 r.p.m., find the shaft diameter if it is to be used for the input-output belts being on the same or opposite sides. **[Ans. 40 mm]**

8. A cast gear wheel is driven by a pinion and transmits 100 kW at 375 r.p.m. The gear has 200 machine cut teeth having 20° pressure angle and is mounted at the centre of a 0.4 m long shaft. The gear weighs 2000 N and its pitch circle diameter is 1.2 m. Design the gear shaft. Assume that the axes of the gear and pinion lie in the same horizontal plane. **[Ans. 80 mm]**

9. Fig. 2.28 shows a shaft from a hand-operated machine. The frictional torque in the journal bearings at *A* and *B* is 15 N-m each. Find the diameter (*d*) of the shaft (on which the pulley is mounted) using maximum distortion energy criterion. The shaft material is 40 C 8 steel for which the yield stress in tension is 380 MPa and the factor of safety is 1.5. **[Ans. 20 mm]**

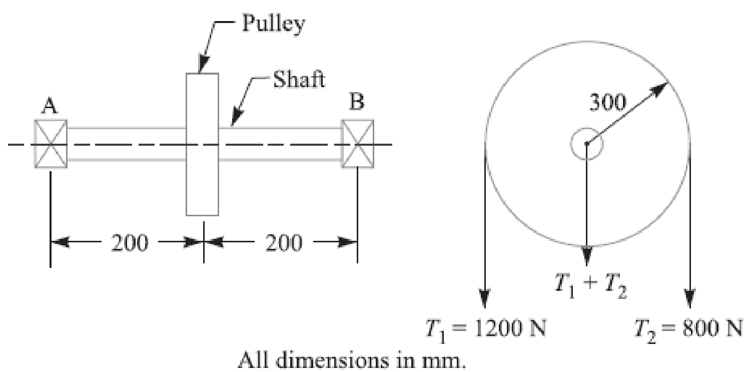


Fig. 2.28

10. A horizontal shaft *AD* supported in bearings at *A* and *B* and carrying pulleys at *C* and *D* is to transmit 75 kW at 500 r.p.m. from drive pulley *D* to off-take pulley *C*, as shown in Fig. 2.29. Calculate the diameter of shaft. The data given is : *P*1 = 2 *P*2 (both horizontal), *Q*1 = 2 *Q*2 (both vertical), radius of pulley *C* = 220 mm, radius of pulley *D* = 160 mm, allowable shear stress = 45 MPa.

[Ans. 100 mm]

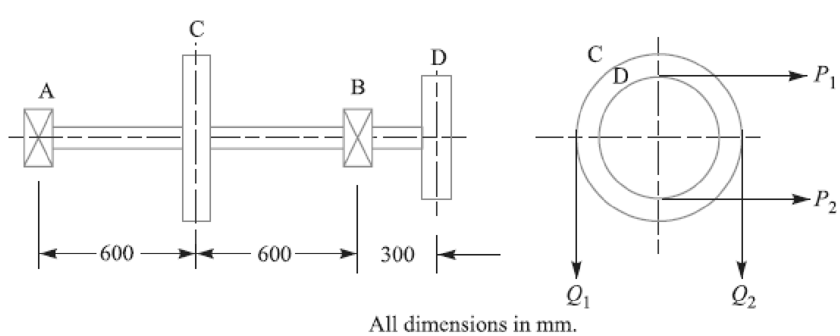


Fig. 2.29

11. A shaft made of steel receives 7.5 kW power at 1500 r.p.m. A pulley mounted on the shaft as shown in Fig. 2.30 has ratio of belt tensions 4. The gear forces are as follows :

Ft = 1590 N; Fr = 580 N

Design the shaft diameter by maximum shear stress theory. The shaft material has the following properties :

Ultimate tensile strength = 720 MPa; Yield strength = 380 MPa; Factor of safety = 1.5.

[Ans. 20 mm]

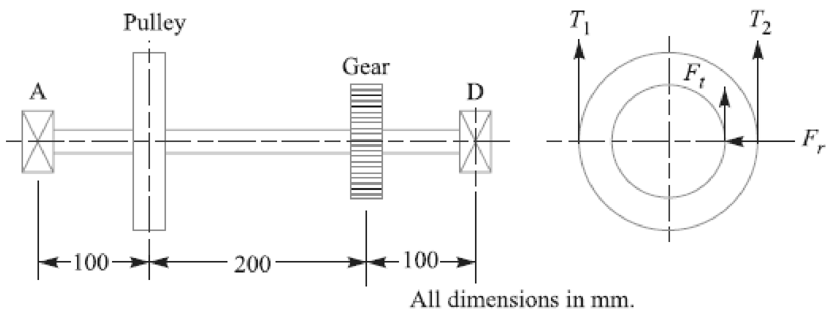


Fig. 2.30

12. A shaft 80 mm diameter transmits power at maximum shear stress of 63 MPa. Find the length of a 20 mm wide key required to mount a pulley on the shaft so that the stress in the key does not exceed 42 MPa.**[Ans. 152 mm]**

13. A shaft 30 mm diameter is transmitting power at a maximum shear stress of 80 MPa. If a pulley is connected to the shaft by means of a key, find the dimensions of the key so that the stress in the key is not to exceed 50 MPa and length of the key is 4 times the width. **[Ans. *l* = 126 mm]**

14. A steel shaft has a diameter of 25 mm. The shaft rotates at a speed of 600 r.p.m. and transmits 30 kW through a gear. The tensile and yield strength of the material of shaft are 650 MPa and 353 MPa respectively. Taking a factor of safety 3, select a suitable key for the gear. Assume that the key and shaft are made of the same material. **[Ans. *l* = 102 mm]**

15. Design a muff coupling to connect two shafts transmitting 40 kW at 120 r.p.m. The permissible shear and crushing stress for the shaft and key material (mild steel) are 30 MPa and 80 MPa respectively. The material of muff is cast iron with permissible shear stress of 15 MPa. Assume that the maximum torque transmitted is 25 per cent greater than the mean torque.

[Ans. *d* = 90 mm ; *w* = 28 mm, *t* = 16 mm, *l* = 157.5 mm ; *D* = 195 mm, *L* = 315 mm]

16. Design a compression coupling for a shaft to transmit 1300 N-m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are 4. The permissible tensile stress for the bolts material is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3. **[Ans. *d* = 55 mm ; *D* = 125 mm ; *L* = 192.5 mm ; *db* = 24 mm]**

17. Design a cast iron protective flange coupling to connect two shafts in order to transmit 7.5 kW at 720 r.p.m. The following permissible stresses may be used :

Permissible shear stress for shaft, bolt and key material = 33 MPa

Permissible crushing stress for bolt and key material = 60 MPa

Permissible shear stress for the cast iron = 15 MPa **[Ans. *d* = 25 mm; *D* = 50 mm]**

18. A flanged protective type coupling is required to transmit 50 kW at 2000 r.p.m.. Find :

(*a*) Shaft diameters if the driving shaft is hollow with di / $d0$ = 0.6 and driven shaft is a solid shaft. Take τ= 100 MPa.

(*b*) Diameter of bolts, if the coupling uses four bolts. Take σ_c = σ_t = 70 MPa and τ= 25 MPa.

Assume pitch circle diameter as about 3 times the outside diameter of the hollow shaft.
(*c*) Thickness of the flange and diameter of the hub. Assume σ_c = 100 MPa and τ = 125 MPa.
(*d*) Make a neat free hand sketch of the assembled coupling showing a longitudinal sectional elevation with the main dimensions. The other dimensions may be assumed suitably.

19. A marine type flange coupling is used to transmit 3.75 MW at 150 r.p.m. The allowable shear stress in the shaft and bolts may be taken as 50 MPa. Determine the shaft diameter and the diameter of the bolts. **[Ans. 300 mm ; 56 mm]**

20. Design a bushed-pin type flexible coupling for connecting a motor shaft to a pump shaft for the following service conditions :
Power to be transmitted = 40 kW ; speed of the motor shaft = 1000 r.p.m. ; diameter of the motor shaft = 50 mm ; diameter of the pump shaft = 45 mm.
The bearing pressure in the rubber bush and allowable stress in the pins are to be limited to 0.45 N/mm^2 and 25 MPa respectively. **[Ans. d_1 = 20 mm; n = 6; d_2 = 40 mm ; l = 152 mm]**

UNIT III TEMPORARY AND PERMANENT JOINTS

3.1 Threaded fasteners

3.1.1 Introduction

A screw thread is formed by cutting a continuous helical groove on a cylindrical surface. A screw made by cutting a single helical groove on the cylinder is known as ***single threaded*** (or single-start) screw and if a second thread is cut in the space between the grooves of the first, a ***double threaded*** (or double-start) screw is formed. Similarly, triple and quadruple (*i.e.* multiple-start) threads may be formed. The helical grooves may be cut either ***right hand*** or ***left hand*.**

A screwed joint is mainly composed of two elements *i.e.* a bolt and nut. The screwed joints are widely used where the machine parts are required to be readily connected or disconnected without damage to the machine or the fastening. This may be for the purpose of holding or adjustment in assembly or service inspection, repair, or replacement or it may be for the manufacturing or assembly reasons.

Advantages and Disadvantages of Screwed Joints

Following are the advantages and disadvantages of the screwed joints.

Advantages

1. Screwed joints are highly reliable in operation.
2. Screwed joints are convenient to assemble and disassemble.
3. A wide range of screwed joints may be adopted to various operating conditions.
4. Screws are relatively cheap to produce due to standardisation and highly efficient manufacturing processes.

Disadvantages

The main disadvantage of the screwed joints is the stress concentration in the threaded portions which are vulnerable points under variable load conditions.

3.1.2 Important Terms Used in Screw Threads

The following terms used in screw threads, as shown in Fig. 3.1, are important from the subject point of view :

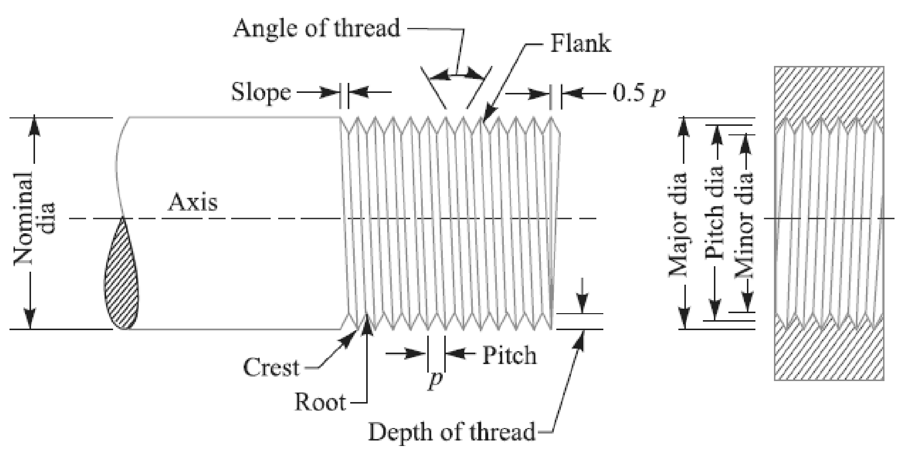


Fig. 3.1. Terms used in screw threads.

1. *Major diameter*. It is the largest diameter of an external or internal screw thread. The screw is specified by this diameter. It is also known as ***outside*** or ***nominal diameter*.**
2. *Minor diameter*. It is the smallest diameter of an external or internal screw thread. It is also known as ***core*** or ***root diameter*.**
3. *Pitch diameter*. It is the diameter of an imaginary cylinder, on a cylindrical screw thread, the surface of which would pass through the thread at such points as to make equal the width of the thread and the width of the spaces between the threads. It is also called an ***effective diameter*.** In a nut and bolt assembly, it is the diameter at which the ridges on the bolt are in complete touch with the ridges of the corresponding nut.
4. *Pitch*. It is the distance from a point on one thread to the corresponding point on the next. This is measured in an axial direction between corresponding points in the same axial plane.
Mathematically,
Pitch =1/ No. of threads per unit length of screw
5. *Lead*. It is the distance between two corresponding points on the same helix. It may also be defined as the distance which a screw thread advances axially in one rotation of the nut. Lead is equal to the pitch in case of single start threads, it is twice the pitch in double start, thrice the pitch in triple start and so on.
6. *Crest*. It is the top surface of the thread.
7. *Root*. It is the bottom surface created by the two adjacent flanks of the thread.
8. *Depth of thread*. It is the perpendicular distance between the crest and root.
9. *Flank*. It is the surface joining the crest and root.
10. *Angle of thread*. It is the angle included by the flanks of the thread.
11. *Slope.* It is half the pitch of the thread.

3.1.3 Forms of Screw Threads
The following are the various forms of screw threads.
1. *British standard whitworth (B.S.W.) thread*. This is a British standard thread profile and has coarse pitches. It is a symmetrical *V*-thread in which the angle between the flankes, measured in an axial plane, is 55°. These threads are found on bolts and screwed fastenings for special purposes. The various proportions of B.S.W. threads are shown in Fig. 3.2.

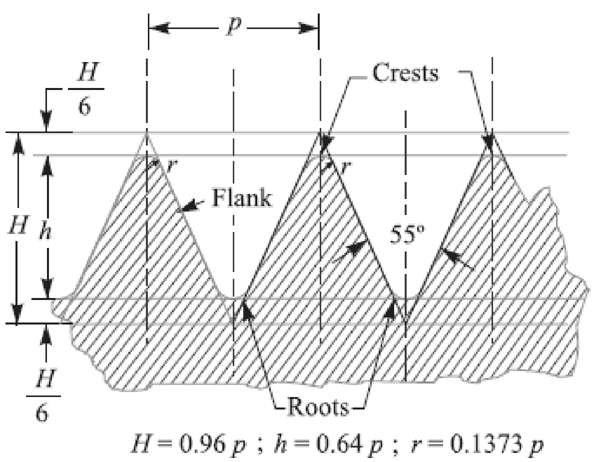


Fig. 3.2. British standard whitworth (B.S.W) thread.

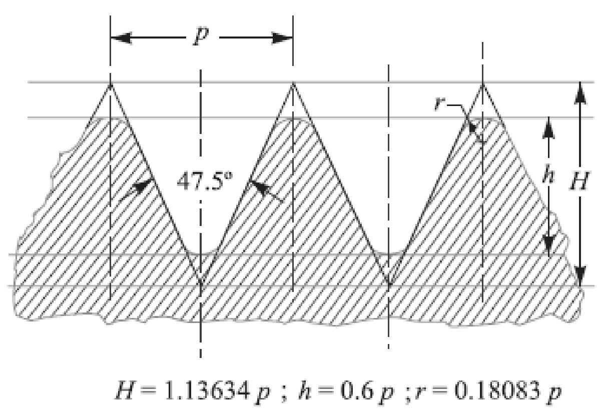


Fig. 3.3. British association (B.A.) thread.

The British standard threads with fine pitches (B.S.F.) are used where great strength at the root is required. These threads are also used for line adjustments and where the connected parts are subjected to increased vibrations as in aero and automobile work. The British standard pipe (B.S.P.) threads with fine pitches are used for steel and iron pipes and tubes carrying fluids. In external pipe threading, the threads are specified by the bore of the pipe.

2. ***British association (B.A.) thread*.** This is a B.S.W. thread with fine pitches. The proportions of the B.A. thread are shown in Fig. 3.3. These threads are used for instruments and other precision works.

3. ***American national standard thread*.** The American national standard or U.S. or Seller's thread has flat crests and roots. The flat crest can withstand more rough usage than sharp *V*-threads. These threads are used for general purposes *e.g*. on bolts, nuts, screws and tapped holes. The various

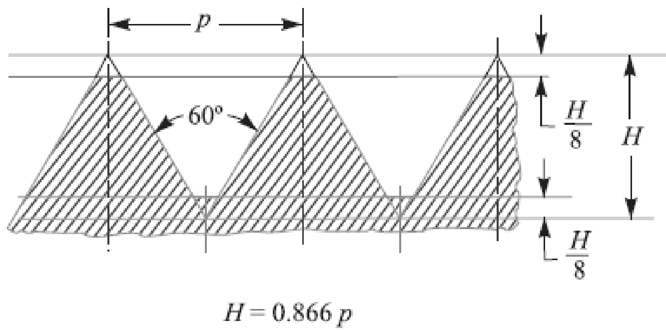


Fig. 3.4. American national standard thread.

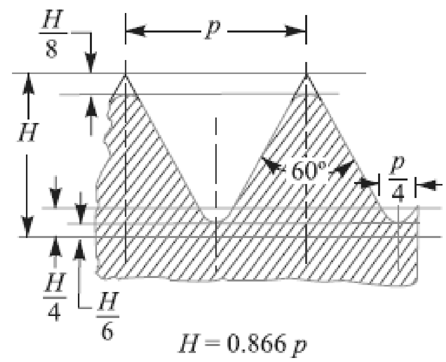


Fig. 3.5. Unified standard thread.

4. ***Unified standard thread*.** The three countries *i.e*., Great Britain, Canada and United States came to an agreement for a common screw thread system with the included angle of 60°, in order to facilitate the exchange of machinery. The thread has rounded crests and roots, as shown in Fig. 3.5.

5. ***Square thread*.** The square threads, because of their high efficiency, are widely used for transmission of power in either direction. Such type of threads are usually found on the feed mechanisms of machine tools, valves, spindles, screw jacks etc. The square threads are not so strong as V-threads but they offer less frictional resistance to motion than Whitworth threads. The pitch of the square thread is often taken twice that of a B.S.W. thread of the same diameter. The proportions of the thread are shown in Fig. 3.6.

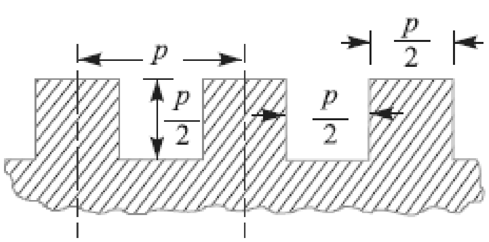


Fig. 3.6. Square thread.

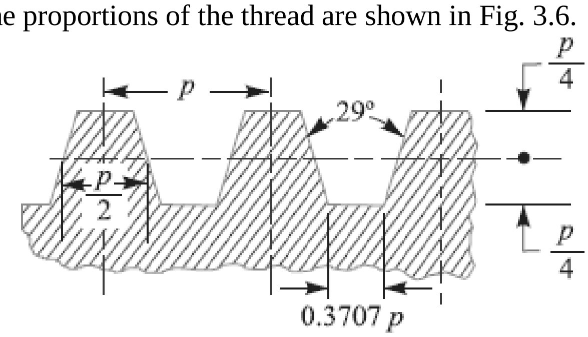


Fig. 3.7. Acme thread.

6. ***Acme thread*.** It is a modification of square thread. It is much stronger than square thread and can be easily produced. These threads are frequently used on screw cutting lathes, brass valves, cocks and bench vices. When used in conjunction with a split nut, as on the lead screw of a lathe, the tapered sides of the thread facilitate ready engagement and disengagement of the halves of the nut when required. The various proportions are shown in Fig. 3.7.

7. ***Knuckle thread*.** It is also a modification of square thread. It has rounded top and bottom. It can be cast or rolled easily and can not economically be made on a machine. These threads are used for rough and ready work. They are usually found on railway carriage couplings, hydrants, necks of glass bottles and large moulded insulators used in electrical trade.

8. ***Buttress thread*.** It is used for transmission of power in one direction only. The force is transmitted almost parallel to the axis. This thread units the advantage of both square and V-threads. It has a low frictional resistance characteristics of the square thread and have the same strength as that of V-thread. The spindles of bench vices are usually provided with buttress thread. The various proportions of buttress thread are shown in Fig. 3.9.

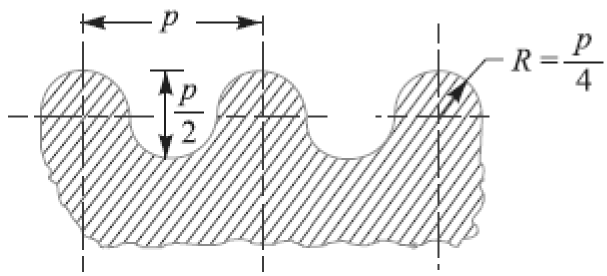


Fig. 3.8. Knuckle thread.

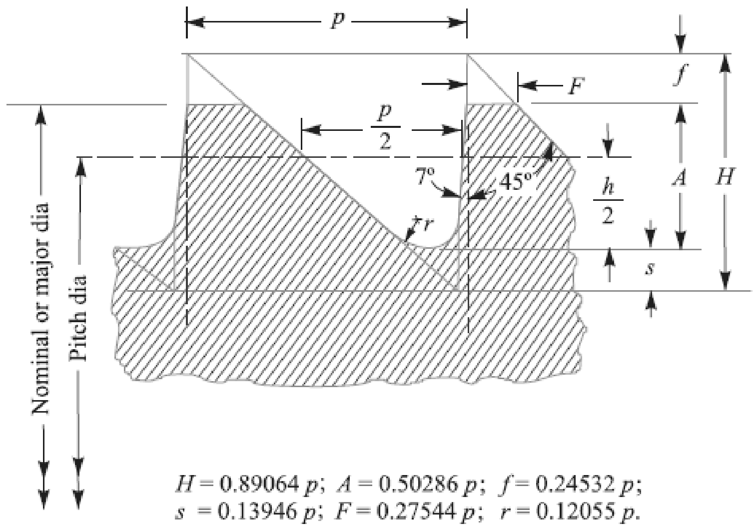


Fig. 3.9. Buttress thread.

9. ***Metric thread*.** It is an Indian standard thread and is similar to B.S.W. threads. It has an included angle of 60° instead of 55°. The basic profile of the thread is shown in Fig. 3.10 and the design profile of the nut and bolt is shown in Fig. 3.11.

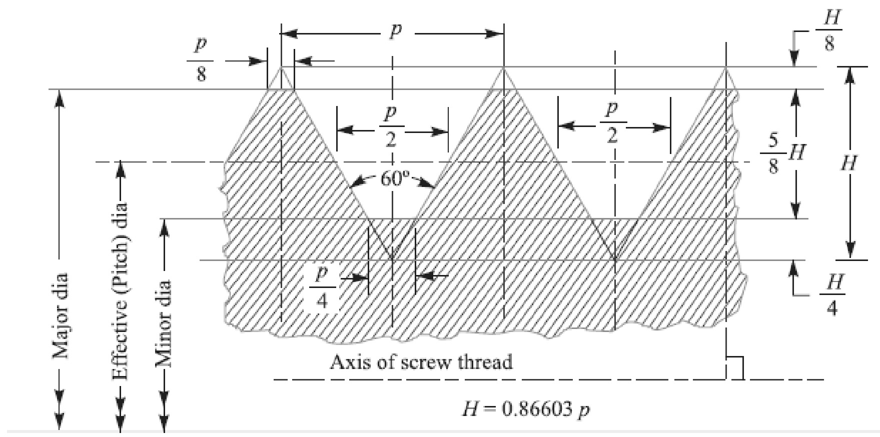


Fig. 3.10. Basic profile of the thread.

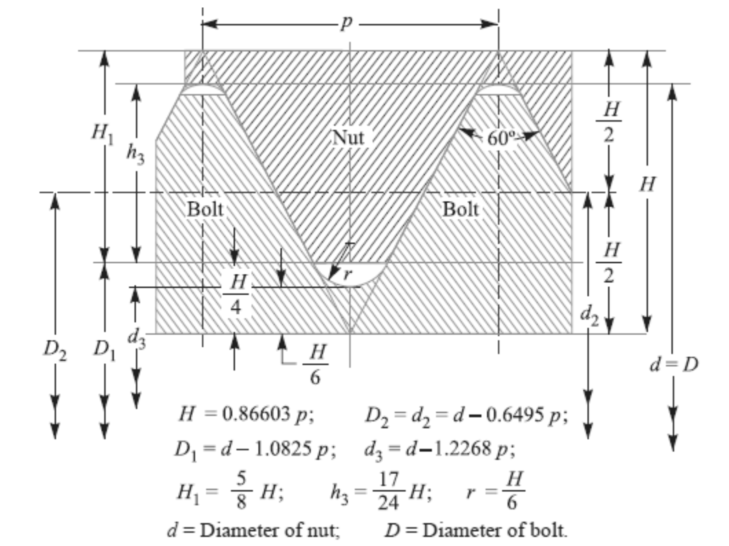


Fig. 3.11. Design profile of the nut and bolt.

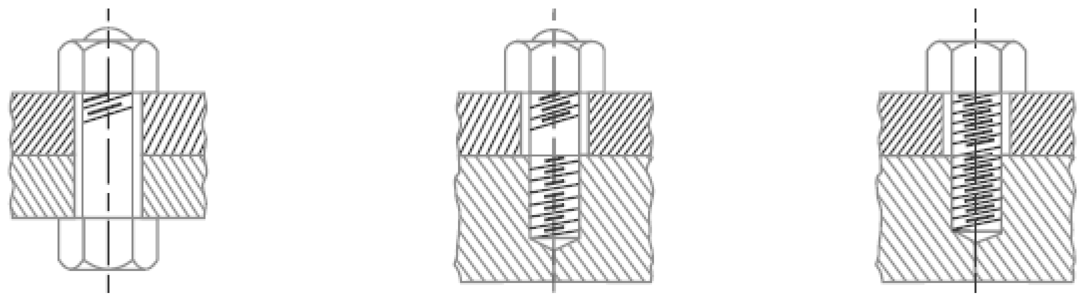
3.1.4 Location of Screwed Joints

The choice of type of fastenings and its location are very important. The fastenings should be located in such a way so that they will be subjected to tensile and/or shear loads and bending of the fastening should be reduced to a minimum. The bending of the fastening due to misalignment, tightening up loads, or external loads are responsible for many failures. In order to relieve fastenings of bending stresses, the use of clearance spaces, spherical seat washers, or other devices may be used.

3.1.5 Common Types of Screw Fastenings

Following are the common types of screw fastenings :

1. *Through bolts*. A through bolt (or simply a bolt) is shown in Fig. 3.12 (*a*). It is a cylindrical bar with threads for the nut at one end and head at the other end. The cylindrical part of the bolt is known as **shank.** It is passed through drilled holes in the two parts to be fastened together and clamped them securely to each other as the nut is screwed on to the threaded end. The through bolts may or may not have a machined finish and are made with either hexagonal or square heads. A through bolt should pass easily in the holes, when put under tension by a load along its axis. If the load acts perpendicular to the axis, tending to slide one of the connected parts along the other end thus subjecting it to shear, the holes should be reamed so that the bolt shank fits snugly there in. The through bolts according to their usage may be known as **machine bolts, carriage bolts, automobile bolts, eye bolts etc.**

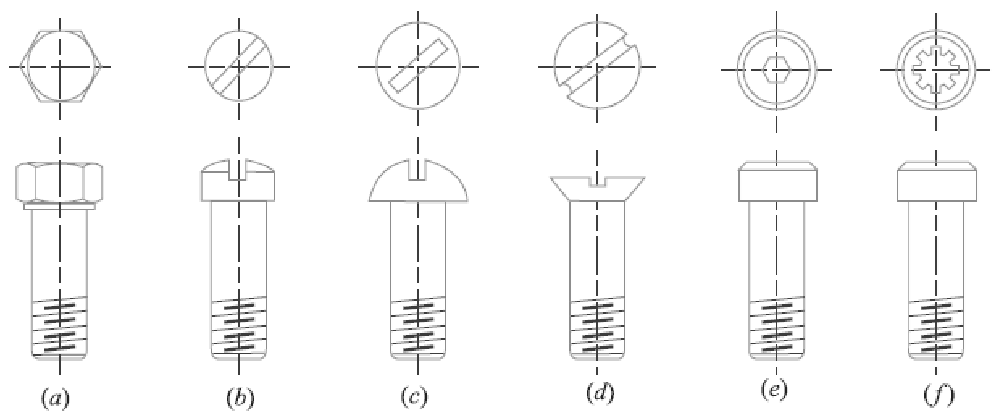


(*a*) Through bolt. (*b*) Tap bolt. (*c*) Stud.

Fig. 3.12

2. ***Tap bolts.*** A tap bolt or screw differs from a bolt. It is screwed into a tapped hole of one of the parts to be fastened without the nut, as shown in Fig. 3.12 (*b*).
3. ***Studs*****.** A stud is a round bar threaded at both ends. One end of the stud is screwed into a tapped hole of the parts to be fastened, while the other end receives a nut on it, as shown in Fig. 3.12 (*c*). Studs are chiefly used instead of tap bolts for securing various kinds of covers *e.g.* covers of engine and pump cylinders, valves, chests etc.

4. ***Cap screws*****.** The cap screws are similar to tap bolts except that they are of small size and a variety of shapes of heads are available as shown in Fig. 3.13.



(*a*) Hexagonal head; (*b*) Fillister head; (*c*) Round head; (*d*) Flat head; (*e*) Hexagonal socket; (*f*) Fluted socket.

Fig. 3.13. Types of cap screws.

5. ***Machine screws*****.** These are similar to cap screws with the head slotted for a screw driver. These are generally used with a nut.
6. ***Set screws*****.** The set screws are shown in Fig. 3.14. These are used to prevent relative motion between the two parts. A set screw is screwed through a threaded hole in one part so that its point (*i.e.* end of the screw) presses against the other part. This resists the relative motion between the two parts by means of friction between the point of the screw and one of the parts. They may be used instead of key to prevent relative motion between a hub and a shaft in light power transmission members. They may also be used in connection with a key, where they prevent relative axial motion

of the shaft, key and hub assembly.

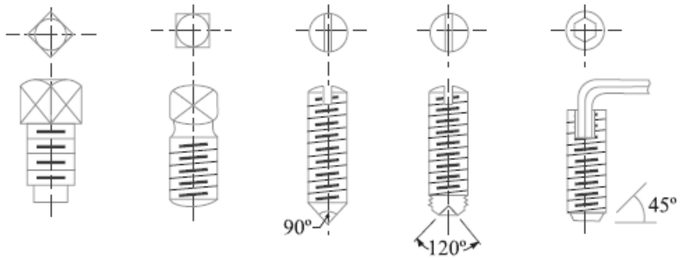


Fig. 3.14. Set screws.

The diameter of the set screw (d) may be obtained from the following expression:
$d = 0.125\,D + 8$ mm
where D is the diameter of the shaft (in mm) on which the set screw is pressed.
The tangential force (in newtons) at the surface of the shaft is given by

$F = 6.6\,(d\,)^{2.3}$

∴ Torque transmitted by a set screw,

$$T = F \times \frac{D}{2} \text{ N-m}$$

and power transmitted (in watts)

$$P = \frac{2\pi N.T}{60}$$

where N is the speed in r.p.m.

3.1.6 Locking Devices

Ordinary thread fastenings, generally, remain tight under static loads, but many of these fastenings become loose under the action of variable loads or when machine is subjected to vibrations. The loosening of fastening is very dangerous and must be prevented. In order to prevent this, a large number of locking devices are available, some of which are discussed below :

1. *Jam nut or lock nut.* A most common locking device is a jam, lock or check nut. It has about one-half to two-third thickness of the standard nut. The thin lock nut is first tightened down with ordinary force, and then the upper nut (*i.e.* thicker nut) is tightened down upon it, as shown in Fig.3.15 (*a*). The upper nut is then held tightly while the lower one is slackened back against it.

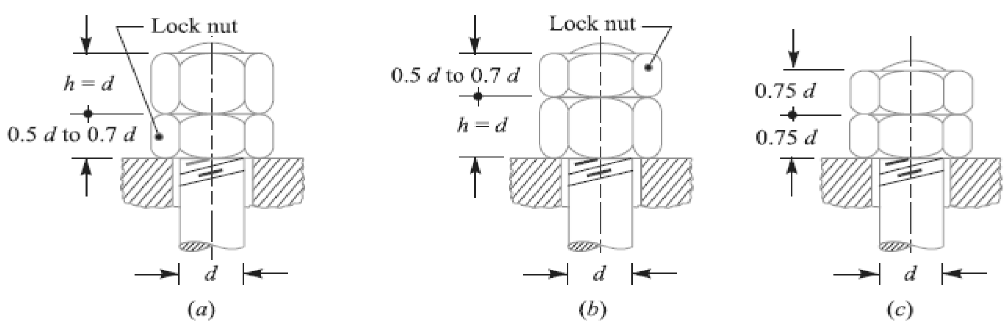


Fig. 11.15. Jam nut or lock nut.

In slackening back the lock nut, a thin spanner is required which is difficult to find in many shops. Therefore to overcome this difficulty, a thin nut is placed on the top as shown in Fig. 3.15 (*b*). If the nuts are really tightened down as they should be, the upper nut carries a greater tensile load than the bottom one. Therefore, the top nut should be thicker one with a thin nut below it because it is desirable to put whole of the load on the thin nut. In order to overcome both the difficulties, both the nuts are made of the same thickness as shown in Fig. 3.15 (*c*).

2. ***Castle nut*.** It consists of a hexagonal portion with a cylindrical upper part which is slotted in line with the centre of each face, as shown in Fig. 3.16. The split pin passes through two slots in the nut and a hole in the bolt, so that a positive lock is obtained unless the pin shears. It is extensively used on jobs subjected to sudden shocks and considerable vibration such as in automobile industry.

3. ***Sawn nut.*** It has a slot sawed about half way through, as shown in Fig. 3.17. After the nut is screwed down, the small screw is tightened which produces more friction between the nut and the bolt. This prevents the loosening of nut.

4. ***Penn, ring or grooved nut*.** It has a upper portion hexagonal and a lower part cylindrical as shown in Fig. 3.18. It is largely used where bolts pass through connected pieces reasonably near their edges such as in marine type connecting rod ends. The bottom portion is cylindrical and is recessed to receive the tip of the locking set screw. The bolt hole requires counter-boring to receive the cylindrical portion of the nut. In order to prevent bruising of the latter by the case hardened tip of the set screw, it is recessed.

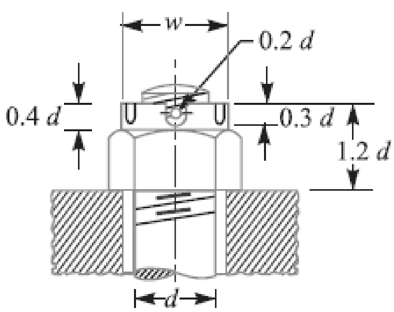


Fig. 3.16. Castle nut.

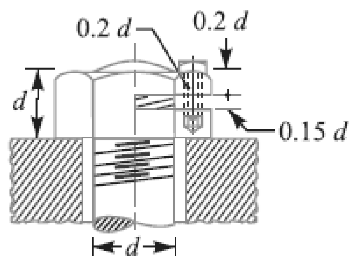


Fig. 3.17. Sawn nut.

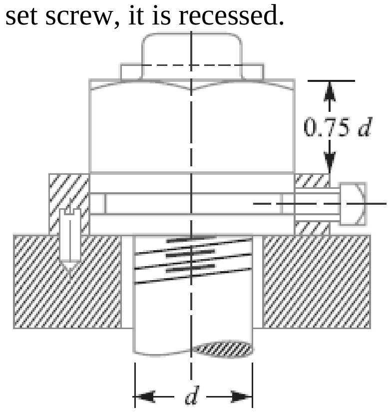


Fig. 3.18. Penn, ring or grooved nut.

5. ***Locking with pin*.** The nuts may be locked by means of a taper pin or cotter pin passing through the middle of the nut as shown in Fig. 3.19 (*a*). But a split pin is often driven through the bolt above the nut, as shown in Fig. 3.19 (*b*).

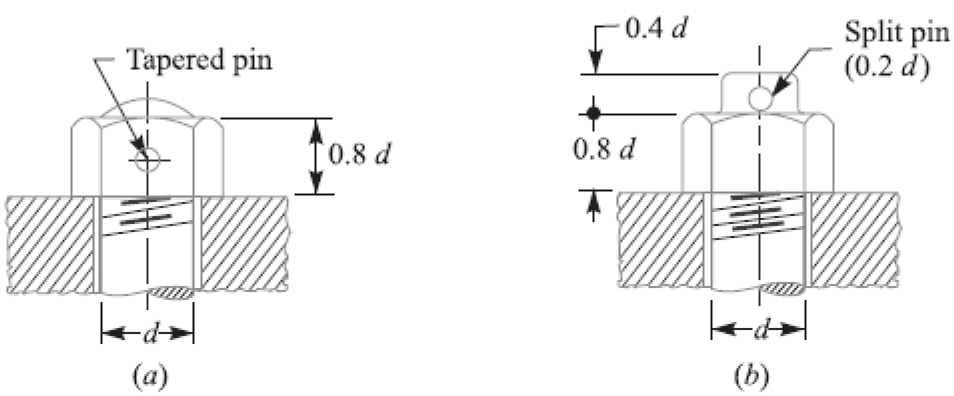
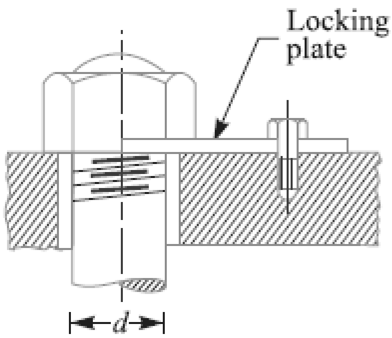


Fig. 3.19. Locking with pin.

6. ***Locking with plate*.**A form of stop plate or locking plate is shown in Fig. 3.20. The nut can be adjusted and subsequently locked through angular intervals of 30° by using these plates.



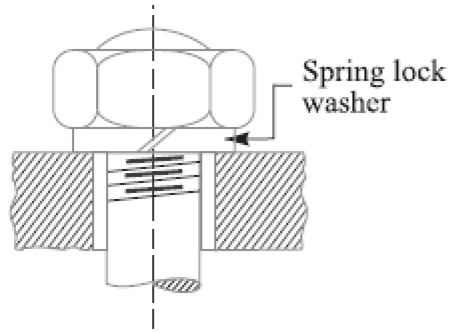


Fig. 3.20. Locking with plate. **Fig. 3.21.** Locking with washer.

Spring lock washer. A spring lock washer is shown in Fig. 3.21. As the nut tightens the washer against the piece below, one edge of the washer is caused to dig itself into that piece, thus increasing the resistance so that the nut will not loosen so easily. There are many kinds of spring lock washers manufactured, some of which are fairly effective.

3.1.7 Designation of Screw Threads

According to Indian standards, IS : 4218 (Part IV) 1976 (Reaffirmed 1996), the complete designation of the screw thread shall include

1. *Size designation*. The size of the screw thread is designated by the letter `*M*' followed by the diameter and pitch, the two being separated by the sign ×. When there is no indication of the pitch, it shall mean that a coarse pitch is implied.

2. *Tolerance designation*. This shall include

(*a*) A figure designating tolerance grade as indicated below:

‘7’ for fine grade, ‘8’ for normal (medium) grade, and ‘9’ for coarse grade.

(*b*) A letter designating the tolerance position as indicated below :

‘*H*’ for unit thread, ‘*d*’ for bolt thread with allowance, and ‘*h*’ for bolt thread without allowance.

For example, A bolt thread of 6 mm size of coarse pitch and with allowance on the threads and normal (medium) tolerance grade is designated as *M*6-8*d*.

3.1.8 Standard Dimensions of Screw Threads

The design dimensions of I.S.O. screw threads for screws, bolts and nuts of coarse and fine series are shown in Table 3.1.

Table 3.1. Design dimensions of screw threads, bolts and nuts according to IS : 4218 (Part III) 1976 (Reaffirmed 1996)

Designation	*Pitch mm*	*Major or nominal diameter Nut and Bolt (d = D) mm*	*Effective or pitch diameter Nut and Bolt (d_p) mm*	*Minor or core diameter (d_c) mm*		*Depth of thread (bolt) mm*	*Stress area mm²*
				Bolt	*Nut*		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Coarse series							
M 0.4	0.1	0.400	0.335	0.277	0.292	0.061	0.074
M 0.6	0.15	0.600	0.503	0.416	0.438	0.092	0.166
M 0.8	0.2	0.800	0.670	0.555	0.584	0.123	0.295
M 1	0.25	1.000	0.838	0.693	0.729	0.153	0.460
M 1.2	0.25	1.200	1.038	0.893	0.929	0.158	0.732
M 1.4	0.3	1.400	1.205	1.032	1.075	0.184	0.983
M 1.6	0.35	1.600	1.373	1.171	1.221	0.215	1.27
M 1.8	0.35	1.800	1.573	1.371	1.421	0.215	1.70
M 2	0.4	2.000	1.740	1.509	1.567	0.245	2.07
M 2.2	0.45	2.200	1.908	1.648	1.713	0.276	2.48
M 2.5	0.45	2.500	2.208	1.948	2.013	0.276	3.39
M 3	0.5	3.000	2.675	2.387	2.459	0.307	5.03
M 3.5	0.6	3.500	3.110	2.764	2.850	0.368	6.78
M 4	0.7	4.000	3.545	3.141	3.242	0.429	8.78
M 4.5	0.75	4.500	4.013	3.580	3.688	0.460	11.3
M 5	0.8	5.000	4.480	4.019	4.134	0.491	14.2
M 6	1	6.000	5.350	4.773	4.918	0.613	20.1

M 7	1	7.000	6.350	5.773	5.918	0.613	28.9
M 8	1.25	8.000	7.188	6.466	6.647	0.767	36.6
M 10	1.5	10.000	9.026	8.160	8.876	0.920	58.3
M 12	1.75	12.000	10.863	9.858	10.106	1.074	84.0
M 14	2	14.000	12.701	11.546	11.835	1.227	115
M 16	2	16.000	14.701	13.546	13.835	1.227	157
M 18	2.5	18.000	16.376	14.933	15.294	1.534	192
M 20	2.5	20.000	18.376	16.933	17.294	1.534	245
M 22	2.5	22.000	20.376	18.933	19.294	1.534	303
M 24	3	24.000	22.051	20.320	20.752	1.840	353
M 27	3	27.000	25.051	23.320	23.752	1.840	459
M 30	3.5	30.000	27.727	25.706	26.211	2.147	561
M 33	3.5	33.000	30.727	28.706	29.211	2.147	694
M 36	4	36.000	33.402	31.093	31.670	2.454	817
M 39	4	39.000	36.402	34.093	34.670	2.454	976
M 42	4.5	42.000	39.077	36.416	37.129	2.760	1104
M 45	4.5	45.000	42.077	39.416	40.129	2.760	1300
M 48	5	48.000	44.752	41.795	42.587	3.067	1465
M 52	5	52.000	48.752	45.795	46.587	3.067	1755
M 56	5.5	56.000	52.428	49.177	50.046	3.067	2022
M 60	5.5	60.000	56.428	53.177	54.046	3.374	2360
Fine series							
M 8 × 1	1	8.000	7.350	6.773	6.918	0.613	39.2
M 10 × 1.25	1.25	10.000	9.188	8.466	8.647	0.767	61.6
M 12 × 1.25	1.25	12.000	11.184	10.466	10.647	0.767	92.1
M 14 × 1.5	1.5	14.000	13.026	12.160	12.376	0.920	125
M 16 × 1.5	1.5	16.000	15.026	14.160	14.376	0.920	167
M 18 × 1.5	1.5	18.000	17.026	16.160	16.376	0.920	216
M 20 × 1.5	1.5	20.000	19.026	18.160	18.376	0.920	272
M 22 × 1.5	1.5	22.000	21.026	20.160	20.376	0.920	333
M 24 × 2	2	24.000	22.701	21.546	21.835	1.227	384
M 27 × 2	2	27.000	25.701	24.546	24.835	1.227	496
M 30 × 2	2	30.000	28.701	27.546	27.835	1.227	621
M 33 × 2	2	33.000	31.701	30.546	30.835	1.227	761
M 36 × 3	3	36.000	34.051	32.319	32.752	1.840	865
M 39 × 3	3	39.000	37.051	35.319	35.752	1.840	1028

3.1.9 Stresses in Screwed Fastening due to Static Loading

The following stresses in screwed fastening due to static loading are important from the subject point of view :

1. Internal stresses due to screwing up forces,

2. Stresses due to external forces, and

3. Stress due to combination of stresses at (1) and (2).

We shall now discuss these stresses, in detail, in the following articles.

3.1.10 Initial Stresses due to Screwing up Forces

The following stresses are induced in a bolt, screw or stud when it is screwed up tightly.

1. ***Tensile stress due to stretching of bolt.*** Since none of the above mentioned stresses are accurately determined, therefore bolts are designed on the basis of direct tensile stress with a large factor of safety in order to account for the indeterminate stresses. The initial tension in a bolt, based on experiments, may be found by the relation

$$P_i = 2840\, d \text{ N}$$

where P_i = Initial tension in a bolt, and

d = Nominal diameter of bolt, in mm.

The above relation is used for making a joint fluid tight like steam engine cylinder cover joints etc. When the joint is not required as tight as fluid-tight joint, then the initial tension in a bolt may be reduced to half of the above value. In such cases

$$P_i = 1420\, d \text{ N}$$

The small diameter bolts may fail during tightening, therefore bolts of smaller diameter (less than M 16 or M 18) are not permitted in making fluid tight joints. If the bolt is not initially stressed, then the maximum safe axial load which may be applied to it, is given by

P = Permissible stress × Cross-sectional area at bottom of the thread (*i.e.* stress area)

The stress area may be obtained from Table 3.1 or it may be found by using the relation

$$\text{Stress area} = \frac{\pi}{4}\left(\frac{d_p + d_c}{2}\right)^2$$

where d_p = Pitch diameter, and

d_c = Core or minor diameter.

2. ***Torsional shear stress caused by the frictional resistance of the threads during its tightening*.**

The torsional shear stress caused by the frictional resistance of the threads during its tightening may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{T}{J} \times r = \frac{T}{\frac{\pi}{32}(d_c)^4} \times \frac{d_c}{2} = \frac{16\,T}{\pi\,(d_c)^3}$$

Where τ = Torsional shear stress,

T = Torque applied, and

d_c = Minor or core diameter of the thread.

It has been shown during experiments that due to repeated unscrewing and tightening of the nut, there is a gradual scoring of the threads, which increases the torsional twisting moment (T).

3. ***Shear stress across the threads*.** The average thread shearing stress for the screw (τ_s) is obtained by using the relation :

$$\tau_s = \frac{P}{\pi d_c \times b \times n}$$

where b = Width of the thread section at the root.

The average thread shearing stress for the nut is

$$\tau_n = \frac{P}{\pi d \times b \times n}$$

where d = Major diameter.

4. *Compression or crushing stress on threads*. The compression or crushing stress between he threads (σ_c) may be obtained by using the relation :

$$\sigma_c = \frac{P}{\pi [d^2 - (d_c)^2]\, n}$$

where d = Major diameter,

d_c = Minor diameter, and

n = Number of threads in engagement.

5. *Bending stress if the surfaces under the head or nut are not perfectly parallel to the bolt axis*.
When the outside surfaces of the parts to be connected are not parallel to each other, then the bolt will be subjected to bending action. The bending stress (□*b*) induced in the shank of the bolt is given by

$$\sigma_b = \frac{x \,.\, E}{2l}$$

where x = Difference in height between the extreme corners of the nut or head,
l = Length of the shank of the bolt, and
E = Young's modulus for the material of the bolt.

Example 3.1. *Determine the safe tensile load for a bolt of M 30, assuming a safe tensile stress of 42 MPa.*

Given :

d = 30 mm

σ_t = 42 MPa = 42 N/mm^2

Solution:

From Table 3.1 (coarse series), we find that the stress area *i.e.* cross-sectional area at the bottom of the thread corresponding to M 30 is 561 mm2.

∴ Safe tensile load = Stress area × σ_t= 561 × 42 = 23 562 N = 23.562 kN **Ans.**

Example 3.2. *Two machine parts are fastened together tightly by means of a 24 mm tap bolt. If the load tending to separate these parts is neglected, find the stress that is set up in the bolt by the initial tightening.*

Given : d = 24 mm

Solution.

From Table 11.1 (coarse series), we find that the core diameter of the thread corresponding to M 24 is d_c= 20.32 mm.

Let
σ_t = Stress set up in the bolt.
We know that initial tension in the bolt,

P = 2840 d = 2840 × 24 = 68 160 N

We also know that initial tension in the bolt (P),

$$68\ 160 = \frac{\pi}{4}(d_c)^2\,\sigma_t = \frac{\pi}{4}(20.30)^2\,\sigma_t = 324\,\sigma_t$$

$$\sigma_t = 68\ 160 / 324 = 210\ \text{N/mm}^2 = 210\ \text{MPa}\ \textbf{Ans.}$$

3.1.11 Bolts of Uniform Strength

When a bolt is subjected to shock loading, as in case of a cylinder head bolt of an internal combustion engine, the resilience of the bolt should be considered in order to prevent breakage at the thread. In an ordinary bolt shown in Fig. 3.22 (*a*), the effect of the impulsive loads applied axially is concentrated on the weakest part of the bolt *i.e*. the cross-sectional area at the root of the threads. In other words, the stress in the threaded part of the bolt will be higher than that in the shank. Hence a great portion of the energy will be absorbed at the region of the threaded part which may fracture the threaded portion because of its small length.

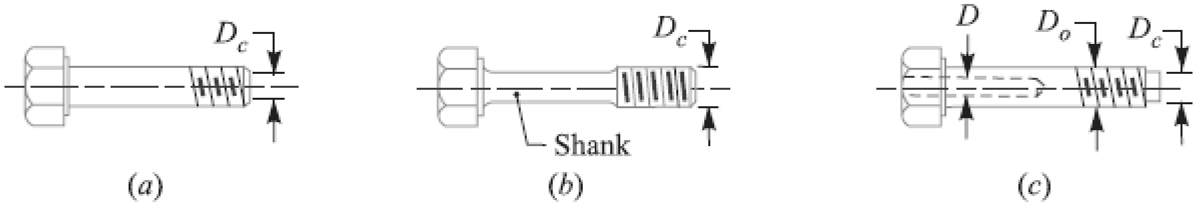


Fig. 3.22. Bolts of uniform strength.

If the shank of the bolt is turned down to a diameter equal or even slightly less than the core diameter of the thread (Dc) as shown in Fig. 3.22 (*b*), then shank of the bolt will undergo a higher stress. This means that a shank will absorb a large portion of the energy, thus relieving the material at the sections near the thread. The bolt, in this way, becomes stronger and lighter and it increases the shock absorbing capacity of the bolt because of an increased modulus of resilience. This gives us ***bolts of uniform strength*.** The resilience of a bolt may also be increased by increasing its length. A second alternative method of obtaining the bolts of uniform strength is shown in Fig. 3.22 (*c*). In this method, an axial hole is drilled through the head as far as the thread portion such that the area of the shank becomes equal to the root area of the thread.

Let D = Diameter of the hole.
Do = Outer diameter of the thread, and
Dc = Root or core diameter of the thread.

$$\therefore \quad \frac{\pi}{4}D^2 = \frac{\pi}{4}\left[(D_o)^2 - (D_c)^2\right]$$

or

$$D^2 = (D_o)^2 - (D_c)^2$$

$$\therefore \quad D = \sqrt{(D_o)^2 - (D_c)^2}$$

Example 3.3. *Determine the diameter of the hole that must be drilled in a M 48 bolt such that the bolt becomes of uniform strength.*
Solution. Given : Do = 48 mm
From Table 11.1 (coarse series), we find that the core diameter of the thread (corresponding to Do = 48 mm) is Dc = 41.795 mm.

We know that for bolts of uniform strength, the diameter of the hole,

$$D = \sqrt{(D_o)^2 - (D_c)^2} = \sqrt{(48)^2 - (41.795)^2} = 23.64 \text{ mm Ans.}$$

3.1.12 Design of a Nut

When a bolt and nut is made of mild steel, then the effective height of nut is made equal to the nominal diameter of the bolt. If the nut is made of weaker material than the bolt, then the height of nut should be larger, such as 1.5 *d* for gun metal, 2 *d* for cast iron and 2.5 *d* for aluminium alloys (where *d* is the nominal diameter of the bolt). In case cast iron or aluminium nut is used, then *V*-threads are permissible only for permanent fastenings, because threads in these materials are damaged due to repeated screwing and unscrewing. When these materials are to be used for parts frequently removed and fastened, a screw in steel bushing for cast iron and cast-in-bronze or monel metal insert should be used for aluminium and should be drilled and tapped in place.

3.1.13 Bolted Joints under Eccentric Loading

There are many applications of the bolted joints which are subjected to eccentric loading such as a wall bracket, pillar crane, etc. The eccentric load may be

1. Parallel to the axis of the bolts,
2. Perpendicular to the axis of the bolts, and
3. In the plane containing the bolts.

We shall now discuss the above cases, in detail, in the following articles.

3.1.14 Eccentric Load Acting Parallel to the Axis of Bolts

Consider a bracket having a rectangular base bolted to a wall by means of four bolts as shown in Fig. 3.23. A little consideration will show that each bolt is subjected to a direct tensile load of

$W_{t1} = W/n$, where *n* is the number of bolts.

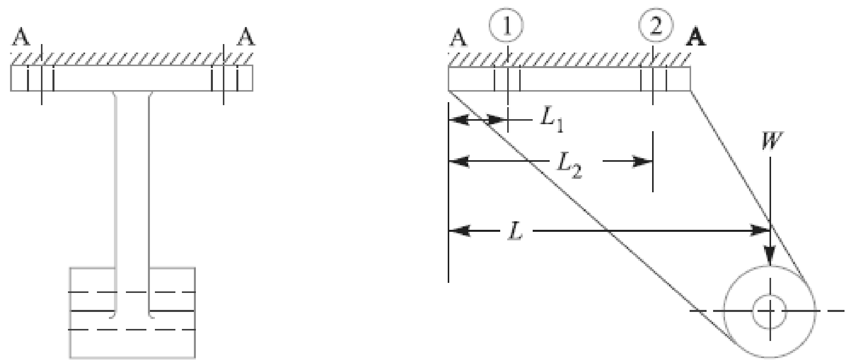


Fig. 3.23. Eccentric load acting parallel to the axis of bolts.

Further the load *W* tends to rotate the bracket about the edge *A*-*A*. Due to this, each bolt is stretched by an amount that depends upon its distance from the tilting edge. Since the stress is a function of elongation, therefore each bolt will experience a different load which also depends upon the distance from the tilting edge. For convenience, all the bolts are made of same size. In case the flange is heavy, it may be considered as a rigid body. Let *w* be the load in a bolt per unit distance due to the turning effect of the bracket and let W_1 and W_2 be the loads on each of the bolts at distances L_1 and L_2 from the tilting edge

∴ Load on each bolt at distance L_1,

$$W_1 = w.L_1$$

and moment of this load about the tilting edge

$$= w_1.L_1 \times L_1 = w\,(L_1)^2$$

Similarly, load on each bolt at distance L_2,

$$W_2 = w.L_2$$

and moment of this load about the tilting edge

$$= w.L_2 \times L_2 = w\,(L_2)^2$$

∴ Total moment of the load on the bolts about the tilting edge

$$= 2w\,(L_1)^2 + 2w\,(L_2)^2 \qquad ...(i)$$

... (∵ There are two bolts each at distance of L_1 and L_2)

Also the moment due to load W about the tilting edge

$$= W.L \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$W.L = 2w\,(L_1)^2 + 2w(L_2)^2 \qquad \text{or} \qquad w = \frac{W.L}{2\,[(L_1)^2 + (L_2)^2]} \qquad ...(iii)$$

It may be noted that the most heavily loaded bolts are those which are situated at the greatest distance from the tilting edge. In the case discussed above, the bolts at distance L_2 are heavily loaded.

∴ Tensile load on each bolt at distance L_2,

$$W_{t2} = W_2 = w.L_2 = \frac{W.L.L_2}{2[(L_1)^2 + (L_2)^2]} \qquad \text{... [From equation (iii)]}$$

and the total tensile load on the most heavily loaded bolt,

$$W_t = W_{t1} + W_{t2} \qquad ...(iv)$$

If d_c is the core diameter of the bolt and σ_t is the tensile stress for the bolt material, then total tensile load,

$$W_t = \frac{\pi}{4}\,(d_c)^2\,\sigma_t \qquad ...(v)$$

From equations (*iv*) and (*v*), the value of d_c may be obtained.

Example 3.4. *A bracket, as shown in Fig. 3.23, supports a load of 30 kN. Determine the size of bolts, if the maximum allowable tensile stress in the bolt material is 60 MPa. The distances are : L_1 = 80 mm, L_2= 250 mm, and L = 500 mm.*

Solution.

Given : W = 30 kN
σ_t = 60 MPa = 60 N/mm^2
L_1 = 80 mm
L_2 = 250 mm
L = 500 mm

We know that the direct tensile load carried by each bolt,

$$W_{t1} = \frac{W}{n} = \frac{30}{4} = 7.5 \text{ kN}$$

and load in a bolt per unit distance,

$$w = \frac{W.L}{2[(L_1)^2 + (L_2)^2]} = \frac{30 \times 500}{2[(80)^2 + (250)^2]} = 0.109 \text{ kN/mm}$$

Since the heavily loaded bolt is at a distance of L_2 mm from the tilting edge, therefore load on the heavily loaded bolt,

$$W_{t2} = w.L_2 = 0.109 \times 250 = 27.25 \text{ kN}$$

∴ Maximum tensile load on the heavily loaded bolt,

$$W_t = W_{t1} + W_{t2} = 7.5 + 27.25 = 34.75 \text{ kN} = 34\ 750 \text{ N}$$

Let *dc* = Core diameter of the bolts.
We know that the maximum tensile load on the bolt (*Wt*),

$$34\ 750 = \frac{\pi}{4}(d_c)^2\ \sigma_t = \frac{\pi}{4}(d_c)^2\ 60 = 47\ (d_c)^2$$

$$\therefore \quad (d_c)^2 = 34\ 750\ /\ 47 = 740$$

$d_c = 27.2$ mm

From Table 3.1 (coarse series), we find that the standard core diameter of the bolt is 28.706 mm and the corresponding size of the bolt is M 33. **Ans**

3.1.15 Eccentric Load Acting Perpendicular to the Axis of Bolts

A wall bracket carrying an eccentric load perpendicular to the axis of the bolts is shown in Fig. 3.24.

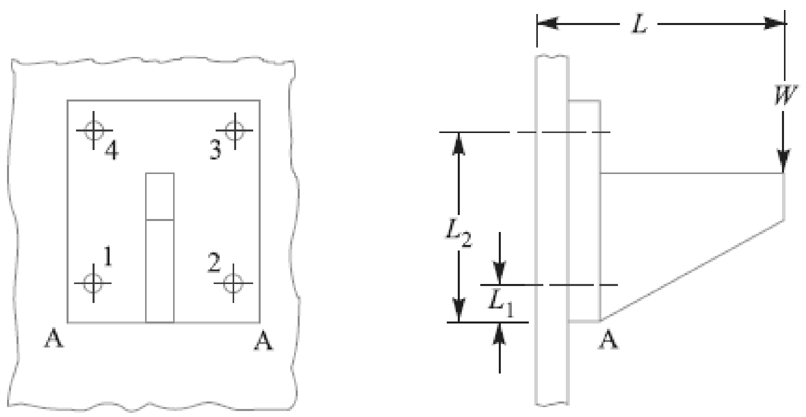


Fig.3.24. Eccentric load perpendicular to the axis of bolts.

In this case, the bolts are subjected to direct shearing load which is equally shared by all the bolts. Therefore direct shear load on each bolts,

$Ws = W/n$, where n is number of bolts.

A little consideration will show that the eccentric load *W* will try to tilt the bracket in the clockwise direction about the edge *A-A*. As discussed earlier, the bolts will be subjected to tensile stress due to the turning moment. The maximum tensile load on a heavily loaded bolt (*Wt*) may be obtained in the similar manner as discussed in the previous article. In this case, bolts 3 and 4 are heavily loaded.

∴ Maximum tensile load on bolt 3 or 4,

$$W_{t2} = W_t = \frac{W.L.L_2}{2[(L_1)^2 + (L_2)^2]}$$

When the bolts are subjected to shear as well as tensile loads, then the equivalent loads may be determined by the following relations :

Equivalent tensile load,

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

and equivalent shear load,

$$W_{se} = \frac{1}{2}\left[\sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

Knowing the value of equivalent loads, the size of the bolt may be determined for the given allowable stresses.

Example 3.5. *For supporting the travelling crane in a workshop, the brackets are fixed on steel columns as shown in Fig. 3.24. The maximum load that comes on the bracket is 12 kN acting vertically at a distance of 400 mm from the face of the column. The vertical face of the bracket is secured to a column by four bolts, in two rows (two in each row) at a distance of 50 mm from the lower edge of the bracket. Determine the size of the bolts if the permissible value of the tensile stress for the bolt material is 84 MPa. Also find the cross-section of the arm of the bracket which is rectangular.*

Solution.

Given : W = 12 kN = 12 × 10^3 N

L = 400 mm

L_1 = 50 mm

L_2 = 375 mm

σ_t = 84 MPa = 84 N/mm2 ;

n = 4

We know that direct shear load on each bolt,

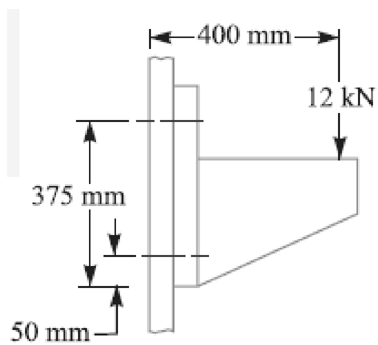


Fig. 3.24

We know that direct shear load on each bolt,

$$W_s = \frac{W}{n} = \frac{12}{4} = 3 \text{ kN}$$

Since the load W will try to tilt the bracket in the clockwise direction about the lower edge, therefore the bolts will be subjected to tensile load due to turning moment. The maximum loaded bolts are 3 and 4 (See Fig. 3.24), because they lie at the greatest distance from the tilting edge A–A (*i.e.* lower edge).

We know that maximum tensile load carried by bolts 3 and 4,

$$W_t = \frac{W.L.L_2}{2[(L_1)^2 + (L_2)^2]} = \frac{12 \times 400 \times 375}{2[(50)^2 + (375)^2]} = 6.29 \text{ kN}$$

Since the bolts are subjected to shear load as well as tensile load, therefore equivalent tensile load,

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right] = \frac{1}{2}\left[6.29 + \sqrt{(6.29)^2 + 4 \times 3^2}\right] \text{ kN}$$

$$= \frac{1}{2}(6.29 + 8.69) = 7.49 \text{ kN} = 7490 \text{ N}$$

Size of the bolt

Let d_c = Core diameter of the bolt.

We know that the equivalent tensile load (W_{te}),

$$7490 = \frac{\pi}{4}(d_c)^2 \sigma_t = \frac{\pi}{4}(d_c)^2 84 = 66 (d_c)^2$$

$\therefore$ $(d_c)^2 = 7490 / 66 = 113.5$ or $d_c = 10.65$ mm

From Table 11.1 (coarse series), the standard core diameter is 11.546 mm and the corresponding size of the bolt is M 14. **Ans.**

Cross-section of the arm of the bracket

Let t and b = Thickness and depth of arm of the bracket respectively.

$\therefore$ Section modulus,

$$Z = \frac{1}{6} t.b^2$$

Assume that the arm of the bracket extends upto the face of the steel column. This assumption gives stronger section for the arm of the bracket.

$\therefore$ Maximum bending moment on the bracket,

$$M = 12 \times 10^3 \times 400 = 4.8 \times 10^6 \text{ N-mm}$$

We know that the bending (tensile) stress (σ_t),

$$84 = \frac{M}{Z} = \frac{4.8 \times 10^6 \times 6}{t.b^2} = \frac{28.8 \times 10^6}{t.b^2}$$

$\therefore$ $t.b^2 = 28.8 \times 10^6 / 84 = 343 \times 10^3$ or $t = 343 \times 10^3 / b^2$

Assuming depth of arm of the bracket, b = 250 mm, we have

$$t = 343 \times 10^3 / (250)^2 = 5.5 \text{ mm } \textbf{Ans.}$$

3.1.16 Eccentric Load on a Bracket with Circular Base

Sometimes the base of a bracket is made circular as in case of a flanged bearing of a heavy machine tool and pillar crane etc. Consider a round flange bearing of a machine tool having four bolts as shown in Fig. 3.25.

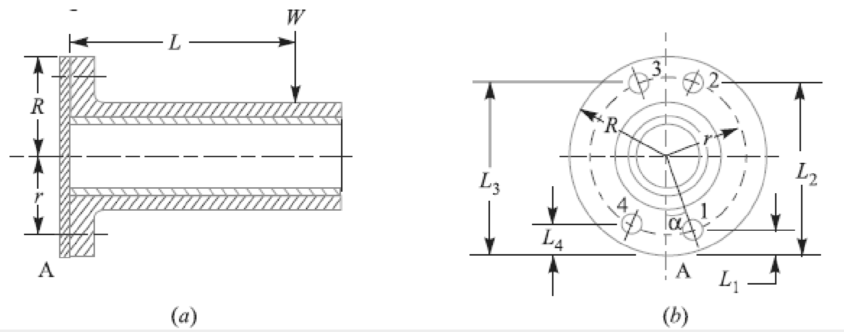


Fig. 3.25. Eccentric load on a bracket with circular base.

Let R = Radius of the column flange,
r = Radius of the bolt pitch circle,
w = Load per bolt per unit distance from the tilting edge,
L = Distance of the load from the tilting edge, and
$L1$, $L2$, $L3$, and $L4$ = Distance of bolt centres from the tilting edge A.

As discussed in the previous article, equating the external moment $W \times L$ to the sum of the resisting moments of all the bolts, we have,

$$WL = w\,[(L_1)^2 + (L_2)^2 + (L_3)^2 + (L_4)^2]$$

$$\therefore \quad w = \frac{W.L}{(L_1)^2 + (L_2)^2 + (L_3)^2 + (L_4)^2} \quad ...(i)$$

Now from the geometry of the Fig. 11.40 (*b*), we find that

$$L_1 = R - r\cos\alpha \qquad L_2 = R + r\sin\alpha$$

$$L_3 = R + r\cos\alpha \quad \text{and} \quad L_4 = R - r\sin\alpha$$

Substituting these values in equation (*i*), we get

$$w = \frac{W.L}{4R^2 + 2r^2}$$

$$\therefore \text{ Load in the bolt situated at } 1 = w.L_1 = \frac{W.L.L_1}{4R^2 + 2r^2} = \frac{W.L\,(R - r\cos\alpha)}{4R^2 + 2r^2}$$

This load will be maximum when cos α is minimum *i.e*. when cos α = – 1 or α= 180^0
Maximum load in a bolt

$$= \frac{W.L\,(R + r)}{4R^2 + 2r^2}$$

In general, if there are n number of bolts, then load in a bolt

$$= \frac{2W.L\,(R - r\cos\alpha)}{n\,(2R^2 + r^2)}$$

and maximum load in a bolt,

$$W_t = \frac{2\,W.L\,(R + r)}{n\,(2R^2 + r^2)}$$

The above relation is used when the direction of the load W changes with relation to the bolts as in the case of pillar crane. But if the direction of load is fixed, then the maximum load on the bolts may be reduced by locating the bolts in such a way that two of them are equally stressed as shown in Fig. 3.26. In such a case, maximum load is given by

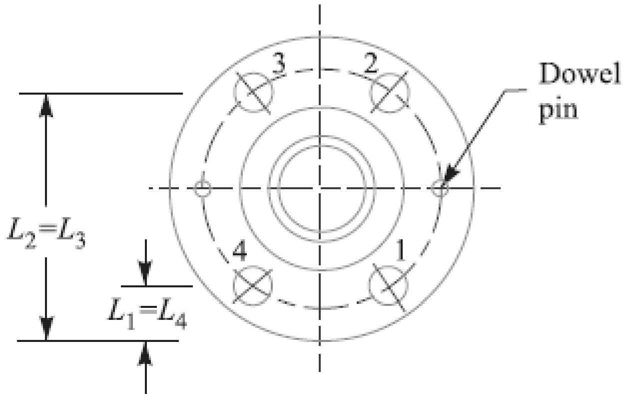


Fig. 3.26

$$W_t = \frac{2\,W.L}{n}\left[\frac{R + r\cos\left(\frac{180}{n}\right)}{2R^2 + r^2}\right]$$

Knowing the value of maximum load, we can determine the size of the bolt.

Example 3.6. *A pillar crane having a circular base of 600 mm diameter is fixed to the foundation of concrete base by means of four bolts. The bolts are of size 30 mm and are equally spaced on a bolt circle diameter of 500 mm.*

Determine : 1. The distance of the load from the centre of the pillar along a line X-X as shown in Fig. 3.27 (a). The load lifted by the pillar crane is 60 kN and the allowable tensile stress for the bolt material is 60 MPa.

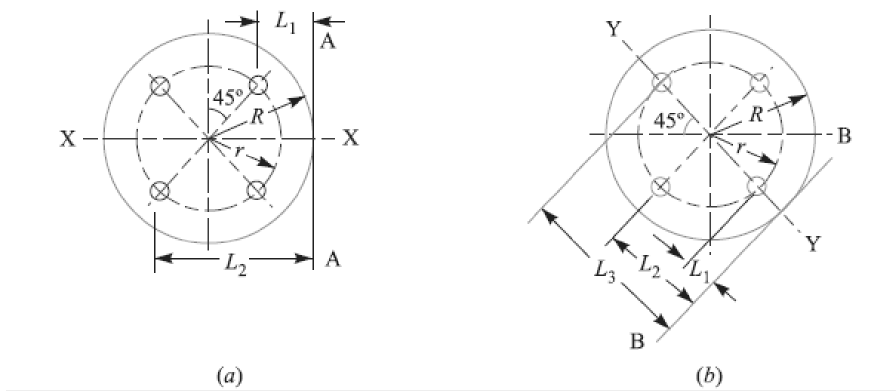


Fig. 3.27

2. The maximum stress induced in the bolts if the load is applied along a line Y-Y of the foundation as shown in Fig. 3.27 (b) at the same distance as in part (1).

Solution. Given :

D = 600 mm or R = 300 mm

n = 4

db = 30 mm

d = 500 mm or r = 250 mm
W = 60 kN
σ_t = 60 MPa = 60 N/mm^2
Since the size of bolt (*i.e.* d_b = 30 mm), is given therefore from Table 3.1, we find that the stress area corresponding to M 30 is 561 mm^2.
We know that the maximum load carried by each bolt
= Stress area × σ_t = 561 × 60 = 33 660 N = 33.66 kN
and direct tensile load carried by each bolt

$$= \frac{W}{n} = \frac{60}{4} = 15 \text{ kN}$$

∴ Total load carried by each bolt at distance $L2$ from the tilting edge *A-A*
= 33.66 + 15 = 48.66 kN ...***(i)***
From Fig. 11.43 (*a*), we find that
$L_1 = R - r \cos 45° = 300 - 250 \times 0.707 = 123$ mm = 0.123 m
and $L_2 = R + r \cos 45° = 300 + 250 \times 0.707 = 477$ mm = 0.477 m
Let w = Load (in kN) per bolt per unit distance.
∴ Total load carried by each bolt at distance $L2$ from the tilting edge *A-A*
= $w.L2 = w \times 0.477$ kN ...***(ii)***
From equations **(*i*)** and **(*ii*),** we have
w = 48.66 / 0.477 = 102 kN/m
∴ Resisting moment of all the bolts about the outer (*i.e.* tilting) edge of the flange along the tangent *A-A*
= $2w [(L1)2 + (L2)2] = 2 \times 102 [(0.123)2 + (0.477)2] = 49.4$ kN-m

1. *Distance of the load from the centre of the pillar*

Let
e = Distance of the load from the centre of the pillar or eccentricity of the load, and
L = Distance of the load from the tilting edge *A-A* = $e - R = e - 0.3$

2. *Maximum stress induced in the bolt*

Since the load is applied along a line *Y-Y* as shown in Fig. 3.27 (*b*), and at the same distance as in part (*1*) *i.e.*
at $L = e - 0.3 = 1.123 - 0.3 = 0.823$ m from the tilting edge *B-B*, therefore
Turning moment due to load W about the tilting edge *B–B*
= $W.L = 60 \times 0.823 = 49.4$ kN-m
From Fig. 11.43 (*b*), we find that
$L1 = R - r = 300 - 250 = 50$ mm = 0.05 m
$L2 = R = 300$ mm = 0.3 m
and $L3 = R + r = 300 + 250 = 550$ mm = 0.55 m
∴ Resisting moment of all the bolts about *B–B*
= $w [(L1)2 + 2(L2)2 + (L3)2] = w[(0.05)2 + 2(0.3)2 + (0.55)2]$ kN-m
= 0.485 w kN-m
Equating resisting moment of all the bolts to the turning moment, we have
0.485 w = 49.4
or w = 49.4 / 0.485 = 102 kN/m
Since the bolt at a distance of $L3$ is heavily loaded, therefore load carried by this bolt
= $w.L3 = 102 \times 0.55 = 56.1$ kN

and net force taken by the bolt

$$= w.L_3 - \frac{W}{n} = 56.1 - \frac{60}{4} = 41.1 \text{ kN} = 41\,100 \text{ N}$$

∴ Maximum stress induced in the bolt

$$= \frac{\text{Force}}{\text{Stress area}} = \frac{41\,000}{516}$$
$$= 79.65 \text{ N/mm}^2 = 79.65 \text{ MPa Ans.}$$

3.1.17 Eccentric Load Acting in the Plane Containing the Bolts

When the eccentric load acts in the plane containing the bolts, as shown in Fig. 3.28, then the same procedure may be followed as discussed for eccentric loaded riveted joints.

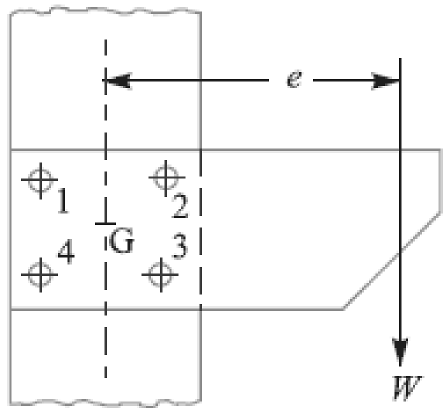


Fig. 3.28. Eccentric load in the plane containing the bolts.

Example 3.7. *Fig. 3.29 shows a solid forged bracket to carry a vertical load of 13.5 kN applied through the centre of hole. The square flange is secured to the flat side of a vertical stanchion through four bolts. Calculate suitable diameter D and d for the arms of the bracket, if the permissible stresses are 110 MPa in tension and 65 MPa in shear. Estimate also the tensile load on each top bolt and the maximum shearing force on each bolt.*

Solution.

Given :
W = 13.5 kN = 13 500 N ;
σ_t= 110 MPa = 110 N/mm2 ;
τ= 65 MPa = 65 N/mm2

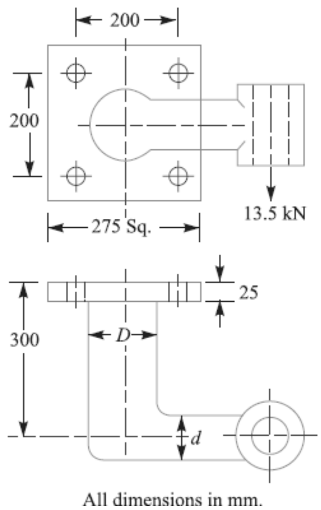


Fig. 3.29

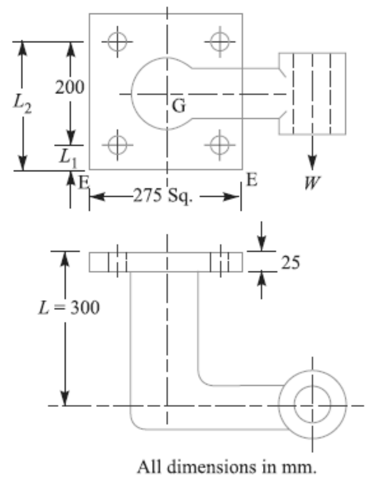


Fig. 3.30

Diameter D for the arm of the bracket

The section of the arm having D as the diameter is subjected to bending moment as well as twisting moment. We know that bending moment,

M = 13 500 × (300 – 25) = 3712.5 × 103 N-mm

and twisting moment, $T = 13\,500 \times 250 = 3375 \times 10^3$ N-mm

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3712.5 \times 10^3)^2 + (3375 \times 10^3)^2} \text{ N-mm}$$

$$= 5017 \times 10^3 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$5017 \times 10^3 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 65 \times D^3 = 12.76\, D^3$$

$$\therefore \quad D^3 = 5017 \times 10^3 / 12.76 = 393 \times 10^3$$

or D = 73.24 say 75 mm **Ans.**

Diameter (d) for the arm of the bracket

The section of the arm having d as the diameter is subjected to bending moment only. We know that bending moment,

$$M = 13\,500 \left(250 - \frac{75}{2}\right) = 2868.8 \times 10^3 \text{ N-mm}$$

and section modulus, $Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$

We know that bending (tensile) stress (σ_t),

$$110 = \frac{M}{Z} = \frac{2868.8 \times 10^3}{0.0982\, d^3} = \frac{29.2 \times 10^6}{d^3}$$

∴ $d^3 = 29.2 \times 10^6 / 110 = 265.5 \times 10^3$ or d = 64.3 say 65 mm **Ans.**

Tensile load on each top bolt

Due to the eccentric load W, the bracket has a tendency to tilt about the edge $E–E$, as shown in Fig. 11.46.

Let w = Load on each bolt per mm distance from the tilting edge due to the tilting effect of the bracket.

Since there are two bolts each at distance L_1 and L_2 as shown in Fig. 11.46, therefore total moment of the load on the bolts about the tilting edge $E–E$

$$= 2\,(w.L_1)\,L_1 + 2(w.L_2)\,L_2 = 2w\,[(L_1)^2 + (L_2)^2]$$

$$= 2w\,[(37.5)^2 + (237.5)^2] = 115\ 625\ w \text{ N-mm} \quad ...(i)$$

...($\because L_1 = 37.5$ mm and $L_2 = 237.5$ mm)

and turning moment of the load about the tilting edge

$$= W.L = 13\ 500 \times 300 = 4050 \times 10^3 \text{ N-mm} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$w = 4050 \times 10^3 / 115\ 625 = 35.03 \text{ N/mm}$$

∴ Tensile load on each top bolt

$$= w.L_2 = 35.03 \times 237.5 = 8320 \text{ N } \textbf{Ans.}$$

Maximum shearing force on each bolt

We know that primary shear load on each bolt acting vertically downwards,

$$W_{s1} = \frac{W}{n} = \frac{13\ 500}{4} = 3375 \text{ N} \quad ...(\because \text{No. of bolts, } n = 4)$$

Since all the bolts are at equal distances from the centre of gravity of the four bolts (G), therefore the secondary shear load on each bolt is same.

Distance of each bolt from the centre of gravity (G) of the bolts,

$$l_1 = l_2 = l_3 = l_4 = \sqrt{(100)^2 + (100)^2} = 141.4 \text{ mm}$$

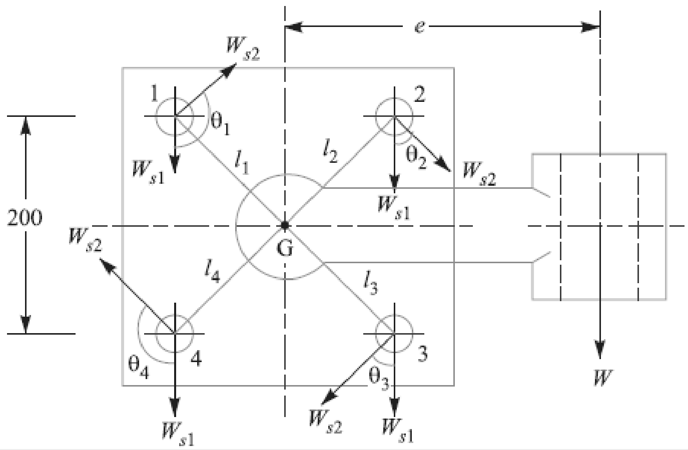


Fig. 3.31

∴ Secondary shear load on each bolt,

$$W_{s2} = \frac{W.e.l_1}{(l_1)^2 + (l_2)^2 + (l_3)^3 + (l_4)^2} = \frac{13\,500 \times 250 \times 141.4}{4\,(141.4)^2} = 5967 \text{ N}$$

Since the secondary shear load acts at right angles to the line joining the centre of gravity of the bolt group to the centre of the bolt as shown in Fig. 3.31, therefore the resultant of the primary and secondary shear load on each bolt gives the maximum shearing force on each bolt.
From the geometry of the Fig. 11.47, we find that

$$\theta_1 = \theta_4 = 135°, \text{ and } \theta_2 = \theta_3 = 45°$$

∴ Maximum shearing force on the bolts 1 and 4

$$= \sqrt{(W_{s1})^2 + (W_{s2})^2 + 2\,W_{s1} \times W_{s2} \times \cos 135°}$$

$$= \sqrt{(3375)^2 + (5967)^2 - 2 \times 3375 \times 5967 \times 0.7071} = 4303 \text{ N Ans.}$$

and maximum shearing force on the bolts 2 and 3

$$= \sqrt{(W_{s1})^2 + (W_{s2})^2 + 2\,W_{s1} \times W_{s2} \times \cos 45°}$$

$$= \sqrt{(3375)^2 + (5967)^2 + 2 \times 3375 \times 5967 \times 0.7071} = 8687 \text{ N Ans.}$$

3.2 Cotter and Knuckle Joints

3.2.1 Introduction

A cotter is a flat wedge shaped piece of rectangular cross-section and its width is tapered (either on one side or both sides) from one end to another for an easy adjustment. The taper varies from 1 in 48 to 1 in 24 and it may be increased up to 1 in 8, if a locking device is provided. The locking device may be a taper pin or a set screw used on the lower end of the cotter. The cotter is usually made of mild steel or wrought iron. A cotter joint is a temporary fastening and is used to connect rigidly two co-axial rods or bars which are subjected to axial tensile or compressive forces. It is usually used in connecting a piston rod to the crosshead of a reciprocating steam engine, a piston rod and its extension as a tail or pump rod, strap end of connecting rod etc.

3.2.2 Types of Cotter Joints

Following are the three commonly used cotter joints to connect two rods by a cotter :

1. Socket and spigot cotter joint,
2. Sleeve and cotter joint, and
3. Gib and cotter joint.

The design of these types of joints are discussed, in detail, in the following pages.

3.2.3 Socket and Spigot Cotter Joint

In a socket and spigot cotter joint, one end of the rods (say *A*) is provided with a socket type of end as shown in Fig. 3.32 and the other end of the other rod (say *B*) is inserted into a socket. The end of the rod which goes into a socket is also called ***spigot.*** *A* rectangular hole is made in the socket and spigot. *A* cotter is then driven tightly through a hole in order to make the temporary connection between the two rods. The load is usually acting axially, but it changes its direction and hence the cotter joint must be designed to carry both the tensile and compressive loads. The compressive load is taken up by the collar on the spigot.

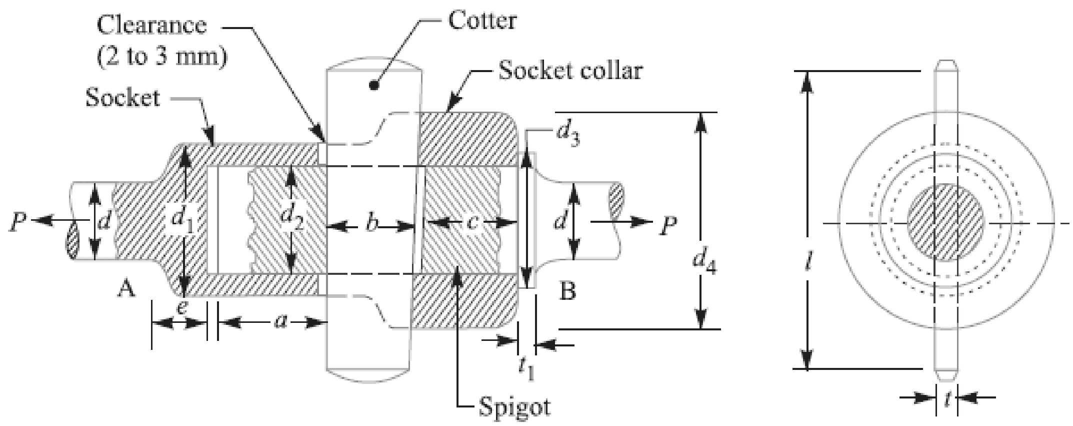


Fig. 3.32. Socket and spigot cotter joint.

3.2.4 Design of Socket and Spigot Cotter Joint

The socket and spigot cotter joint is shown in Fig. 3.32.

Let P = Load carried by the rods,

d = Diameter of the rods,

$d1$ = Outside diameter of socket,

$d2$ = Diameter of spigot or inside diameter of socket,

$d3$ = Outside diameter of spigot collar,

$t1$ = Thickness of spigot collar,

$d4$ = Diameter of socket collar,

c = Thickness of socket collar,

b = Mean width of cotter,

t = Thickness of cotter,

l = Length of cotter,

a = Distance from the end of the slot to the end of rod,

σ_t = Permissible tensile stress for the rods material,

τ= Permissible shear stress for the cotter material, and

σ_c= Permissible crushing stress for the cotter material.

The dimensions for a socket and spigot cotter joint may be obtained by considering the various modes of failure as discussed below :

1. *Failure of the rods in tension*

The rods may fail in tension due to the tensile load P. We know that Area resisting tearing

$$= \frac{\pi}{4} \times d^2$$

∴ Tearing strength of the rods,

$$= \frac{\pi}{4} \times d^2 \times \sigma_t$$

Equating this to load (P), we have

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (d) may be determined.

2. *Failure of spigot in tension across the weakest section (or slot)*
Since the weakest section of the spigot is that section which has a slot in it for the cotter, as shown in Fig. 3.33, therefore Area resisting tearing of the spigot across the slot

$$= \frac{\pi}{4}(d_2)^2 - d_2 \times t$$

and tearing strength of the spigot across the slot

$$= \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t$$

Equating this to load (P), we have

$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t$$

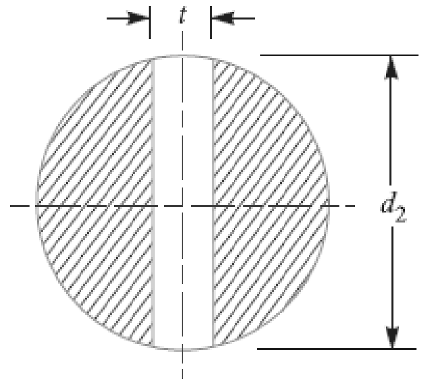


Fig. 3.33.

3. *Failure of the rod or cotter in crushing*

We know that the area that resists crushing of a rod or cotter

$= d2 \times t$

∴ Crushing strength = $d2 \times t \times \sigma_c$

Equating this to load (P), we have

$P = d2 \times t \times \sigma_c$

From this equation, the induced crushing stress may be checked.

4. *Failure of the socket in tension across the slot*
We know that the resisting area of the socket across the slot, as shown in Fig. 3.34

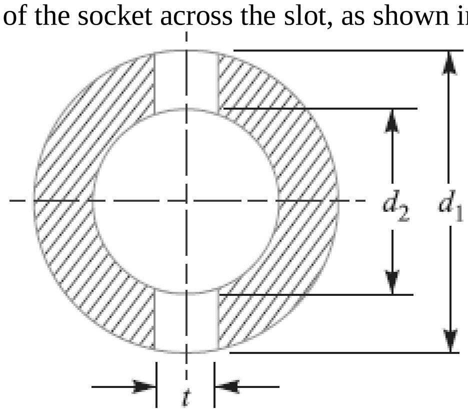


Fig. 3.34

$$= \frac{\pi}{4}\left[(d_1)^2 - (d_2)^2\right] - (d_1 - d_2)\, t$$

∴ Tearing strength of the socket across the slot

$$= \left\{\frac{\pi}{4}\left[(d_1)^2 - (d_2)^2\right] - (d_1 - d_2)\, t\right\} \sigma_t$$

Equating this to load (P), we have

$$P = \left\{\frac{\pi}{4}\left[(d_1)^2 - (d_2)^2\right] - (d_1 - d_2)\, t\right\} \sigma_t$$

From this equation, outside diameter of socket ($d1$) may be determined.

5. *Failure of cotter in shear*

Considering the failure of cotter in shear as shown in Fig. 3.35. Since the cotter is in double shear, therefore shearing area of the cotter

$$= 2\, b \times t$$

and shearing strength of the cotter

$$= 2\, b \times t \times \tau$$

Equating this to load (P), we have

$$P = 2\, b \times t \times \tau$$

From this equation, width of cotter (b) is determined

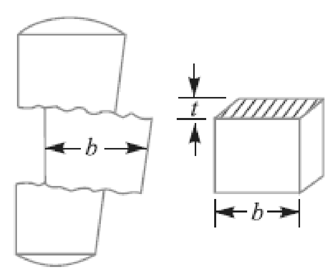


Fig. 3.35

6. *Failure of the socket collar in crushing*

Considering the failure of socket collar in crushing as shown in Fig. 3.36.

We know that area that resists crushing of socket collar

$$= (d_4 - d_2)\, t$$

and crushing strength $= (d_4 - d_2)\, t \times \sigma_c$

Equating this to load (P), we have

$$P = (d_4 - d_2)\, t \times \sigma_c$$

From this equation, the diameter of socket collar (d_4) may be obtained.

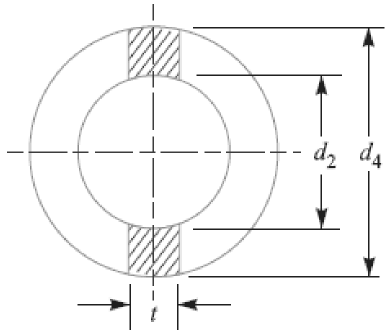


Fig. 3.36

7. ***Failure of socket end in shearing***

Since the socket end is in double shear, therefore area that resists shearing of socket collar

$$= 2\,(d_4 - d_2)\,c$$

and shearing strength of socket collar

$$= 2\,(d_4 - d_2)\,c \times \tau$$

Equating this to load (P), we have

$$P = 2\,(d_4 - d_2)\,c \times \tau$$

From this equation, the thickness of socket collar (c) may be obtained.

9. ***Failure of spigot collar in crushing***

Considering the failure of the spigot collar in crushing as shown in Fig. 3.37. We know that area that resists crushing of the collar

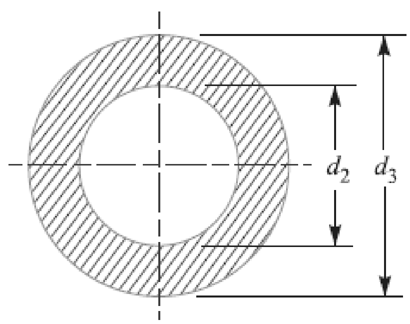


Fig. 3.37

$$= \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]$$

and crushing strength of the collar

$$= \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c$$

Equating this to load (P), we have

$$P = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c$$

From this equation, the diameter of the spigot collar (d_3) may be obtained

10. *Failure of the spigot collar in shearing*

Considering the failure of the spigot collar in shearing as shown in Fig. 3.38. We know that area that resists shearing of the collar

$$= \pi d_2 \times t_1$$

and shearing strength of the collar,

$$= \pi d_2 \times t_1 \times \tau$$

Equating this to load (P) we have

$$P = \pi d_2 \times t_1 \times \tau$$

From this equation, the thickness of spigot collar (t_1) may be obtained.

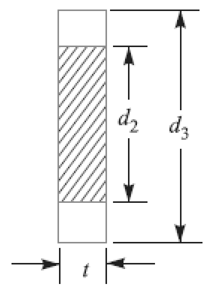


Fig. 3.38

11. *Failure of cotter in bending*

In all the above relations, it is assumed that the load is uniformly distributed over the various cross-sections of the joint. But in actual practice, this does not happen and the cotter is subjected to bending. In order to find out the bending stress induced, it is assumed that the load on the cotter in the rod end is uniformly distributed while in the socket end it varies from zero at the outer diameter ($d4$) and maximum at the inner diameter (d_2), as shown in Fig. 3.39.

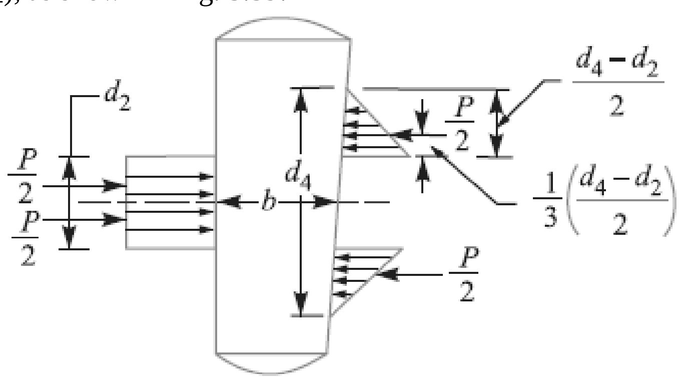


Fig. 3.39

The maximum bending moment occurs at the centre of the cotter and is given by

$$M_{max} = \frac{P}{2}\left(\frac{1}{3} \times \frac{d_4 - d_2}{2} + \frac{d_2}{2}\right) - \frac{P}{2} \times \frac{d_2}{4}$$
$$= \frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{2} - \frac{d_2}{4}\right) = \frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)$$

We know that section modulus of the cotter,

$$Z = t \times b^2 / 6$$

∴ Bending stress induced in the cotter,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{\frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)}{t \times b^2 / 6} = \frac{P\,(d_4 + 0.5\,d_2)}{2\,t \times b^2}$$

This bending stress induced in the cotter should be less than the allowable bending stress of the cotter.
12. The length of cotter (*l*) is taken as 4 *d*.
13. The taper in cotter should not exceed 1 in 24. In case the greater taper is required, then a locking device must be provided.
14. The draw of cotter is generally taken as 2 to 3 mm.

Example 3.8. *Design and draw a cotter joint to support a load varying from 30 kN in compression to 30 kN in tension. The material used is carbon steel for which the following allowable stresses may be used. The load is applied statically.*
Tensile stress = compressive stress = 50 MPa
shear stress = 35 MPa and crushing stress = 90 MPa.
Solution. Given :
P = 30 kN = 30 × 103 N
σ_t = 50 MPa = 50 N / mm^2
τ= 35 MPa = 35 N / mm ^{2}
σ_c = 90 MPa = 90 N/mm^2

The cotter joint is shown in Fig. 3.32. The joint is designed as discussed below :
1. ***Diameter of the rods***
Let *d* = Diameter of the rods.
Considering the failure of the rod in tension. We know that load (*P*),

$$30 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 50 = 39.3\,d^2$$

$$\therefore \quad d^2 = 30 \times 10^3 / 39.3 = 763 \quad \text{or} \quad d = 27.6 \text{ say } 28 \text{ mm} \quad \textbf{Ans.}$$

2. ***Diameter of spigot and thickness of cotter***
Let d_2 = Diameter of spigot or inside diameter of socket, and
t = Thickness of cotter. It may be taken as d_2 / 4.
Considering the failure of spigot in tension across the weakest section. We know that load (*P*),

$$30 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right]50 = 26.8\,(d_2)^2$$

$$\therefore \quad (d_2)^2 = 30 \times 10^3 / 26.8 = 1119.4 \quad \text{or} \quad d_2 = 33.4 \text{ say } 34 \text{ mm}$$

and thickness of cotter, $t = \frac{d_2}{4} = \frac{34}{4} = 8.5$ mm

Let us now check the induced crushing stress. We know that load (P),

$$30 \times 10^3 = d_2 \times t \times \sigma_c = 34 \times 8.5 \times \sigma_c = 289\,\sigma_c$$

$$\therefore \quad \sigma_c = 30 \times 10^3 / 289 = 103.8 \text{ N/mm}^2$$

Since this value of σ_c is more than the given value of $\sigma_c = 90$ N/mm^2, therefore the dimensions d_2 = 34 mm and t = 8.5 mm are not safe. Now let us find the values of d_2 and t by substituting the value of $\sigma_c = 90$ N/mm^2 in the above expression, *i.e.*

$$30 \times 10^3 = d_2 \times \frac{d_2}{4} \times 90 = 22.5\,(d_2)^2$$

$$\therefore \quad (d_2)^2 = 30 \times 10^3 / 22.5 = 1333 \quad \text{or} \quad d_2 = 36.5 \text{ say } 40 \text{ mm } \textbf{Ans.}$$

and $\quad t = d_2 / 4 = 40 / 4 = 10$ mm **Ans.**

3. *Outside diameter of socket*

Let $\quad d_1$ = Outside diameter of socket.

Considering the failure of the socket in tension across the slot. We know that load (P),

$$30 \times 10^3 = \left[\frac{\pi}{4}\left\{(d_1)^2 - (d_2)^2\right\} - (d_1 - d_2)\,t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}\left\{(d_1)^2 - (40)^2\right\} - (d_1 - 40)\,10\right]50$$

$$30 \times 10^3/50 = 0.7854\,(d_1)^2 - 1256.6 - 10\,d_1 + 400$$

$$\text{or} \quad (d_1)^2 - 12.7\,d_1 - 1854.6 = 0$$

$$\therefore \quad d_1 = \frac{12.7 \pm \sqrt{(12.7)^2 + 4 \times 1854.6}}{2} = \frac{12.7 \pm 87.1}{2}$$

$$= 49.9 \text{ say } 50 \text{ mm } \textbf{Ans.} \qquad \text{...(Taking +ve sign)}$$

4. *Width of cotter*

Let $\quad b$ = Width of cotter.

Considering the failure of the cotter in shear. Since the cotter is in double shear, therefore load (P),

$$30 \times 10^3 = 2\,b \times t \times \tau = 2\,b \times 10 \times 35 = 700\,b$$

$$b = 30 \times 10^3 / 700 = 43 \text{ mm } \textbf{Ans.}$$

5. *Diameter of socket collar*

Let $\quad d_4$ = Diameter of socket collar.

Considering the failure of the socket collar and cotter in crushing. We know that load (P),

$$30 \times 10^3 = (d_4 - d_2)\,t \times \sigma_c = (d_4 - 40)10 \times 90 = (d_4 - 40)\,900$$

$$\therefore \quad d_4 - 40 = 30 \times 10^3 / 900 = 33.3 \quad \text{or} \quad d_4 = 33.3 + 40 = 73.3 \text{ say } 75 \text{ mm } \textbf{Ans.}$$

6. *Thickness of socket collar*

Let c = Thickness of socket collar.

Considering the failure of the socket end in shearing. Since the socket end is in double shear, therefore load (P),

$$30 \times 10^3 = 2(d_4 - d_2)\, c \times \tau = 2\,(75 - 40)\, c \times 35 = 2450\, c$$

$$\therefore \quad c = 30 \times 10^3 / 2450 = 12 \text{ mm } \textbf{Ans.}$$

7. *Distance from the end of the slot to the end of the rod*

Let a = Distance from the end of slot to the end of the rod.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$$30 \times 10^3 = 2\, a \times d_2 \times \tau = 2a \times 40 \times 35 = 2800\, a$$

$$\therefore \quad a = 30 \times 10^3 / 2800 = 10.7 \text{ say } 11 \text{ mm } \textbf{Ans.}$$

8. *Diameter of spigot collar*

Let d_3 = Diameter of spigot collar.

Considering the failure of spigot collar in crushing. We know that load (P),

$$30 \times 10^3 = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right] \sigma_c = \frac{\pi}{4}\left[(d_3)^2 - (40)^2\right] 90$$

or

$$(d_3)^2 - (40)^2 = \frac{30 \times 10^3 \times 4}{90 \times \pi} = 424$$

$$\therefore \quad (d_3)^2 = 424 + (40)^2 = 2024 \quad \text{or} \quad d_3 = 45 \text{ mm } \textbf{Ans.}$$

9. *Thickness of spigot collar*

Let t_1 = Thickness of spigot collar.

Considering the failure of spigot collar in shearing. We know that load (P),

$$30 \times 10^3 = \pi\, d_2 \times t_1 \times \tau = \pi \times 40 \times t_1 \times 35 = 4400\, t_1$$

$$\therefore \quad t_1 = 30 \times 10^3 / 4400 = 6.8 \text{ say } 8 \text{ mm } \textbf{Ans.}$$

10. The length of cotter (l) is taken as 4 d.

$$\therefore \quad l = 4\,d = 4 \times 28 = 112 \text{ mm } \textbf{Ans.}$$

11. The dimension e is taken as 1.2 d.

$$\therefore \quad e = 1.2 \times 28 = 33.6 \text{ say } 34 \text{ mm } \textbf{Ans.}$$

3.2.5 Sleeve and Cotter Joint

Sometimes, a sleeve and cotter joint as shown in Fig. 3.40, is used to connect two round rods or bars. In this type of joint, a sleeve or muff is used over the two rods and then two cotters (one on each rod end) are inserted in the holes provided for them in the sleeve and rods. The taper of cotter is usually 1 in 24. It may be noted that the taper sides of the two cotters should face each other as shown in Fig. 3.40. The clearance is so adjusted that when the cotters are driven in, the two rods come closer to each other thus making the joint tight.

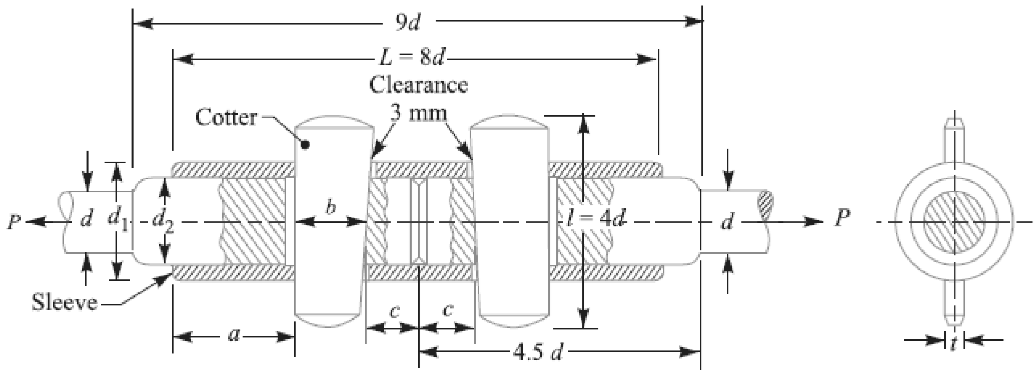


Fig. 3.40. Sleeve and cotter joint.

The various proportions for the sleeve and cotter joint in terms of the diameter of rod (*d*) are as follows :
Outside diameter of sleeve,

$D_1 = 2.5\ d$

Diameter of enlarged end of rod,

D_2 = Inside diameter of sleeve = 1.25 *d*

Length of sleeve, $L = 8\ d$

Thickness of cotter, $t = d2/4$ or 0.31 *d*

Width of cotter, $b = 1.25\ d$

Length of cotter, $l = 4\ d$

Distance of the rod end (*a*) from the beginning to the cotter hole (inside the sleeve end)

= Distance of the rod end (*c*) from its end to the cotter hole

= 1.25 *d*

3.2.6 Design of Sleeve and Cotter Joint

The sleeve and cotter joint is shown in Fig. 3.40.

Let *P* = Load carried by the rods,

d = Diameter of the rods,

*d*1 = Outside diameter of sleeve,

*d*2 = Diameter of the enlarged end of rod,

t = Thickness of cotter,

l = Length of cotter,

b = Width of cotter,

a = Distance of the rod end from the beginning to the cotter hole (inside the sleeve end),

c = Distance of the rod end from its end to the cotter hole,

σ_t ,$\tau$and σ_c = Permissible tensile, shear and crushing stresses respectively for the material of the rods and cotter.

The dimensions for a sleeve and cotter joint may be obtained by considering the various modes of failure as discussed below :

1. *Failure of the rods in tension*
The rods may fail in tension due to the tensile load *P*. We know that

1. *Failure of the rods in tension*

The rods may fail in tension due to the tensile load *P*. We know that

$$\text{Area resisting tearing} = \frac{\pi}{4} \times d^2$$

∴ Tearing strength of the rods

$$= \frac{\pi}{4} \times d^2 \times \sigma_t$$

Equating this to load (*P*), we have

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (*d*) may be obtained.

2. *Failure of the rod in tension across the weakest section (i.e. slot)*

Since the weakest section is that section of the rod which has a slot in it for the cotter, therefore area resisting tearing of the rod across the slot

$$= \frac{\pi}{4} (d_2)^2 - d_2 \times t$$

and tearing strength of the rod across the slot

$$= \left[\frac{\pi}{4} (d_2)^2 - d_2 \times t\right] \sigma_t$$

Equating this to load (*P*), we have

$$P = \left[\frac{\pi}{4} (d_2)^2 - d_2 \times t\right] \sigma_t$$

From this equation, the diameter of enlarged end of the rod (d_2) may be obtained.

3. *Failure of the rod or cotter in crushing*

We know that the area that resists crushing of a rod or cotter

$$= d_2 \times t$$

$\therefore$ Crushing strength $= d_2 \times t \times \sigma_c$

Equating this to load (P), we have

$$P = d_2 \times t \times \sigma_c$$

From this equation, the induced crushing stress may be checked.

4. *Failure of sleeve in tension across the slot*

We know that the resisting area of sleeve across the slot

$$= \frac{\pi}{4}\left[(d_1)^2 - (d_2)^2 \right] - (d_1 - d_2)\, t$$

$\therefore$ Tearing strength of the sleeve across the slot

$$= \left[\frac{\pi}{4} [(d_1)^2 - (d_2)^2] - (d_1 - d_2)\, t \right] \sigma_t$$

Equating this to load (P), we have

$$P = \left[\frac{\pi}{4} [(d_1)^2 - (d_2)^2] - (d_1 - d_2)\, t \right] \sigma_t$$

From this equation, the outside diameter of sleeve (d_1) may be obtained.

5. *Failure of cotter in shear*

Since the cotter is in double shear, therefore shearing area of the cotter

$$= 2b \times t$$

and shear strength of the cotter

$$= 2b \times t \times \tau$$

Equating this to load (P), we have

$$P = 2b \times t \times \tau$$

From this equation, width of cotter (b) may be determined.

6. *Failure of rod end in shear*

Since the rod end is in double shear, therefore area resisting shear of the rod end

$$= 2\, a \times d_2$$

and shear strength of the rod end

$$= 2\, a \times d_2 \times \tau$$

Equating this to load (P), we have

$$P = 2\, a \times d_2 \times \tau$$

From this equation, distance (a) may be determined.

7. *Failure of sleeve end in shear*

Since the sleeve end is in double shear, therefore the area resisting shear of the sleeve end

$$= 2\,(d_1 - d_2)\, c$$

and shear strength of the sleeve end

$$= 2\,(d_1 - d_2)\, c \times \tau$$

Equating this to load (P), we have

$$P = 2\,(d_1 - d_2)\, c \times \tau$$

From this equation, distance (c) may be determined.

Example 3.9.

Design a sleeve and cotter joint to resist a tensile load of 60 kN. All parts of the joint are made of the same material with the following allowable stresses:
σ_t = 60 MPa ; τ= 70 MPa ; and σ_c = 125 MPa.

Solution.

Given : P = 60 kN = 60 × 10^3 N

σ_t = 60 MPa = 60 N/mm^2

τ = 70 MPa = 70 N/mm^2

σ_c= 125 MPa = 125 N/mm^2

1. ***Diameter of the rods***

Let d = Diameter of the rods.

Considering the failure of the rods in tension. We know that load (P),

$$60 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 60 = 47.13\, d^2$$

$\therefore$ $d^2 = 60 \times 10^3 / 47.13 = 1273$ or $d = 35.7$ say 36 mm **Ans.**

2. *Diameter of enlarged end of rod and thickness of cotter*

Let d_2 = Diameter of enlarged end of rod, and

t = Thickness of cotter. It may be taken as $d_2 / 4$.

Considering the failure of the rod in tension across the weakest section (*i.e.* slot). We know that load (P),

$$60 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right]60 = 32.13\,(d_2)^2$$

$\therefore$ $(d_2)^2 = 60 \times 10^3 / 32.13 = 1867$ or $d_2 = 43.2$ say 44 mm **Ans.**

and thickness of cotter,

$$t = \frac{d_2}{4} = \frac{44}{4} = 11 \text{ mm } \textbf{Ans.}$$

Let us now check the induced crushing stress in the rod or cotter. We know that load (P),

$$60 \times 10^3 = d_2 \times t \times \sigma_c = 44 \times 11 \times \sigma_c = 484\, \sigma_c$$

$\therefore$ $\sigma_c = 60 \times 10^3 / 484 = 124 \text{ N/mm}^2$

Since the induced crushing stress is less than the given value of 125 N/mm^2, therefore the dimensions d_2 and t are within safe limits.

3. *Outside diameter of sleeve*

Let d_1 = Outside diameter of sleeve.

Considering the failure of sleeve in tension across the slot. We know that load (P)

$$60 \times 10^3 = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}[(d_1)^2 - (44)^2] - (d_1 - 44)\,11\right]60$$

$\therefore$ $60 \times 10^3 / 60 = 0.7854\,(d_1)^2 - 1520.7 - 11\,d_1 + 484$

or $(d_1)^2 - 14\,d_1 - 2593 = 0$

$$\therefore \quad d_1 = \frac{14 \pm \sqrt{(14)^2 + 4 \times 2593}}{2} = \frac{14 \pm 102.8}{2}$$

$= 58.4$ say 60 mm **Ans.** ...(Taking +ve sign)

4. *Width of cotter*

Let b = Width of cotter.

Considering the failure of cotter in shear. Since the cotter is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,b \times t \times \tau = 2 \times b \times 11 \times 70 = 1540\,b$$

$\therefore$ $b = 60 \times 10^3 / 1540 = 38.96$ say 40 mm **Ans.**

5. *Distance of the rod from the beginning to the cotter hole (inside the sleeve end)*

Let a = Required distance.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,a \times d_2 \times \tau = 2\,a \times 44 \times 70 = 6160\,a$$

$\therefore$ $a = 60 \times 10^3 / 6160 = 9.74$ say 10 mm **Ans.**

6. Distance of the rod end from its end to the cotter hole

Let c = Required distance.

Considering the failure of the sleeve end in shear. Since the sleeve end is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,(d_1 - d_2)\,c \times \tau = 2\,(60 - 44)\,c \times 70 = 2240\,c$$

$$\therefore \quad c = 60 \times 10^3 / 2240 = 26.78 \text{ say } 28 \text{ mm} \quad \textbf{Ans.}$$

3.2.7 Knuckle Joint

A knuckle joint is used to connect two rods which are under the action of tensile loads. However, if the joint is guided, the rods may support a compressive load. A knuckle joint may be readily disconnected for adjustments or repairs. Its use may be found in the link of a cycle chain, tie rod joint for roof truss, valve rod joint with eccentric rod, pump rod joint, tension link in bridge structure and lever and rod connections of various types.

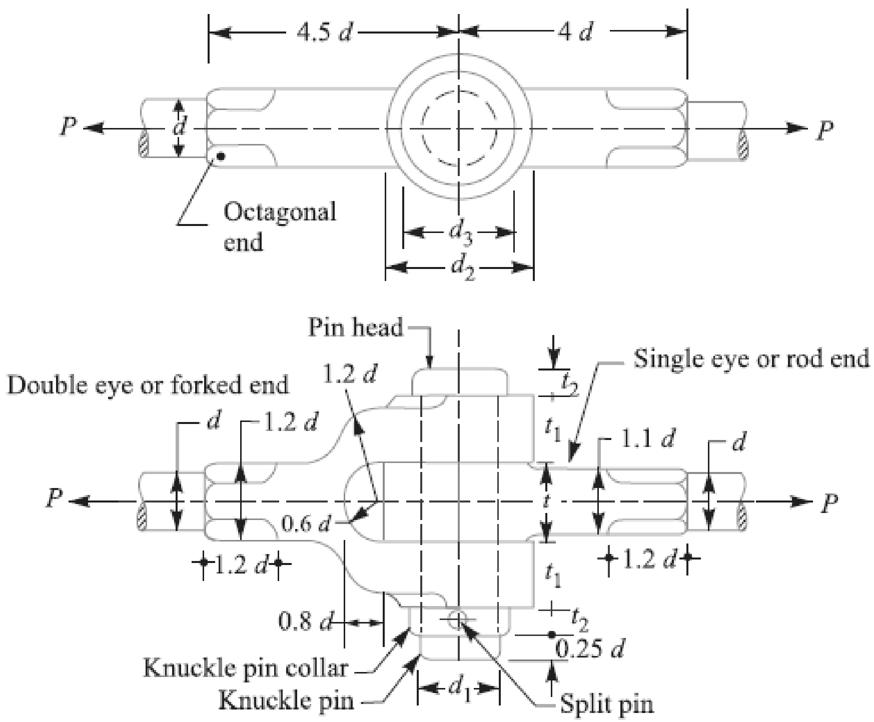


Fig. 3.41. Kunckle joint.

In knuckle joint (the two views of which are shown in Fig. 3.41), one end of one of the rods is made into an eye and the end of the other rod is formed into a fork with an eye in each of the fork leg. The knuckle pin passes through both the eye hole and the fork holes and may be secured by means of a collar and taper pin or spilt pin. The knuckle pin may be prevented from rotating in the fork by means of a small stop, pin, peg or snug. In order to get a better quality of joint, the sides of the fork and eye are machined, the hole is accurately drilled and pin turned. The material used for the joint may be steel or wrought iron.

3.2.8 Dimensions of Various Parts of the Knuckle Joint

The dimensions of various parts of the knuckle joint are fixed by empirical relations as given below. It may be noted that all the parts should be made of the same material *i.e.* mild steel or wrought iron.

If d is the diameter of rod, then diameter of pin,

$$d_1 = d$$

Outer diameter of eye,

$d_2 = 2\ d$

Diameter of knuckle pin head and collar,

$d_3 = 1.5\ d$

Thickness of single eye or rod end,

$t = 1.25\ d$

Thickness of fork, $t1 = 0.75\ d$

Thickness of pin head, $t2 = 0.5\ d$

Other dimensions of the joint are shown in Fig. 3.41.

3.2.9 Methods of Failure of Knuckle Joint

Consider a knuckle joint as shown in Fig. 3.41.

Let P = Tensile load acting on the rod,

d = Diameter of the rod,

$d1$ = Diameter of the pin,

$d2$ = Outer diameter of eye,

t = Thickness of single eye,

$t1$ = Thickness of fork.

σ_t ,$\tau$and σ_c = Permissible stresses for the joint material in tension, shear and crushing respectively.

In determining the strength of the joint for the various methods of failure, it is assumed that

1. There is no stress concentration, and

2. The load is uniformly distributed over each part of the joint.

Due to these assumptions, the strengths are approximate, however they serve to indicate a well proportioned joint. Following are the various methods of failure of the joint :

1. *Failure of the solid rod in tension*

Since the rods are subjected to direct tensile load, therefore tensile strength of the rod,

$$= \frac{\pi}{4} \times d^2 \times \sigma_t$$

Equating this to the load (P) acting on the rod, we have

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rod (d) is obtained.

2. *Failure of the knuckle pin in shear*

Since the pin is in double shear, therefore cross-sectional area of the pin under shearing

$$= 2 \times \frac{\pi}{4} (d_1)^2$$

and the shear strength of the pin

$$= 2 \times \frac{\pi}{4} (d_1)^2\ \tau$$

Equating this to the load (P) acting on the rod, we have

$$P = 2 \times \frac{\pi}{4} (d_1)^2\ \tau$$

From this equation, diameter of the knuckle pin (d_1) is obtained. This assumes that there is no slack and clearance between the pin and the fork and hence there is no bending of the pin. But, in actual practice, the knuckle pin is loose in forks in order to permit angular movement of one with respect to the other, therefore the pin is subjected to bending in addition to shearing. By making the diameter of knuckle pin equal to the diameter of the rod (*i.e.*, $d1 = d$), a margin of strength is provided to allow for the bending of the pin. In case, the stress due to bending is taken into account, it is assumed that the load on the pin is uniformly distributed along the middle portion (*i.e.* the eye end) and varies uniformly over the forks as shown in Fig. 3.42. Thus in the forks, a load $P/2$ acts through a distance of $t1 / 3$ from the inner edge and the bending moment will be maximum at the centre of the pin. The value of maximum bending moment is given by

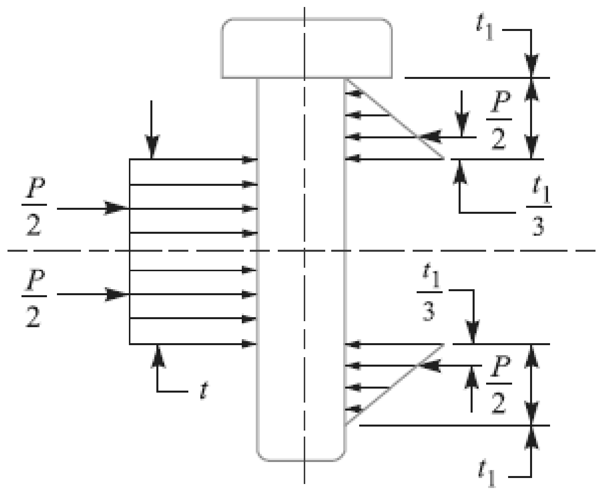


Fig. 3.42. Distribution of load on the pin.

$$M = \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{2}\right) - \frac{P}{2} \times \frac{t}{4}$$

$$= \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{2} - \frac{t}{4}\right)$$

$$= \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{4}\right)$$

and section modulus, $Z = \frac{\pi}{32}(d_1)^3$

∴ Maximum bending (tensile) stress,

$$\sigma_t = \frac{M}{Z} = \frac{\frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{4}\right)}{\frac{\pi}{32}(d_1)^3}$$

From this expression, the value of $d1$ may be obtained.

3. *Failure of the single eye or rod end in tension*

The single eye or rod end may tear off due to the tensile load. We know that area resisting tearing $= (d_2 - d_1)\, t$

∴ Tearing strength of single eye or rod end

$$= (d_2 - d_1)\, t \times \sigma_t$$

Equating this to the load (P) we have

$$P = (d_2 - d_1)\, t \times \sigma_t$$

From this equation, the induced tensile stress (σ_t) for the single eye or rod end may be checked. In case the induced tensile stress is more than the allowable working stress, then increase the outer diameter of the eye (d_2).

4. *Failure of the single eye or rod end in shearing*

The single eye or rod end may fail in shearing due to tensile load. We know that area resisting shearing $= (d_2 - d_1)\, t$

∴ Shearing strength of single eye or rod end

$$= (d_2 - d_1)\, t \times \tau$$

Equating this to the load (P), we have

$$P = (d_2 - d_1)\, t \times \tau$$

From this equation, the induced shear stress (τ) for the single eye or rod end may be checked.

5. *Failure of the single eye or rod end in crushing*

The single eye or pin may fail in crushing due to the tensile load. We know that area resisting crushing $= d_1 \times t$

∴ Crushing strength of single eye or rod end

$$= d_1 \times t \times \sigma_c$$

Equating this to the load (P), we have

$$\therefore \quad P = d_1 \times t \times \sigma_c$$

From this equation, the induced crushing stress (σ_c) for the single eye or pin may be checked. In case the induced crushing stress in more than the allowable working stress, then increase the thickness of the single eye (t).

6. *Failure of the forked end in tension*

The forked end or double eye may fail in tension due to the tensile load. We know that area resisting tearing

$$= (d_2 - d_1) \times 2\, t_1$$

∴ Tearing strength of the forked end

$$= (d_2 - d_1) \times 2\, t_1 \times \sigma_t$$

Equating this to the load (P), we have

$$P = (d_2 - d_1) \times 2t_1 \times \sigma_t$$

From this equation, the induced tensile stress for the forked end may be checked.

6. *Failure of the forked end in tension*

The forked end or double eye may fail in tension due to the tensile load. We know that area resisting tearing

$$= (d_2 - d_1) \times 2\, t_1$$

$\therefore$ Tearing strength of the forked end

$$= (d_2 - d_1) \times 2\, t_1 \times \sigma_t$$

Equating this to the load (*P*), we have

$$P = (d_2 - d_1) \times 2t_1 \times \sigma_t$$

From this equation, the induced tensile stress for the forked end may be checked.

7. *Failure of the forked end in shear*

The forked end may fail in shearing due to the tensile load. We know that area resisting shearing

$$= (d_2 - d_1) \times 2t_1$$

$\therefore$ Shearing strength of the forked end

$$= (d_2 - d_1) \times 2t_1 \times \tau$$

Equating this to the load (*P*), we have

$$P = (d_2 - d_1) \times 2t_1 \times \tau$$

From this equation, the induced shear stress for the forked end may be checked. In case, the induced shear stress is more than the allowable working stress, then thickness of the fork (t_1) is increased.

8. *Failure of the forked end in crushing*

The forked end or pin may fail in crushing due to the tensile load. We know that area resisting crushing $= d_1 \times 2\, t_1$

$\therefore$ Crushing strength of the forked end

$$= d_1 \times 2\, t_1 \times \sigma_c$$

Equating this to the load (*P*), we have

$$P = d_1 \times 2\, t_1 \times \sigma_c$$

From this equation, the induced crushing stress for the forked end may be checked.

3.2.10 Design Procedure of Knuckle Joint

The following procedure may be adopted :

1. First of all, find the diameter of the rod by considering the failure of the rod in tension. We know that tensile load acting on the rod,

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

where *d* = Diameter of the rod, and

σ_t = Permissible tensile stress for the material of the rod.

2. After determining the diameter of the rod, the diameter of pin (*d*1) may be determined by considering the failure of the pin in shear. We know that load,

$$P = 2 \times \frac{\pi}{4}(d_1)^2\ \tau$$

A little consideration will show that the value of $d1$ as obtained by the above relation is less than the specified value (*i.e.* the diameter of rod). So fix the diameter of the pin equal to the diameter of the rod.
3. Other dimensions of the joint are fixed by empirical relations as discussed in Art. 3.2.8
4. The induced stresses are obtained by substituting the empirical dimensions in the relations as discussed in Art. 3.2.9
In case the induced stress is more than the allowable stress, then the corresponding dimension may be increased.

Example 3.10. *Design a knuckle joint to transmit 150 kN. The design stresses may be taken as 75 MPa in tension, 60 MPa in shear and 150 MPa in compression.*

Solution. Given : P = 150 kN = 150 × 10^3 N ;
σ_t= 75 MPa = 75 N/mm^2
τ= 60 MPa = 60 N/mm^2
σ_c = 150 MPa = 150 N/mm^2
The knuckle joint is shown in Fig. 3.41. The joint is designed by considering the various methods of failure as discussed below :

1. *Failure of the solid rod in tension*

Let d = Diameter of the rod.

We know that the load transmitted (P),

$$150 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 75 = 59\, d^2$$

$\therefore$ $d^2 = 150 \times 10^3 / 59 = 2540$ or $d = 50.4$ say 52 mm **Ans.**

Now the various dimensions are fixed as follows :

Diameter of knuckle pin,

$$d_1 = d = 52 \text{ mm}$$

Outer diameter of eye, $d_2 = 2\,d = 2 \times 52 = 104$ mm

Diameter of knuckle pin head and collar,

$$d_3 = 1.5\,d = 1.5 \times 52 = 78 \text{ mm}$$

Thickness of single eye or rod end,

$$t = 1.25\,d = 1.25 \times 52 = 65 \text{ mm}$$

Thickness of fork, $t_1 = 0.75\,d = 0.75 \times 52 = 39$ say 40 mm

Thickness of pin head, $t_2 = 0.5\,d = 0.5 \times 52 = 26$ mm

2. *Failure of the knuckle pin in shear*

Since the knuckle pin is in double shear, therefore load (P),

$$150 \times 10^3 = 2 \times \frac{\pi}{4} \times (d_1)^2\, \tau = 2 \times \frac{\pi}{4} \times (52)^2\, \tau = 4248\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 4248 = 35.3 \text{ N/mm}^2 = 35.3 \text{ MPa}$$

3. *Failure of the single eye or rod end in tension*

The single eye or rod end may fail in tension due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, t \times \sigma_t = (104 - 52)\, 65 \times \sigma_t = 3380\, \sigma_t$$

$$\therefore \quad \sigma_t = 150 \times 10^3 / 3380 = 44.4 \text{ N / mm}^2 = 44.4 \text{ MPa}$$

4. *Failure of the single eye or rod end in shearing*

The single eye or rod end may fail in shearing due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, t \times \tau = (104 - 52)\, 65 \times \tau = 3380\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

5. *Failure of the single eye or rod end in crushing*

The single eye or rod end may fail in crushing due to the load. We know that load (P),

$$150 \times 10^3 = d_1 \times t \times \sigma_c = 52 \times 65 \times \sigma_c = 3380\, \sigma_c$$

$$\therefore \quad \sigma_c = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

6. *Failure of the forked end in tension*

The forked end may fail in tension due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, 2\, t_1 \times \sigma_t = (104 - 52)\, 2 \times 40 \times \sigma_t = 4160\, \sigma_t$$

$$\therefore \quad \sigma_t = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

7. *Failure of the forked end in shear*

The forked end may fail in shearing due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, 2\, t_1 \times \tau = (104 - 52)\, 2 \times 40 \times \tau = 4160\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

8. *Failure of the forked end in crushing*

The forked end may fail in crushing due to the load. We know that load (P),

$$150 \times 10^3 = d_1 \times 2\, t_1 \times \sigma_c = 52 \times 2 \times 40 \times \sigma_c = 4160\, \sigma_c$$

$$\therefore \quad \sigma_c = 150 \times 10^3 / 4180 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

From above, we see that the induced stresses are less than the given design stresses, therefore the joint is safe.

3.2 WELDED JOINTS

3.3.1 Introduction

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding.
Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints. It is also used as a repair medium *e.g*. to reunite metal at a crack, to build up a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface.

3.3.2 Advantages and Disadvantages of Welded Joints over Riveted Joints

Following are the advantages and disadvantages of welded joints over riveted joints.

Advantages

1. The welded structures are usually lighter than riveted structures. This is due to the reason,
that in welding, gussets or other connecting components are not used.
2. The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
3. Alterations and additions can be easily made in the existing structures.
4. As the welded structure is smooth in appearance, therefore it looks pleasing.
5. In welded connections, the tension members are not weakened as in the case of riveted joints.
6. A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
7. Sometimes, the members are of such a shape (*i.e*. circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.
8. The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.
9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.
10. The process of welding takes less time than the riveting.

Disadvantages

1. Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.
2. It requires a highly skilled labour and supervision.
3. Since no provision is kept for expansion and contraction in the frame, therefore there is a
possibility of cracks developing in it.
4. The inspection of welding work is more difficult than riveting work.

3.3.3 Welding Processes

The welding processes may be broadly classified into the following two groups:
1. Welding processes that use heat alone *e.g*. fusion welding.
2. Welding processes that use a combination of heat and pressure
e.g. forge welding.
These processes are discussed in detail, in the following pages

3.3.4 Fusion Welding

In case of fusion welding, the parts to be jointed are held in position while the molten metal is supplied to the joint. The molten metal may come from the parts themselves (*i.e*. parent metal) or filler metal
which normally have the composition of the parent metal. The joint surface become

plastic or even molten because of the heat from the molten filler metal or other source. Thus, when the molten metal solidifies or fuses, the joint is formed. The fusion welding, according to the method of heat generated, may be classified as:

1. Thermit welding, **2.** Gas welding, and **3.** Electric arc welding.

3.3.5 Thermit Welding

In thermit welding, a mixture of iron oxide and aluminium called ***thermit*** is ignited and the iron oxide is reduced to molten iron. The molten iron is poured into a mould made around the joint and fuses with the parts to be welded. A major advantage of the thermit welding is that all parts of weld section are molten at the same time and the weld cools almost uniformly. This results in a minimum problem with residual stresses. It is fundamentally a melting and casting process. The thermit welding is often used in joining iron and steel parts that are too large to be manufactured in one piece, such as rails, truck frames, locomotive frames, other large sections used on steam and rail roads, for stern frames, rudder frames etc. In steel mills, thermit electric welding is employed to replace broken gear teeth, to weld new necks on rolls and pinions, and to repair broken shears

3.3.6 Gas Welding

A gas welding is made by applying the flame of an oxy-acetylene or hydrogen gas from a welding torch upon the surfaces of the prepared joint. The intense heat at the white cone of the flame heats up the local surfaces to fusion point while the operator manipulates a welding rod to supply the metal for the weld. A flux is being used to remove the slag. Since the heating rate in gas welding is slow, therefore it can be used on thinner materials.

3.3.7 Electric Arc Welding

In electric arc welding, the work is prepared in the same manner as for gas welding. In this case the filler metal is supplied by metal welding electrode. The operator, with his eyes and face protected, strikes an arc by touching the work of base metal with the electrode. The base metal in the path of the arc stream is melted, forming a pool of molten metal, which seems to be forced out of the pool by the blast from the arc, as shown in Fig. 3.45. A small depression is formed in the base metal and the molten metal is deposited around the edge of this depression, which is called the ***arc crater.*** The slag is brushed off after the joint has cooled. The arc welding does not require the metal to be preheated and since the temperature of the arc is quite high, therefore the fusion of the metal is almost instantaneous. There are two kinds of arc weldings depending upon the type of electrode.

1. Un-shielded arc welding, and
2. Shielded arc welding.

When a large electrode or filler rod is used for welding, it is then said to be ***un-shielded arc welding.*** In this case, the deposited weld metal while it is hot will absorb oxygen and nitrogen from the atmosphere. This decreases the strength of weld metal and lower its ductility and resistance to corrosion. In ***shielded arc welding,*** the welding rods coated with solid material are used, as shown in Fig. 3.45. The resulting projection of coating focuses a concentrated arc stream, which protects the globules of metal from the air and prevents the absorption of large amounts of harmful oxygen and nitrogen.

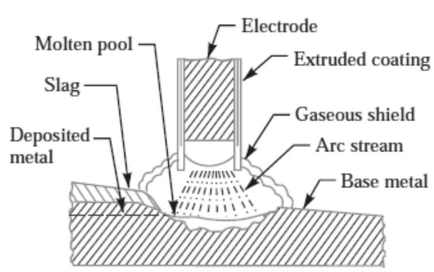


Fig. 3.45. Shielded electric arc welding

3.3.8 Forge Welding

In forge welding, the parts to be jointed are first heated to a proper temperature in a furnace or forge and then hammered. This method of welding is rarely used now-a-days. An ***electric-resistance welding*** is an example of forge welding. In this case, the parts to be joined are pressed together and an electric current is passed from one part to the other until the metal is heated to the fusion temperature of the joint. The principle of applying heat and pressure, either sequentially or simultaneously, is widely used in the processes known as ***spot, seam, projection, upset and flash welding.***

3.3.9 Types of Welded Joints

Following two types of welded joints are important from the subject point of view:

1. Lap joint or fillet joint, and **2.** Butt joint.

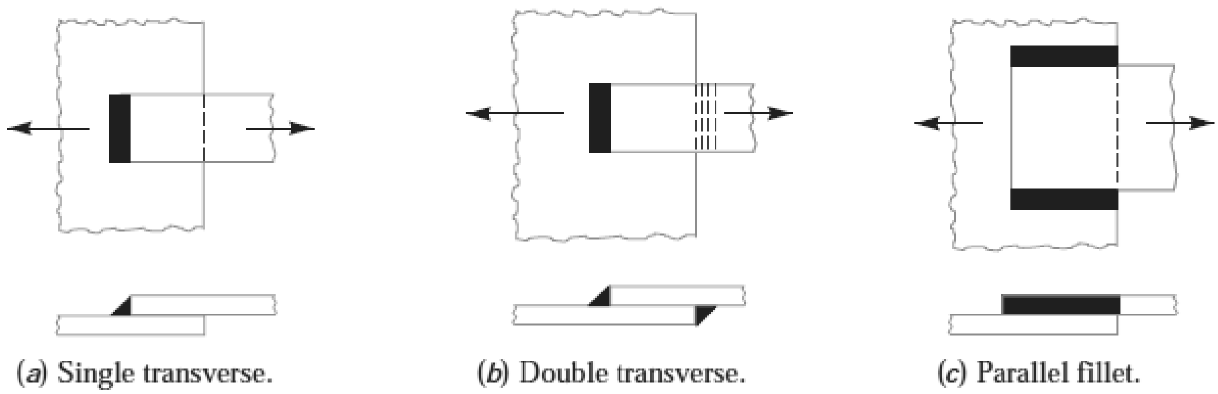


Fig. 3.46. Types of lap or fillet joints.

Lap Joint

The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be

1. Single transverse fillet, **2.** Double transverse fillet, and **3.** Parallel fillet joints.

The fillet joints are shown in Fig. 3.46. A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.

Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in Fig. 3.47. In butt welds, the plate edges do not require bevelling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U-groove on both sides.

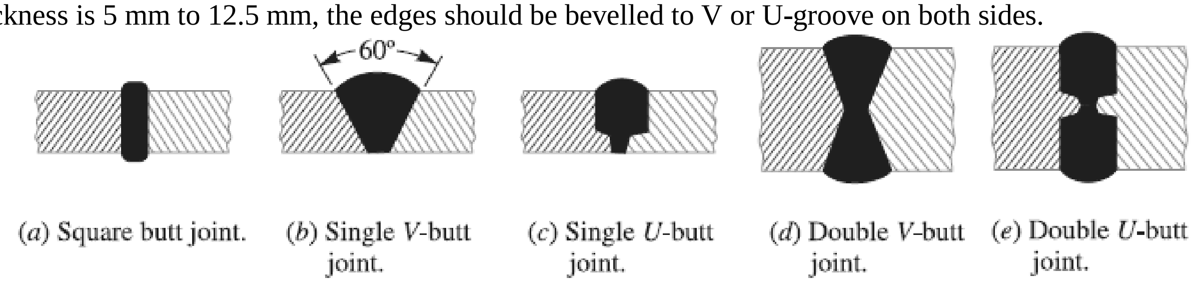


Fig. 3.47. Types of butt joints.

The butt joints may be

1. Square butt joint, **2.** Single V-butt joint **3.** Single U-butt joint, **4.** Double V-butt joint, and **5.** Double U-butt joint.

These joints are shown in Fig. 3.48.
The other type of welded joints are corner joint, edge joint and T-joint as shown in Fig. 3.48.

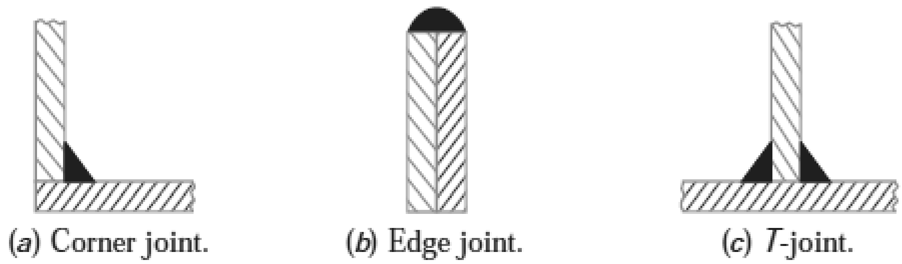


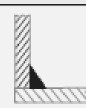
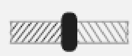
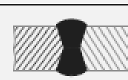
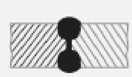
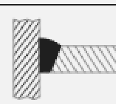
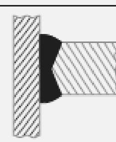
Fig. 3.48. Other types of welded joints.

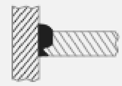
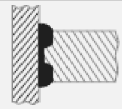
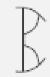
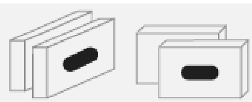
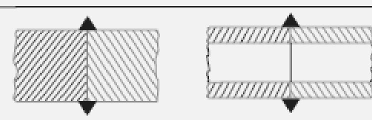
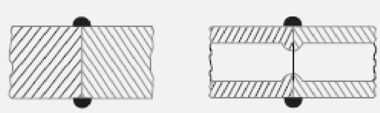
The main considerations involved in the selection of weld type are:
1. The shape of the welded component required,
2. The thickness of the plates to be welded, and
3. The direction of the forces applied.

3.3.10 Basic Weld Symbols

The basic weld symbols according to IS : 813 – 1961 (Reaffirmed 1991) are shown in the following table.

Table 3.2. Basic weld symbols.

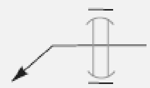
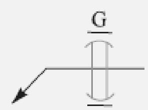
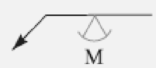
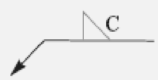
S. No.	*Form of weld*	*Sectional representation*	*Symbol*
1.	Fillet		
2.	Square butt		
3.	Single-*V* butt		
4.	Double-*V* butt		
5.	Single-*U* butt		
6.	Double-*U* butt		
7.	Single bevel butt		
8.	Double bevel butt		

S. No.	*Form of weld*	*Sectional representation*	*Symbol*
9.	Single-*J* butt		
10.	Double-*J* butt		
11.	Bead (edge or seal)		
12.	Stud		⊥
13.	Sealing run		○
14.	Spot		
15.	Seam		
16.	Mashed seam	 Before After	
17.	Plug		
18.	Backing strip		=
19.	Stitch		
20.	Projection	 Before After	△
21.	Flash	 Rod or bar Tube	
22.	Butt resistance or pressure (upset)	 Rod or bar Tube	|

3.3.11 Supplementary Weld Symbols

In addition to the above symbols, some supplementary symbols, according to IS:813 – 1961 (Reaffirmed 1991), are also used as shown in the following table.

Table 3.3. Supplementary weld symbols.

S. No.	*Particulars*	*Drawing representation*	*Symbol*
1.	Weld all round		○
2.	Field weld		●
3.	Flush contour		—
4.	Convex contour		⌒
5.	Concave contour		◡
6.	Grinding finish		G
7.	Machining finish		M
8.	Chipping finish		C

3.3.12 Elements of a Welding Symbol

A welding symbol consists of the following eight elements:

1. Reference line, **2.** Arrow,
3. Basic weld symbols, **4.** Dimensions and other data,
5. Supplementary symbols, **6.** Finish symbols,
7. Tail, and **8.** Specification, process or other references.

3.3.13 Standard Location of Elements of a Welding Symbol

According to Indian Standards, IS: 813 – 1961 (Reaffirmed 1991), the elements of a welding symbol shall have standard locations with respect to each other. The arrow points to the location of weld, the basic symbols with dimensions are located on one or both sides of reference line. The specification if any is placed in the tail of arrow. Fig. 3.49 shows the standard locations of welding symbols represented on drawing.

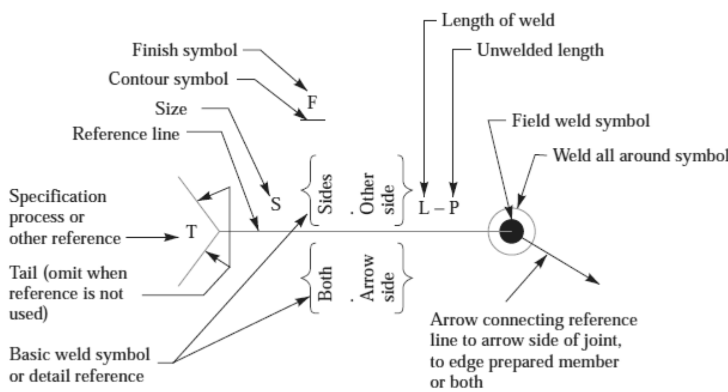
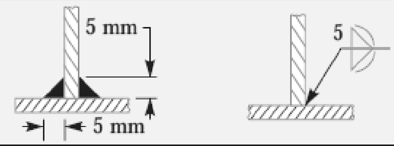
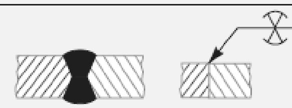
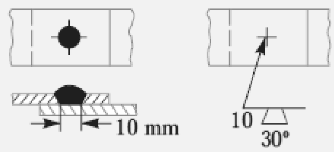
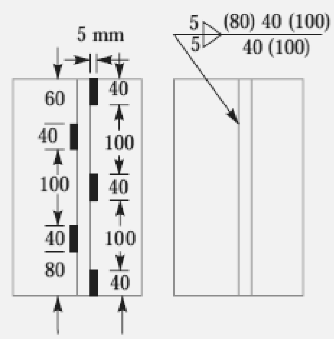


Fig. 3.49. Standard location of welding symbols.

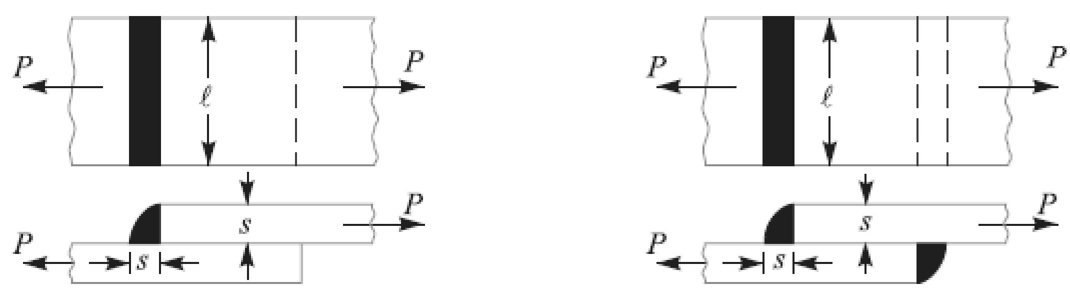
Some of the examples of welding symbols represented on drawing are shown in the following table.

Table 3.4. Representation of welding symbols.

S. No.	Desired weld	Representation on drawing
1.	Fillet-weld each side of Tee- convex contour	
2.	Single V-butt weld -machining finish	
3.	Double V- butt weld	
4.	Plug weld - 30° Groove-angle-flush contour	
5.	Staggered intermittent fillet welds	

3.3.14 Strength of Transverse Fillet Welded Joints

We have already discussed that the fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates. The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse fillet welds as shown in Fig. 3.50 (*a*) and (*b*) respectively.



(*a*) Single transverse fillet weld. (*b*) Double transverse fillet weld.

Fig. 3.50. Transverse fillet welds.

In order to determine the strength of the fillet joint, it is assumed that the section of fillet is a right angled triangle *ABC* with hypotenuse *AC* making equal angles with other two sides *AB* and *BC*. The enlarged view of the fillet is shown in Fig. 3.50. The length of each side is known as ***leg*** or ***size of the weld*** and the perpendicular distance of the hypotenuse from the intersection of legs (*i.e. BD*) is known as ***throat thickness.*** The minimum area of the weld is obtained at the throat *BD*, which is given by the product of the throat thickness and length of weld.

Let t = Throat thickness (*BD*),
s = Leg or size of weld,
= Thickness of plate, and
l = Length of weld,
From Fig. 3.50, we find that the throat thickness,
$t = s \times \sin 45° = 0.707\ s$
∴ Minimum area of the weld or throat area,
A = Throat thickness ×Length of weld
$= t \times l = 0.707\ s \times l$

If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

P = Throat area × Allowable tensile stress = $0.707\ s \times l \times \sigma_t$

and tensile strength of the joint for double fillet weld,

$P = 2 \times 0.707\ s \times l \times \sigma_t = 1.414\ s \times l \times \sigma_t$

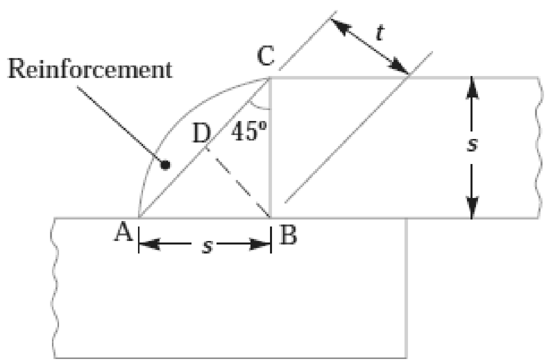


Fig. 3.51. Enlarged view of a fillet weld.

3.3.15 Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint as shown in Fig. 3.52 (*a*). We have already discussed in the previous article, that the minimum area of weld or the throat area,

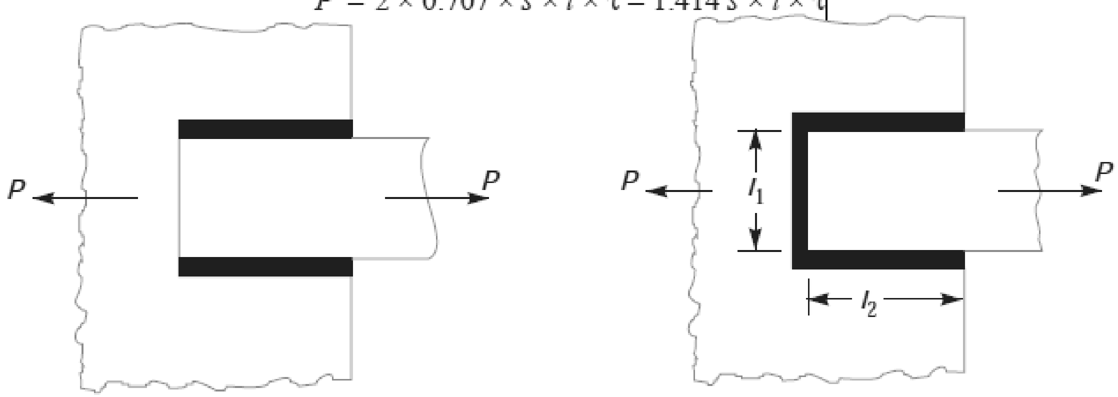
$$A = 0.707\ s \times l$$

If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

$$P = \text{Throat area} \times \text{Allowable shear stress} = 0.707\, s \times l \times \tau$$

and shear strength of the joint for double parallel fillet weld,

$$P = 2 \times 0.707 \times s \times l \times \tau = 1.414\, s \times l \times \tau$$



(*a*) Double parallel fillet weld.

(*b*) Combination of transverse and parallel fillet weld.

Fig. 3.52

Example 3.11. *A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.*

Solution. Given:
Width = 100 mm
Thickness = 10 mm
$P = 80$ kN $= 80 \times 10^3$ N
$\tau = 55$ MPa $= 55$ N/mm^2

Let l =Length of weld, and
s = Size of weld = Plate thickness = 10 mm ... (Given)

We know that maximum load which the plates can carry for double parallel fillet weld (P),

$80 \times 103 = 1.414 \times s \times l \times \tau = 1.414 \times 10 \times l \times 55 = 778\, l$

$\therefore \quad l = 80 \times 103 / 778 = 103$ mm

Adding 12.5 mm for starting and stopping of weld run, we have

$l = 103 + 12.5 = 115.5$ mm **Ans.**

3.3.16 Special Cases of Fillet Welded Joints

The following cases of fillet welded joints are important from the subject point of view.

1. *Circular fillet weld subjected to torsion.* Consider a circular rod connected to a rigid plate by a fillet weld as shown in Fig. 3.53.

Let d = Diameter of rod,
r = Radius of rod,
T = Torque acting on the rod,
s = Size (or leg) of weld,
t = Throat thickness,
J = Polar moment of inertia of the

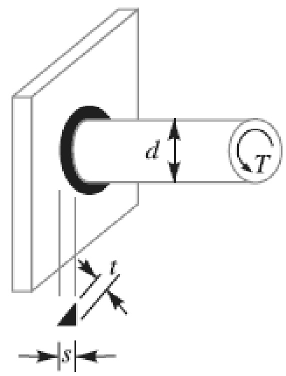


Fig. 3.53. Circular fillet weld subjected to torsion.

weld section = $\frac{\pi t d^3}{4}$

We know that shear stress for the material,

$$\tau = \frac{T.r}{J} = \frac{T \times d/2}{J}$$

$$= \frac{T \times d/2}{\pi t d^3/4} = \frac{2T}{\pi t d^2}$$

This shear stress occurs in a horizontal plane along a leg of the fillet weld. The maximum shear occurs on the throat of weld which is inclined at 45° to the horizontal plane.

∴ Length of throat, $t = s \sin 45° = 0.707\ s$

and maximum shear stress,

$$\tau_{max} = \frac{2T}{\pi \times 0.707\ s \times d^2} = \frac{2.83\ T}{\pi\ s\ d^2}$$

2. *Circular fillet weld subjected to bending moment.* Consider a circular rod connected to a rigid plate by a fillet weld as shown in Fig. 3.54.

Let d = Diameter of rod,

M = Bending moment acting on the rod,

s = Size (or leg) of weld,

t = Throat thickness,

Z = Section modulus of the weld section

$$= \frac{\pi t d^2}{4}$$

We know that the bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{M}{\pi t d^2/4} = \frac{4M}{\pi t d^2}$$

This bending stress occurs in a horizontal plane along a leg of the fillet weld. The maximum bending stress occurs on the throat of the weld which is inclined at 45° to the horizontal plane.

∴ Length of throat, $t = s \sin 45° = 0.707\ s$

and maximum bending stress,

$$\sigma_{b(max)} = \frac{4\,M}{\pi \times 0.707\,s \times d^2} = \frac{5.66\,M}{\pi\,s\,d^2}$$

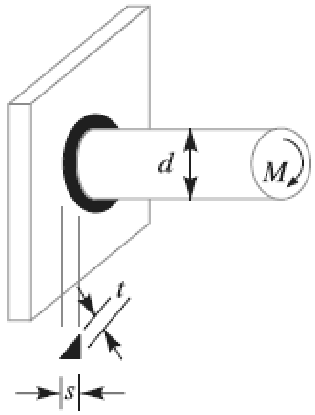


Fig. 3.54. Circular fillet weld subjected to bending moment.

3. ***Long fillet weld subjected to torsion.*** Consider a vertical plate attached to a horizontal plate by two identical fillet welds as shown in Fig. 3.55.

Let T = Torque acting on the vertical plate,
l = Length of weld,
s = Size (or leg) of weld,
t = Throat thickness, and
J = Polar moment of inertia of the weld section

$$= 2 \times \frac{t \times l^3}{12} = \frac{t \times l^3}{6} \quad \ldots \quad (\because \text{ of both sides weld})$$

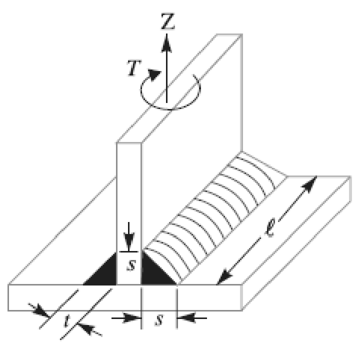


Fig. 3.55. Long fillet weld subjected to torsion.

It may be noted that the effect of the applied torque is to rotate the vertical plate about the Z-axis through its mid point. This rotation is resisted by shearing stresses developed between two fillet welds and the horizontal plate. It is assumed that these horizontal shearing stresses vary from zero at the Z-axis and maximum at the ends of the plate. This variation of shearing stress is analogous to the variation of normal stress over the depth (l) of a beam subjected to pure bending.

$$\therefore \text{ Shear stress, } \quad \tau = \frac{T \times l/2}{t \times l^3/6} = \frac{3\,T}{t \times l^2}$$

The maximum shear stress occurs at the throat and is given by

$$\tau_{max} = \frac{3T}{0.707\,s \times l^2} = \frac{4.242\,T}{s \times l^2}$$

Example 3.12. *A 50 mm diameter solid shaft is welded to a flat plate by 10 mm fillet weld as shown in Fig. 3.56. Find the maximum torque that the welded joint can sustain if the maximum shear stress intensity in the weld material is not to exceed 80 MPa.*

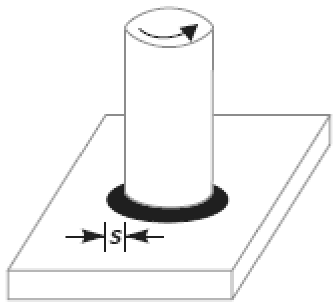


Fig. 3.56

Solution.
Given : d = 50 mm
s = 10 mm
τmax = 80 MPa = 80 N/mm^2
Let T = Maximum torque that the welded joint can sustain.
We know that the maximum shear stress (τmax),

$$80 = \frac{2.83\,T}{\pi s \times d^2} = \frac{2.83T}{\pi \times 10\,(50)^2} = \frac{2.83\,T}{78550}$$

$$T = 80 \times 78\,550/2.83$$

$$= 2.22 \times 10^6 \text{ N-mm} = 2.22 \text{ kN-m} \quad \text{Ans.}$$

Example 3.13. *A plate 1 m long, 60 mm thick is welded to another plate at right angles to each other by 15 mm fillet weld, as shown in Fig. 3.57. Find the maximum torque that the welded joint can sustain if the permissible shear stress intensity in the weld material is not to exceed 80 MPa.*

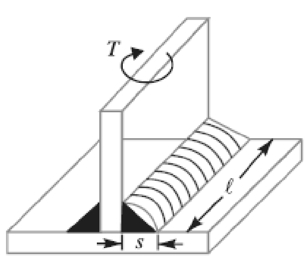


Fig. 3.57

Solution. Given: l = 1m = 1000 mm
Thickness = 60 mm
s = 15 mm
τ_{max} = 80 MPa = 80 N/mm^2
Let T = Maximum torque that the welded joint can sustain.
We know that the maximum shear stress (τ_{max}),

$$80 = \frac{4.242\,T}{s \times l^2} = \frac{4.242\,T}{15\,(1000)^2} = \frac{0.283\,T}{10^6}$$

$$\therefore \quad T = 80 \times 10^6 / 0.283 = 283 \times 10^6 \text{ N-mm} = 283 \text{ kN-m} \quad \text{Ans.}$$

3.3 RIVETED JOINTS

3.4. 1 Introduction

A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called ***shank*** or ***body***
and lower portion of shank is known as ***tail,*** as shown in Fig. 3.58. The rivets are used to make permanent fastening between the plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals. The fastenings (*i.e.* joints) may be classified into the following two groups :

1. Permanent fastenings, and **2.** Temporary or detachable fastenings

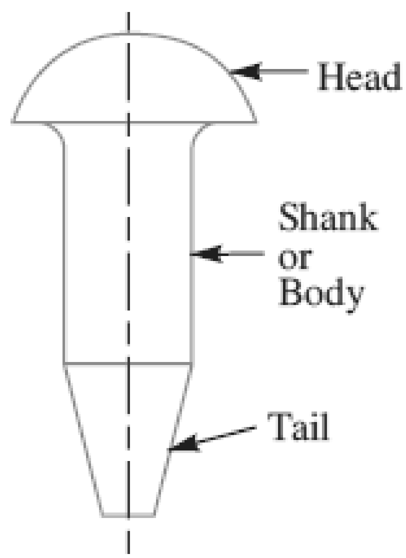


Fig. 3.58. Rivet parts.

The ***permanent fastenings*** are those fastenings which can not be disassembled without destroying the connecting components. The examples of permanent fastenings in order of strength are soldered, brazed, welded and riveted joints. The ***temporary*** or ***detachable fastenings*** are those fastenings which can be disassembled without destroying the connecting components. The examples of temporary fastenings are screwed, keys, cotters, pins and splined joints.

3.4.2 Methods of Riveting

The function of rivets in a joint is to make a connection that has strength and tightness. The strength is necessary to prevent failure of the joint. The tightness is necessary in order to contribute to strength and to prevent leakage as in a boiler or in a ship hull. When two plates are to be fastened together by a rivet as shown in Fig. 3.59 (*a*), the holes in the plates are punched and reamed or drilled. Punching is the cheapest method and is used for relatively thin plates and in structural work. Since punching injures the material around the hole, therefore drilling is used in most pressure-vessel work. In structural and pressure vessel riveting, the diameter of the rivet hole is usually 1.5 mm larger than the nominal diameter of the rivet.

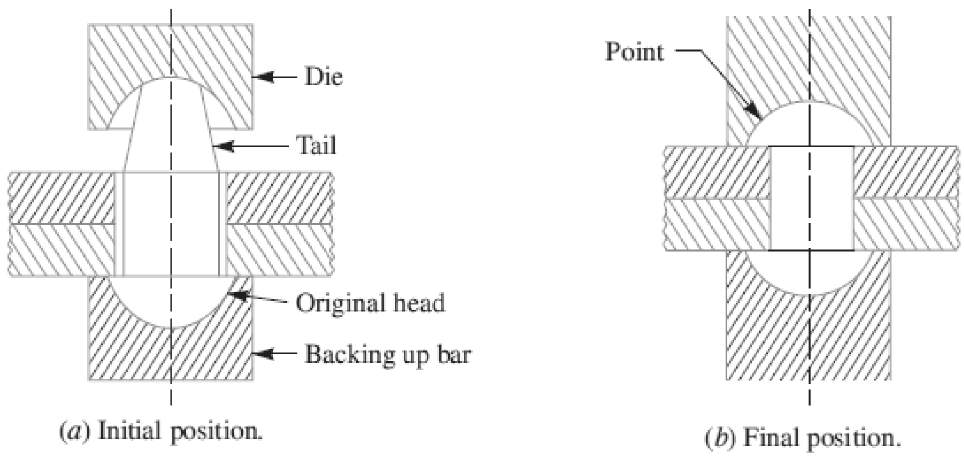


Fig. 3.59. Methods of riveting

The plates are drilled together and then separated to remove any burrs or chips so as to have a tight flush joint between the plates. A cold rivet or a red hot rivet is introduced into the plates and the ***point*** (*i.e.* second head) is then formed. When a cold rivet is used, the process is known as ***cold riveting*** and when a hot rivet is used, the process is known as ***hot riveting*****.** The cold riveting process is used for structural joints while hot riveting is used to make leak proof joints The riveting may be done by hand or by a riveting machine. In hand riveting, the original rivet head is backed up by a hammer or heavy bar and then the die or set, as shown in Fig. 3.59 (*a*), is placed against the end to be headed and the blows are applied by a hammer. This causes the shank to expand thus filling the hole and the tail is converted into a ***point*** as shown in Fig. 3.59 (*b*). As the rivet cools, it tends to contract. The lateral contraction will be slight, but there will be a longitudinal tension introduced in the rivet which holds the plates firmly together. In machine riveting, the die is a part of the hammer which is operated by air, hydraulic or steam pressure.

3.4.3 Material of Rivets

The material of the rivets must be tough and ductile. They are usually made of steel (low carbon steel or nickel steel), brass, aluminium or copper, but when strength and a fluid tight joint is the main consideration, then the steel rivets are used. The rivets for general purposes shall be manufactured from steel conforming to the following
Indian Standards :

(***a***) IS : 1148–1982 (Reaffirmed 1992) – Specification for hot rolled rivet bars (up to 40 mm diameter) for structural purposes; or

(***b***) IS : 1149–1982 (Reaffirmed 1992) – Specification for high tensile steel rivet bars for structural purposes.

The rivets for boiler work shall be manufactured from material conforming to IS : 1990 – 1973 (Reaffirmed 1992) – Specification for steel rivets and stay bars for boilers.

3.4.4 Essential Qualities of a Rivet

According to Indian standard, IS : 2998 – 1982 (Reaffirmed 1992), the material of a rivet must have a tensile strength not less than 40 N/mm2 and elongation not less than 26 percent. The material must be of such quality that when in cold condition, the shank shall be bent on itself through 180° without cracking and after being heated to 650°C and quenched, it must pass the same test. The rivet when hot must flatten without cracking to a diameter 2.5 times the diameter of shank
.

3.4.5 Manufacture of Rivets

According to Indian standard specifications, the rivets may be made either by cold heading or by hot forging. If rivets are made by the cold heading process, they shall subsequently be adequately heat treated so that the stresses set up in the cold heading process are eliminated. If they are made by hot forging process, care shall be taken to see that the finished rivets cool gradually.

3.4.6 Types of Rivet Heads

According to Indian standard specifications, the rivet heads are classified into the following three types :

1. Rivet heads for general purposes (below 12 mm diameter) as shown in Fig. 3.60, according to IS : 2155 – 1982 (Reaffirmed 1996).

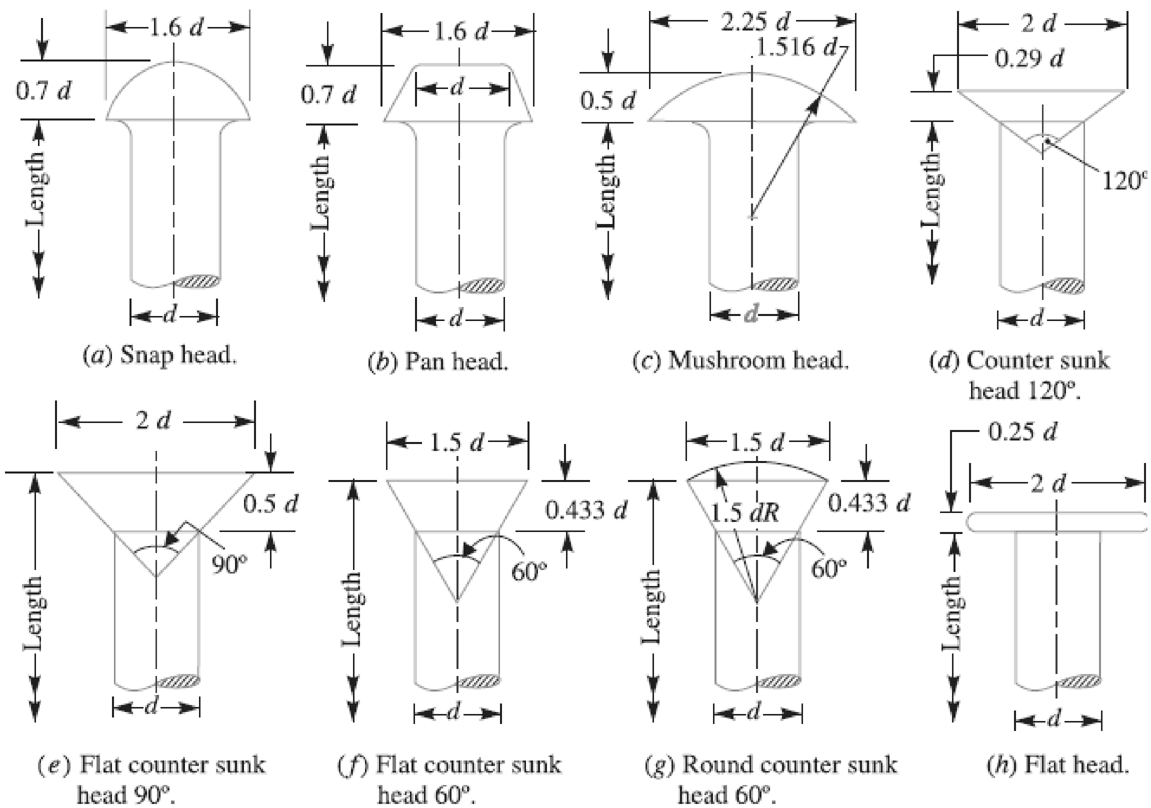


Fig. 3.60. Rivet heads for general purposes (below 12 mm diameter).

2. Rivet heads for general purposes (From 12 mm to 48 mm diameter) as shown in Fig. 3.61, according to IS : 1929 – 1982 (Reaffirmed 1996).

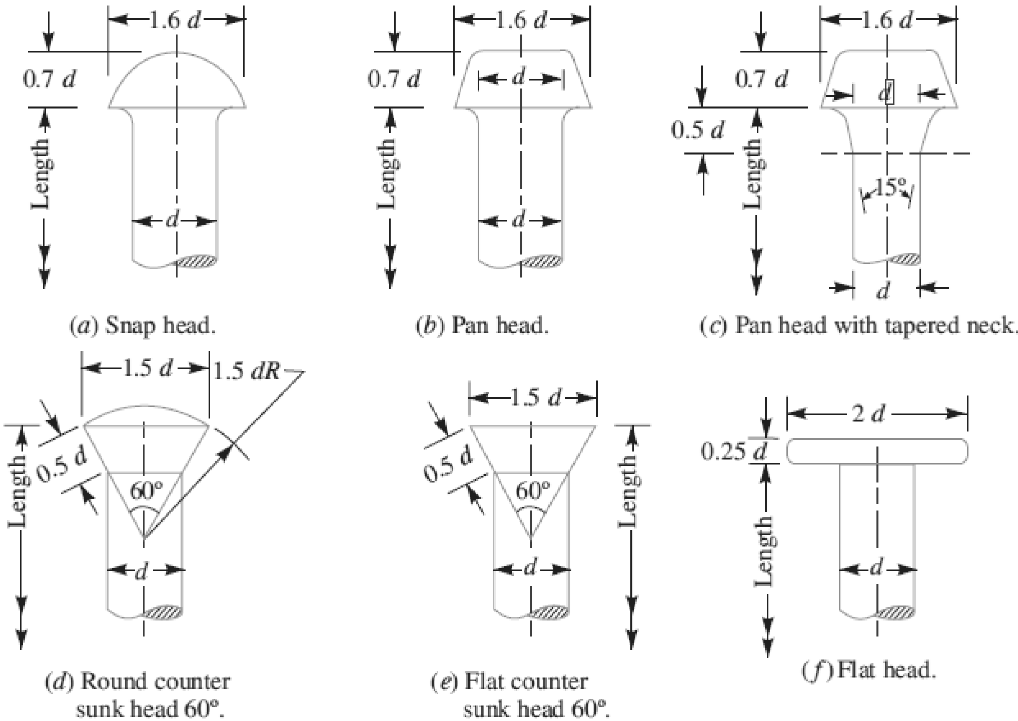
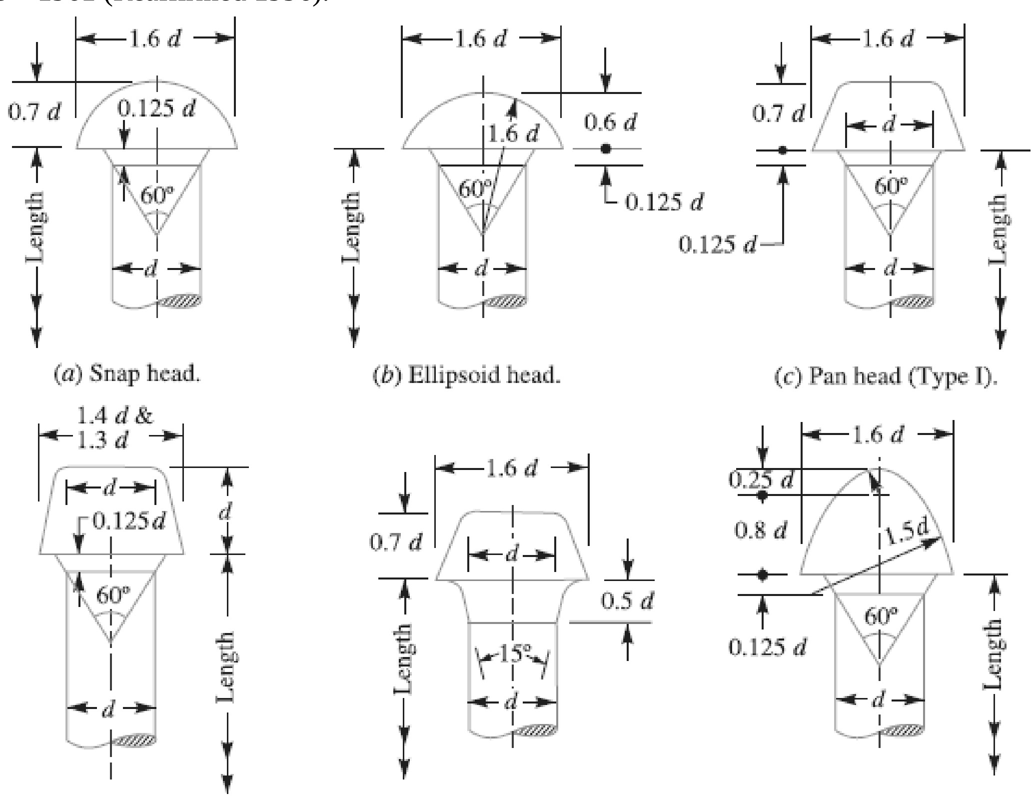


Fig. 3.61. Rivet heads for general purposes (from 12 mm to 48 mm diameter)

3. Rivet heads for boiler work (from 12 mm to 48 mm diameter, as shown in Fig. 3.62, according to IS : 1928 – 1961 (Reaffirmed 1996).



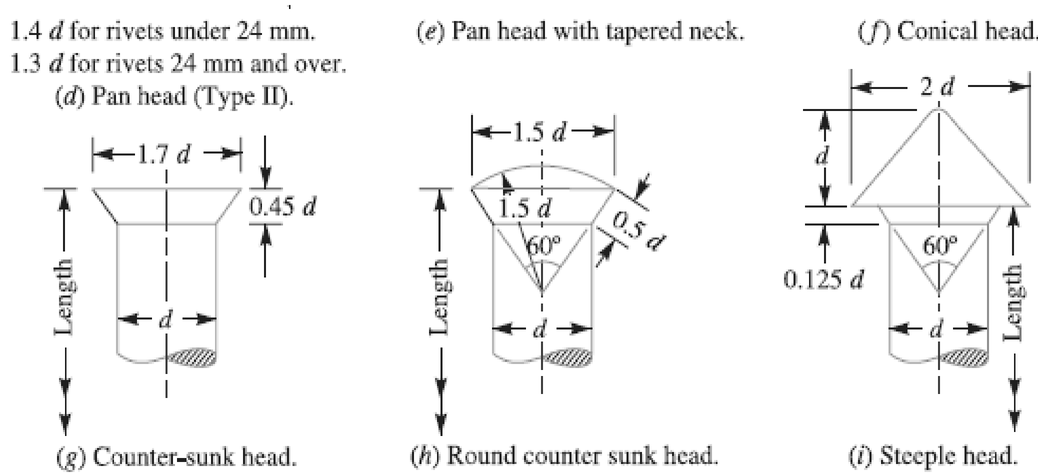


Fig. 3.62. Rivet heads for boiler work.

The **snap heads** are usually employed for structural work and machine riveting. The **counter sunk heads** are mainly used for ship building where flush surfaces are necessary. The **conical heads** (also known as **conoidal heads)** are mainly used in case of hand hammering. The **pan heads** have maximum strength, but these are difficult to shape.

3.4.7 Types of Riveted Joints

Following are the two types of riveted joints, depending upon the way in which the plates are connected.

1. Lap joint, and **2.** Butt joint.

Lap Joint

A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

Butt Joint

A butt joint is that in which the main plates are kept in alignment butting (*i.e.* touching) each other and a cover plate (*i.e.* strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with the main plates. Butt joints are of the following two types :

1. Single strap butt joint, and **2.** Double strap butt joint.

In a ***single strap butt joint,*** the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

In a ***double strap butt joint,*** **t**he edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together. In addition to the above, following are the types of riveted joints depending upon the number of rows of the rivets.

1. Single riveted joint, and **2.** Double riveted joint.

A ***single riveted joint*** is that in which there is a single row of rivets in a lap joint as shown in Fig. 3.63 (*a*) and there is a single row of rivets on each side in a butt joint as shown in Fig. 3.65.

A ***double riveted joint*** is that in which there are two rows of rivets in a lap joint as shown in Fig. 3.66 (*b*) and (*c*) and there are two rows of rivets on each side in a butt joint as shown in Fig. 3.66.

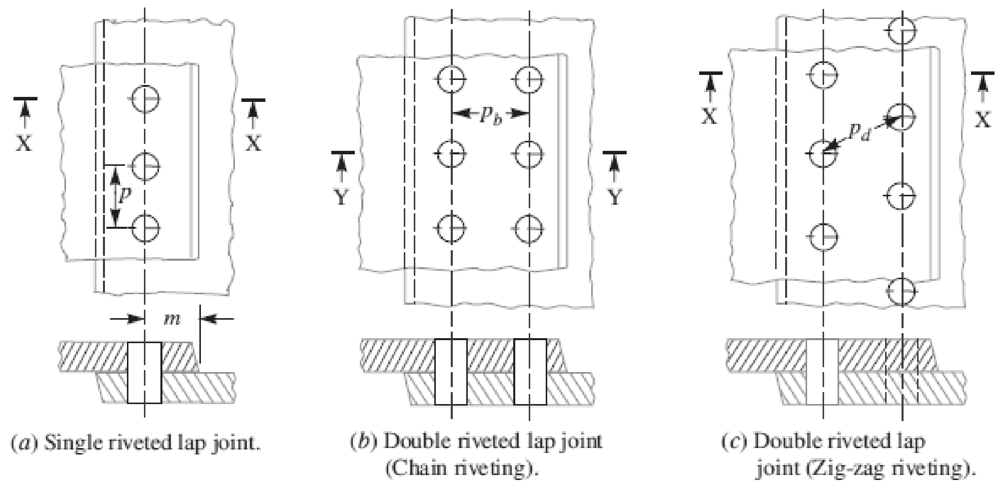


Fig. 3.63. Single and double riveted lap joints.

Similarly the joints may be ***triple riveted*** or ***quadruple riveted*** every rivet is in the middle of the two rivets of the opposite row as shown in Fig. 3.63 (*c*), then the joint is said to be ***zig-zag riveted*.**

2. Since the plates overlap in lap joints, therefore the force *P*, *P* acting on the plates [See Fig. 9.15 (*a*)] are not in the same straight line but they are at a distance equal to the thickness of the plate. These forces will form a couple which may bend the joint. Hence the lap joints may be used only where small loads are to be transmitted. On the other hand, the forces *P*, *P* in a butt joint act in the same straight line, therefore there will be no couple. Hence the butt joints are used where heavy loads are to be transmitted.

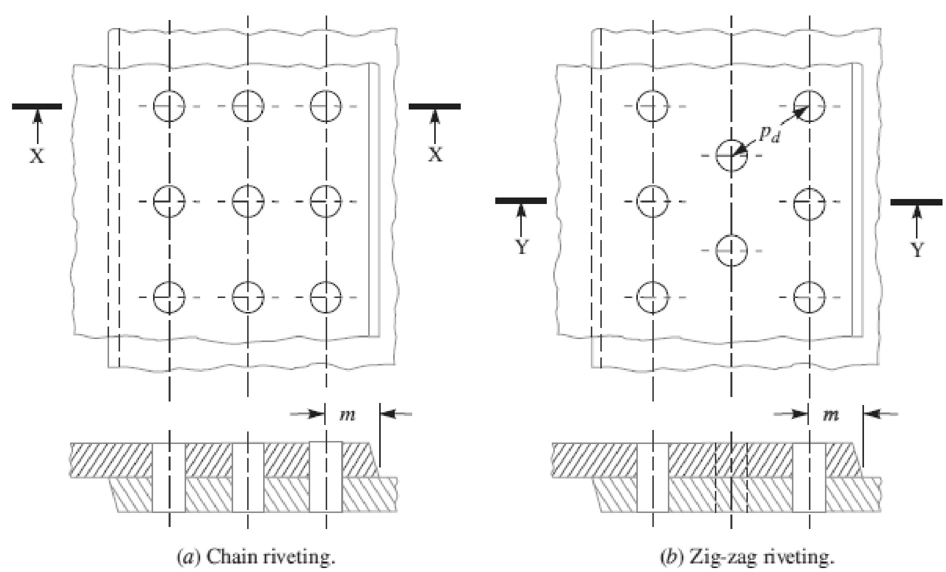


Fig. 3.64. Triple riveted lap joint.

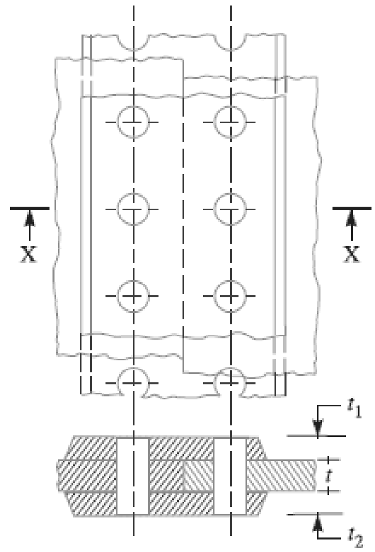


Fig. 3.65. Single riveted double strap butt joint.

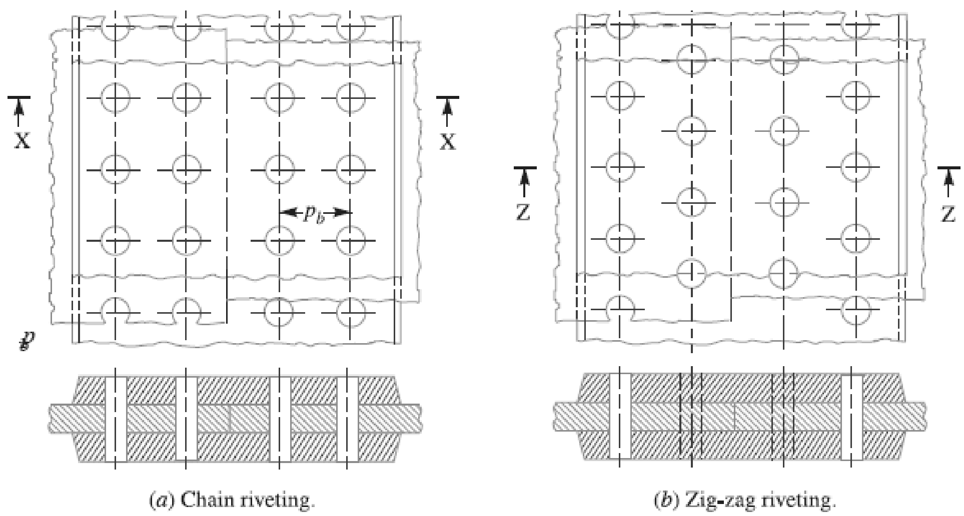


Fig. 3.66. Double riveted double strap (equal) butt joints.
Fig. 3.67. Double riveted double strap (unequal) butt joint with zig-zag riveting.

3.4.8 Important Terms Used in Riveted Joints

The following terms in connection with the riveted joints are important from the subject point of view :

1. *Pitch*. It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the seam as shown in Fig. 3.63. It is usually denoted by *p*.
2. *Back pitch*. It is the perpendicular distance between the centre lines of the successive rows as shown in Fig. 3.63. It is usually denoted by *pb*.

3. *Diagonal pitch*. It is the distance between the centres of the rivets in adjacent rows of zig-zag riveted joint as shown in Fig. 3.63. It is usually denoted by *pd*.
4. *Margin or marginal pitch*. It is the distance between the centre of rivet hole to the nearest edge of the plate as shown in Fig. 3.63. It is usually denoted by *m*.

EXERCISES

1. Determine the safe tensile load for bolts of M 20 and M 36. Assume that the bolts are not initially stressed and take the safe tensile stress as 200 MPa. **[Ans. 49 kN; 16.43 kN]**

2. An eye bolt carries a tensile load of 20 kN. Find the size of the bolt, if the tensile stress is not to exceed 100 MPa. Draw a neat proportioned figure for the bolt. **[Ans. M 20]**

3. An engine cylinder is 300 mm in diameter and the steam pressure is 0.7 N/mm^2. If the cylinder head is held by 12 studs, find the size. Assume safe tensile stress as 28 MPa. **[Ans. M 24]**

4. Find the size of 14 bolts required for a C.I. steam engine cylinder head. The diameter of the cylinder is 400 mm and the steam pressure is 0.12 N/mm^2. Take the permissible tensile stress as 35 MPa. **[Ans. M 24]**

5. The cylinder head of a steam engine is subjected to a pressure of 1 N/mm^2. It is held in position by means of 12 bolts. The effective diameter of the cylinder is 300 mm. A soft copper gasket is used to make the joint leak proof. Determine the size of the bolts so that the stress in the bolts does not exceed 100 MPa. **[Ans. M 36]**

6. A steam engine cylinder of 300 mm diameter is supplied with steam at 1.5 N/mm^2. The cylinder cover is fastened by means of 8 bolts of size M 20. The joint is made leak proof by means of suitable gaskets. Find the stress produced in the bolts. **[Ans. 249 MPa]**

7. The effective diameter of the cylinder is 400 mm. The maximum pressure of steam acting on the cylinder cover is 1.12 N/mm^2. Find the number and size of studs required to fix the cover. Draw a neat proportioned sketch for the elevation of the cylinder cover. **[Ans. 14; M 24]**

8. Specify the size and number of studs required to fasten the head of a 400 mm diameter cylinder containing steam at 2 N/mm^2. A hard gasket (gasket constant = 0.3) is used in making the joint. Draw a neat sketch of the joint also. Other data may be assumed. **[Ans. M 30; 12]**

9. A steam engine cylinder has an effective diameter of 200 mm. It is subjected to a maximum steam pressure of 1.75 N/mm^2. Calculate the number and size of studs required to fix the cylinder cover onto the cylinder flange assuming the permissible stress in the studs as 30 MPa. Take the pitch circle diameter of the studs as 320 mm and the total load on the studs as 20% higher than the external load on the joint. Also check the circumferential pitch of the studs so as to give a leak proof joint. **[Ans. 16; M 16]**

15. A pulley bracket, as shown in Fig. 3.68, is supported by 4 bolts, two at *A*-*A* and two at *B*-*B*. Determine the size of bolts using an allowable shear stress of 25 MPa for the material of the bolts. **[Ans. M 27]**

16. A wall bracket, as shown in Fig. 3.69, is fixed to a wall by means of four bolts. Find the size of the bolts and the width of bracket. The safe stress in tension for the bolt and bracket may be assumed as 70 MPa.

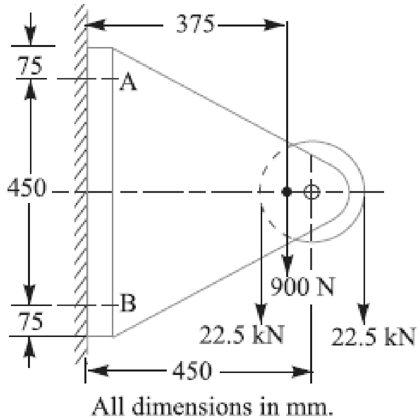


Fig. 3.68

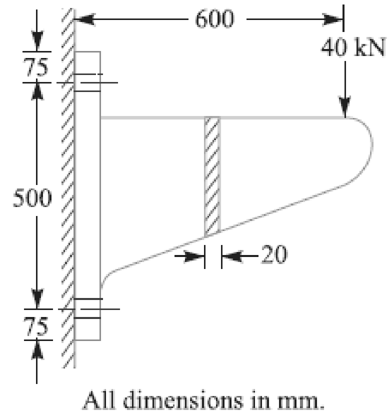


Fig. 3.69

20. A bracket, as shown in Fig. 3.70, is fixed to a vertical steel column by means of five standard bolts. Determine : (*a*) The diameter of the fixing bolts, and (*b*) The thickness of the arm of the bracket. Assume safe working stresses of 70 MPa in tension and 50 MPa in shear. **[Ans. M 18; 50 mm]**

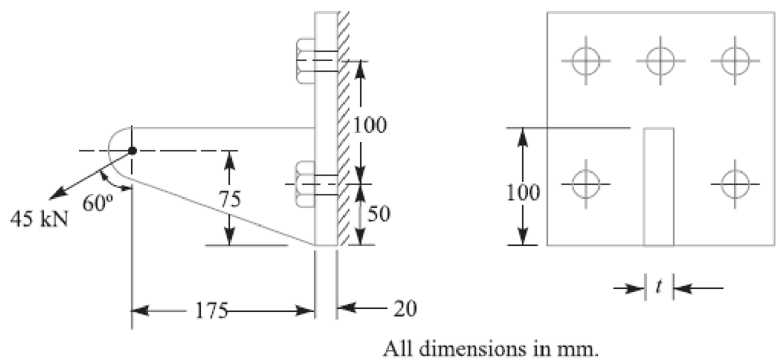


Fig. 3.70

21. Design a cotter joint to connect two mild steel rods for a pull of 30 kN. The maximum permissible stresses are 55 MPa in tension ; 40 MPa in shear and 70 MPa in crushing. Draw a neat sketch of the joint designed.
[Ans. d = 22 mm; $d2$ = 32 mm ; t = 14 mm ; $d1$ = 44 mm ; b = 30 mm ; a = 12 mm ; d_4 = 65 mm ; c = 12 mm ; d_3 = 40 mm ; t_1 = 8 mm]

22. Two rod ends of a pump are joined by means of a cotter and spigot and socket at the ends. Design the joint for an axial load of 100 kN which alternately changes from tensile to compressive. The allowable stresses for the material used are 50 MPa in tension, 40 MPa in shear and 100 MPa in crushing.
[Ans. d = 51 mm ; d_2 = 62 mm ; t = 16 mm ; d_1 = 72 mm ; b = 78 mm ; a = 20 mm ; d_3 = 83 mm ; d_4 = 125 mm ; c = 16 mm ; t_1 = 13 mm]

23. Two mild steel rods 40 mm diameter are to be connected by a cotter joint. The thickness of the cotter is 12 mm. Calculate the dimensions of the joint, if the maximum permissible stresses are: 46 MPa in

tension ; 35 MPa in shear and 70 MPa in crushing.
[Ans. d_2 = 30 mm ; d_1 = 48 mm ; b = 70 mm ; a = 27.5 mm ; d_4 = 100 mm ; c = 12 mm ; d_3 = 44.2 mm ; t = 35 mm ; t_1 = 13.5 mm]

24. Design and draw a cotter foundation bolt to take a load of 90 kN. Assume the permissible stresses as follows :
σ_t = 50 MPa, τ= 60 MPa and σ_c = 100 MPa.
[Ans. d = 50 mm ; $d1$ = 60 mm ; t = 15 mm ; b = 60 mm]

25. Design a knuckle joint to connect two mild steel bars under a tensile load of 25 kN. The allowable stresses are 65 MPa in tension, 50 MPa in shear and 83 MPa in crushing.
[Ans. $d = d_1$ = 23 mm ; d_2 = 46 mm ; d_3 = 35 mm ; t = 29 mm ; t_1 = 18 mm]

26. A knuckle joint is required to withstand a tensile load of 25 kN. Design the joint if the permissible stresses are :
σ_t = 56 MPa ; τ = 40 MPa and σ_c = 70 MPa.
[Ans. $d = d_1$ = 28 mm ; d_2 = 56 mm ; d_3 = 42 mm ; t_1 = 21 mm]

27. The pull in the tie rod of a roof truss is 44 kN. Design a suitable adjustable screw joint. The permissible tensile and shear stresses are 75 MPa and 37.5 MPa respectively. Draw full size two suitable views of the joint. **[Ans. d = 36 mm ; l = 11 mm ; D = 45 mm ; D_2 = 58 mm]**

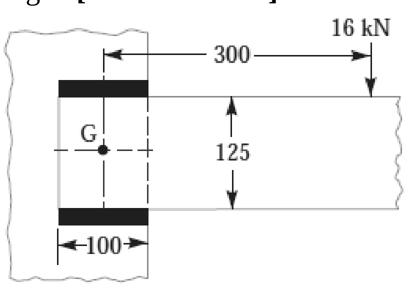
28. A plate 100 mm wide and 10 mm thick is to be welded with another plate by means of transverse welds at the ends. If the plates are subjected to a load of 70 kN, find the size of weld for static as well as fatigue load. The permissible tensile stress should not exceed 70 MPa. **[Ans. 83.2 mm; 118.5 mm]**

29. If the plates in Ex. 1, are joined by double parallel fillets and the shear stress is not to exceed 56 MPa, find the length of weld for (*a*) Static loading, and (*b*) Dynamic loading. **[Ans. 91 mm; 259 mm]**

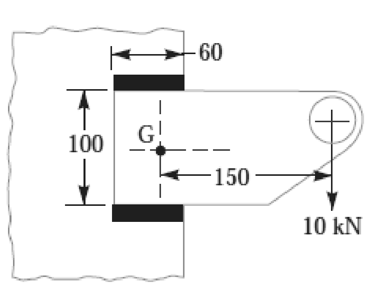
30. A 125 × 95 × 10 mm angle is joined to a frame by two parallel fillet welds along the edges of 150 mm leg. The angle is subjected to a tensile load of 180 kN. Find the lengths of weld if the permissible static load per mm weld length is 430 N. **[Ans. 137 mm and 307 mm]**

31. A circular steel bar 50 mm diameter and 200 mm long is welded perpendicularly to a steel plate to form a cantilever to be loaded with 5 kN at the free end. Determine the size of the weld, assuming the allowable stress in the weld as 100 MPa. **[Ans. 7.2 mm]**

32. A 125 × 95 × 10 mm angle is welded to a frame by two 10 mm fillet welds, as shown in Fig. 3.71. A load of 16 kN is apsplied normal to the gravity axis at a distance of 300 mm from the centre of\ gravity of welds. Find maximum shear stress in the welds, assuming each weld to be 100 mm long and parallel to the axis of the angle. **[Ans. 45.5 MPa]**



All dimensions in mm.

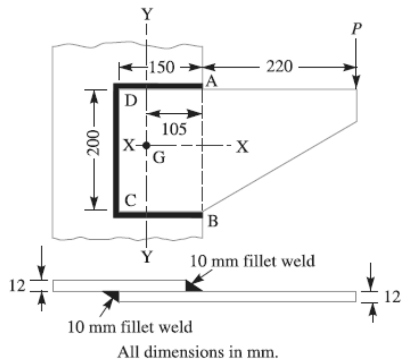


All dimensions in mm.

Fig. 3.71 **Fig. 3.72**

33. A bracket, as shown in Fig. 3.72, carries a load of 10 kN. Find the size of the weld if the allowable shear stress is not to exceed 80 MPa. **[Ans. 10.83 mm]**

34. A bracket is welded to the side of a column and carries a vertical load *P*, as shown in Fig. 3.73. Evaluate *P* so that the maximum shear stress in the 10 mm fillet welds is 80 MPa.
[Ans. 50.7 kN]



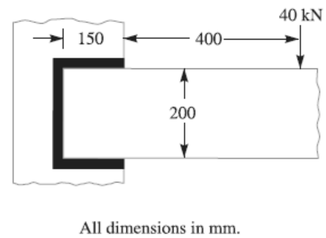


Fig. 3.73 **Fig. 3.74**

35. A bracket, as shown in Fig. 10.39, carries a load of 40 kN. Calculate the size of weld, if the allowable shear stress is not to exceed 80 MPa. **[Ans. 7 mm]**

36. A single riveted lap joint is made in 15 mm thick plates with 20 mm diameter rivets. Determine the strength of the joint, if the pitch of rivets is 60 mm. Take σ_t = 120 MPa; τ = 90 MPa and σ_c = 160 MPa.
[Ans. 28 280 N]

37. Two plates 16 mm thick are joined by a double riveted lap joint. The pitch of each row of rivets is 90 mm. The rivets are 25 mm in diameter. The permissible stresses are as follows :
σ_t= 140 MPa ; τ= 110 MPa and σ_c= 240 MPa Find the efficiency of the joint. **[Ans. 53.5%]**

38. A single riveted double cover butt joint is made in 10 mm thick plates with 20 mm diameter rivets with a pitch of 60 mm. Calculate the efficiency of the joint, if
σ_t = 100 MPa ; τ = 80 MPa and σ_c= 160 MPa. **[Ans. 53.8%]**

UNIT 4

ENERGY STORING ELEMENTS AND ENGINE COMPONENTS

4.1 SPRINGS

4.1.1 Introduction

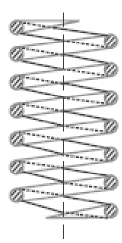
A spring is defined as an elastic body, whose functionis to distort when loaded and to recover its original shape
when the load is removed. The various importantapplications of springs are as follows :

1. To cushion, absorb or control energy due to eithershock or vibration as in car springs, railwaybuffers, air-craft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and springloadedvalves.
3. To control motion by maintaining contact betweentwo elements as in cams and followers.
4. To measure forces, as in spring balances andengine indicators.
5. To store energy, as in watches, toys, etc.

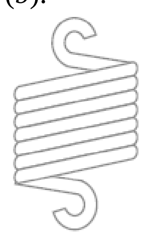
4.1.2 Types of Springs

Though there are many types of the springs, yet thefollowing, according to their shape, are important from the
subject point of view.

Helical springs*.** The helical springs are made up of a wire coiled in the form of a helix andis primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compressionhelical spring as shown in Fig. 4.1 (*a*) and ***tension helical spring*** as shown in Fig. 4.1 (*b*).



(*a*) Compression helical spring.



(*b*) Tension helical spring.

Fig.4.1 Helical springs.

In ***open coiled helical springs,*** the spring wire is coiled in such a way that there is a gap between
the two consecutive turns, as a result of which the helix angle is large.

The helical springs have the following advantages:
(*a*) These are easy to manufacture.
(*b*) These are available in wide range.
(*c*) These are reliable.
(*d*) These have constant spring rate.
(*e*) Their performance can be predicted more accurately.
(*f*) Their characteristics can be varied by changing dimensions.

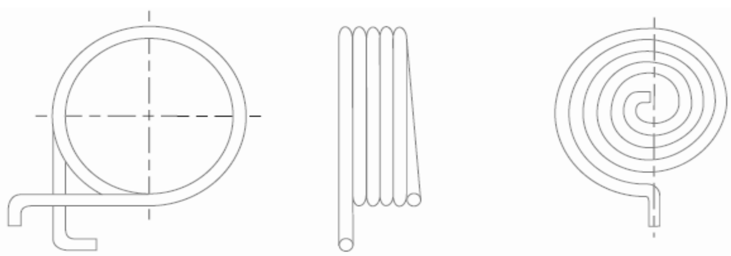
2. *Conical and volute springs*.
The conical spring, as shown in Fig. 4.2 (*a*), is wound with a uniform pitch whereasthe volute springs, as shown in Fig. 4.2 (*b*), are wound in the form of paraboloid with constant pitch. The major stresses produced in conical and volute springs are also shear stresses due to twisting.

(*a*) Conical spring. (*b*) Volute spring.

Fig.4.2 Conical and volute springs.

3. *Torsion springs*. These springs may be of ***helical*** or ***spiral*** type as shown in Fig. 4.3. The**helical type** may be used only in applications where the load tends to wind up the spring and are usedin various electrical mechanisms. The **spiral type** is also used where the load tends to increase thenumber of coils and when made of flat strip are used in watches and clocks.

The major stresses produced in torsion springs are tensile and compressive due to bending.



(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

Fig.4.3 Torsion springs.

4. *Laminated or leaf springs*. The laminated or leaf spring (also known as ***flat spring*** or ***carriagespring***) consists of a number of flat plates (known as leaves) of varying lengths held together bymeans of clamps and bolts, as shown in Fig. 4.4. These are mostly used in automobiles.The major stresses produced in leaf springs are tensile and compressive stresses.

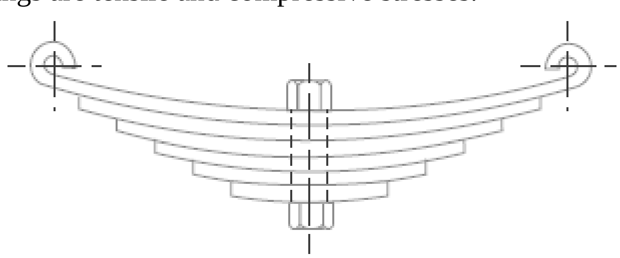


Fig.4.4 Laminated or leaf springs

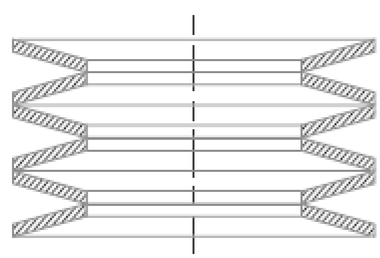


Fig.4.5 Disc or bellevile springs.

5. ***Disc or bellevile springs*.** These springs consist of a number of conical discs held togetheragainst slipping by a central bolt or tube as shown in Fig. 4.5. These springs are used in applicationswhere high spring rates and compact spring units are required.The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

4.1.3 Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience andit should be creep resistant. It largely depends upon the service for which they are used *i.e*. severeservice, average service or light service.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 percent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrousmaterials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in specialcases to increase fatigue resistance, temperature resistance and corrosion resistance.

4.1.4 Terms used in Compression Springs

The following terms used in connection with compression springs are important.

1. ***Solid length*.** When the compression spring is compressed until the coils come in contactwith each other, then the spring is said to be ***solid*.** The solid length of a spring is the product of totalnumber of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$L_S = n'.d$

where n' = Total number of coils, and

d = Diameter of the wire.

2. ***Free length*.** The free length of a compression spring, as shown in Fig. 4.6, is the length ofthe spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection \or compression of the spring and the clearance between the adjacent coils (when fully compressed).
Mathematically,

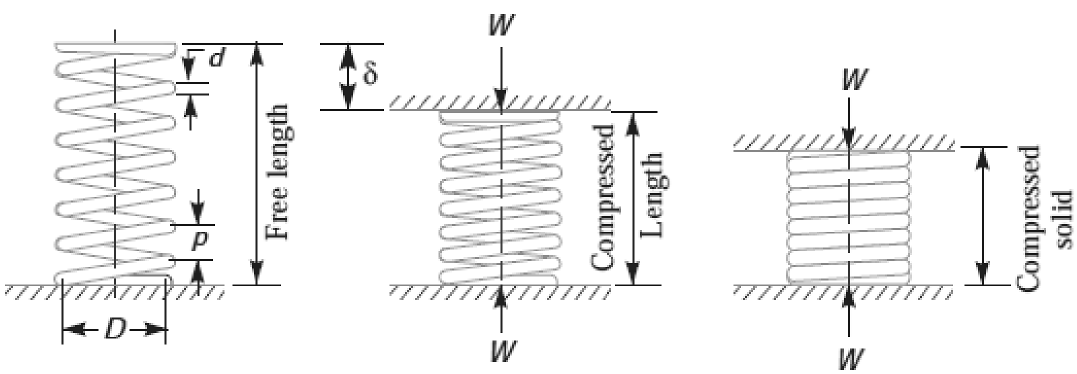


Fig 4.6. Compression spring nomenclature.

Free length of the spring,

L_F = Solid length + Maximum compression + Clearance between adjacent coils (or clash allowance)

$= n'.d + \delta max + 0.15\ \delta max$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta max + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. *Spring index*. The spring index is defined as the ratio of the mean diameter of the coil to thediameter of the wire. Mathematically,
Spring index, $C = D / d$

where D = Mean diameter of the coil, and
d = Diameter of the wire.

4. *Spring rate*. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$
where W = Load, and
δ = Deflection of the spring.

5. *Pitch*. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state.
Mathematically,

Pitch of the coil, $p = \frac{FreeLength}{n^2 - 1}$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $p = \frac{L_{F-L_S}}{n'} + d$

where L_F = Free length of the spring,
L_S = Solid length of the spring,
n' = Total number of coils, and
d = Diameter of the wire.

4.1.5 End Connections for Compression Helical Springs

The end connections for compression helical springs are suitably formed in order to apply theload. Various forms of end connections are shown in Fig. 4.7.

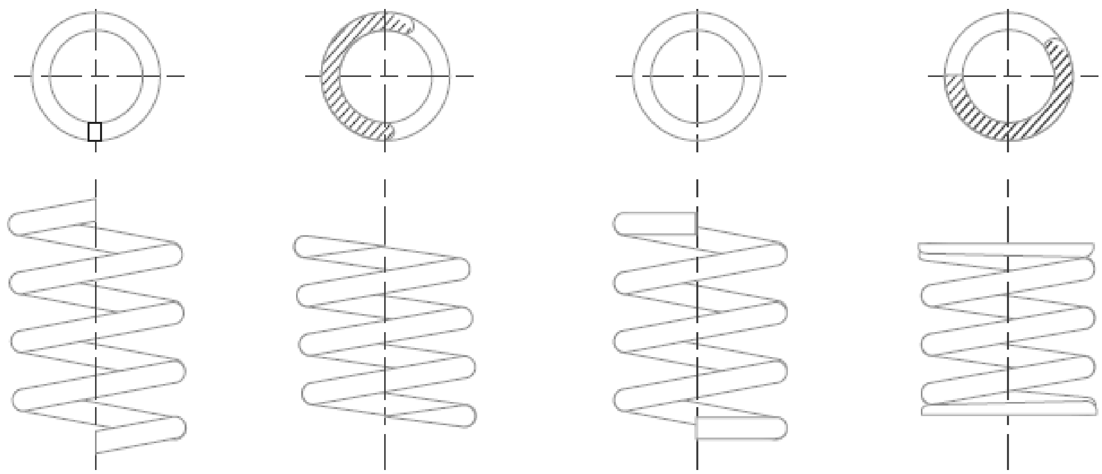


Fig 4.7 End connections for compression helical spring.

In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. It may be noted that part of the coil which is in contact with the seat does not contribute to spring action and
hence are termed as ***inactive coils***. The turns which impart spring action are known as ***active turns*.** As the load increases, the number of inactive coils also increases due to seating of the end coils andthe amount of increase varies from 0.5 to 1 turn at the usual working loads. The following table showsthe total number of turns, solid length and free length for different types of end connections.

Table 4.1. Total number of turns, solid length and free length for different types of end connections.

Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n + 1)\,d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n + 2$	$(n + 3)\,d$	$p \times n + 3d$
4. Squared and ground ends	$n + 2$	$(n + 2)\,d$	$p \times n + 2d$

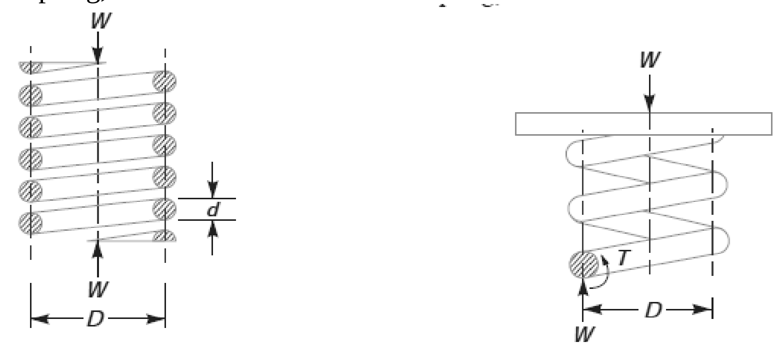
where n = Number of active turns,
p = Pitch of the coils, and
d = Diameter of the spring wire.

4.1.6 Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig. 23.10 (*a*).

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire,
n = Number of active coils,
G = Modulus of rigidity for the spring material,
W = Axial load on the spring,
τ= Maximum shear stress induced in the wire,
C = Spring index = D/d,
p = Pitch of the coils, and
δ= Deflection of the spring, as a result of an axial load W.



(a) Axially loaded helical spring. (b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Fig 4.8.

Now consider a part of the compression spring as shown in Fig. 23.10 (*b*). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire.
A little consideration will show that part of the spring, as shown in Fig. 23.10 (*b*), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8 W.D}{\pi d^3} \quad \text{...}(i)$$

The torsional shear stress diagram is shown in Fig. 23.11 (*a*).
In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

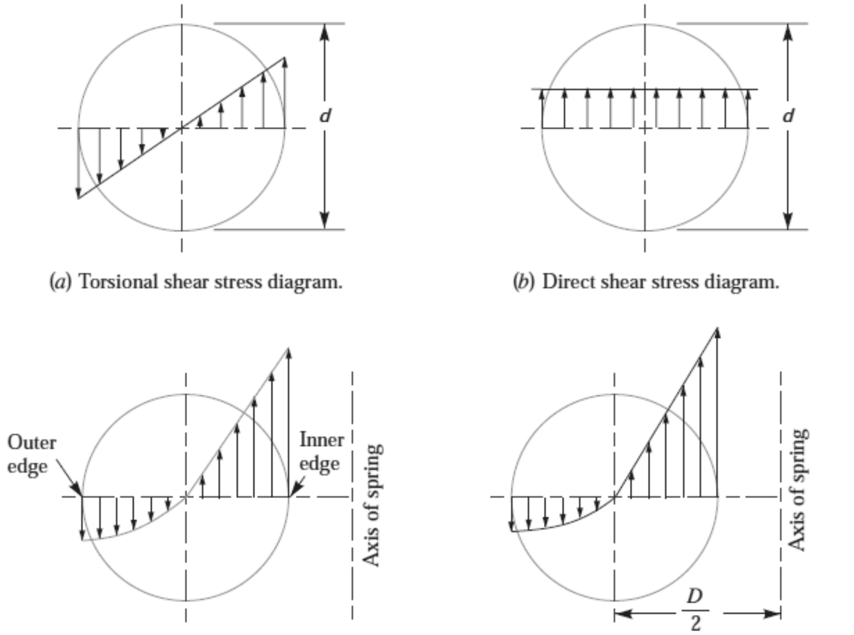
1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire.

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2} \quad \text{...}(ii)$$

The direct shear stress diagram is shown in Fig. 4.9 (*b*) and the resultant diagram of torsionalshear stress and direct shear stress is shown in Fig. 4.9 (*c*).



(*a*) Torsional shear stress diagram.

(*b*) Direct shear stress diagram.

(*c*) Resultant torsional shear and direct shear stress diagram.

(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

Fig. 4.9. Superposition of stresses in a helical spring.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The ***positive*** sign is used for the inner edge of the wire and ***negative*** sign is used for the outeredge of the wire. Since the stress is maximum at the inner edge of the wire, thereforeMaximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi d^3} \quad ...(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \frac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\frac{8\,WD}{\pi d^3} \times \frac{1}{2C}\right)$ is appreciable for springs of small spring index *C*. Also we have neglected the effect of wire curvaturein equation ***(iii)***. It may be noted that when the springs are subjected to static loads, the effect of wirecurvature may be neglected, because yielding of the material will relieve the stresses.In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl'sstress factor (*K*) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear,direct shear and curvature shear stress is shown in Fig. 4.9 (*d*). Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\, W.D}{\pi\, d^3} = K \times \frac{8\, W.C}{\pi\, d^2} \quad ...(iv)$$

where $$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

The values of *K* for a given spring index (*C*) may be obtained from the graph as shown in Fig. 4.10.

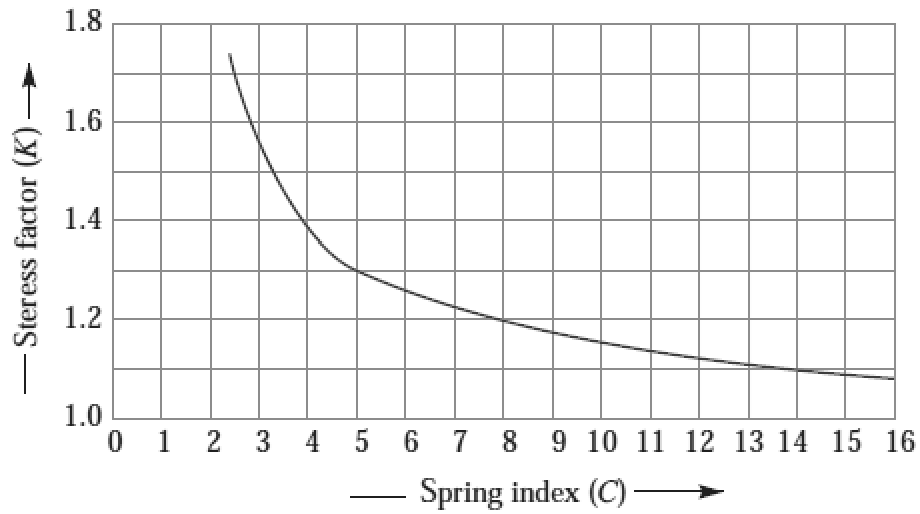


Fig. 4.10. Wahl's stress factor for helical springs.

We see from Fig. 4.10 that Wahl's stress factor increases very rapidly as the spring indexdecreases. The spring mostly used in machinery have spring index above 3.

Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \text{...}(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad \text{...}\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$$= \frac{\pi}{32} \times d^4, \ d \text{ being the diameter of spring wire.}$$

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi D.n}{\frac{\pi}{32} \times d^4 G} = \frac{16 W.D^2.n}{G.d^4} \qquad \text{...}(ii)$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \qquad \text{... } (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

4.1.7 Eccentric Loading of Springs

Sometimes, the load on the springs does not coincide with the axis of the spring, *i.e*. the springis subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, thestiffness of the spring is also affected. The eccentric load on the spring increases the stress on one sideof the spring and decreases on the other side. When the load is offset by a distance *e* from the springaxis, then the safe load on the spring may be obtained by multiplying the axial load by the factor

$$\frac{D}{2\,e + D}, \text{ where } D \text{ is the mean diameter of the spring.}$$

4.1.8 Buckling of Compression Springs

It has been found experimentally that when the free length of the spring (*LF*) is more than fourtimes the mean or pitch diameter (*D*), then the spring behaves like a column and may fail by bucklingat a comparatively low load as shown in Fig. 4.11. The critical axial load (*Wcr*) that causes bucklingmay be calculated by using the following relation, *i.e*.

$$W_{cr} = k \times K_B \times L_F$$

where k = Spring rate or stiffness of the spring = W/δ,

L_F = Free length of the spring, and

K_B = Buckling factor depending upon the ratio L_F / D.

The buckling factor (*K*B) for the hinged end and built-in end springs may be taken from the following table.

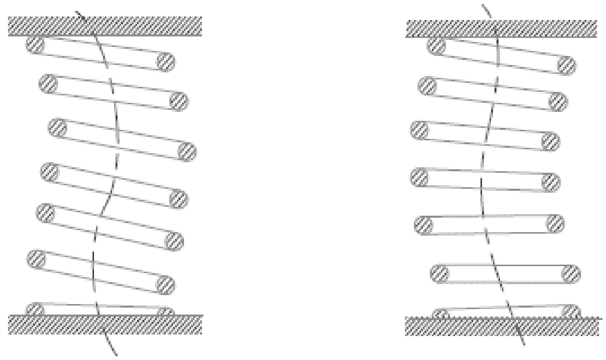


Fig. 4.11. Buckling of compression springs.

Table 4.2. Values of buckling factor (*K*B).

L_F/D	*Hinged end spring*	*Built-in end spring*	L_F / D	*Hinged end spring*	*Built-in end spring*
1	0.72	0.72	5	0.11	0.53
2	0.63	0.71	6	0.07	0.38
3	0.38	0.68	7	0.05	0.26
4	0.20	0.63	8	0.04	0.19

It may be noted that a ***hinged end spring*** is one which is supported on pivots at both ends as incase of springs having plain ends where as a ***built-in end spring*** is one in which a squared and groundend spring is compressed between two rigid and parallel flat plates.It order to avoid the buckling of spring, it is either mounted on a central rod or located on a tube.When the spring is located on a tube, the clearance between the tube walls and the spring should bekept as small as possible, but it must be sufficient to allow for increase in spring diameter duringcompression.

4.1.9 Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end is loadedsuddenly, then all the coils of the spring will not suddenly deflect equally, because some time isrequired for the propagation of stress along the spring wire. A little consideration will show that in thebeginning, the end coils of the spring in contact with the applied load takes up whole of the deflectionand then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compressionpropagates through the coils to the supported end from where it is reflected back to the deflected end.This wave of compression travels along the spring indefinitely. If the applied load is of fluctuatingtype as in the case of valve spring in internal combustion engines and if the time interval between theload applications is equal to the time required for the wave to travel from one end to the other end,then resonance will occur. This results in very large deflections of the coils and correspondingly veryhigh stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon iscalled ***surge*.**

It has been found that the natural frequency of spring should be atleast twenty times the frequencyof application of a periodic load in order to avoid resonance with all harmonic frequencies uptotwentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n}\sqrt{\frac{6\,G . g}{\rho}} \text{ cycles/s}$$

where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.

2. By using springs of high natural frequency.

3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Example 4.1. *A compression coil spring made of an alloy steel is having the followingspecifications : Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.*

If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Solution. Given :

D = 50 mm

d = 5 mm

n = 20

W = 500 N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$$= 534.7 \text{ MPa} \textbf{ Ans.}$$

Example 4.2. *Design a spring for a balance to measure 0 to 1000 N over a scale of length80 mm. The spring is to be enclosed in a casing of 25 mm diameter. The approximate number of turnsis 30. The modulus of rigidity is 85 kN/mm2. Also calculate the maximum shear stress induced.*

Solution.

Given :

W = 1000 N

δ = 80 mm

n = 30

G = 85 kN/mm^2 = 85 × 10^3 N/mm^2

Design of spring

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire, and

C = Spring index = D/d.

Since the spring is to be enclosed in a casing of 25 mm diameter, therefore the outer diameter of the spring coil ($Do = D + d$) should be less than 25 mm.

We know that deflection of the spring (δ),

$$80 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000 \times C^3 \times 30}{85 \times 10^3 \times d} = \frac{240\ C^3}{85\ d}$$

$$\therefore \qquad \frac{C^3}{d} = \frac{80 \times 85}{240} = 28.3$$

Let us assume that d = 4 mm. Therefore

$$C^3 = 28.3\,d = 28.3 \times 4 = 113.2 \quad \text{or} \quad C = 4.84$$

and $D = C.d = 4.84 \times 4 = 19.36$ mm **Ans.**

We know that outer diameter of the spring coil,

$$D_o = D + d = 19.36 + 4 = 23.36 \text{ mm Ans.}$$

Since the value of D_o = 23.36 mm is less than the casing diameter of 25 mm, therefore the assumed dimension, d = 4 mm is correct.

Maximum shear stress induced

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 4.84 - 1}{4 \times 4.84 - 4} + \frac{0.615}{4.84} = 1.322$$

∴ Maximum shear stress induced,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.322 \times \frac{8 \times 1000 \times 4.84}{\pi \times 4^2}$$

$$= 1018.2 \text{ N/mm}^2 = 1018.2 \text{ MPa Ans.}$$

Example 4.3. *Design a helical spring for a spring loaded safety valve (Ramsbottom safetyvalve) for the following conditions :*

Diameter of valve seat = 65 mm ; Operating pressure = 0.7N/mm^2; Maximum pressure when the valve blows off freely = 0.75N/mm^2; Maximum lift of the valve when the pressure rises from 0.7 to0.75 N/mm^2 = 3.5 mm ; Maximum allowable stress = 550 MPa ;Modulus of rigidity = 84 kN/mm^2; Spring index = 6. Draw a neat sketch of the free spring showing the maindimensions.

Solution. Given :

$D1$ = 65 mm

P_1 = 0.7 N/mm^2

P_2 = 0.75N/mm^2

δ = 3.5 mm

τ = 550 MPa = 550 N/mm^2

G = 84 kN/mm^2 = 84 × 103 N/mm^2
C = 6

1. ***Mean diameter of the spring coil***

Let D = Mean diameter of the spring coil, and
d = Diameter of the spring wire.

Since the safety valve is a Ramsbottom safety valve, therefore thespring will be under tension. We know that initial tensile force actingon the spring (*i.e.* before the valve lifts),

$$W_1 = \frac{\pi}{4}(D_1)^2\, p_1 = \frac{\pi}{4}(65)^2\, 0.7 = 2323 \text{ N}$$

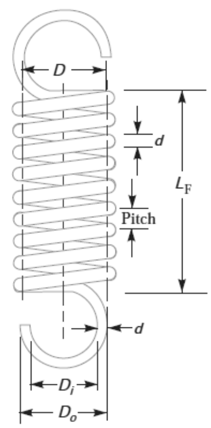


Fig. 4.12

and maximum tensile force acting on the spring (*i.e.* when the valve blows off freely),

$$W_2 = \frac{\pi}{4}(D_1)^2\, p_2 = \frac{\pi}{4}(65)^2\, 0.75 = 2489 \text{ N}$$

∴ Force which produces the deflection of 3.5 mm,

$$W = W_2 - W_1 = 2489 - 2323 = 166 \text{ N}$$

Since the diameter of the spring wire is obtained for the maximum spring load (W_2), therefore maximum twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2489 \times \frac{6\,d}{2} = 7467\,d \qquad \text{... } (\because C = D/d = 6)$$

We know that maximum twisting moment (T),

$$7467\,d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 550 \times d^3 = 108\,d^3$$

$$\therefore \qquad d^2 = 7467 / 108 = 69.14 \quad \text{or} \quad d = 8.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 2/0 having diameter (d) = 8.839 mm **Ans.**

∴ Mean diameter of the coil,

$$D = 6\,d = 6 \times 8.839 = 53.034 \text{ mm } \textbf{Ans.}$$

Outside diameter of the coil,

$$D_o = D + d = 53.034 + 8.839 = 61.873 \text{ mm } \textbf{Ans.}$$

and inside diameter of the coil,

$$D_i = D - d = 53.034 - 8.839 = 44.195 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the coil*

Let n = Number of active turns of the coil.

We know that the deflection of the spring (δ),

$$3.5 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 166 \times 6^3 \times n}{84 \times 10^3 \times 8.839} = 0.386\,n$$

∴ $n = 3.5 / 0.386 = 9.06$ say 10 **Ans.**

For a spring having loop on both ends, the total number of turns,

$$n' = n + 1 = 10 + 1 = 11 \text{ Ans.}$$

3. *Free length of the spring*

Taking the least gap between the adjacent coils as 1 mm when the spring is in free state, the free length of the tension spring,

$$L_F = n.d + (n - 1)\,1 = 10 \times 8.839 + (10 - 1)\,1 = 97.39 \text{ mm Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n - 1} = \frac{97.39}{10 - 1} = 10.82 \text{ mm Ans.}$$

The tension spring is shown in Fig. 4.12.

4.2 Stress and Deflection in Helical Springs of Non-circular Wire

This expression is applicable when the longer side (*i.e. t* >*b*) is parallelto the axis of the spring. But when the shorter side (*i.e. t* <*b*) is parallel to theaxis of the spring, then maximum shear stress,

$$\delta = \frac{2.45\, W.D^3.n}{G.b^3\ (t - 0.56\, b)}$$

For springs made of square wire, the dimensions b and t are equal. Therefore, the maximum shear stress is given by

$$\tau = K \times \frac{2.4\, W.D}{b^3}$$

and deflection of the spring,

$$\delta = \frac{5.568\, W.D^3.n}{G.b^4} = \frac{5.568\, W.C^3.n}{G.b} \qquad \ldots\left(\because C = \frac{D}{b}\right)$$

where b = Side of the square.

Note : In the above expressions,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}, \text{ and } C = \frac{D}{b}$$

Example 4.4. *A loaded narrow-gauge car of mass 1800 kg and moving at a velocity72 m/min., is brought to rest by a bumper consisting of two helical steel springs of square section.The mean diameter of the coil is six times the side of the square section. In bringing the car to rest,the springs are to be compressed 200 mm. Assuming the allowable shear stress as 365 MPa andspring index of 6, find :*

1. *Maximum load on each spring, 2. Side of the square section of the wire, 3. Mean diameter ofcoils, and 4. Number of active coils.Take modulus of rigidity as 80 kN/mm^2.*

Solution.

Given :
m = 1800 kg
v = 72 m/min = 1.2 m/s
δ = 200 mm ;
τ = 365 MPa = 365 N/mm^2
C = 6
G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

To Find
1.Maximum load on each spring,
2. Side of the square section of the wire,
3. Mean diameter of coils, and
4. Number of active coils

1. ***Maximum load on each spring,***

Let W = Maximum load on each spring.

We know that kinetic energy of the car

$$= \frac{1}{2} m.v^2 = \frac{1}{2} \times 1800\ (1.2)^2 = 1296 \text{ N-m} = 1296 \times 10^3 \text{ N-mm}$$

This energy is absorbed in the two springs when compressed to 200 mm. If the springs are loaded gradually from 0 to W, then

$$\left(\frac{0 + W}{2}\right) 2 \times 200 = 1296 \times 10^3$$

$$\therefore \quad W = 1296 \times 10^3 / 200 = 6480 \text{ N } \textbf{Ans.}$$

2. *Side of the square section of the wire*

Let b = Side of the square section of the wire, and

D = Mean diameter of the coil = 6 b ... ($\because C = D/b = 6$)

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stres (τ),

$$365 = K \times \frac{2.4\ W.D}{b^3} = 1.2525 \times \frac{2.4 \times 6480 \times 6\ b}{b^3} = \frac{116\ 870}{b^2}$$

$$\therefore \quad b^2 = 116\ 870 / 365 = 320 \quad \text{or} \quad b = 17.89 \text{ say } 18 \text{ mm } \textbf{Ans.}$$

3. *Mean diameter of the coil*

We know that mean diameter of the coil,

$$D = 6\ b = 6 \times 18 = 108 \text{ mm } \textbf{Ans.}$$

4. *Number of active coils*

Let n = Number of active coils.

We know that the deflection of the spring (δ),

$$200 = \frac{5.568\ W.C^3.n}{G.b} = \frac{5.568 \times 6480 \times 6^3 \times n}{80 \times 10^3 \times 18} = 5.4\ n$$

$$\therefore \quad n = 200 / 5.4 = 37 \textbf{ Ans.}$$

4.3 Concentric or Composite Springs

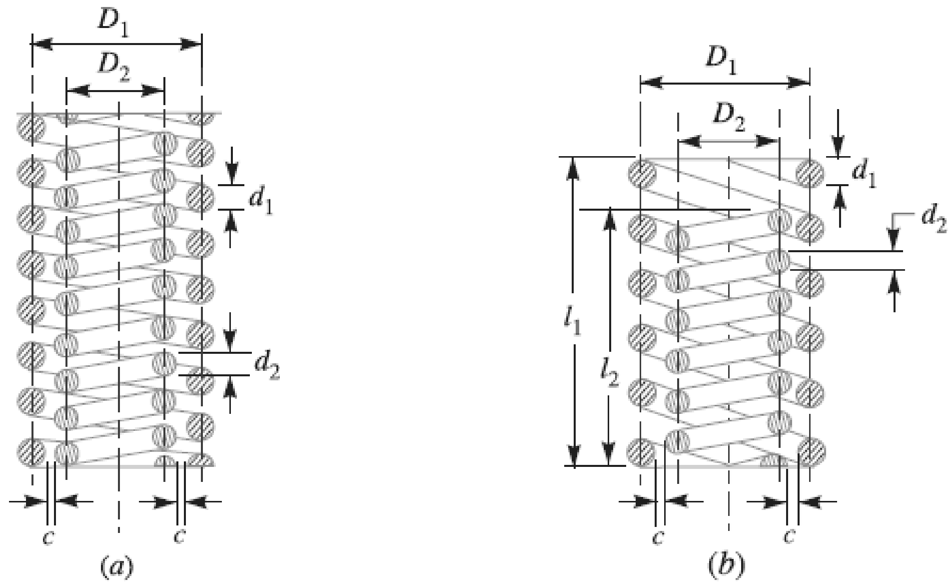


Fig. 4.15. Concentric springs.

Consider a concentric spring as shown in Fig. 4.15(*a*).

Let W = Axial load,

$W1$ = Load shared by outer spring,

$W2$ = Load shared by inner spring,

$d1$ = Diameter of spring wire of outer spring,

$d2$ = Diameter of spring wire of inner spring,

$D1$ = Mean diameter of outer spring,

$D2$ = Mean diameter of inner spring,

$\delta 1$ = Deflection of outer spring,

$\delta 2$ = Deflection of inner spring,

$n1$ = Number of active turns of outer spring, and

$n2$ = Number of active turns of inner spring.

Assuming that both the springs are made of same material, then the maximum shear stressinduced in both the springs is approximately same, *i.e.*

$$\tau_1 = \tau_2$$

$$\frac{8\,W_1 . D_1 . K_1}{\pi\,(d_1)^3} = \frac{8\,W_2 . D_2 . K_2}{\pi\,(d_2)^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 . D_1}{(d_1)^3} = \frac{W_2 . D_2}{(d_2)^3} \quad ...(i)$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e.*

$$\delta_1 = \delta_2$$

or $$\frac{8W_1\,(D_1)^3\,n_1}{(d_1)^4\,G} = \frac{8W_2\,(D_2)^3\,n_2}{(d_2)^4\,G} \quad \text{or} \quad \frac{W_1\,(D_1)^3\,n_1}{(d_1)^4} = \frac{W_2\,(D_2)^3\,n_2}{(d_2)^4} \quad ...(ii)$$

When both the springs are compressed until the adjacent coils meet, then the solid length ofboth the springs is equal, *i.e.*

$$n1.d1 = n2.d2$$

The equation ***(ii)*** may be written as

$$\frac{W_1\ (D_1)^3}{(d_1)^5} = \frac{W_2\ (D_2)^3}{(d_2)^5} \quad ...(iii)$$

Now dividing equation (*iii*) by equation (*i*), we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \quad ...(iv)$$

i.e. the springs should be designed in such a way that the spring index for both the springs is same.

From equations (*i*) and (*iv*), we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \quad ..(v)$$

From Fig. 23.22 (*a*), we find that the radial clearance between the two springs,

$$*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance between the two springs is taken as $\frac{d_1 - d_2}{2}$.

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or

$$\frac{D_1 - D_2}{2} = d_1 \quad ...(vi)$$

From equation (*iv*), we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of D_1 and D_2 in equation (*vi*), we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\ d_1 = C.d_2$$

From equation (*iv*), we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of D_1 and D_2 in equation (*vi*), we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\ d_1 = C.d_2$$

$$\therefore \quad d_1\ (C - 2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C - 2} \quad ...(vii)$$

Example 4.5. *A concentric spring for an aircraft engine valve is to exert a maximum force of5000 N under an axial deflection of 40 mm. Both the springs have same free length, same solidlength and are subjected to equal maximum shear stress of 850 MPa. If the spring index for both thesprings is 6, find (a) the load shared by each spring, (b) the main dimensions of both the springs, and(c) the number of active coils in each spring.*
Assume G = 80 kN/mm2 and diametral clearance to be equal to the difference between the wirediameters.

Solution. Given :
W = 5000 N
δ = 40 mm

$\tau_1 = \tau_2$ = 850 MPa = 850 N/mm^2
C = 6
G = 80 kN/mm^2 = 80 × 103 N/mm^2

The concentric spring is shown in Fig. 4.15(*a*).

(*a*) *Load shared by each spring*

Let $W1$ and $W2$ = Load shared by outer and inner spring respectively,
$d1$ and $d2$ = Diameter of spring wires for outer and inner springs respectively, and
$D1$ and $D2$ = Mean diameter of the outer and inner springs respectively.

Since the diametral clearance is equal to the difference between the wire diameters, therefore

$$(D_1 - D_2) - (d_1 + d_2) = d_1 - d_2$$

or

$$D_1 - D_2 = 2\,d_1$$

We know that $D_1 = C.d_1$, and $D_2 = C.d_2$

$$\therefore \quad C.d_1 - C.d_2 = 2\,d_1$$

or

$$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5 \qquad ...(i)$$

We also know that

$$\frac{W_1}{W_2} = \left(\frac{d_1}{d_2}\right)^2 = (1.5)^2 = 2.25 \qquad ...(ii)$$

and

$$W_1 + W_2 = W = 5000 \text{ N} \qquad ...(iii)$$

From equations (*ii*) and (*iii*), we find that

$$W_1 = 3462 \text{ N, and } W_2 = 1538 \text{ N } \textbf{Ans.}$$

(b) Main dimensions of both the springs

We know that Wahl's stress factor for both the springs,

$$K_1 = K_2 = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stress induced in the outer spring (τ_1),

$$850 = K_1 \times \frac{8\,W_1.C}{\pi (d_1)^2} = 1.2525 \times \frac{8\times 3462\times 6}{\pi (d_1)^2} = \frac{66\ 243}{(d_1)^2}$$

$$\therefore \quad (d_1)^2 = 66\ 243 / 850 = 78 \text{ or } d_1 = 8.83 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

and

$$D_1 = C.d_1 = 6\,d_1 = 6 \times 10 = 60 \text{ mm } \textbf{Ans.}$$

Similarly, maximum shear stress induced in the inner spring (τ_2),

$$850 = K_2 \times \frac{8W_2.C}{\pi (d_2)^2} = 1.2525 \times \frac{8\times 1538\times 6}{\pi (d_2)^2} = \frac{29\ 428}{(d_2)^2}$$

$$\therefore \quad (d_2)^2 = 29\ 428 / 850 = 34.6 \text{ or } {}^*d_2 = 5.88 \text{ say } 6 \text{ mm } \textbf{Ans.}$$

and

$$D_2 = C.d_2 = 6 \times 6 = 36 \text{ mm } \textbf{Ans.}$$

(*c*) *Number of active coils in each spring*

Let n_1 and n_2 = Number of active coils of the outer and inner spring respectively.

We know that the axial deflection for the outer spring (δ),

$$40 = \frac{8\, W_1 . C^3 . n_1}{G . d_1} = \frac{8 \times 3462 \times 6^3 \times n_1}{80 \times 10^3 \times 10} = 7.48\, n_1$$

∴ $n_1 = 40 / 7.48 = 5.35$ say 6 **Ans.**

Assuming square and ground ends for the spring, the total number of turns of the outer spring,

$$n_1' = 6 + 2 = 8$$

∴ Solid length of the outer spring,

$$L_{S1} = n_1' . d_1 = 8 \times 10 = 80 \text{ mm}$$

Let n_2' be the total number of turns of the inner spring. Since both the springs have the same solid length, therefore,

$$n_2' . d_2 = n_1' . d_1$$

or $$n_2' = \frac{n_1' . d_1}{d_2} = \frac{8 \times 10}{6} = 13.3 \text{ say } 14$$

and $n_2 = 14 - 2 = 12$ **Ans.** ... ($\because n_2' = n_2 + 2$)

Since both the springs have the same free length, therefore

Free length of outer spring

= Free length of inner spring

$= L_{S1} + \delta + 0.15\, \delta = 80 + 40 + 0.15 \times 40 = 126$ mm **Ans.**

Other dimensions of the springs are as follows:

Outer diameter of the outer spring

$= D_1 + d_1 = 60 + 10 = 70$ mm **Ans.**

Inner diameter of the outer spring

$= D_1 - d_1 = 60 - 10 = 50$ mm **Ans.**

Outer diameter of the inner spring

$= D_2 + d_2 = 36 + 6 = 42$ mm **Ans.**

Inner diameter of the inner spring

$= D_2 - d_2 = 36 - 6 = 30$ mm **Ans.**

4.1.12 Helical Torsion Springs

The helical torsion springs as shown in Fig. 4.16, may be made from round, rectangular orsquare wire. These are wound in a similar manner as helical compression or tension springs but theends are shaped to transmit torque. The primarystress in helical torsion springs is bending stresswhereas in compression or tension springs, thestresses are torsional shear stresses. The helicaltorsion springs are widely used for transmittingsmall torques as in door hinges, brush holders inelectric motors, automobile starters etc.

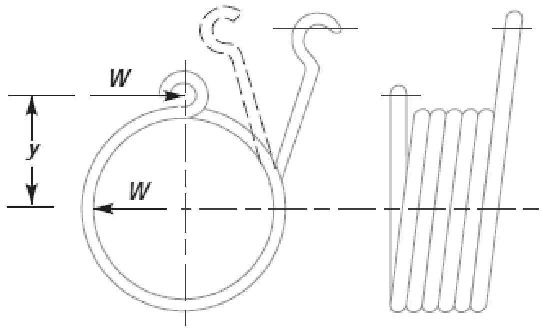


Fig. 4.16. Helical torsion spring.

A little consideration will show that theradius of curvature of the coils changes when thetwisting moment is applied to the spring. Thus,the wire is under pure bending. According to A.M.Wahl, the bending stress in a helical torsion springmade of round wire is

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = K \times \frac{32\,W.y}{\pi\,d^3}$$

where K = Wahl's stress factor = $\dfrac{4C^2 - C - 1}{4C^2 - 4C}$,

C = Spring index,

M = Bending moment = $W \times y$,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

and total angle of twist or angular deflection,

$$^*\theta = \frac{M.l}{E.I} = \frac{M \times \pi\,D.n}{E \times \pi\,d^4 / 64} = \frac{64\,M.D.n}{E.d^4}$$

where l = Length of the wire = $\pi.D.n$,

E = Young's modulus,

I = Moment of inertia = $\dfrac{\pi}{64} \times d^4$,

D = Diameter of the spring, and

n = Number of turns.

and total angle of twist or angular deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\, d^4 / 64} = \frac{64\, M.D.n}{E.d^4}$$

where l = Length of the wire = $\pi.D.n$,

E = Young's modulus,

I = Moment of inertia = $\frac{\pi}{64} \times d^4$,

D = Diameter of the spring, and

n = Number of turns.

and deflection, $$\delta = \theta \times y = \frac{64\, M.D.n}{E.d^4} \times y$$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6\, M}{t.b^2} = K \times \frac{6\, W \times y}{t.b^2}$$

where $$K = \frac{3C^2 - C - 0.8}{3C^2 - 3C}$$

Angular deflection, $$\theta = \frac{12\, \pi M.D.n}{E.t.b^3}; \quad \text{and} \quad \delta = \theta.y = \frac{12\, \pi M.D.n}{E.t.b^3} \times y$$

In case the spring is made of square wire with each side equal to b, then substituting $t = b$, in the above relation, we have

$$\sigma_b = K \times \frac{6\, M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\, \pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\, \pi M.D.n}{E.b^4} \times y$$

Example 4.6. *A helical torsion spring of mean diameter 60 mm is made of a round wire of6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and theangular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for thespring material is 200 kN/mm^2. The number of effective turns may be taken as 5.5.*

Solution.

Given :
D = 60 mm ;
d = 6 mm
M = 6 N-m = 6000 N-mm
C = 10 ;E = 200 kN/mm^2 = 200 × 103 N/mm^2
n = 5.5

Bending stress induced

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\ M}{\pi\ d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa } \textbf{Ans.}$$

Angular deflection of the spring

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\ M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28^\circ \textbf{ Ans.}$$

4.4 Leaf Springs

We know that the maximum deflection for a cantilever with concentrated load at the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\,W.L^3}{E.b.t^3} \qquad ...(ii)$$

$$= \frac{2\,\sigma.L^2}{3\,E.t} \qquad ...\left(\because \sigma = \frac{6W.L}{b.t^2}\right)$$

It may be noted that due to bending moment, top fibres will be in tension and the bottom fibresare in compression, but the shear stress is zero at the extreme fibres and maximum at the centre, asshown in Fig. 4.18. Hence for analysis, both stresses need not to be taken into account simultaneously.We shall consider the bending stress only.

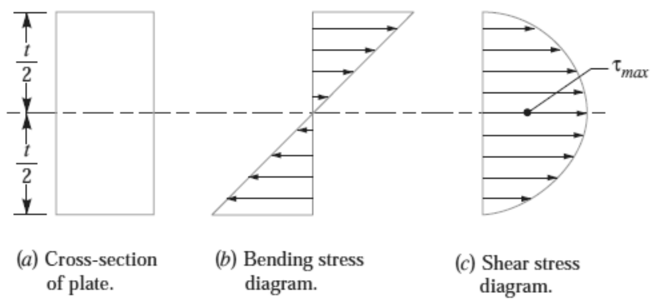


Fig. 4.18

If the spring is not of cantilever type but it is likea simply supported beam, with length 2L and load 2W in the centre, as shown in Fig. 4.19,

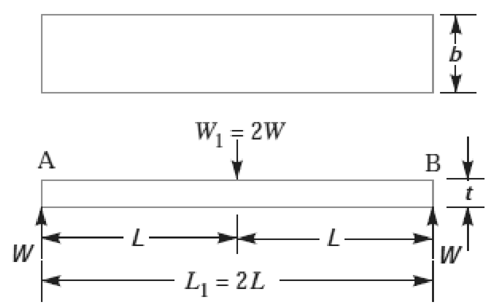


Fig. 4.19. Flat spring (simply supportedbeam type).

ThenMaximum bending moment in the centre

$$M = W.L$$

Section modulus, $$Z = b.t^2/6$$

$\therefore$ Bending stress, $$\sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6} = \frac{6\,W.L}{b.t^2}$$

We know that maximum deflection of a simplysupported beam loaded in the centre is given by

$$\delta = \frac{W_1 (L_1)^3}{48\ E.I} = \frac{(2W)\ (2L)^3}{48\ E.I} = \frac{W.L^3}{3\ E.I}$$

...($\because$ In this case, $W_1 = 2W$, and $L_1 = 2L$)

From above we see that a spring such as automobile spring (semi-elliptical spring) with length $2L$ and loaded in the centre by a load $2W$, may be treated as a double cantilever.

If the plate of cantilever is cut into a series of n strips of width b and these are placed as shown in Fig. 4.20, then equations ***(i)*** and ***(ii)*** may be written as

$$\sigma = \frac{6\ W.L}{n.b.t^2} \qquad \text{...}(iii)$$

and

$$\delta = \frac{4\ W.L^3}{n.E.b.t^3} = \frac{2\ \sigma.L^2}{3\ E.t} \qquad \text{...}(iv)$$

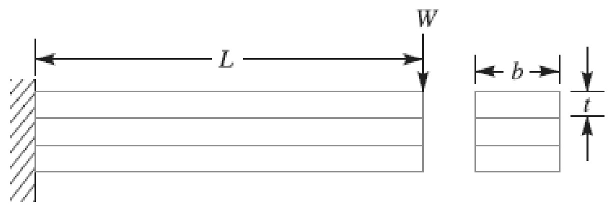


Fig. 4.20

The above relations give the stress and deflection of a leaf spring of uniform cross-section. The stress at such a spring is maximum at the support.

If a triangular plate is used as shown in Fig. 4.21 (*a*), the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. 4.21 (*b*) to form a graduated or laminated leaf spring, then

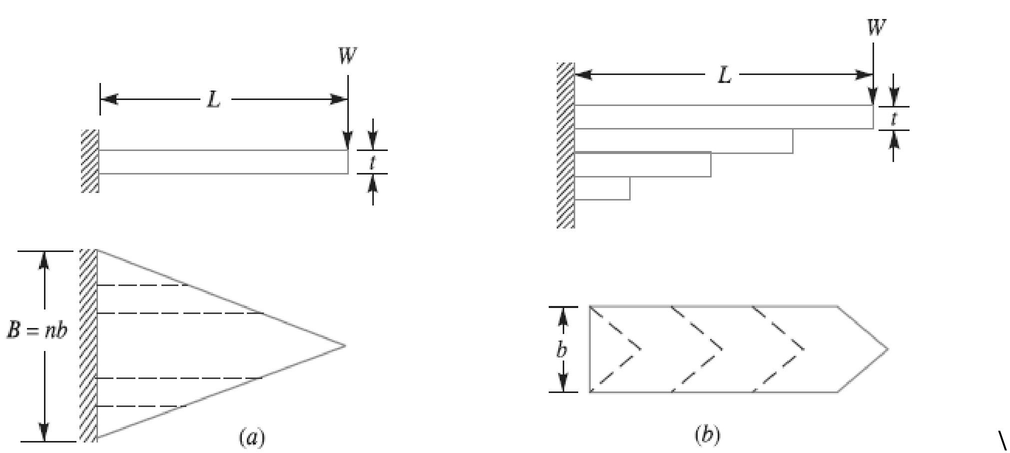


Fig. 4.21. Laminated leaf spring.

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \ldots(v)$$

and
$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad \ldots(vi)$$

Where n = Number of graduated leaves.
A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

When bending stress alone is considered, the graduated leaves may have zero width at theloaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessaryto have one or more leaves of uniform cross-section extending clear to the end. We see from equations**(*iv*)** and **(*vi*)** that for the same deflection, the stress in the uniform cross-section leaves (*i.e*. full lengthleaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects accordingto its own elastic curve. If the suffixes F and G are used to indicate the full length (or uniform crosssection) and graduated leaves, thenThe deflection in full length and graduated leaves is given by equation

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$

4.1.14 Construction of Leaf Spring:

A leaf spring commonly used in automobiles is of semi-elliptical form as shown in Fig. 4.22. It is built up of a number of plates (known as leaves). The leaves are usually given an initial curvatureor cambered so that they will tend to straighten under the load. The leaves are held together by meansof a band shrunk around them at the centre or by a bolt passing through the centre. Since the bandexerts a stiffening and strengthening effect, therefore the effective length of the spring for bendingwill be overall length of the spring *minus* width of band. In case of a centre bolt, two-third distancebetween centres of *U*-bolt should be subtracted from the overall length of the spring in order to findeffective length. The spring is clamped to the axle housing by means of *U*-bolts.

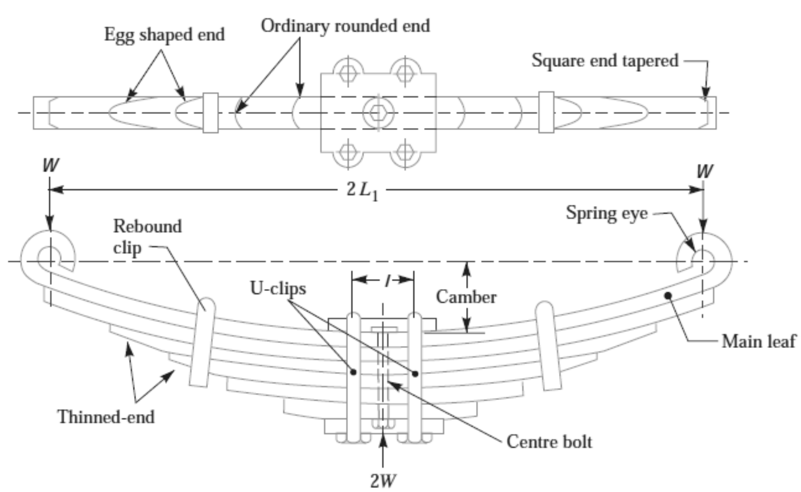


Fig. 4.22. Semi-elliptical leaf spring.

The longest leaf known as ***main leaf*** or ***master leaf***has its ends formed in the shape of an eye through whichthe bolts are passed to secure the spring to its supports.Usually the eyes, through which the spring is attached tothe hanger or shackle, are provided with bushings of someantifriction material such as bronze or rubber. The otherleaves of the spring are known as ***graduated leaves.*** Inorder to prevent digging in the adjacent leaves, the ends ofthe graduated leaves are trimmed in various forms as shownin Fig. 4.22. Since the master leaf has to with stand verticalbending loads as well as loads due to sideways of thevehicle and twisting, therefore due to the presence ofstresses caused by these loads, it is usual to provide twofull length leaves and the rest graduated leaves as shownin Fig. 4.22.Rebound clips are located at intermediate positionsin the length of the spring, so that the graduated leavesalso share the stresses induced in the full length leaveswhen the spring rebounds.

4.1.15Equalised Stress in Spring Leaves(Nipping)

We have already discussed that the stress in the fulllength leaves is 50% greater than the stress in the graduatedleaves. In order to utilise the material to the best advantage, all the leaves should be equally stressed.This condition may be obtained in the following two ways :

1. By making the full length leaves of smaller thickness than the graduated leaves. In this way,the full length leaves will induce smaller bending stress due to small distance from the neutral axis tothe edge of the leaf.

2. By giving a greater radius of curvature to the full length leaves than graduated leaves, as shownin Fig. 4.23, before the leaves are assembled to form a spring. By doing so, a gap or clearance will beleft between the leaves. This initial gap, as shown by *C* in Fig. 4.23, is called ***nip.*** When the central bolt,holding the various leaves together, is tightened, the full length leaf will bend back as shown dotted in
Fig. 4.23 and have an initial stress in a direction opposite to that of the normal load. The graduated

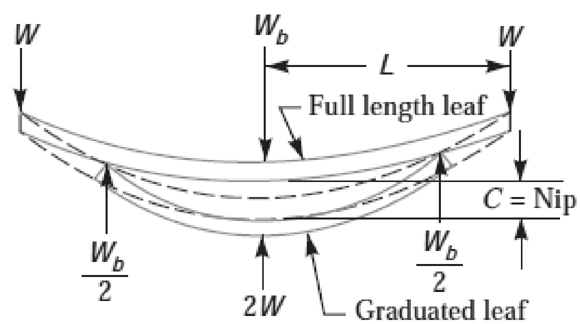


Fig. 4.23

leaves will have an initial stress in the same direction as that of the normal load. When the load isgradually applied to the spring, the full length leaf is first relieved of this initial stress and then stressed in opposite direction. Consequently, the full length leaf will be stressed less than the graduated leaf. The initial gap between the leaves may be adjusted so that under maximum load condition the stress in all the leaves is equal, or if desired, the full length leaves may have the lower stress. This is desirable inautomobile springs in which full length leaves are designed for lower stress because the full length leaves carry additional loads caused by the swaying of the car, twisting and in some cases due to driving the car through the rear springs. Let us now find the value of initial gap or nip *C*.

Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the fulllength leaves by an amount equal to the initial gap *C*. In other words,

$$\delta_G = \delta_F + C$$

$$\therefore \quad C = \delta_G - \delta_F = \frac{6\,W_G\,.\,L^3}{n_G\,E.b.t^3} - \frac{4\,W_F.L^3}{n_F.E.b.t^3} \qquad \ldots(i)$$

Since the stresses are equal, therefore

$$\sigma_G = \sigma_F$$

$$\frac{6\,W_G.L}{n_G.b.t^2} = \frac{6\,W_F.L}{n_F.b.t^2} \quad \text{or} \quad \frac{W_G}{n_G} = \frac{W_F}{n_F}$$

$$\therefore \quad W_G = \frac{n_G}{n_F} \times W_F = \frac{n_G}{n} \times W$$

and

$$W_F = \frac{n_F}{n_G} \times W_G = \frac{n_F}{n} \times W$$

Substituting the values of W_G and W_F in equation (*i*), we have

$$C = \frac{6W.L^3}{n.E.b.t^3} - \frac{4W.L^3}{n.E.b.t^3} = \frac{2W.L^3}{n.E.b.t^3} \qquad \ldots(ii)$$

The load on the clip bolts (*Wb*) required to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$$\therefore \quad C = \delta_F + \delta_G$$

$$\frac{2W.L^3}{n.E.b.t^3} = \frac{4L^3}{n_F.E.b.t^3} \times \frac{W_b}{2} + \frac{6\,L^3}{n_G.E.b.t^3} \times \frac{W_b}{2}$$

or

$$\frac{W}{n} = \frac{W_b}{n_F} + \frac{3\,W_b}{2\,n_G} = \frac{2\,n_G.W_b + 3\,n_F.W_b}{2\,n_F.n_G} = \frac{W_b\,(2\,n_G + 3\,n_F)}{2\,n_F.n_G}$$

$$\therefore \quad W_b = \frac{2\,n_F.n_G.W}{n\,(2\,n_G + 3\,n_F)} \qquad \ldots(iii)$$

The final stress in spring leaves will be the stress in the full length leaves due to the applied load ***minus*** the initial stress.

$\therefore$ Final stress,

$$\sigma = \frac{6\,W_F.L}{n_F.b.t^2} - \frac{6\,L}{n_F.b.t^2} \times \frac{W_b}{2} = \frac{6\,L}{n_F.b.t^2}\left(W_F - \frac{W_b}{2}\right)$$

$$= \frac{6\,L}{n_F.b.t^2}\left[\frac{3n_F}{2n_G + 3n_F} \times W - \frac{n_F.n_G.W}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6W.L}{b.t^2}\left[\frac{3}{2n_G + 3n_F} - \frac{n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3n - n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3(n_F + n_G) - n_G}{n\,(2n_G + 3n_F)}\right] = \frac{6\,W.L}{n.b.t^2} \qquad \ldots(iv)$$

... (Substituting $n = n_F + n_G$)

4.1.16 Length of Leaf Spring Leaves

The length of the leaf spring leaves may be obtained as discussed below :

Let $2L_1$ = Length of span or overall length of the spring,

l = Width of band or distance between centres of *U*-bolts. It is theineffective length of the spring,

n_F = Number of full length leaves,

n_G = Number of graduated leaves, and

n = Total number of leaves = $n_F + n_G$.

We have already discussed that the effective length of the Spring,

$$2L = 2L_1 - l \quad \text{...(When band is used)}$$

$$= 2L_1 - \frac{2}{3} l \quad \text{... (When } U\text{-bolts are used)}$$

It may be noted that when there is only one full length leaf (*i.e.* master leaf only), then thenumber of leaves to be cut will be n and when there are two full length leaves (including one masterleaf), then the number of leaves to be cut will be $(n - 1)$. If a leaf spring has two full length leaves, thenthe length of leaves is obtained as follows :

$$\text{Length of smallest leaf} = \frac{\text{Effective length}}{n-1} + \text{Ineffective length}$$

$$\text{Length of next leaf} = \frac{\text{Effective length}}{n-1} \times 2 + \text{Ineffective length}$$

Similarly, length of $(n - 1)$th leaf

$$= \frac{\text{Effective length}}{n-1} \times (n-1) + \text{Ineffective length}$$

The nth leaf will be the master leaf and it is of full length. Since the master leaf has eyes on both sides, therefore

$$\text{Length of master leaf} = 2L_1 + \pi(d + t) \times 2$$

where d = Inside diameter of eye, and

t = Thickness of master leaf.

The approximate relation between the radius of curvature (R) and the camber (y) of the spring is given by

$$R = \frac{(L_1)^2}{2y}$$

The exact relation is given by

$$y(2R + y) = (L_1)^2$$

where L_1 = Half span of the spring.

4.1.17 Standard Sizes of Automobile Suspension Springs

Following are the standard sizes for the automobile suspension springs:

1. Standard nominal widths are : 32, 40*, 45, 50*, 55, 60*, 65, 70*, 75, 80, 90, 100 and 125 mm. (Dimensions marked* are the preferred widths)

2. Standard nominal thicknesses are : 3.2, 4.5, 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11, 12, 14 and 16 mm.

3. At the eye, the following bore diameters are recommended :

19, 20, 22, 23, 25, 27, 28, 30, 32, 35, 38, 50 and 55 mm.

4. Dimensions for the centre bolts, if employed, shall be as given in the following table.

Table 4.3. Dimensions for centre bolts.

Width of leaves in mm	*Dia. of centre bolt in mm*	*Dia. of head in mm*	*Length of bolt head in mm*
Upto and including 65	8 or 10	12 or 15	10 or 11
Above 65	12 or 16	17 or 20	11

5. Minimum clip sections and the corresponding sizes of rivets and bolts used with the clipsshall be as given in the following table (See Fig. 4.24).

Table 4.6. Dimensions of clip, rivet and bolts.

Spring width (B) in mm	*Clip section (b × t) in mm × mm*	*Dia. of rivet (d_1) in mm*	*Dia. of bolt (d_2) in mm*
Under 50	20 × 4	6	6
50, 55 and 60	25 × 5	8	8
65, 70, 75 and 80	25 × 6	10	8
90, 100 and 125	32 × 6	10	10

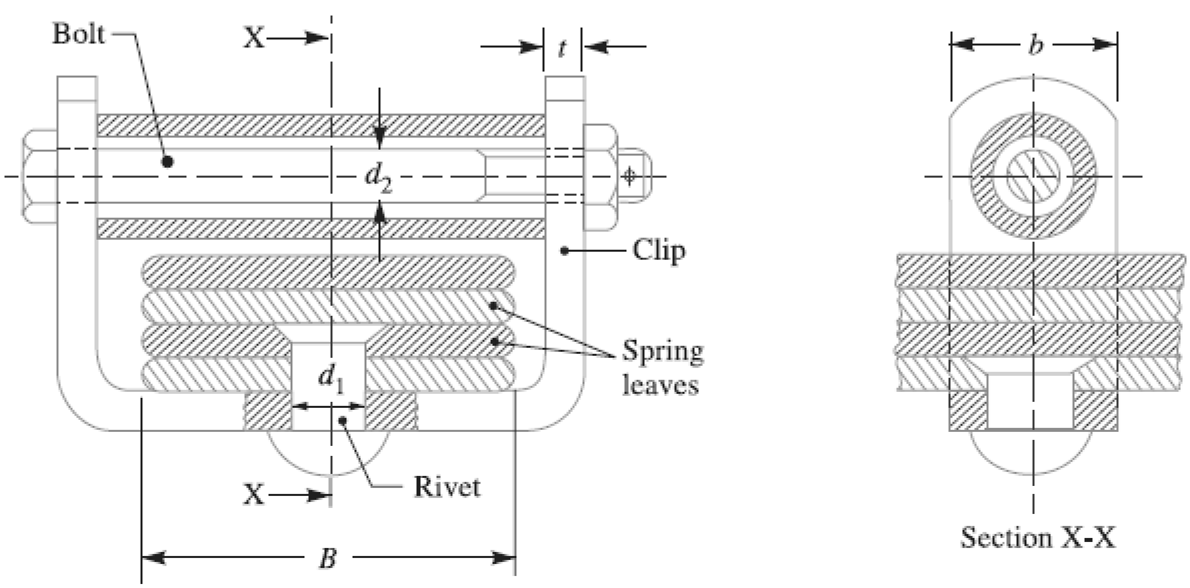


Fig. 4.24. Spring clip.

4.1.18 Materials for Leaf Springs

The material used for leaf springs is usually a plain carbon steel having 0.90 to 1.0% carbon.
The leaves are heat treated after the forming process. The heat treatment of spring steel produces greater strength and therefore greater load capacity, greater range of deflection and better fatigue properties.
According to Indian standards, the recommended materials are :
1. For automobiles : 50 Cr 1, 50 Cr 1 V 23, and 55 Si 2 Mn 90 all used in hardened and tempered state.

2. For rail road springs : C 55 (water-hardened), C 75 (oil-hardened), 40 Si 2 Mn 90 (waterhardened) and 55 Si 2 Mn 90 (oil-hardened).

3. The physical properties of some of these materials are given in the following table. Allvalues are for oil quenched condition and for single heat only.

Table 4.4. Physical properties of materials commonly used for leaf springs.

Material	*Condition*	*Ultimate tensile strength (MPa)*	*Tensile yield strength (MPa)*	*Brinell hardness number*
50 Cr 1	Hardened	1680 – 2200	1540 – 1750	461 – 601
50 Cr 1 V 23	and	1900 – 2200	1680 – 1890	534 – 601
55 Si 2 Mn 90	tempered	1820 – 2060	1680 – 1920	534 – 601

Example 4.7. *Design a leaf spring for the following specifications :Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves= 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm.Take Young's modulus, E = 200 kN/mm² and allowable stress in spring material as 600 MPa*

Solution. Given :

Total load = 140 kN

No. of springs = 4; n = 10

$2L$ = 1000 mm or L = 500 mm

δ = 80 mm

E = 200 kN/mm^2 = 200 × 103 N/mm^2

σ = 600 MPa = 600 N/mm^2

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$\therefore$ $W = 35 / 2 = 17.5$ kN = 17 500 N

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$\therefore$ $n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3$ ...(*i*)

and deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$\therefore$ $n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6$ ...(*ii*)

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$$b = 87.5 \text{ say } 90 \text{ mm Ans.}$$

Example 4.8. *A semi-elliptical laminated vehicle spring to carry a load of 6000 N is toconsist of seven leaves 65 mm wide, two of the leaves extending the full length of the spring. Thespring is to be 1.1 m in length and attached to the axle by two U-bolts 80 mm apart. The bolts holdthe central portion of the spring so rigidly that they may be considered equivalent to a band havinga width equal to the distance between the bolts. Assume a design stress for spring material as 350MPa. Determine :*
1. Thickness of leaves, 2. Deflection of spring, 3. Diameter of eye, 4. Length of leaves, and 5.
Radius to which leaves should be initially bent.
Sketch the semi-elliptical leaf-spring arrangement.
The standard thickness of leaves are : 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11 etc. in mm.

Solution.

Given :

$2W = 6000$ N
$W = 3000$ N
$n = 7$
$b = 65$ mm
$n_F = 2$
$2L_1 = 1.1$ m= 1100 mm or $L_1 = 550$ mm
$l = 80$ mm
$\sigma = 350$ MPa = 350 N/mm^2

1. *Thickness of leaves*

Let t = Thickness of leaves.

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1100 - 80 = 1020 \text{ mm}$$

$$\therefore \quad L = 1020 / 2 = 510 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 7 - 2 = 5$$

Assuming that the leaves are not initially stressed, the maximum stress (σ_F),

$$350 = \frac{18\, W.L}{b.t^2\ (2n_G + 3n_F)} = \frac{18 \times 3000 \times 510}{65 \times t^2\ (2 \times 5 + 3 \times 2)} = \frac{26\,480}{t^2} \quad \text{... } (\sigma_F = \sigma)$$

$$\therefore \quad t^2 = 26\,480 / 350 = 75.66 \quad \text{or} \quad t = 8.7 \text{ say 9 mm Ans.}$$

2. *Deflection of spring*

We know that deflection of spring,

$$\delta = \frac{12\, W.L^3}{E.b.t^3\ (2n_G + 3n_F)} = \frac{12 \times 3000\ (510)^3}{210 \times 10^3 \times 65 \times 9^3\ (2 \times 5 + 3 \times 2)}$$

$$= 30 \text{ mm Ans.} \quad \text{... (Taking } E = 210 \times 10^3 \text{ N/mm}^2\text{)}$$

3. *Diameter of eye*

The inner diameter of eye is obtained by considering the pin in the eye in bearing, because the inner diameter of the eye is equal to the diameter of the pin.

Let d = Inner diameter of the eye or diameter of the pin,

l_1 = Length of the pin which is equal to the width of the eye or leaf (*i.e.* b) = 65 mm ...(Given)

p_b = Bearing pressure on the pin which may be taken as 8 N/mm^2.

We know that the load on pin (W),

$$3000 = d \times l_1 \times p_b$$

$$= d \times 65 \times 8 = 520\, d$$

$$\therefore \quad d = 3000 / 520$$

$$= 5.77 \text{ say 6 mm}$$

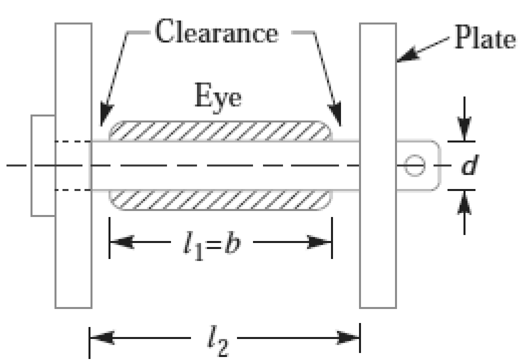


Fig. 4.25

Let us now consider the bending of the pin. Since thereis a clearance of about 2 mm between the shackle (or plate)and eye as shown in Fig. 4.25, therefore length of the pinunder bending,

$L_2 = l_1 + 2 \times 2 = 65 + 4 = 69$ mm

Maximum bending moment on the pin,

$$M = \frac{W \times l_2}{4} = \frac{3000 \times 69}{4} = 51\ 750 \text{ N-mm}$$

and section modulus, $Z = \frac{\pi}{32} \times d^3 = 0.0982\ d^3$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{51\ 750}{0.0982\ d^3} = \frac{527 \times 10^3}{d^3} \quad \text{... (Taking } \sigma_b = 80 \text{ N/mm}^2\text{)}$$

$\therefore$ $d^3 = 527 \times 10^3 / 80 = 6587$ or $d = 18.7$ say 20 mm **Ans.**

We shall take the inner diameter of eye or diameter of pin (d) as 20 mm **Ans.**

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin (W),

$$3000 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2\ \tau = 628.4\ \tau$$

$\therefore$ $\tau = 3000 / 628.4 = 4.77$ N/mm^2, which is safe.

4. *Length of leaves*

We know that ineffective length of the spring

$= l = 80$ mm ... ($\because$ *U*-bolts are considered equivalent to a band)

$$\therefore \text{Length of the smallest leaf} = \frac{\text{Effective length}}{n-1} + \text{Ineffective length}$$

$$= \frac{1020}{7-1} + 80 = 250 \text{ mm} \quad \textbf{Ans.}$$

Length of the 2nd leaf $= \frac{1020}{7-1} \times 2 + 80 = 420$ mm **Ans.**

Length of the 3rd leaf $= \frac{1020}{7-1} \times 3 + 80 = 590$ mm **Ans.**

Length of the 4th leaf $= \frac{1020}{7-1} \times 4 + 80 = 760$ mm **Ans.**

Length of the 5th leaf $= \frac{1020}{7-1} \times 5 + 80 = 930$ mm **Ans.**

Length of the 6th leaf $= \frac{1020}{7-1} \times 6 + 80 = 1100$ mm **Ans.**

The 6th and 7th leaves are full length leaves and the 7th leaf (*i.e.* the top leaf) will act as a master leaf.

We know that length of the master leaf

$$= 2L_1 + \pi (d + t)\ 2 = 1100 + \pi (20 + 9)2 = 1282.2 \text{ mm } \textbf{Ans.}$$

5. *Radius to which the leaves should be initially bent*

Let R = Radius to which the leaves should be initially bent, and

y = Camber of the spring.

We know that

$$y(2R - y) = (L_1)^2$$

$$30(2R - 30) = (550)^2 \text{ or } 2R - 30 = (550)^2/30 = 10\ 083 \quad \ldots (\because y = \delta)$$

$$\therefore \quad R = \frac{10\ 083 + 30}{2} = 5056.5 \text{ mm } \textbf{Ans.}$$

4.2 FLYWHEEL

4.2.1 Introduction

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply ofenergy is more than the requirement and releases it duringthe period when the requirement of energy is more thansupply.In case of steam engines, internal combustion engines,reciprocating compressors and pumps, the energy isdeveloped during one stroke and the engine is to run forthe whole cycle on the energy produced during this onestroke. For example, in I.C. engines, the energy is developedonly during power stroke which is much more than theengine load, and no energy is being developed duringsuction, compression and exhaust strokes in case of fourstroke engines and during compression in case of two strokeengines. The excess energy developed during power strokeis absorbed by the flywheel and releases it to the crankshaftduring other strokes in which no energy is developed, thusrotating the crankshaft at a uniform speed. A little consideration will show that when the flywheelabsorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel doesnot maintain a constant speed, it simply reduces the fluctuation of speed.In machines where the operation is intermittent like punching machines, shearing machines,riveting machines, crushers etc., the flywheel stores energy from the power source during the greaterportion of the operating cycle and gives it up during a small period of the cycle. Thus the energy fromthe power source to the machines is supplied practically at a constant rate throughout the operation.

4.2.2 Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the ***maximumfluctuation of speed.*** The ratio of the maximum fluctuation of speed to the mean speed is called ***coefficient of fluctuation of speed*.**

Let $N1$ = Maximum speed in r.p.m. during the cycle,

$N2$ = Minimum speed in r.p.m. during the cycle, and

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

$\therefore$ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It variesdepending upon the nature of service to which the flywheel is employed. Table 4.26 shows the permissiblevalues for coefficient of fluctuation of speed for some machines.

Table 4.5. Permissible values for coefficient of fluctuation of speed (*C*S).

S.No.	*Type of machine or class of service*	*Coefficient of fluctuation of speed* (C_S)
1.	Crushing machines	0.200
2.	Electrical machines	0.003
3.	Electrical machines (direct drive)	0.002
4.	Engines with belt transmission	0.030
5.	Gear wheel transmission	0.020
6.	Hammering machines	0.200
7.	Pumping machines	0.03 to 0.05
8.	Machine tools	0.030
9.	Paper making, textile and weaving machines	0.025
10.	Punching, shearing and power presses	0.10 to 0.15
11.	Spinning machinery	0.10 to 0.020
12.	Rolling mills and mining machines	0.025

4.2.3 Fluctuation of Energy

The fluctuation of energy may be determined by the turning moment diagram for one completecycle of operation. Consider a turning moment diagram for a single cylinder double acting steamengine as shown in Fig. 4.26. The vertical ordinate represents the turning moment and the horizontalordinate (abscissa) represents the crank angle.

A little consideration will show that the turning moment is zero when the crank angle is zero. Itrises to a maximum value when crank angle reaches 90º and it is again zero when crank angle is 180º.This is shown by the curve *abc*in Fig. 4.26 and it represents the turning moment diagram for outstroke.The curve *cde*is the turning moment diagram for instroke and is somewhat similar to the curve *abc*.Since the work done is the product of the turning moment and the angle turned, therefore thearea of the turning moment diagram represents the work done per revolution. In actual practice, theengine is assumed to work against the mean resisting torque, as shown by a horizontal line *AF*. Theheight of the ordinate *aA*represents the mean height of the turning moment diagram. Since it isassumed that the work done by the turning moment per revolution is equal to the work done againstthe mean resisting torque, therefore the area of the rectangle *aA Fe* is proportional to the work doneagainst the mean resisting torque.

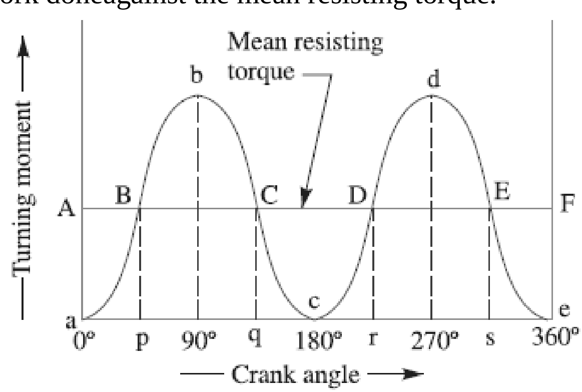


Fig. 4.26. Turning moment diagram for a single cylinder double acting steam engine.

We see in Fig. 4.26, that the mean resisting torque line *AF* cuts the turning moment diagram atpoints *B*, *C*, *D* and *E*. When the crank moves from '*a*' to '*p*' the work done by the engine is equal tothe area *aBp*, whereas the energy required is represented by the area *aABp*. In other words, the enginehas done less work (equal to the area *aAB*) than the requirement. This amount of energy is taken fromthe flywheel and hence the speed of the flywheel decreases. Now the crank moves from *p* to *q*, thework done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is representedby the area *pBCq*. Therefore the engine has done more work than the requirement. This excess work(equal to the area *BbC*) is stored in the flywheel and hence the speed of the flywheel increases whilethe crank moves from *p* to *q*.

Similarly when the crank moves from *q* to *r*, more work is taken from the engine than is developed.This loss of work is represented by the area *CcD*. To supply this loss, the flywheel gives up some ofits energy and thus the speed decreases while the crank moves from *q* to *r*. As the crank moves from*r* to *s*, excess energy is again developed given by the area *DdE*and the speed again increases. As thepiston moves from *s* to *e*, again there is a loss of work and the speed decreases. The variations ofenergy above and below the mean resisting torque line are called ***fluctuation of energy.*** The areas*BbC*, *CcD*, *DdE*etc. represent fluctuations of energy.

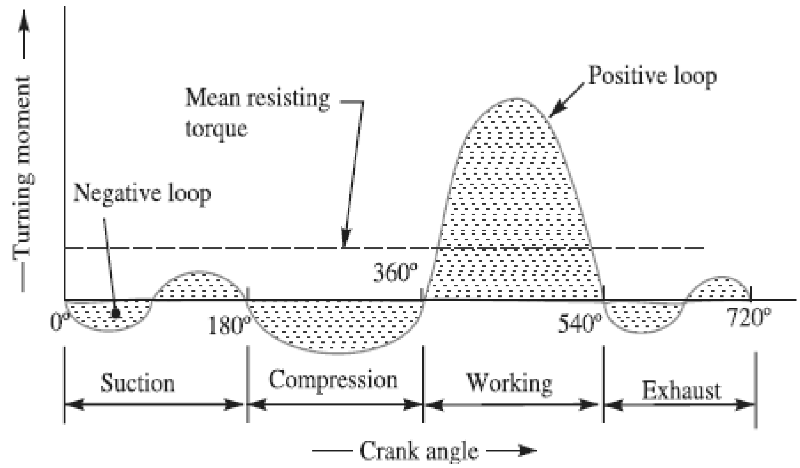


Fig. 4.27. Tunring moment diagram for a four stroke internal combustion engine.

A little consideration will show that the engine hasa maximum speed either at *q* or at *s*. This is due to the fact that the flywheel absorbs energy while the crankmoves from *p* to *q* and from *r* to *s*. On the other hand,the engine has a minimum speed either at *p* or at *r*. Thereason is that the flywheel gives out some of its energywhen the crank moves from *a* to*p* and from *q* to *r*. Thedifference between the maximum and the minimumenergies is known as ***maximum fluctuation of energy.***A turning moment diagram for a four strokeinternal combustion engine is shown in Fig. 4.27. Weknow that in a four stroke internal combustion engine,there is one working stroke after the crank has turnedthrough 720º (or 4π radians). Since the pressure inside the engine cylinder is less than the atmosphericpressure during suction stroke, therefore a negative loop is formed as shown in Fig. 4.27. During thecompression stroke, the work is done on the gases, therefore a higher negative loop is obtained. In theworking stroke, the fuel burns and the gases expand, therefore a large positive loop is formed. Duringexhaust stroke, the work is done on the gases, therefore a negative loop is obtained.A turning moment diagram for a compound steam engine having three cylinders and the resultantturning moment diagram is shown in Fig. 4.28. The resultant turning moment diagram is the sum ofthe turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the highpressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressurecylinder. The cranks, in case of three cylinders are usually placed at 120º to each other.

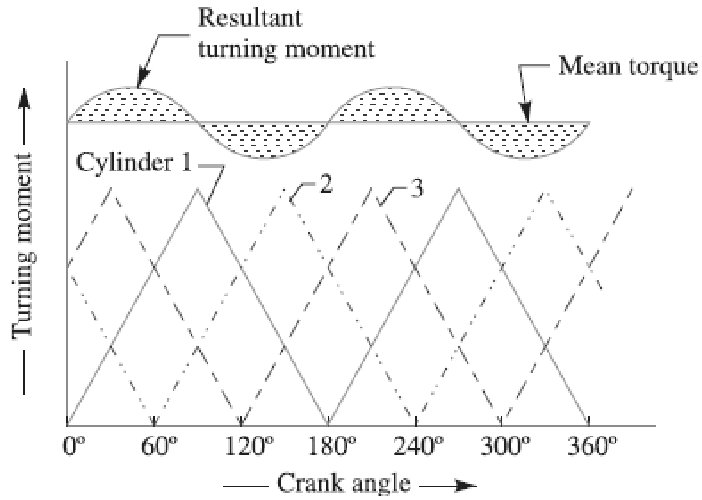
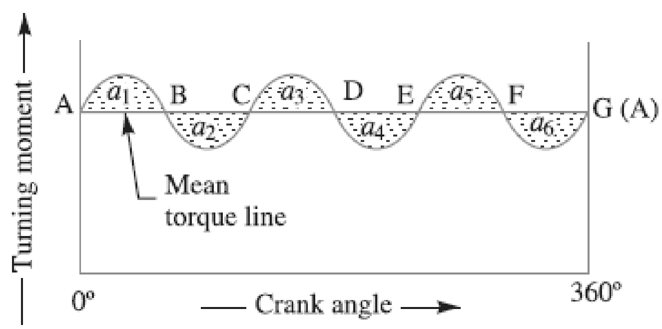


Fig. 4.28. Turning moment diagram for a compound steam engine.

4.2.4 Maximum Fluctuation of Energy

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. 22.4.The horizontal line *AG* represents the mean torque line. Let *a*1, *a*3, *a*5 be the areas above the mean torque line and *a*2, *a*4 and *a*6 be the areas below the mean torque line. These areas represent somequantity of energy which is either added or subtracted from the energy of the moving parts of theengine.



Let the energy in the flywheel at $A = E$, then from Fig. 22.4, we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$ = Energy at A

Let us now suppose that the maximum of these energies is at B and minimum at E.

$\therefore$ Maximum energy in the flywheel

$$= E + a_1$$

and minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

∴ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

4.2.5 Coefficient of Fluctuation of Energy

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It isusually denoted by *C*E. Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The workdone per cycle may be obtained by using the following relations:

1. Workdone / cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned in radians per revolution

= 2π, in case of steam engines and two stroke internal combustion engines.

= 4π, in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation *i.e.*

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = $2\pi N / 60$

2. The workdone per cycle may also be obtained by using the following relation:

$$\text{Workdone / cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute.

= N, in case of steam engines and two stroke internal combustion engines.

= $N/2$, in case of four stroke internal combustion engines.

The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

Table 4.6. Coefficient of fluctuation of energy (*C*E) for steam and internal combustion engines.

S.No.	*Type of engine*	*Coefficient of fluctuation of energy* (C_E)
1.	Single cylinder, double acting steam engine	0.21
2.	Cross-compound steam engine	0.096
3.	Single cylinder, single acting, four stroke gas engine	1.93
4.	Four cylinder, single acting, four stroke gas engine	0.066
5.	Six cylinder, single acting, four stroke gas engine	0.031

4.2.6 Energy Stored in a Flywheel
A flywheel is shown in Fig. 4.29. We have already discussed that when a flywheel absorbs energy its speed increases and when it gives up energy its speed decreases.

Let m = Mass of the flywheel inkg,
k = Radius of gyration of theflywheel in metres,
I = Mass moment of inertia ofthe flywheel about the axis of rotation in kg-m^2= $m.k^2$,
N_1 and N_2 = Maximum and minimum
speeds during the cycle inr.p.m.,
ω1 and ω2 = Maximum and minimumangular speeds during the cycle in rad / s,

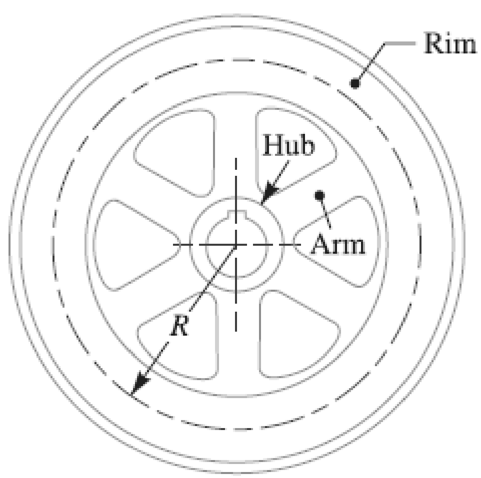


Fig. 4.29. Flywheel.

$$N = \text{Mean speed during the cycle in r.p.m.} = \frac{N_1 + N_2}{2},$$

$$\omega = \text{Mean angular speed during the cycle in rad / s} = \frac{\omega_1 + \omega_2}{2}$$

$$C_S = \text{Coefficient of fluctuation of speed} = \frac{N_1 - N_2}{N} \text{ or } \frac{\omega_1 - \omega_2}{\omega}$$

We know that mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \text{ (in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.} = \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2$$

$$= \frac{1}{2} \times I \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= I.\omega(\omega_1 - \omega_2) \qquad \ldots\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right) \ldots(i)$$

$$= I.\omega^2 \left(\frac{\omega_1 - \omega_2}{\omega}\right) \qquad \ldots\text{[Multiplying and dividing by } \omega]$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \qquad \ldots(\because I = m.k^2) \ldots(ii)$$

$$= 2\,E.C_S \qquad \ldots\left(\because E = \frac{1}{2} \times I.\omega^2\right) \ldots(iii)$$

The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because thethickness of rim is very small as compared to the diameter of rim. Therefore substituting $k = R$ inequation **(*ii*)**, we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S \qquad \ldots(\because v = \omega.R)$$

From this expression, the mass of the flywheel rim may be determined.

Example 4.9. *The intercepted areas between the output torque curve and the mean resistanceline of a turning moment diagram for a multicylinder engine, taken in order from one end are asfollows:*

– 35, + 410, – 285, + 325, – 335, + 260, – 365, + 285, – 260 mm2.

The diagram has been drawn to a scale of 1 mm = 70 N-m and 1 mm = 4.5°. The engine speedis 900 r.p.m. and the fluctuation in speed is not to exceed 2% of the mean speed.Find the mass and cross-section of the flywheel rim having 650 mm mean diameter. The densityof the material of the flywheel may be taken as 7200 kg / m3. The rim is rectangular with the width2 times the thickness. Neglect effect of arms, etc.

Solution.
Given :
N = 900 r.p.m. or $\omega = 2\pi \times 900 / 60 = 94.26$ rad/s
$\omega_1 - \omega_2 = 2\%\ \omega$ or
$((\omega1 - \omega2)/2))=CS = 2\% = 0.02$
D = 650 mm or R = 325 mm = 0.325 m
$\rho = 7200$ kg / m^3

Mass of the flywheel rim

Let m = Mass of the flywheel rim in kg.
First of all, let us find the maximum fluctuation of energy. The turning moment diagram for amulti-cylinder engine is shown in Fig. 4.30.

Since the scale of turning moment is 1 mm = 70 N-m and scale of the crank angle is 1 mm = 4.5º = π / 40 rad, therefore 1 mm2 on the turning moment diagram.

= 70 × π / 40 = 5.5 N-m

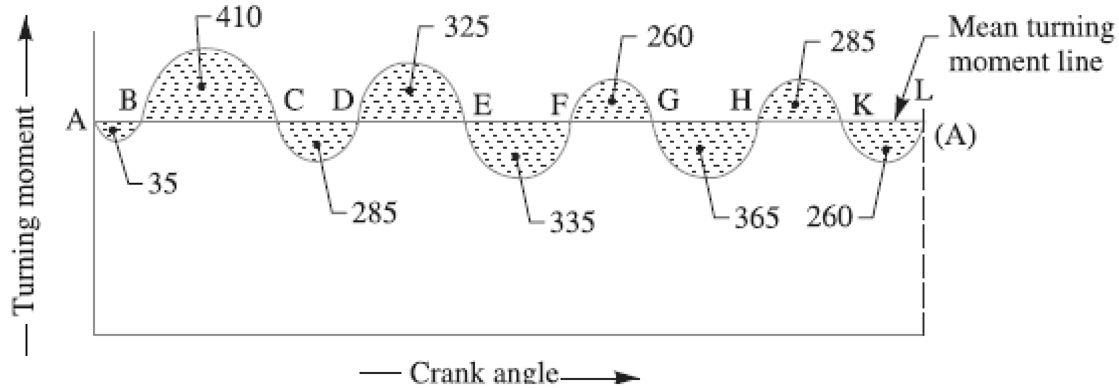


Fig. 4.30

Let the total energy at $A = E$. Therefore from Fig. 4.30, we find that

Energy at $B = E - 35$

Energy at $C = E - 35 + 410 = E + 375$

Energy at $D = E + 375 - 285 = E + 90$

Energy at $E = E + 90 + 325 = E + 415$

Energy at $F = E + 415 - 335 = E + 80$

Energy at $G = E + 80 + 260 = E + 340$

Energy at $H = E + 340 - 365 = E - 25$

Energy at $K = E - 25 + 285 = E + 260$

Energy at $L = E + 260 - 260 = E$ = Energy at A

From above, we see that the energy is maximum at E and minimum at B.

∴ Maximum energy = $E + 415$

and minimum energy = $E - 35$

We know that maximum fluctuation of energy,

$= (E + 415) - (E - 35) = 450 \text{ mm}^2$

= 450 × 5.5 = 2475 N-m

We also know that maximum fluctuation of energy (ΔE),

$$2475 = m.R^2.\omega^2.C_S = m\,(0.325)^2\,(94.26)^2\,0.02 = 18.77\,m$$

$\therefore$ $m = 2475 / 18.77 = 132$ kg **Ans.**

Cross-section of the flywheel rim

Let t = Thickness of the rim in metres, and

b = Width of the rim in metres = $2\,t$ ...(Given)

$\therefore$ Area of cross-section of the rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$132 = A \times 2\,\pi\,R \times \rho = 2\,t^2 \times 2\,\pi \times 0.325 \times 7200 = 29\,409\,t^2$$

$\therefore$ $t^2 = 132 / 29\,409 = 0.0044$ or $t = 0.067$ m = 67 mm **Ans.**

and $b = 2t = 2 \times 67 = 134$ mm **Ans.**

4.2.7 Stresses in a Flywheel Rim

A flywheel, as shown in Fig. 4.31, consists of arim at which the major portion of the mass or weightof flywheel is concentrated, a boss or hub for fixingthe flywheel on to the shaft and a number of arms for supporting the rim on the hub.The following types of stresses are induced inthe rim of a flywheel:

1. Tensile stress due to centrifugal force,

2. Tensile bending stress caused by the restraintof the arms, and

3. The shrinkage stresses due to unequal rateof cooling of casting. These stresses may bevery high but there is no easy method of determining.This stress is taken care of by afactor of safety.

We shall now discuss the first two types ofstresses as follows:

1. ***Tensile stress due to the centrifugal force***

The tensile stress in the rim due to the centrifugal force, assuming that the rim is unstrained bythe arms, is determined in a similar way as a thin cylinder subjected to internal pressure.

Let

b = Width of rim,

t = Thickness of rim,

A = Cross-sectional area of rim = $b \times t$,

D = Mean diameter of flywheel

R = Mean radius of flywheel,

ρ = Density of flywheel material,

ω = Angular speed of flywheel,

v = Linear velocity of flywheel, and

σ_t = Tensile or hoop stress.

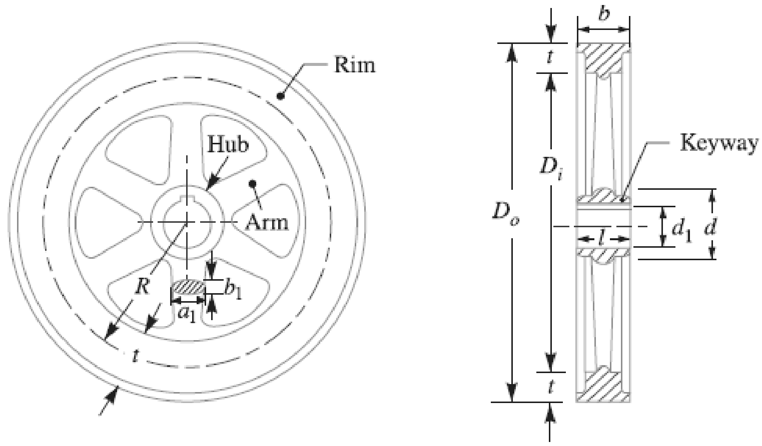


Fig. 4.31. Flywheel.

Consider a small element of the rim as shown shaded in Fig. 4.32. Let it subtends an angle ∂_θ at the centre of the flywheel.

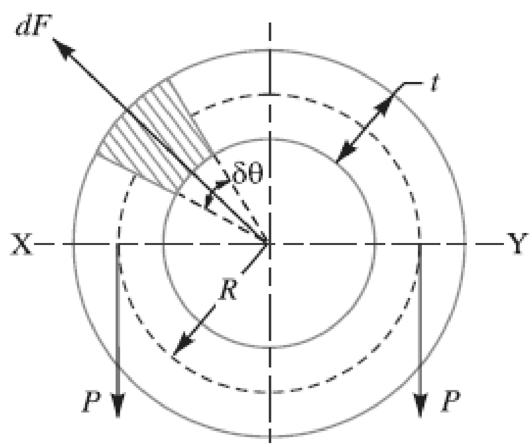


Fig. 4.32. Cross-section of a flywheel rim.

Volume of the small element

$$= A.R.\delta\theta$$

∴ Mass of the small element,

$$dm = \text{Volume} \times \text{Density}$$
$$= A.R.\delta\theta.\rho = \rho.A.R.\delta\theta$$

and centrifugal force on the element,

$$dF = dm.\omega^2.R = \rho.A.R.\delta\theta.\omega^2.R$$
$$= \rho.A.R^2.\omega^2.\delta\theta$$

Vertical component of dF

$$= dF.\sin\theta$$
$$= \rho.A.R^2.\omega^2.\delta\theta \sin\theta$$

$\therefore$ Total vertical bursting force across the rim diameter X-Y,

$$= \rho.A\, R^2.\omega^2 \int_0^{\pi} \sin\theta\, d\theta$$

$$= \rho.A.R^2.\omega^2 \left[-\cos\theta\right]_0^{\pi} = 2\,\rho.A.R^2.\omega^2 \quad ...(i)$$

This vertical force is resisted by a force of $2P$, such that

$$2P = 2\sigma_t \times A \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$2\rho A.R^2.\omega^2 = 2\,\sigma_t \times A$$

$$\therefore \quad \sigma_t = \rho.R^2.\omega^2 = \rho.v^2 \qquad ...(\because v = \omega.R) \quad ...(iii)$$

when ρ is in kg / m^3 and v is in m / s, then σ_t will be in N / m^2 or Pa.

2. *Tensile bending stress caused by restraint of the arms*

The tensile bending stress in the rim due to the restraint of the arms is based on the assumptionthat each portion of the rim between a pair of arms behaves like a beam fixed at both ends anduniformly loaded, as shown in Fig. 4.33, such that length between fixed ends,

$$l = \frac{\pi D}{n} = \frac{2\pi R}{n}, \text{ where } n = \text{Number of arms.}$$

The uniformly distributed load (w) per metre length will be equal to the centrifugal force between a pair of arms.

$$\therefore \quad w = b.t.\rho.\omega^2.R \text{ N/m}$$

We know that maximum bending moment,

$$M = \frac{w\,.\,l^2}{12} = \frac{b\,.\,t\,\rho\,.\,\omega^2\,.\,R}{12}\left(\frac{2\pi R}{n}\right)^2$$

and section modulus, $\quad Z = \frac{1}{6}\, b \times t^2$

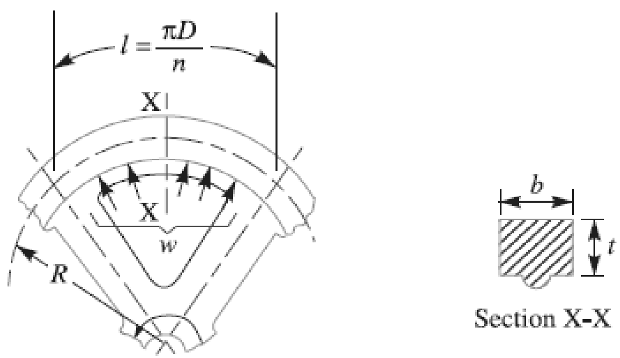


Fig. 4.33

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{b\,.\,t\,\rho\,.\,\omega^2\,.\,R}{12}\left(\frac{2\,\pi\,R}{n}\right)^2 \times \frac{6}{b \times t^2}$$

$$= \frac{19.74\,\rho\,.\,\omega^2\,.\,R^3}{n^2\,.\,t} = \frac{19.74\,\rho\,.\,v^2\,.\,R}{n^2\,.\,t} \quad \text{...(iv)}$$

...(Substituting $\omega = v/R$)

Now total stress in the rim,

$$\sigma = \sigma_t + \sigma_b$$

If the arms of a flywheel do not stretch at all and are placed very close together, then centrifugal force will not set up stress in the rim. In other words, σ_t will be zero. On the other hand, if the arms are stretched enough to allow free expansion of the rim due to centrifugal action, there will be no restraint due to the arms, *i.e.* σ_b will be zero.

It has been shown by G. Lanza that the arms of a flywheel stretch about $\frac{3}{4}$ th of the amount necessary for free expansion. Therefore the total stress in the rim,

$$= \frac{3}{4}\,\sigma_t + \frac{1}{4}\,\sigma_b = \frac{3}{4}\,\rho.v^2 + \frac{1}{4} \times \frac{19.74\,\rho\,.\,v^2\,.\,R}{n^2\,.\,t} \quad \text{...(v)}$$

$$= \rho.v^2\left(0.75 + \frac{4.935\,R}{n^2\,.\,t}\right)$$

Example 4.10. *A multi-cylinder engine isto run at a constant load at a speed of 600 r.p.m.On drawing the crank effort diagram to a scaleof 1 m = 250 N-m and 1 mm = 3º, the areas in sqmm above and below the mean torque line are asfollows:*
+ 160, – 172, + 168, – 191, + 197, – 162 sq mm
The speed is to be kept within ± 1% of themean speed of the engine. Calculate the necessarymoment of inertia of the flywheel.
Determine suitable dimensions for cast ironflywheel with a rim whose breadth is twice its radial thickness. The density of cast iron is 7250kg / m^3, and its working stress in tension is 6 MPa.Assume that the rim contributes 92% of theflywheel effect.

Solution.
Given :
N = 600 r.p.m. or$\omega = 2\pi \times 600 / 60 = 62.84$ rad / s
$\rho = 7250$ kg / m^3
σ_t= 6 MPa = 6 × 106 N/m^2

Moment of inertia of the flywheel

Let

I = Moment of inertia of the flywheel.
First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig. 4.34.

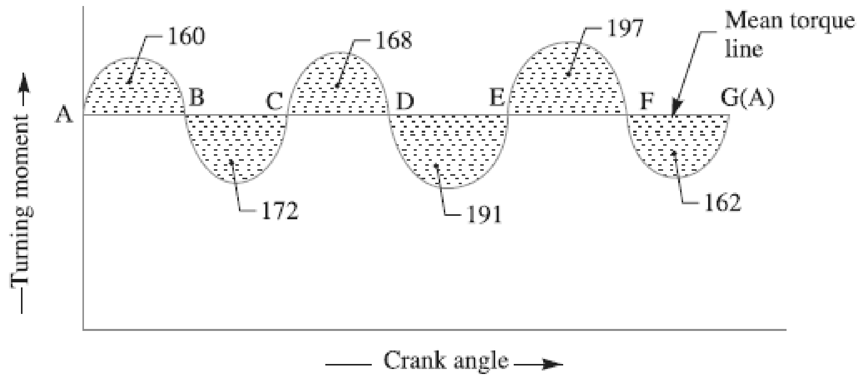


Fig. 4.34

Since the scale for the turning moment is 1 mm = 250 N-m and the scale for the crank angle is

$1 \text{ mm} = 3° = \frac{\pi}{60}$ rad, therefore

1 mm^2 on the turning moment diagram

$$= 250 \times \frac{\pi}{60} = 13.1 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 4.34, we find that

Energy at B = $E + 160$

Energy at C = $E + 160 - 172$ = $E - 12$

Energy at D = $E - 12 + 168$ = $E + 156$

Energy at E = $E + 156 - 191$ = $E - 35$

Energy at F = $E - 35 + 197$ = $E + 162$

Energy at G = $E + 162 - 162$ = E = Energy at A

From above, we find that the energy is maximum at F and minimum at E.

$\therefore$ Maximum energy $= E + 162$

and minimum energy $= E - 35$

We know that the maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 162) - (E - 35) = 197 \text{ mm}^2 = 197 \times 13.1 = 2581 \text{ N-m}$$

Since the fluctuation of speed is ± 1% of the mean speed (ω), therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 2\% \ \omega = 0.02 \ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.02$$

We know that the maximum fluctuation of energy (ΔE),

$$2581 = I.\omega^2.C_S = I\,(62.84)^2\, 0.02 = 79\, I$$

$$\therefore \quad I = 2581 / 79 = 32.7 \text{ kg-m}^2 \textbf{ Ans.}$$

Dimensions of a flywheel rim

Let t = Thickness of the flywheel rim in metres, and

b = Breadth of the flywheel rim in metres = $2t$ ...(Given)

First of all let us find the peripheral velocity (v) and mean diameter (D) of the flywheel.

We know that tensile stress (σ_t),

$$6 \times 10^6 = \rho . v^2 = 7250 \times v^2$$

$$\therefore \quad v^2 = 6 \times 10^6 / 7250 = 827.6 \quad \text{or} \quad v = 28.76 \text{ m/s}$$

We also know that peripheral velocity (v),

$$28.76 = \frac{\pi D . N}{60} = \frac{\pi D \times 600}{60} = 31.42 D$$

$$\therefore \quad D = 28.76 / 31.42 = 0.915 \text{ m} = 915 \text{ mm } \textbf{Ans.}$$

Now let us find the mass of the flywheel rim. Since the rim contributes 92% of the flywheel effect, therefore the energy of the flywheel rim (E_{rim}) will be 0.92 times the total energy of the flywheel (E). We know that maximum fluctuation of energy (ΔE),

$$2581 = E \times 2 C_S = E \times 2 \times 0.02 = 0.04 E$$

$$\therefore \quad E = 2581 / 0.04 = 64\,525 \text{ N-m}$$

and energy of the flywheel rim,

$$E_{rim} = 0.92 E = 0.92 \times 64\,525 = 59\,363 \text{ N-m}$$

Let m = Mass of the flywheel rim.

We know that energy of the flywheel rim (E_{rim}),

$$59\,363 = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times m\,(28.76)^2 = 413.6\, m$$

$$\therefore \quad m = 59\,363 / 413.6 = 143.5 \text{ kg}$$

We also know that mass of the flywheel rim (m),

$$143.5 = b \times t \times \pi D \times \rho = 2t \times t \times \pi \times 0.915 \times 7250 = 41\,686\, t^2$$

$$\therefore \quad t^2 = 143.5 / 41\,686 = 0.003\,44$$

or $t = 0.0587$ say 0.06 m = 60 mm **Ans.**

and $b = 2t = 2 \times 60 = 120$ mm **Ans.**

4.3 Connecting Rod and Crank Shaft

4.3.1 Introduction

The connecting rod is the intermediate member between the piston and the crankshaft. Its primaryfunction is to transmit the push and pull from the piston pin to the crankpin and thus convert thereciprocating motion of the piston into the rotary motion of the crank. The usual form of the connecting rod in internal combustion engines is shown in Fig. 4.35. It consists of a long shank, a small end and abig end. The cross-section of the shank may be rectangular, circular, tubular, *I*-section or *H*-section.Generally circular section is used for low speed engines while *I*-section is preferred for high speed engines.

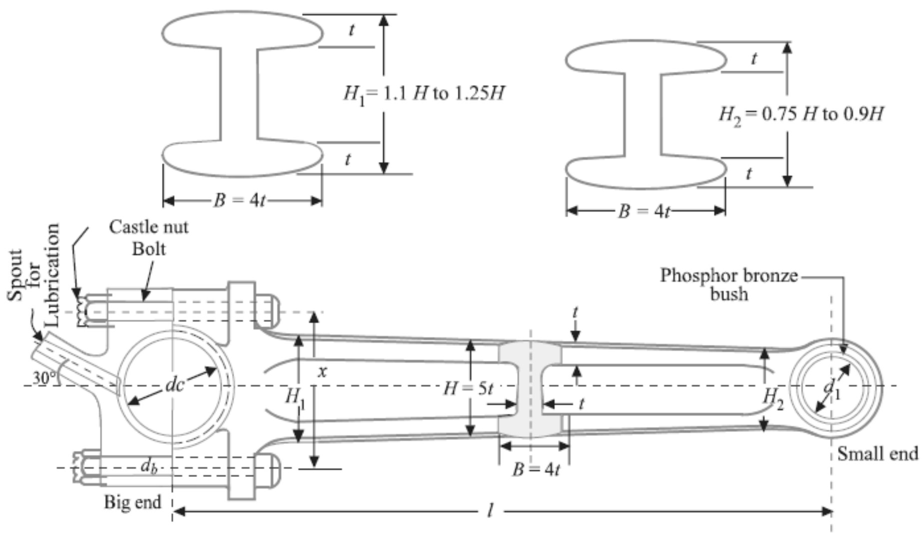


Fig. 4.35. Connecting rod.

The length of the connecting rod (*l*) depends upon the ratio of *l* / *r*, where *r* is the radius ofcrank. It may be noted that the smaller length will decrease the ratio *l* / *r*. This increases the angularityof the connecting rod which increases the side thrust of the piston against the cylinder liner which inturn increases the wear of the liner. The larger length of the connecting rod will increase the ratio*l* / *r*. This decreases the angularity of the connecting rod and thus decreases the side thrust and theresulting wear of the cylinder. But the larger length of the connecting rod increases the overall heightof the engine. Hence, a compromise is made and the ratio *l* / *r* is generally kept as 4 to 5.The small end of the connecting rod is usually made in the form of an eye and is provided witha bush of phosphor bronze. It is connected to the piston by means of a piston pin.The big end of the connecting rod is usually made split (in two halves) so that it can bemounted easily on the crankpin bearing shells. The split cap is fastened to the big end with two capbolts. The bearing shells of the big end are made of steel, brass or bronze with a thin lining (about0.75 mm) of white metal or babbit metal. The wear of the big end bearing is allowed for by insertingthin metallic strips (known as ***shims***) about 0.04 mm thick between the cap and the fixed half of theconnecting rod. As the wear takes place, one or more strips are removed and the bearing is trued up.

The connecting rods are usually manufactured by drop forging process and it should have adequatestrength, stiffness and minimum weight. The material mostly used for connecting rods varies frommild carbon steels (having 0.35 to 0.45 percent carbon) to alloy steels (chrome-nickel or chromemolybdenum steels). The carbon steel having 0.35 percent carbon has an ultimate tensile strength ofabout 650 MPa when properly heat treated and a carbon steel with 0.45 percent carbon has a ultimatetensile strength of 750 MPa. These steels are used for connecting rods of industrial engines. The alloysteels have an ultimate tensile strength of about 1050 MPa and are used for connecting rods ofaeroengines and automobile engines.The bearings at the two ends of the connecting rod are either splash lubricated or pressurelubricated. The big end bearing is usually splash lubricated while the small end bearing is pressurelubricated. In the **splash lubrication system**, the cap at the big end is provided with a dipper or spoutand set at an angle in such a way that when the connecting rod moves downward, the spout will dip into the lubricating oil contained in the sump. The oil is forced up the spout and then to the big end bearing. Now when the connecting rod moves upward, a splash of oil is produced by the spout. This splashed up lubricant find its way into the small end bearing through the widely chamfered holesprovided on the upper surface of the small end.

In the **pressure lubricating system,** the lubricating oil is fed under pressure to the big end bearingthrough the holes drilled in crankshaft, crankwebs and crank pin. From the big end bearing, the oil is fed tosmall end bearing through a fine hole drilled in the shank of the connecting rod. In some cases, the smallend bearing is lubricated by the oil scrapped from the walls of the cyinder liner by the oil scraper rings.

4.3.2 Forces Acting on the Connecting Rod

The various forces acting on the connecting rod are as follows :

1. Force on the piston due to gas pressure and inertia of the reciprocating parts,

2. Force due to inertia of the connecting rod or inertia bending forces,

3. Force due to friction of the piston rings and of the piston, and

4. Force due to friction of the piston pin bearing and the crankpin bearing.

We shall now derive the expressions for the forces acting on a vertical engine, as discussed below.

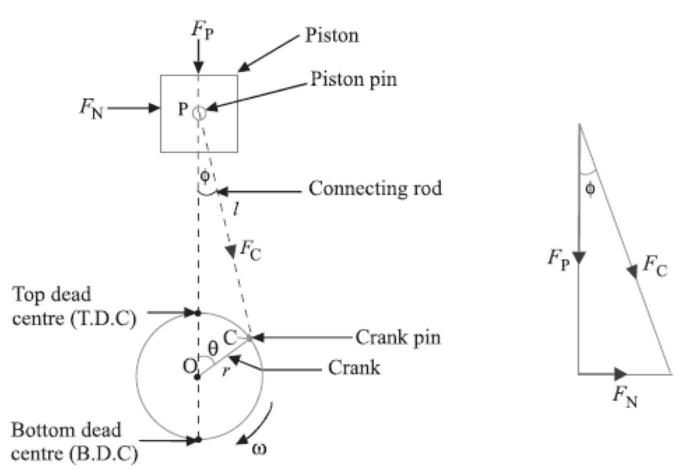
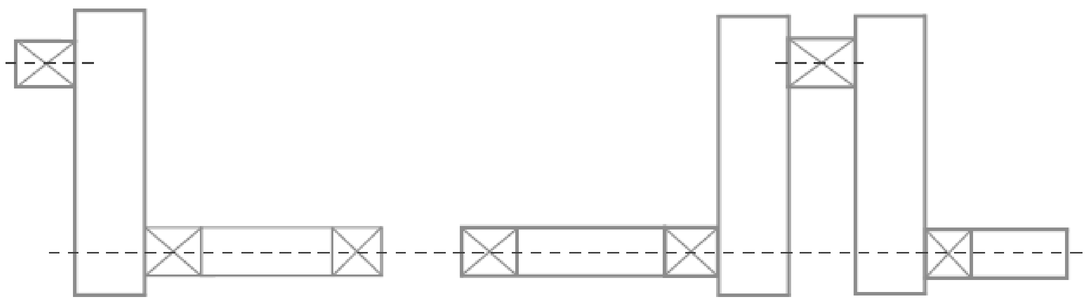


Fig. 4.36. Forces on the connecting rod.

4.3.3 Crankshaft

A crankshaft (*i.e.* a shaft with a crank) is used to convert reciprocating motion of the piston intorotatory motion or vice versa. The crankshaft consists of the shaft parts which revolve in the mainbearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms orwebs (also called cheeks) which connect the crankpins and the shaft parts. The crankshaft, dependingupon the position of crank, may be divided into the following two types :
1. Side crankshaft or overhung crankshaft, as shown in Fig. 4.37 (*a*), and
2. Centre crankshaft, as shown in Fig. 4.37 (*b*).



(*a*) Side Crankshaft.

(*a*) Centre Crankshaft.

Fig. 4.37. Types of crankshafts.

The crankshaft, depending upon the number of cranks in the shaft, may also be classfied assingle throw or multi-throw crankshafts. A crankhaft with only one side crank or centre crank iscalled a ***single throw crankshaft*** whereas the crankshaft with two side cranks, one on each end orwith two or more centre cranks is known as ***multi-throw crankshaft.***The side crankshafts are used for medium and large size horizontal engines.

4.3.4 Material and manufacture of Crankshafts

The crankshafts are subjected to shock and fatigue loads. Thus material of the crankshaft shouldbe tough and fatigue resistant. The crankshafts are generally made of carbon steel, special steel orspecial cast iron. In industrial engines, the crankshafts are commonly made from carbon steel such as 40 C 8,55 C 8 and 60 C 4. In transport engines, manganese steel such as 20 Mn 2, 27 Mn 2 and 37 Mn 2 aregenerally used for the making of crankshaft. In aero engines, nickel chromium steel such as35 Ni 1 Cr 60 and 40 Ni 2 Cr 1 Mo 28 are extensively used for the crankshaft.The crankshafts are made by drop forging or casting process but the former method is morecommon. The surface of the crankpin is hardened by case carburizing, nitriding or inductionhardening.

4.3.5 Bearing Pressures and Stresses in Crankshaft

The bearing pressures are very important in the design of crankshafts. The *maximum permissible bearing pressure depends upon the maximum gas pressure, journal velocity, amount and method of lubrication and change of direction of bearing pressure.
The following two types of stresses are induced in the crankshaft.

1. Bending stress ; and **2.** Shear stress due to torsional moment on the shaft.
2. Most crankshaft failures are caused by a progressive fracture due to repeated bending or reversed
3. torsional stresses. Thus the crankshaft is under fatigue loading and, therefore, its design should be
4. based upon endurance limit. Since the failure of a crankshaft is likely to cause a serious engine

5. destruction and neither all the forces nor all the stresses acting on the crankshaft can be determined
6. accurately, therefore a high factor of safety from 3 to 4, based on the endurance limit, is used.

EXERCISES

1. A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 600 N to 1600 N. The spring index is 6 and the design factor of safety is 1.43. If the yield shearstress is 700 MPa and the endurance stress is 350 MPa, find the size of the spring wire and meandiameter of the spring coil. **[Ans. 10 mm ; 60 mm]**

2. A helical spring *B* is placed inside the coils of a second helical spring *A*, having the same number of coils and free length. The springs are made of the same material. The composite spring is compressed by an axial load of 2300 N which is shared between them. The mean diameters of the spring *A* and *B* are 100 mm and 70 mm respectively and wire diameters are 13 mm and 8 mm respectively. Find the load taken and the maximum stress in each spring.

[Ans. *W*A = 1670 N ;*W*B = 630 N ; σA = 230 MPa ; σB = 256 MPa]

3. Design a concentric spring for an air craft engine valve to exert a maximum force of 5000 N under a deflection of 40 mm. Both the springs have same free length, solid length and are subjected to equal maximum shear stress of 850 MPa. The spring index for both the springs is 6.
[Ans. d_1 = 8 mm ;d_2 = 6 mm ; n = 4]

4. The free end of a torsional spring deflects through 90° when subjected to a torque of 4 N-m. The spring indexis 6. Determine the coil wire diameter and number of turns with the following data :
Modulus of rigidity = 80 GPa ; Modulus of elasticity = 200 GPa; Allowable stress = 500 MPa.
[Ans. 5 mm ; 26]

5. A flat spiral steel spring is to give a maximum torque of 1500 N-mm for a maximum stress of 1000 MPa.Find the thickness and length of the spring to give three complete turns of motion, when the stress decreases from 1000 to zero. The width of the spring strip is 12 mm. The Young's modulus for the material of the strip is 200 kN/mm^2. **[Ans. 1.225 mm ; 4.6 m]**

6. A semi-elliptical spring has ten leaves in all, with the two full length leaves extending 625 mm. It is 62.5 mm wide and 6.25 mm thick. Design a helical spring with mean diameter of coil 100 mm which will have approximately the same induced stress and deflection for any load. The Young's modulus for the material of the semi-elliptical spring may be taken as 200 kN/mm^2 and modulus of rigidity for the material of helical spring is 80 kN/mm^2.

7. A carriage spring 800 mm long is required to carry a proof load of 5000 N at the centre. The spring is made of plates 80 mm wide and 7.5 mm thick. If the maximum permissible stress for the material of the plates is not to exceed 190 MPa, determine :
1. The number of plates required, 2. The deflection of the spring, and 3. The radius to which the plates must be initially bent.
The modulus of elasticity may be taken as 205 kN/mm^2. **[Ans. 6 ; 23 mm ; 3.5 m]**

8. A semi-elliptical laminated spring 900 mm long and 55 mm wide is held together at the centre by a band 50 mm wide. If the thickness of each leaf is 5 mm, find the number of leaves required to carry a load of 4500 N. Assume a maximum working stress of 490 MPa.If the two of these leaves extend the full length of the spring, find the deflection of the spring. The

Young's modulus for the spring material may be taken as 210 kN/mm^2.
[Ans. 9 ; 71.8 mm]

9. A semi-elliptical laminated spring is made of 50 mm wide and 3 mm thick plates. The length between the supports is 650 mm and the width of the band is 60 mm. The spring has two full length leaves and five graduated leaves. If the spring carries a central load of 1600 N, find :
1. Maximum stress in full length and graduated leaves for an initial condition of no stress in the leaves.
2. The maximum stress if the initial stress is provided to cause equal stress when loaded.
3. The deflection in parts (1) and (2). **[Ans. 590 MPa ; 390 MPa ; 450 MPa ; 54 mm]**

10. A machine has to carry out punching operation at the rate of 10 holes/min. It does 6 N-m of work per sq mm of the sheared area in cutting 25 mm diameter holes in 20 mm thick plates. A flywheel is fitted to the machine shaft which is driven by a constant torque. The fluctuation of speed is between 180 and 200 r.p.m. Actual punching takes 1.5 seconds. Frictional losses are equivalent to 1/6 of the workdone during punching. Find:
(*a*) Power required to drive the punching machine, and
(*b*) Mass of the flywheel, if radius of gyration of the wheel is 450 mm.

11. A single cylinder internal combustion engine working on the four stroke cycle develops 75 kW at 360 r.p.m. The fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle. If thefluctuation of speed is not to exceed 1 per cent and the maximum centrifugal stress in the flywheel isto be 5.5 MPa, estimate the mean diameter and the cross-sectional area of the rim. The material of therim has a density of 7200 kg / m^3. **[Ans. 1.464 m ; 0.09 m^2]**

12. Design a cast iron flywheel for a four stroke cycle engine to develop 110 kW at 150 r.p.m. The work done in the power stroke is 1.3 times the average work done during the whole cycle. Take the meandiameter of the flywheel as 3 metres. The total fluctuation of speed is limited to 5 per cent of the meanspeed. The material density is 7250 kg / m^3. The permissible shear stress for the shaft material is 40 MPa and flexural stress for the arms of the flywheel is 20 MPa.

13. A punching press is required to punch 40 mm diameter holes in a plate of 15 mm thickness at the rate of 30 holes per minute. It requires 6 N-m of energy per mm^2 of sheared area. Determine the momentof inertia of the flywheel if the punching takes one-tenth of a second and the r.p.m. of the flywheelvaries from 160 to 140.

14. A punch press is fitted with a flywheel capable of furnishing 3000 N-m of energy during quarter of a revolution near the bottom dead centre while blanking a hole on sheet metal. The maximum speed ofthe flywheel during the operation is 200 r.p.m. and the speed decreases by 10% during the cuttingstroke. The mean radius of the rim is 900 mm. Calculate the approximate mass of the flywheel rimassuming that it contributes 90% of the energy requirements.

UNIT 5

BEARINGS

5.1 SLIDING CONTACT BEARINGS

5.1.1 Introduction

A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may be used.

5.1.2 Classification of Bearings

1. ***Depending upon the direction of load to be supported*.** The bearings under this group are classified as:
(*a*) Radial bearings, and **(*b*)** Thrust bearings.

In ***radial bearings***, the load acts perpendicular to the direction of motion of the moving element as shown in Fig. 5.1 (*a*) and (*b*).

In ***thrust bearings,*** the load acts along the axis of rotation as shown in Fig. 5.1 (*c*).

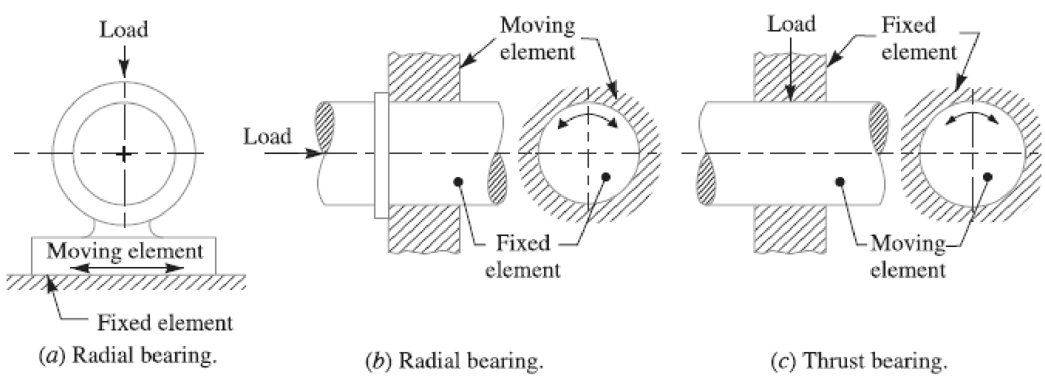


Fig.5.1Radial and thrust bearings.

2. ***Depending upon the nature of contact.*** The bearings under this group are classified as :
(*a*) Sliding contact bearings, and **(*b*)** Rolling contact bearings.

In ***sliding contact bearings,*** as shown in Fig. 26.2 (*a*), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as ***plain bearings.***

In ***rolling contact bearings,*** as shown in Fig. 26.2 (*b*), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.

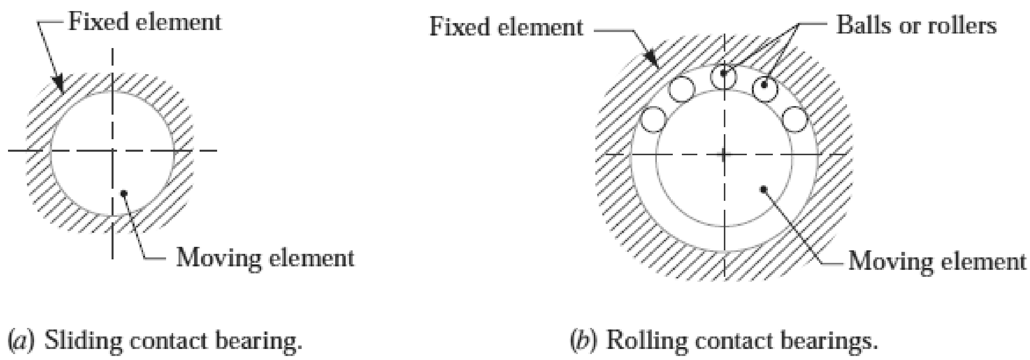


Fig 5.2 Sliding and rolling contact bearings.

5.1.3 Types of Sliding Contact Bearings

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig. 5.1 (*a*), may be called ***slipper*** or ***guide bearings.*** Such type of bearings are usually found in cross-head of steam engines.

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as ***journal*** or ***sleeve bearings.*** When the angle of contact of the bearing with the journal is 360° as shown in Fig. 4.3 (*a*), then the bearing is called a ***full journal bearing.*** This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.

When the angle of contact of the bearing with the journal is 120°, as shown in Fig. 5.3 (*b*), then the bearing is said to be ***partial journal bearing*****.** This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as ***clearance bearings*** because the diameter of the journal is less than that of bearing.

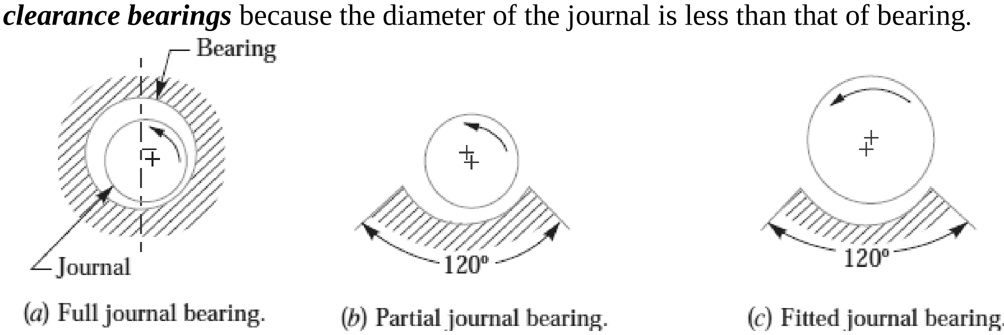


Fig. 5.3. Journal or sleeve bearings.

When a partial journal bearing has no clearance *i.e*. the diameters of the journal and bearing are equal, then the bearing is called a ***fitted bearing,*** as shown in Fig. 5.3 (*c*).

The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows :

1. *Thick film bearings*. The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as ***hydrodynamic lubricated bearings.***

2. *Thin film bearings.* The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other atleast part of the time. Such type of bearings are also called ***boundary lubricated bearings.***

3. *Zero film bearings.* The zero film bearings are those which operate without any lubricant present.

4. *Hydrostatic or externally pressurized lubricated bearings.* The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

5.1.4 Hydrodynamic Lubricated Bearings

We have already discussed that in hydrodynamic lubricated bearings, there is a thick film of lubricant between the journal and the bearing. A little consideration will show that when the bearing is supplied with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is eccentric with the bearing axis. The load can be supported by this fluid pressure without any actual contact between the journal and bearing. The load carrying ability of a hydrodynamic bearing arises simply because a viscous fluid resists being
pushed around. Under the proper conditions, this resistance to motion will develop a pressure distribution in the lubricant film that can support a useful load. The load supporting pressure in hydrodynamic bearings arises from either

1. the flow of a viscous fluid in a converging channel (known as ***wedge film lubrication***), or
2. the resistance of a viscous fluid to being squeezed out from between approaching surfaces (known as ***squeeze film lubrication***).

5.1.5 Assumptions in Hydrodynamic Lubricated Bearings

The following are the basic assumptions used in the theory of hydrodynamic lubricated bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.
4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional, *i.e.* the side leakage is neglected.

5.1.6 Important Factors for the Formation of Thick Oil Film in Hydrodynamic Lubricated Bearings

According to Reynolds, the following factors are essential for the formation of a thick film of oil in hydrodynamic lubricated bearings :

1. A continuous supply of oil.
2. A relative motion between the two surfaces in a direction approximately tangential to the surfaces.
3. The ability of one of the surfaces to take up a small inclination to the other surface in the direction of the relative motion.
4. The line of action of resultant oil pressure must coincide with the line of action of the external load between the surfaces.

5.1.7 Wedge Film Journal Bearings

The load carrying ability of a wedge-film journal bearing results when the journal and/or the bearing rotates relative to the load. The most common case is that of a steady load, a fixed (nonrotating) bearing and a rotating journal. Fig. 5.4 (*a*) shows a journal at rest with metal to metal contact at *A* on the line of action of the supported load. When the journal rotates slowly in the anticlockwise direction, as shown in Fig. 5.4 (*b*), the point of contact will move to *B*, so that the angle *AOB* is the angle of sliding friction of the surfaces in contact at *B*. In the absence of a lubricant, there will be dry metal to metal friction. If a lubricant is present in the clearance space of the bearing and journal, then a thin absorbed film of the lubricant may partly separate the surface, but a continuous fluid film completely separating the surfaces will not exist because of slow speed.

When the speed of the journal is increased, a continuous fluid film is established as in Fig. 5.4(*c*). The centre of the journal has moved so that the minimum film thickness is at *C*. It may be noted that from *D* to *C* in the direction of motion, the film is continually narrowing and hence is a converging film. The curved converging film may be considered as a wedge shaped film of a slipper bearing wrapped around the journal. A little consideration will show that from *C* to *D* in the direction of rotation, as shown in Fig. 5.4 (*c*), the film is diverging and cannot give rise to a positive pressure or a supporting action.

Fig. 5.5 shows the two views of the bearing shown in Fig. 26.4 (*c*), with the variation of pressure in the converging film. Actually, because of side leakage, the angle of contact on which pressure acts is less than 180°.

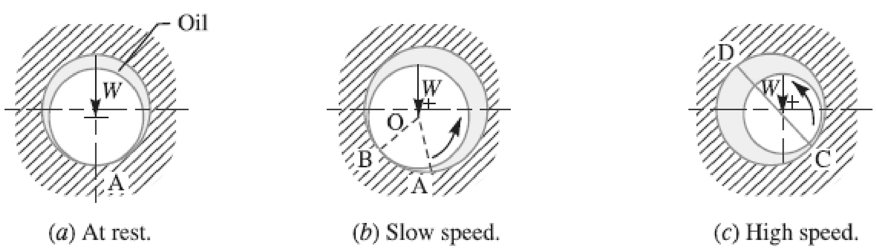


Fig 5.4. Wedge film journal bearing.

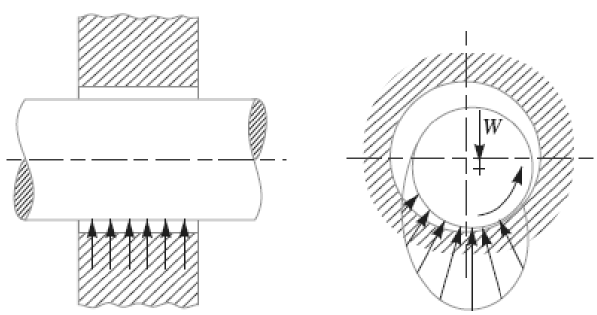


Fig 5.5. Variation of pressure in the converging film.

5.1.8 Materials used for Sliding Contact Bearings

The materials commonly used for sliding contact bearings are discussed below :

1. *Babbit metal.* The tin base and lead base babbits are widely used as a bearing material, because they satisfy most requirements for general applications. The babbits are recommended where the maximum bearing pressure (on projected area) is not over 7 to 14 N/mm^2.
2. *Bronzes.* The bronzes (alloys of copper, tin and zinc) are generally used in the form of machined bushes pressed into the shell. The bush may be in one or two pieces. The bronzes commonly used for bearing material are gun metal and phosphor bronzes.
3. *Cast iron.* The cast iron bearings are usually used with steel journals. Such type of bearings are fairly successful where lubrication is adequate and the pressure is limited to 3.5 N/mm^2 and speed to 40 metres per minute.
4. *Silver.* The silver and silver lead bearings are mostly used in aircraft engines where the fatigue strength is the most important consideration.
5. *Non-metallic bearings.* The various non-metallic bearings are made of carbon-graphite, rubber, wood and plastics.

5.1.9 *Viscosity*.

The viscocity of the lubricant is measured by Saybolt universal viscometer. It determines the time required for a standard volume of oil at a certain temperature to flow under a certain head through a tube of standard diameter and length. The time so determined in seconds is the Saybolt universal viscosity. In order to convert Saybolt universal viscosity in seconds to absolute viscosity (in kg / m-s), the following formula may be used:

$$Z = \text{Sp. gr. of oil} \left(0.000\,22\,S - \frac{0.18}{S}\right) \text{kg/m-s} \quad ...(i)$$

where Z = Absolute viscosity at temperature t in kg / m-s, and

S = Saybolt universal viscosity in seconds.

The variation of absolute viscosity with temperature for commonly used lubricating oils is shown in Table 5.1

Table 5.1. Absolute viscosity of commonly used lubricating oils.

S. No.	*Type of oil*	*Absolute viscosity in kg / m-s at temperature in °C*											
		30	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*	*90*
1.	SAE 10	0.05	0.036	0.027	0.0245	0.021	0.017	0.014	0.012	0.011	0.009	0.008	0.005
2.	SAE 20	0.069	0.055	0.042	0.034	0.027	0.023	0.020	0.017	0.014	0.011	0.010	0.0075
3.	SAE 30	0.13	0.10	0.078	0.057	0.048	0.040	0.034	0.027	0.022	0.019	0.016	0.010
4.	SAE 40	0.21	0.17	0.12	0.096	0.78	0.06	0.046	0.04	0.034	0.027	0.022	0.013
5.	SAE 50	0.30	0.25	0.20	0.17	0.12	0.09	0.076	0.06	0.05	0.038	0.034	0.020
6.	SAE 60	0.45	0.32	0.27	0.20	0.16	0.12	0.09	0.072	0.057	0.046	0.040	0.025
7.	SAE 70	1.0	0.69	0.45	0.31	0.21	0.165	0.12	0.087	0.067	0.052	0.043	0.033

5.1.10 Bearing Characteristic Number and Bearing Modulus for Journal Bearings

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables,

i.e.

(*i*) $\frac{ZN}{p}$; (*ii*) $\frac{d}{c}$; and (*iii*) $\frac{l}{d}$

Therefore the coefficient of friction may be expressed as

$$\mu = \phi\left(\frac{ZN}{p}, \frac{d}{c}, \frac{l}{d}\right)$$

where μ = Coefficient of friction,

ϕ = A functional relationship,

Z = Absolute viscosity of the lubricant, in kg / m-s,

N = Speed of the journal in r.p.m.,

p = Bearing pressure on the projected bearing area in N/mm^2,

= Load on the journal ÷ $l \times d$

d = Diameter of the journal,

l = Length of the bearing, and

c = Diametral clearance.

The factor *ZN* / *p* is termed as ***bearing characteristic number*** and is a dimensionless number.

5.1.11 Coefficient of Friction for Journal Bearings

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used.

Coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k \quad \text{... (when Z is in kg / m-s and } p \text{ is in N / mm}^2\text{)}$$

where *Z, N, p, d* and *c* have usual meanings as discussed in previous article, and

= Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (*i.e*. *l* / *d*).

=0.002 for *l* / *d* ratios of 0.75 to 2.8.

The operating values of *ZN* / *p* should be compared with values given in Table 5.2 to ensure safe margin between operating conditions and the point of film breakdown

Table 5.2. Design values for journal bearings.

Machinery	Bearing	Maximum bearing pressure (p) in N/mm^2	Operating values			
			Absolute Viscosity (Z) in kg/m-s	ZN/p Z in kg/m-s p in N/mm^2	$\frac{c}{d}$	$\frac{l}{d}$
Automobile and air-craft engines	Main Crank pin Wrist pin	5.6 – 12 10.5 – 24.5 16 – 35	0.007 0.008 0.008	2.1 1.4 1.12	—	0.8 – 1.8 0.7 – 1.4 1.5 – 2.2
Four stroke-Gas and oil engines	Main Crank pin Wrist pin	5 – 8.5 9.8 – 12.6 12.6 – 15.4	0.02 0.04 0.065	2.8 1.4 0.7	0.001	0.6 – 2 0.6 – 1.5 1.5 – 2
Two stroke-Gas and oil engines	Main Crank pin Wrist pin	3.5 – 5.6 7 – 10.5 8.4 – 12.6	0.02 0.04 0.065	3.5 1.8 1.4	0.001	0.6 – 2 0.6 – 1.5 1.5 – 2
Marine steam engines	Main Crank pin Wrist pin	3.5 4.2 10.5	0.03 0.04 0.05	2.8 2.1 1.4	0.001	0.7 – 1.5 0.7 – 1.2 1.2 – 1.7
Stationary, slow speed steam engines	Main Crank pin Wrist pin	2.8 10.5 12.6	0.06 0.08 0.06	2.8 0.84 0.7	0.001	1 – 2 0.9 – 1.3 1.2 – 1.5
Stationary, high speed steam engine	Main Crank pin Wrist pin	1.75 4.2 12.6	0.015 0.030 0.025	3.5 0.84 0.7	0.001	1.5 – 3 0.9 – 1.5 13 – 1.7
Reciprocating pumps and compressors	Main Crank pin Wrist pin	1.75 4.2 7.0	0.03 0.05 0.08	4.2 2.8 1.4	0.001	1 – 2.2 0.9 – 1.7 1.5 – 2.0
Steam locomotives	Driving axle Crank pin Wrist pin	3.85 14 28	0.10 0.04 0.03	4.2 0.7 0.7	0.001	1.6 – 1.8 0.7 – 1.1 0.8 – 1.3

Machinery	*Bearing*	*Maximum bearing pressure (p) in N/mm²*	*Operating values*			
			Absolute Viscosity (Z) in kg/m-s	*ZN/p Z in kg/m-s p in N/mm²*	$\frac{c}{d}$	$\frac{l}{d}$
Railway cars	Axle	3.5	0.1	7	0.001	1.8 – 2
Steam turbines	Main	0.7 – 2	0.002 – 0.016	14	0.001	1 – 2
Generators, motors, centrifugal pumps	Rotor	0.7 – 1.4	0.025	28	0.0013	1 – 2
Transmission shafts	Light, fixed Self -aligning Heavy	0.175 1.05 1.05	0.025- 0.060	7 2.1 2.1	0.001	2 – 3 2.5 – 4 2– 3
Machine tools	Main	2.1	0.04	0.14	0.001	1–4
Punching and shearing machines	Main Crank pin	28 56	0.10	—	0.001	1–2
Rolling Mills	Main	21	0.05	1.4	0.0015	1–1.5

5.1.12 Critical Pressure of the Journal Bearing

The pressure at which the oil film breaks down so that metal to metal contact begins, is known as **critical pressure** or the **minimum operating pressure** of the bearing. It may be obtained by the following empirical relation, *i.e.*

Critical pressure or minimum operating pressure,

$$p = \frac{ZN}{4.75 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{d+l}\right) \text{ N/mm}^2 \qquad \text{...(when } Z \text{ is in kg / m-s)}$$

5.1.13 Sommerfeld Number

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically,

$$\text{Sommerfeld number} = \frac{ZN}{p}\left(\frac{d}{c}\right)^2$$

For design purposes, its value is taken as follows :

$$\frac{ZN}{p}\left(\frac{d}{c}\right)^2 = 14.3 \times 10^6 \qquad \text{... (when } Z \text{ is in kg / m-s and } p \text{ is in N / mm}^2\text{)}$$

5.1.14 Heat Generated in a Journal Bearing

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$$Q_g = \mu.W.V \text{ N-m/s or J/s or watts} \qquad \text{...}(\textit{i})$$

where μ = Coefficient of friction,

W = Load on the bearing in N,

= Pressure on the bearing in N/mm^2 × Projected area of the bearing in mm^2 = p (l × d),

V = Rubbing velocity in m/s = $\frac{\pi d.N}{60}$, d is in metres, and

N = Speed of the journal in r.p.m.

After the thermal equilibrium has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing,

$$Q_d = C.A\,(t_b - t_a) \text{ J/s or W} \quad \ldots (\because 1 \text{ J/s} = 1 \text{ W}) \ldots (ii)$$

where C = Heat dissipation coefficient in W/m^2/°C,

= Projected area of the bearing in m^2 = $l \times d$,

t_b = Temperature of the bearing surface in °C, and

t_a = Temperature of the surrounding air in °C.

The value of C have been determined experimentally by O. Lasche. The values depend upon the type of bearing, its ventilation and the temperature difference. The average values of C (in W/m^2/°C), for journal bearings may be taken as follows :

For unventilated bearings (Still air)

=140 to 420 W/m^2/°C

For well ventilated bearings

=490 to 1400 W/m^2/°C

It has been shown by experiments that the temperature of the bearing (t_b) is approximately mid-way between the temperature of the oil film (t_0) and the temperature of the outside air (t_a). In other words,

$$t_b - t_a = \frac{1}{2}(t_0 - t_a)$$

5.1.15 Design Procedure for Journal Bearing

The following procedure may be adopted in designing journal bearings, when the bearing load,the diameter and the speed of the shaft are known.

1. Determine the bearing length by choosing a ratio of l / d from Table 5.2.
2. Check the bearing pressure, $p = W / l.d$ from Table 5.2 for probable satisfactory value.
3. Assume a lubricant from Table 5.1 and its operating temperature (t_o). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations such as steam turbines.
4. Determine the operating value of ZN / p for the assumed bearing temperature and check this value with corresponding values in Table 5.2, to determine the possibility of maintaining fluid film operation.
5. Assume a clearance ratio c / d from Table 5.2.
6. Determine the coefficient of friction (μ) by using the relation as discussed in Art. 5.11.

7. Determine the heat generated by using the relation as discussed in Art. 5.14.
8. Determine the heat dissipated by using the relation as discussed in Art. 5.14.
9. Determine the thermal equilibrium to see that the heat dissipated becomes at least equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

Example 5.1. *Design a journal bearing for a centrifugal pump from the following data : Load on the journal = 20 000 N; Speed of the journal = 900 r.p.m.; Type of oil is SAE 10, for which the absolute viscosity at 55°C = 0.017 kg / m-s; Ambient temperature of oil = 15.5°C ; Maximum bearing pressure for the pump = 1.5 N / mm². Calculate also mass of the lubricating oil required for artificial cooling, if rise of temperature of oil be limited to 10°C. Heat dissipation coefficient = 1232 W/m²/°C.*

Given Data :

W = 20 000 N
N = 900 r.p.m.
t_0 = 55°C
Z = 0.017 kg/m-s
t_a = 15.5°C ;
p = 1.5 N/mm²
t = 10°C
C = 1232 W/m²/°C

To Find
Design a journal bearing

Solution.

The journal bearing is designed as discussed in the following steps :

1, First of all, let us find the length of the journal (l). Assume the diameter of the journal (d) as 100 mm. From Table 26.3, we find that the ratio of l / d for centrifugal pumps varies from 1 to 2. Let us take l / d = 1.6.

$$l = 1.6\, d = 1.6 \times 100 = 160 \text{ mm } \textbf{Ans.}$$

2,We know that bearing pressure,

$$p = \frac{w}{l.d} = \frac{20000}{160X100} = 1.25$$

Since the given bearing pressure for the pump is 1.5 N/mm², therefore the above value of p is safe and hence the dimensions of l and d are safe.

3,

$$\frac{Z.N}{p} = \frac{0.017X900}{1.25} = 12.24$$

From Table 5.2, we find that the operating value of

$$\frac{Z.N}{p} = 28$$

We have discussed in Art. 26.14, that the minimum value of the bearing modulus at which the oil film will break is given by

$$3\,K = \frac{ZN}{p}$$

∴ Bearing modulus at the minimum point of friction,

$$K = \frac{1}{3}\left(\frac{Z.N}{p}\right) = \frac{1}{3} \times 28 = 9.33$$

Since the calculated value of bearing characteristic number $\left(\frac{Z.N}{p} = 12.24\right)$ is more than 9.33, therefore the bearing will operate under hydrodynamic conditions.

4. From Table 5.2, we find that for centrifugal pumps, the clearance ratio (*c/d*) = 0.0013
5. We know that coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8} \times 12.24 \times \frac{1}{0.0013} + 0.002$$

$$= 0.0031 + 0.002 = 0.0051$$

... [From Art. 26.13, k = 0.002]

6. Heat generated,

$$Q_g = \mu\, W\, V = \mu\, W\left(\frac{\pi\, d.N}{60}\right) \text{W} \qquad \ldots\left(\because V = \frac{\pi\, d.N}{60}\right)$$

$$= 0.0051 \times 20\,000\left(\frac{\pi \times 0.1 \times 900}{60}\right) = 480.7 \text{ W}$$

... (*d* is taken in metres)

7. Heat dissipated,

$$Q_d = C.A\,(t_b - t_a) = C.l.d\,(t_b - t_a) \text{ W} \qquad \ldots(\because A = l \times d)$$

We know that

$$(t_b - t_a) = \tfrac{1}{2}\,(t_0 - t_a) = \tfrac{1}{2}\,(55° - 15.5°) = 19.75°\text{C}$$

$$\therefore \quad Q_d = 1232 \times 0.16 \times 0.1 \times 19.75 = 389.3 \text{ W}$$

... (*l* and *d* are taken in metres)

We see that the heat generated is greater than the heat dissipated which indicates that the bear-ing is warming up. Therefore, either the bearing should be redesigned by taking t_0 = 63°C or the bearing should be cooled artificially.

We know that the amount of artificial cooling required

= Heat generated – Heat dissipated = $Q_g – Q_d$
= 480.7 – 389.3 = 91.4 W

Mass of lubricating oil required for artificial cooling

Let m = Mass of the lubricating oil required for artificial cooling in kg / s.

We know that the heat taken away by the oil,

$Q_t = m.S.t = m \times 1900 \times 10 = 19\,000\, m$ W

... [Specific heat of oil (S) = 1840 to 2100 J/kg/°C]

Equating this to the amount of artificial cooling required, we have

$19\,000\, m = 91.4$

$m = 91.4 / 19\,000 = 0.0048$ kg / s = 0.288 kg / min **Ans.**

Example 5.2. *A full journal bearing of 50 mm diameter and 100 mm long has a bearing pressure of 1.4 N/mm2. The speed of the journal is 900 r.p.m. and the ratio of journal diameter to the diametral clearance is 1000. The bearing is lubricated with oil whose absolute viscosity at the operating temperature of 75°C may be taken as 0.011 kg/m-s. The room temperature is 35°C. Find :*

1. The amount of artificial cooling required, and 2. The mass of the lubricating oil required, if the difference between the outlet and inlet temperature of the oil is 10°C. Take specific heat of the oil as 1850 J / kg / °C.

Given Data :

d = 50 mm = 0.05 m
l = 100 mm = 0.1 m
p = 1.4 N/mm^2
N = 900 r.p.m.
d / c = 1000
Z = 0.011 kg / m-s

t_0 = 75°C
t_a = 35°C
t = 10°C
S = 1850 J/kg / °C

Solution.

1. *Amount of artificial cooling required*

We know that the coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8}\left(\frac{0.011 \times 900}{1.4}\right)(1000) + 0.002$$

$$= 0.002\,33 + 0.002 = 0.004\,33$$

Load on the bearing,

$$W = p \times d.l = 1.4 \times 50 \times 100 = 7000 \text{ N}$$

and rubbing velocity,

$$V = \frac{\pi d.N}{60} = \frac{\pi \times 0.05 \times 900}{60} = 2.36 \text{ m/s}$$

∴ Heat generated,

$$Q_g = \mu.W.V = 0.004\ 33 \times 7000 \times 2.36 = 71.5 \text{ J/s}$$

Let t_b = Temperature of the bearing surface.

We know that

$$(t_b - t_a) = \frac{1}{2}(t_0 - t_a) = \frac{1}{2}(75 - 35) = 20°\text{C}$$

Since the value of heat dissipation coefficient (C) for unventilated bearing varies from 140 to 420 W/m^2/°C, therefore let us take

$$C = 280 \text{ W/m}^2/°\text{ C}$$

We know that heat dissipated,

$$Q_d = C.A\,(t_b - t_a) = C.l.d\,(t_b - t_a)$$
$$= 280 \times 0.05 \times 0.1 \times 20 = 28 \text{ W} = 28 \text{ J/s}$$

∴ Amount of artificial cooling required

$$= \text{Heat generated} - \text{Heat dissipated} = Q_g - Q_d$$
$$= 71.5 - 28 = 43.5 \text{ J/s or W}$$ **Ans.**

2. ***Mass of the lubricating oil required***

Let m = Mass of the lubricating oil required in kg / s.

We know that heat taken away by the oil,

$$Q_t = m.S.t = m \times 1850 \times 10 = 18\ 500\ m \text{ J/s}$$

Since the heat generated at the bearing is taken away by the lubricating oil, therefore equating

$$Q_g = Q_t \text{ or } 71.5 = 18\ 500\ m$$

∴ $m = 71.5 / 18\ 500 = 0.003\ 86$ kg / s $= 0.23$ kg / min **Ans.**

5.2 ROLLING CONTACT BEARINGS

5.2.1 Introduction

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction. Due to this low friction offered by rolling contact bearings, these are called ***antifriction bearings.***

5.2.2 Advantages and Disadvantages of Rolling Contact Bearings Over Sliding Contact Bearings

The following are some advantages and disadvantages of rolling contact bearings over sliding contact bearings.

Advantages

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

Disadvantages

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.

5.2.3 Types of Rolling Contact Bearings

Following are the two types of rolling contact bearings:
1. Ball bearings; and **2.** Roller bearings.

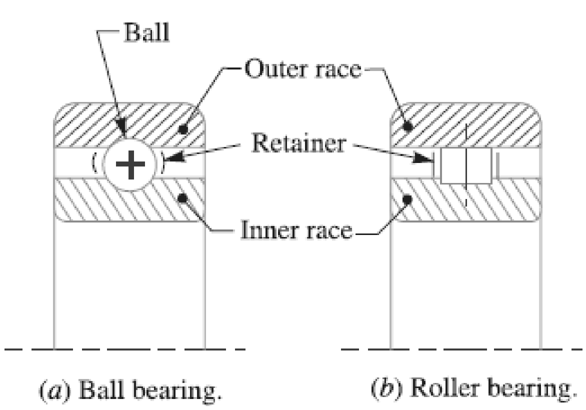


Fig.5.6. Ball and roller bearings.

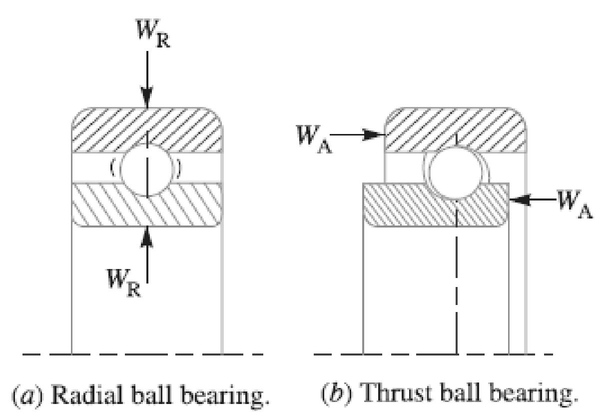


Fig.5.7. Radial and thrust ball bearings.

The ***ball and roller bearings*** consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig.5.6. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads.

The rolling contact bearings, depending upon the load to be carried, are classified as :
(*a*) Radial bearings, and **(*b*)** Thrust bearings.
The radial and thrust ball bearings are shown in Fig. 5.7 (*a*) and (*b*) respectively. When a ball bearing supports only a radial load (WR), the plane of rotation of the ball is normal to the centre line of the bearing, as shown in Fig. 5.7 (*a*). The action of thrust load (WA) is to shift the plane of rotation of the balls, as shown in Fig. 5.7 (*b*). The radial and thrust loads both may be carried simultaneously.

5.2.4 Types of Radial Ball Bearings

Following are the various types of radial ball bearings:
1. *Single row deep groove bearing.* A single row deep groove bearing is shown in Fig. 5.8(*a*).

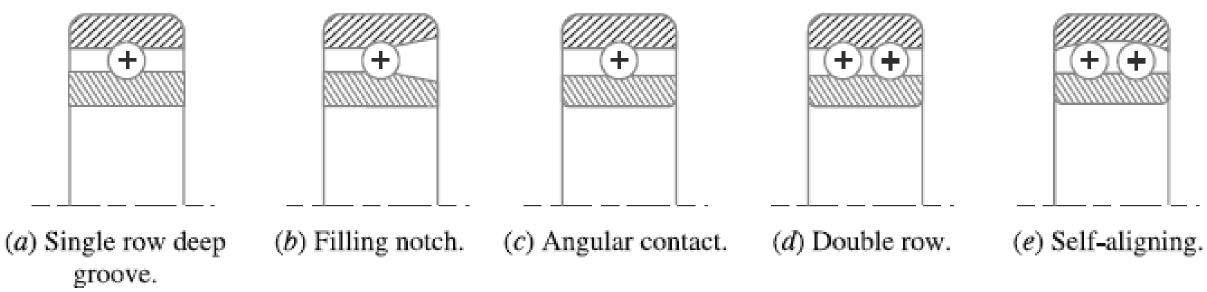


Fig 5.8. Types of radial ball bearing

During assembly of this bearing, the races are offset and the maximum number of balls are placed between the races. The races are then centred and the balls are symmetrically located by the use of a retainer or cage. The deep groove ball bearings are used due to their high load carrying capacity and suitability for high running speeds. The load carrying capacity of a ball bearing is related to the size and number of the balls.

2. *Filling notch bearing.* A filling notch bearing is shown in Fig. 5.8 (*b*). These bearings have notches in the inner and outer races which permit more balls to be inserted than in a deep groove ball bearings. The notches do not extend to the bottom of the race way and therefore the balls inserted through the notches must be forced in position. Since this type of bearing contains larger number of balls than a corresponding unnotched one, therefore it has a larger bearing load capacity.

3. *Angular contact bearing*. An angular contact bearing is shown in Fig. 5.8 (*c*). These bearings have one side of the outer race cut away to permit the insertion of more balls than in a deep groove bearing but without having a notch cut into both races. This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load. The angular contact bearings are usually used in pairs so that thrust loads may be carried in either direction.

4. ***Double row bearing.*** A double row bearing is shown in Fig. 5.8 (*d*). These bearings may be made with radial or angular contact between the balls and races. The double row bearing is appreciably narrower than two single row bearings. The load capacity of such bearings is slightly less than twice that of a single row bearing.

5. ***Self-aligning bearing.*** A self-aligning bearing is shown in Fig. 5.8 (*e*). These bearings permit shaft deflections within 2-3 degrees. It may be noted that normal clearance in a ball bearing are too small to accommodate any appreciable misalignment of the shaft relative to the housing. If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur.

Following are the two types of self-aligning bearings :

(a) Externally self-aligning bearing, and **(*b*)** Internally self-aligning bearing.

In an ***externally self-aligning bearing,*** the outside diameter of the outer race is ground to a spherical surface which fits in a mating spherical surface in a housing, as shown in Fig. 5.8 (*e*). In case of ***internally self-aligning bearing,*** the inner surface of the outer race is ground to a s surface. Consequently, the outer race may be displaced through a small angle without interfering with the normal operation of the bearing. The internally self-aligning ball bearing is interchangeable with other ball bearings.

5.2.5 Standard Dimensions and Designations of Ball Bearings

The dimensions that have been standardised on an international basis are shown in Fig. 5.9. These dimensions are a function of the bearing bore and the series of bearing. The standard dimensions are given in millimetres. There is no standard for the size and number of steel balls. The bearings are designated by a number. In general, the number consists of atleast three digits. Additional digits or letters are used to indicate special features *e.g*. deep groove, filling notch etc. The last three digits give the series and the bore of the bearing. The last two digits from 04 onwards, when multiplied by 5, give the bore diameter in millimetres. The third from the last digit designates the series of the bearing.
The most common ball bearings are available in four series as follows :

1. Extra light (100), **2.** Light (200),
3. Medium (300), **4.** Heavy (400)

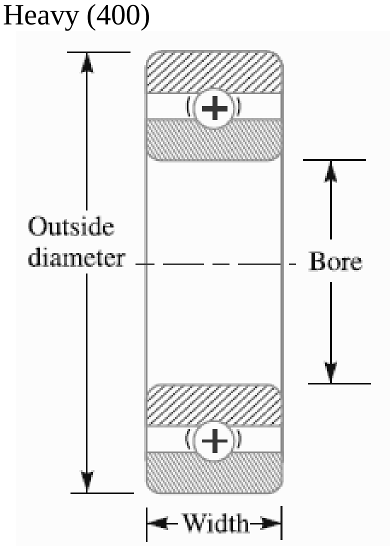
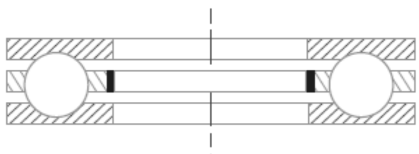


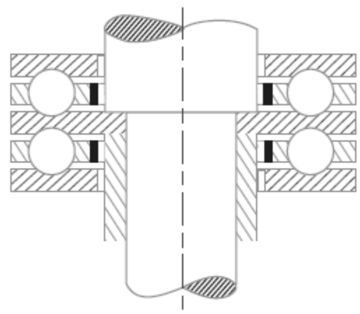
Fig. 5.9. Standard designations of ball bearings.

5.2.6 Thrust Ball Bearings

The thrust ball bearings are used for carrying thrust loads exclusively and at speeds below 2000 r.p.m. At high speeds, centrifugal force causes the balls to be forced out of the races. Therefore at high speeds, it is recommended that angular contact ball bearings should be used in place of thrust ball bearings. thrust ball bearing may be a single direction, flat face as shown in Fig.5.10 (*a*) or a double direction with flat face as shown in Fig. 5.10 (*b*).



(*a*) Single direction thrust ball bearing.



(*b*) Double direction thrust ball bearing.

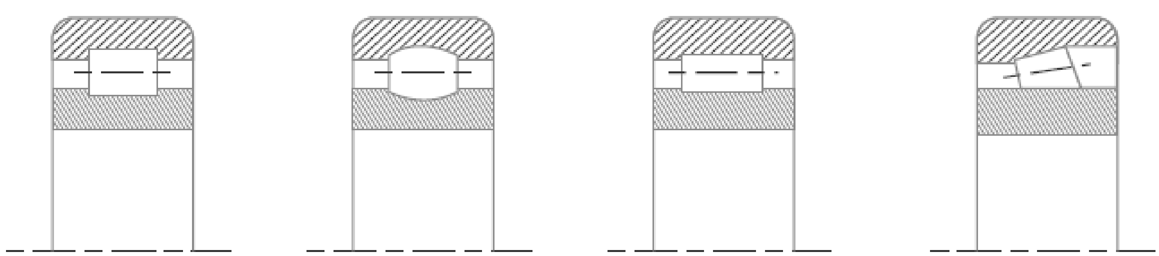
Fig. 5.10. Thrust ball bearing

5.2.7 Types of Roller Bearings

Following are the principal types of roller bearings :

1. *Cylindrical roller bearings.* A cylindrical roller bearing is shown in Fig. 5.11 (*a*). These bearings have short rollers guided in a cage. These bearings are relatively rigid against radial motion and have the lowest coefficient of friction of any form of heavy duty rolling-contact bearings. Such type of bearings are used in high speed service.

2. *Spherical roller bearings.* A spherical roller bearing is shown in Fig. 5.11 (*b*). These bearings are self-aligning bearings. The self-aligning feature is achieved by grinding one of the races in the form of sphere.



(*a*) Cylindrical roller. (*b*) Spherical roller. (*c*) Needle roller. (*d*) Tapered roller.

Fig. 5.11. Types of roller bearings.

3. *Needle roller bearings.* A needle roller bearing is shown in Fig. 5.11. (*c*). These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed. These bearings are used when heavy loads are to be carried with an oscillatory motion, *e.g.* piston pin bearings in heavy duty diesel engines, where the reversal of motion tends to keep the rollers in correct alignment.

4. *Tapered roller bearings.* A tapered roller bearing is shown in Fig. 5.11 (*d*). The rollers and race ways of these bearings are truncated cones whose elements intersect at a common point. Such type of bearings can carry both radial and thrust loads. These bearings are available in various combinations as double row bearings and with different cone angles for use with different relative magnitudes of radial and thrust loads.

5.2.8 Life of a Bearing

The ***life*** of an individual ball (or roller) bearing may be defined as the number of revolutions (or hours at some given constant speed) which the bearing runs before the first evidence of fatigue develops in the material of one of the rings or any of the rolling elements. The ***rating life*** of a group of apparently identical ball or roller bearings is defined as the number of revolutions (or hours at some given constant speed) that 90 per cent of a group of bearings will complete or exceed before the first evidence of fatigue develops (*i.e.* only 10 per cent of a group of bearings fail due to fatigue).

The term ***minimum life*** is also used to denote the rating life. It has been found that the life which 50 per cent of a group of bearings will complete or exceed is approximately 5 times the life which 90 per cent of the bearings will complete or exceed. In other words, we may say that the average life of a bearing is 5 times the rating life (or minimum life). It may be noted that the longest life of a single bearing is seldom longer than the 4 times the average life and the maximum life of a single bearing is about 30 to 50 times the minimum life.

The life of bearings for various types of machines is given in the following table.

Table 5.3. Life of bearings for various types of machines.

S. No.	Application of bearing	Life of bearing, in hours
1.	Instruments and apparatus that are rarely used	
	(*a*) Demonstration apparatus, mechanism for operating sliding doors	500
	(*b*) Aircraft engines	1000 – 2000
2.	Machines used for short periods or intermittently and whose breakdown would not have serious consequences *e.g.* hand tools, lifting tackle in workshops, and operated machines, agricultural machines, cranes in erecting shops, domestic machines.	4000 – 8000
3.	Machines working intermittently whose breakdown would have serious consequences *e.g.* auxillary machinery in power stations, conveyor plant for flow production, lifts, cranes for piece goods, machine tools used frequently.	8000 – 12 000
4.	Machines working 8 hours per day and not always fully utilised *e.g.* stationary electric motors, general purpose gear units.	12 000 – 20 000
5.	Machines working 8 hours per day and fully utilised *e.g.* machines for the engineering industry, cranes for bulk goods, ventilating fans, counter shafts.	20 000 – 30 000
6.	Machines working 24 hours per day *e.g.* separators, compressors, pumps, mine hoists, naval vessels.	40 000 – 60 000
7.	Machines required to work with high degree of reliability 24 hours per day *e.g.* pulp and paper making machinery, public power plants, mine-pumps, water works.	100 000 – 200 000

5.2.9 Dynamic Equivalent Load for Rolling Contact Bearings

The dynamic equivalent load may be defined as the constant stationary radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied to a bearing with rotating inner ring and stationary outer ring, would give the same life as that which the bearing will attain under the actual conditions of load and rotation. The dynamic equivalent radial load (W) for radial and angular contact bearings, except the filling slot types, under combined constant radial load (W_R) and constant axial or thrust load (W_A) is given by

$$W = X \cdot V \cdot W_R + Y \cdot W_A$$

where V = A rotation factor,

= 1, for all types of bearings when the inner race is rotating,

= 1, for self-aligning bearings when inner race is stationary,

= 1.2, for all types of bearings except self-aligning, when inner race is stationary.

The values of radial load factor (X) and axial or thrust load factor (Y) for the dynamically loaded bearings may be taken from the following table:

Table 5.4. Values of X and Y for dynamically loaded bearings.

Type of bearing	*Specifications*	$\frac{W_A}{W_R} \leq e$		$\frac{W_A}{W_R} > e$		e
		X	Y	X	Y	
Deep groove ball bearing	$\frac{W_A}{C_0} = 0.025$				2.0	0.22
	= 0.04				1.8	0.24
	= 0.07				1.6	0.27
	= 0.13	1	0	0.56	1.4	0.31
	= 0.25				1.2	0.37
	= 0.50				1.0	0.44
Angular contact ball bearings	Single row		0	0.35	0.57	1.14
	Two rows in tandem		0	0.35	0.57	1.14
	Two rows back to back	1	0.55	0.57	0.93	1.14
	Double row		0.73	0.62	1.17	0.86
Self-aligning bearings	Light series : for bores					
	10 – 20 mm		1.3		2.0	0.50
	25 – 35	1	1.7	6.5	2.6	0.37
	40 – 45		2.0		3.1	0.31
	50 – 65		2.3		3.5	0.28
	70 – 100		2.4		3.8	0.26
	105 – 110		2.3		3.5	0.28
	Medium series : for bores					
	12 mm		1.0	0.65	1.6	0.63
	15 – 20		1.2		1.9	0.52
	25 – 50		1.5		2.3	0.43
	55 – 90		1.6		2.5	0.39
Spherical roller bearings	For bores :					
	25 – 35 mm		2.1		3.1	0.32
	40 – 45	1	2.5	0.67	3.7	0.27
	50 – 100		2.9		4.4	0.23
	100 – 200		2.6		3.9	0.26
Taper roller bearings	For bores :					
	30 – 40 mm				1.60	0.37
	45 – 110	1	0	0.4	1.45	0.44
	120 – 150				1.35	0.41

5.2.10 Reliability of a Bearing.

The reliability (R) is defined as the ratio of the number of bearings which have successfully completed L million revolutions to the total number of bearings under test. Sometimes, it becomes necessary to select a bearing having a reliability of more than 90%. According to Wiebull, the relation between the bearing life and the reliability is given as

$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b \quad \text{or} \quad \frac{L}{a} = \left[\log_e\left(\frac{1}{R}\right)\right]^{1/b} \qquad ...(i)$$

where L is the life of the bearing corresponding to the desired reliability R and a and b are constants whose values are

$a = 6.84$, and $b = 1.17$

If L_{90} is the life of a bearing corresponding to a reliability of 90% (*i.e.* R_{90}), then

$$\frac{L_{90}}{a} = \left[\log_e\left(\frac{1}{R_{90}}\right)\right]^{1/b} \qquad ...(ii)$$

Dividing equation (*i*) by equation (*ii*), we have

$$\frac{L}{L_{90}} = \left[\frac{\log_e(1/R)}{\log_e(1/R_{90})}\right]^{1/b} = *6.85\,[\log_e(1/R)]^{1/1.17} \qquad ...(\because\ b = 1.17)$$

Example 5.3. *A shaft rotating at constant speed is subjected to variable load. The bearings supporting the shaft are subjected to stationary equivalent radial load of 3 kN for 10 per cent of time, 2 kN for 20 percent of time, 1 kN for 30 per cent of time and no load for remaining time of cycle. If the total life expected for the bearing is 20 × 106 revolutions at 95 per cent reliability, calculate dynamic load rating of the ball bearing.*

Given Data : $W_1 = 3$ kN

$n_1 = 0.1\,n$
$W_2 = 2$ kN
$n_2 = 0.2\,n$
$W_3 = 1$ kN
$n_3 = 0.3\,n$
$W_4 = 0$
$n_4 = (1 – 0.1 – 0.2 – 0.3)\,n = 0.4\,n$
$L_{95} = 20 \times 10^6$ rev

Solution

Let

L_{90} = Life of the bearing corresponding to reliability of 90 per cent,

L_{95} = Life of the bearing corresponding to reliability of 95 per cent

$= 20 \times 10^6$ revolutions …….(Given)

We know that

$$\frac{L_{95}}{L_{90}} = \left[\frac{\log_e (1/R_{95})}{\log_e (1/R_{90})}\right]^{1/b} = \left[\frac{\log_e (1/0.95)}{\log_e (1/0.90)}\right]^{1/1.17} \quad \text{... } (\because \; b = 1.17)$$

$$= \left(\frac{0.0513}{0.1054}\right)^{0.8547} = 0.54$$

$$\therefore \quad L_{90} = L_{95} / 0.54 = 20 \times 10^6 / 0.54 = 37 \times 10^6 \text{ rev}$$

We know that equivalent radial load,

$$W = \left[\frac{n_1 (W_1)^3 + n_2 (W_2)^3 + n_3 (W_3)^3 + n_4 (W_4)^3}{n_1 + n_2 + n_3 + n_4}\right]^{1/3}$$

$$= \left[\frac{0.1 n \times 3^3 + 0.2\, n \times 2^3 + 0.3 n \times 1^3 + 0.4\, n \times 0^3}{0.1\, n + 0.2 n + 0.3 n + 0.4 n}\right]^{1/3}$$

$$= (2.7 + 1.6 + 0.3 + 0)^{1/3} = 1.663 \text{ kN}$$

We also know that dynamic load rating,

$$C = W \left(\frac{L_{90}}{10^6}\right)^{1/k} = 1.663 \left(\frac{37 \times 10^6}{10^6}\right)^{1/3} = 5.54 \text{ kN Ans.}$$

$$\text{... } (\because \; k = 3, \text{ for ball bearing})$$

5.2.11 Selection of Radial Ball Bearings

In order to select a most suitable ball bearing, first of all, the basic dynamic radial load is calculated. It is then multiplied by the service factor (*K*S) to get the design basic dynamic radial load capacity. The service factor for the ball bearings is shown in the following table

.

Table 5.5. Values of service factor (*K*S).

S.No.	*Type of service*	*Service factor (K_S) for radial ball bearings*
1.	Uniform and steady load	1.0
2.	Light shock load	1.5
3.	Moderate shock load	2.0
4.	Heavy shock load	2.5
5.	Extreme shock load	3.0

Table 5.6. Basic static and dynamic capacities of various types of radial ball bearings.

Bearing No.	*Basic capacities in kN*							
	Single row deep groove ball bearing		*Single row angular contact ball bearing*		*Double row angular contact ball bearing*		*Self-aligning ball bearing*	
	Static (C_0)	*Dynamic (C)*	*Static (C_0)*	*Dynamic (C)*	*Static (C_0)*	*Dynamic (C)*	*Static (C_0)*	*Dynamic (C)*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*
200	2.24	4	—	—	4.55	7.35	1.80	5.70
300	3.60	6.3	—	—	—	—	—	—
201	3	5.4	—	—	5.6	8.3	2.0	5.85
301	4.3	7.65	—	—	—	—	3.0	9.15
202	3.55	6.10	3.75	6.30	5.6	8.3	2.16	6
302	5.20	8.80	—	—	9.3	14	3.35	9.3
203	4.4	7.5	4.75	7.8	8.15	11.6	2.8	7.65
303	6.3	10.6	7.2	11.6	12.9	19.3	4.15	11.2
403	11	18	—	—	—	—	—	—
204	6.55	10	6.55	10.4	11	16	3.9	9.8
304	7.65	12.5	8.3	13.7	14	19.3	5.5	14
404	15.6	24	—	—	—	—	—	—
205	7.1	11	7.8	11.6	13.7	17.3	4.25	9.8
305	10.4	16.6	12.5	19.3	20	26.5	7.65	19
405	19	28	—	—	—	—	—	—
206	10	15.3	11.2	16	20.4	25	5.6	12
306	14.6	22	17	24.5	27.5	35.5	10.2	24.5
406	23.2	33.5	—	—	—	—	—	—
207	13.7	20	15.3	21.2	28	34	8	17
307	17.6	26	20.4	28.5	36	45	13.2	30.5
407	30.5	43	—	—	—	—	—	—
208	16	22.8	19	25	32.5	39	9.15	17.6
308	22	32	25.5	35.5	45.5	55	16	35.5
408	37.5	50	—	—	—	—	—	—
209	18.3	25.5	21.6	28	37.5	41.5	10.2	18
309	30	41.5	34	45.5	56	67	19.6	42.5
409	44	60	—	—	—	—	—	—
210	21.2	27.5	23.6	29	43	47.5	10.8	18
310	35.5	48	40.5	53	73.5	81.5	24	50
410	50	68	—	—	—	—	—	—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
211	26	34	30	36.5	49	53	12.7	20.8
311	42.5	56	47.5	62	80	88	28.5	58.5
411	60	78	—	—	—	—	—	—
212	32	40.5	36.5	44	63	65.5	16	26.5
312	48	64	55	71	96.5	102	33.5	68
412	67	85	—	—	—	—	—	—
213	35.5	44	43	50	69.5	69.5	20.4	34
313	55	72	63	80	112	118	39	75
413	76.5	93	—	—	—	—	—	—
214	39	48	47.5	54	71	69.5	21.6	34.5
314	63	81.5	73.5	90	129	137	45	85
414	102	112	—	—	—	—	—	—
215	42.5	52	50	56	80	76.5	22.4	34.5
315	72	90	81.5	98	140	143	52	95
415	110	120	—	—	—	—	—	—
216	45.5	57	57	63	96.5	93	25	38
316	80	96.5	91.5	106	160	163	58.5	106
416	120	127	—	—	—	—	—	—
217	55	65.5	65.5	71	100	106	30	45.5
317	88	104	102	114	180	180	62	110
417	132	134	—	—	—	—	—	—
218	63	75	76.5	83	127	118	36	55
318	98	112	114	122	—	—	69.5	118
418	146	146	—	—	—	—	—	—
219	72	85	88	95	150	137	43	65.5
319	112	120	125	132	—	—	—	—
220	81.5	96.5	93	102	160	146	51	76.5
320	132	137	153	150	—	—	—	—
221	93	104	104	110	—	—	56	85
321	143	143	166	160	—	—	—	—
222	104	112	116	120	—	—	64	98
322	166	160	193	176	—	—	—	—

Example 5.4. *Select a single row deep groove ball bearing for a radial load of 4000 N and an axial load of 5000 N, operating at a speed of 1600 r.p.m. for an average life of 5 years at 10 hours per day. Assume uniform and steady load.*

Given Data :

W_R = 4000 N

W_A = 5000 N ;

N = 1600 r.p.m.

Solution

Since the average life of the bearing is 5 years at 10 hours per day, therefore life of the bearing in hours,

$L_H = 5 \times 300 \times 10 = 15\,000$ hours .. (Assuming 300 working days per year)

and life of the bearing in revolutions,

$L = 60\, N \times L_H = 60 \times 1600 \times 15\,000 = 1440 \times 10^6$ rev

We know that the basic dynamic equivalent radial load,

$W = X.V.W_R + Y.W_A$ ...**(*i*)**

In order to determine the radial load factor (X) and axial load factor (Y), we require W_A / W_R and W_A / C_0. Since the value of basic static load capacity (C_0) is not known, therefore let us take $W_A / C_0 = 0.5$. Now from Table 5.4, we find that the values of X and Y corresponding to W_A / C_0

= 0.5 and $W_A/W_R = 5000 / 4000 = 1.25$ (which is greater than $e = 0.44$) are

$X = 0.56$ and $Y = 1$

Since the rotational factor (V) for most of the bearings is 1, therefore basic dynamic equivalent radial load,

$W = 0.56 \times 1 \times 4000 + 1 \times 5000 = 7240$ N

From Table 5.5, we find that for uniform and steady load, the service factor (K_S) for ball bearings is 1. Therefore the bearing should be selected for W = 7240 N.

We know that basic dynamic load rating,

$$C = W\left(\frac{L}{10^6}\right)^{1/k} = 7240\left(\frac{1440 \times 10^6}{10^6}\right)^{1/3} = 81\,760 \text{ N}$$

= 81.76 kN ... (k = 3, for ball bearings)

From Table 5.6, let us select the bearing No. 315 which has the following basic capacities,

$C_0 = 72$ kN = 72 000 N and $C = 90$ kN = 90 000 N

Now $W_A / C_0 = 5000 / 72\,000 = 0.07$

From Table 5.4, the values of X and Y are

$X = 0.56$ and $Y = 1.6$

Substituting these values in equation **(*i*)**, we have dynamic equivalent load,

$W = 0.56 \times 1 \times 4000 + 1.6 \times 5000 = 10\,240$ N

Basic dynamic load rating,

$$C = 10\ 240\left(\frac{1440 \times 10^6}{10^6}\right)^{1/3}$$

= 115 635 N = 115.635 kN

From Table 5.6, the bearing number 319 having C = 120 kN, may be selected. **Ans.**

Example 5.5. *Design a self-aligning ball bearing for a radial load of 7000 N and a thrust load of 2100 N. The desired life of the bearing is 160 millions of revolutions at 300 r.p.m. Assume uniform and steady load,*

Given Data:
WR = 7000 N
WA = 2100 N
L = 160 × 106 rev
N = 300 r.p.m.

Solution.

From Table 5.4,
we find that for a self-aligning ball bearing, the values of radial factor (X)
And thrust factor (Y) for WA / WR = 2100 / 7000 = 0.3, are as follows :

X = 0.65 and Y = 3.5

Since the rotational factor (V) for most of the bearings is 1, therefore dynamic equivalent load,

$W = X.V.W$R + $Y.W$A = 0.65 × 1 × 7000 + 3.5 × 2100 = 11 900 N

From Table 5.5, we find that for uniform and steady load, the service factor KS for ball bearingsis 1.
Therefore the bearing should be selected for W = 11 900 N.
We know that the basic dynamic load rating,

$$C = W\left(\frac{L}{10^6}\right)^{1/K} = 11900\left(\frac{160\ X\ 10^6}{10^6}\right)^{1/3}$$

= 64 600 N = 64.6 kN ... (k = 3, for ball bearings)

From Table 5.6, let us select bearing number 219 having C = 65.5 kN **Ans.**

Example 5.6. *Select a single row deep groove ball bearing with the operating cycle listed below, which will have a life of 15 000 hours.*

Fraction of cycle	*Type of load*	*Radial (N)*	*Thrust (N)*	*Speed (R.P.M.)*	*Service factor*
1/10	*Heavy shocks*	*2000*	*1200*	*400*	*3.0*
1/10	*Light shocks*	*1500*	*1000*	*500*	*1.5*
1/5	*Moderate shocks*	*1000*	*1500*	*600*	*2.0*

3/5	*No shock*	*1200*	*2000*	*800*	*1.0*

Assume radial and axial load factors to be 1.0 and 1.5 respectively and inner race rotates.

Given Data :
L_H = 15 000 hours
W_{R1} = 2000 N ; W_{A1} = 1200 N ; N_1 = 400 r.p.m. ; K_{S1} = 3
W_{R2} = 1500 N ; W_{A2} = 1000 N ; N_2 = 500 r.p.m. ; K_{S2} = 1.5
W_{R3} = 1000 N ; W_{A3} = 1500 N ; N_3 = 600 r.p.m. ; K_{S3} = 2
W_{R4} = 1200 N ; W_{A4} = 2000 N ; N_4 = 800 r.p.m. ; K_{S4} = 1
X = 1 ; Y = 1.5

Solution

We know that basic dynamic equivalent radial load considering service factor is

$W = [X.V.W_R + Y.W_A]\ K_S$ ...***(i)***

It is given that radial load factor (X) = 1 and axial load factor (Y) = 1.5. Since the rotational factor (V) for most of the bearings is 1, therefore equation ***(i)*** may be written as

$W = (W_R + 1.5\ W_A)\ K_S$

Now, substituting the values of W_R, W_A and K_S for different operating cycle, we have

$W_1 = (W_{R1} + 1.5\ W_{A1})\ K_{S1} = (2000 + 1.5 \times 1200)\ 3 = 11\ 400$ N

$W_2 = (W_{R2} + 1.5\ W_{A2})\ K_{S2} = (1500 + 1.5 \times 1000)\ 1.5 = 4500$ N

$W_3 = (W_{R3} + 1.5\ W_{A3})\ K_{S3} = (1000 + 1.5 \times 1500)\ 2 = 6500$ N

and $W_4 = (W_{R4} + 1.5\ W_{A4})\ K_{S4} = (1200 + 1.5 \times 2000)\ 1 = 4200$ N

We know that life of the bearing in revolutions

$$L = 60\ N.L_H = 60\ N \times 15\ 000 = 0.9 \times 10^6\ N \text{ rev}$$

∴ Life of the bearing for 1/10 of a cycle,

$$L_1 = \frac{1}{10} \times 0.9 \times 10^6\ N_1 = \frac{1}{10} \times 0.9 \times 10^6 \times 400 = 36 \times 10^6 \text{ rev}$$

Similarly, life of the bearing for the next 1/10 of a cycle,

$$L_2 = \frac{1}{10} \times 0.9 \times 10^6\ N_2 = \frac{1}{10} \times 0.9 \times 10^6 \times 500 = 45 \times 10^6 \text{ rev}$$

Life of the bearing for the next 1/5 of a cycle,

$$L_3 = \frac{1}{5} \times 0.9 \times 10^6\ N_3 = \frac{1}{5} \times 0.9 \times 10^6 \times 600 = 108 \times 10^6 \text{ rev}$$

and life of the bearing for the next 3/5 of a cycle,

$$L_4 = \frac{3}{5} \times 0.9 \times 10^6\ N_4 = \frac{3}{5} \times 0.9 \times 10^6 \times 800 = 432 \times 10^6 \text{ rev}$$

We know that equivalent dynamic load,

$$W = \left[\frac{L_1 (W_1)^3 + L_2 (W_2)^3 + L_3 (W_3)^3 + L_4 (W_4)^3}{L_1 + L_2 + L_3 + L_4}\right]^{1/3}$$

$$= \left[\frac{36\times10^6\,(11\,400)^3 + 45\times10^6\,(4500)^3 + 108\times10^6\,(6500)^3 + 432\times10^6\,(4200)^3}{36\times10^6 + 45\times10^6 + 108\times10^6 + 423\times10^6}\right]^{1/3}$$

$$= \left[\frac{1.191\times10^8\times10^{12}}{621\times10^6}\right]^{1/3} = (0.1918\times10^{12})^{1/3} = 5767\text{ N}$$

and $L = L_1 + L_2 + L_3 + L_4$

$$= 36\times10^6 + 45\times10^6 + 108\times10^6 + 432\times10^6 = 621\times10^6\text{ rev}$$

We know that dynamic load rating,

$$C = W\left(\frac{L}{10^6}\right)^{1/k} = 5767\left(\frac{621\times10^6}{10^6}\right)^{1/3}$$

$$= 5767\times8.53 = 49\,193\text{ N} = 49.193\text{ kN}$$

From Table 5.6, the single row deep groove ball bearing number 215 having C = 52 kN may be selected. **Ans.**

EXERCISES

1. The main bearing of a steam engine is 100 mm in diameter and 175 mm long. The bearing supports a load of 28 kN at 250 r.p.m. If the ratio of the diametral clearance to the diameter is 0.001 and the absolute viscosity of the lubricating oil is 0.015 kg/m-s, find : 1. The coefficient of friction ; and 2. The heat generated at the bearing due to friction. **[Ans. 0.002 77 ; 101.5 J/s]**

2. A journal bearing is proposed for a steam engine. The load on the journal is 3 kN, diameter 50 mm, length 75 mm, speed 1600 r.p.m., diametral clearance 0.001 mm, ambient temperature 15.5°C. Oil SAE 10 is used and the film temperature is 60°C. Determine the heat generated and heat dissipated. Take absolute viscosity of SAE10 at 60°C = 0.014 kg/m-s. **[Ans. 141.3 J/s ; 25 J/s]**

3. A 100 mm long and 60 mm diameter journal bearing supports a load of 2500 N at 600 r.p.m. If the room temperature is 20°C, what should be the viscosity of oil to limit the bearing surface temperature to 60°C? The diametral clearance is 0.06 mm and the energy dissipation coefficient based on projected area of bearing is 210 W/m^2/°C. **[Ans. 0.0183 kg/m-s]**

4. A tentative design of a journal bearing results in a diameter of 75 mm and a length of 125 mm for supporting a load of 20 kN. The shaft runs at 1000 r.p.m. The bearing surface temperature is not to exceed 75°C in a room temperature of 35°C. The oil used has an absolute viscosity of 0.01 kg/m-s at the operating temperature. Determine the amount of artificial cooling required in watts. Assume d/c = 1000. **[Ans. 146 W]**

5. A ball bearing subjected to a radial load of 5 kN is expected to have a life of 8000 hours at 1450 r.p.m. with a reliability of 99%. Calculate the dynamic load capacity of the bearing so that it can be selected from the manufacturer's catalogue based on a reliability of 90%. **[Ans. 86.5 kN]**

6. A ball bearing subjected to a radial load of 4000 N is expected to have a satisfactory life of 12 000 hours at 720 r.p.m. with a reliability of 95%. Calculate the dynamic load carrying capacity of the bearing, so that it can be selected from manufacturer's catalogue based on 90% reliability. If there are four such bearings each with a reliability of 95% in a system, what is the reliability of the complete system? **[Ans. 39.5 kN ; 81.45%]**