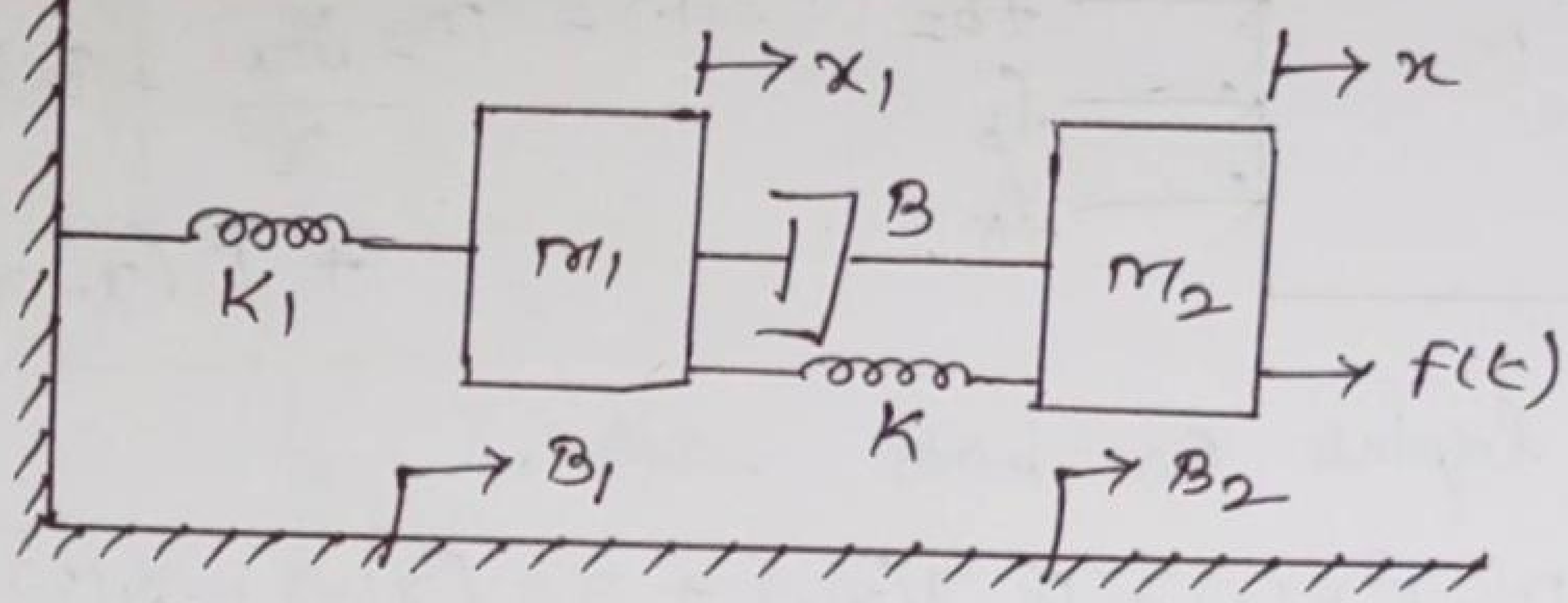
UNIT - 1 - INTRODUCTION.

Mathematical Modeling of Systems.

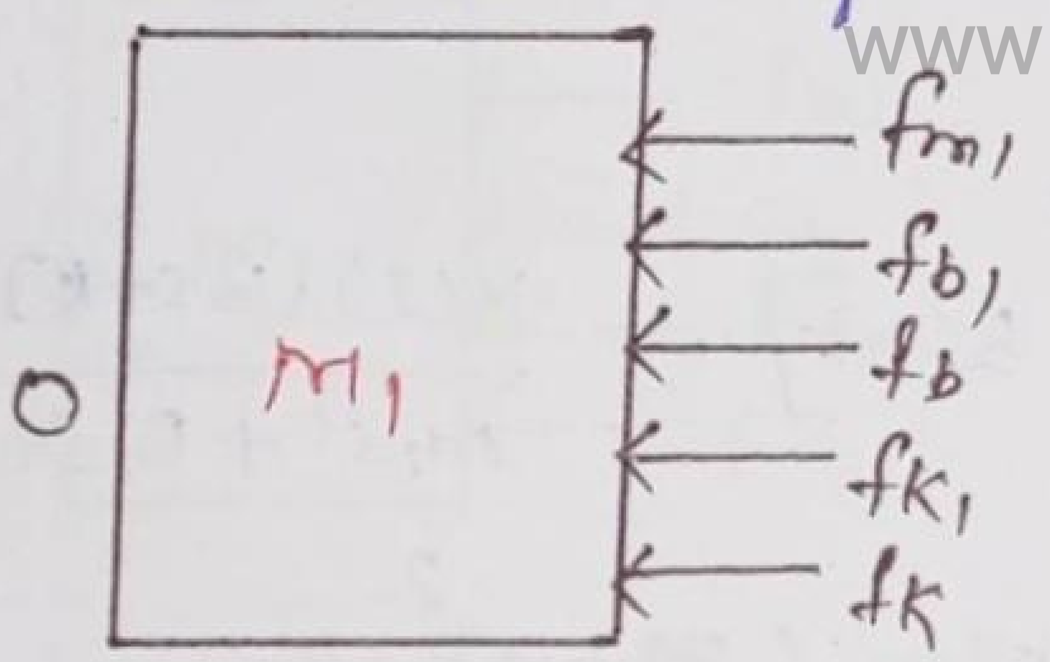
Compute the differential equation governing the mechanical system shown below and determine the transfer function.



Transfer function = $\frac{L.T \text{ of } O/P}{L.T \text{ of } I/P} = \frac{X(s)}{F(s)}$.

Force balanced equation of M_1.

Free body diagram of M_1



$$0 = f_{m_1} + f_{b_1} + f_b + f_{k_1} + f_k \quad \text{--- (1)}$$

$$0 = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x) + k_1 x_1 + k(x_1 - x) \quad \text{--- (2)}$$

Taking laplace on both sides

$$0 = M_1 s^2 x_1(s) + B_1 s x_1(s) + Bs[x_1(s) - x(s)] + k_1 x_1(s) + k[x_1(s) - x(s)] \quad \text{--- (3)}$$

$$x_1(s)[M_1 s^2 + B_1 s + Bs + k_1 + k] - X(s)[Bs + k] = 0 \quad \text{--- (4)}$$

$$\therefore X_1(s)[M_1 s^2 + B_1 s + Bs + k_1 + k] = X(s)[Bs + k] \quad \text{--- (5)}$$

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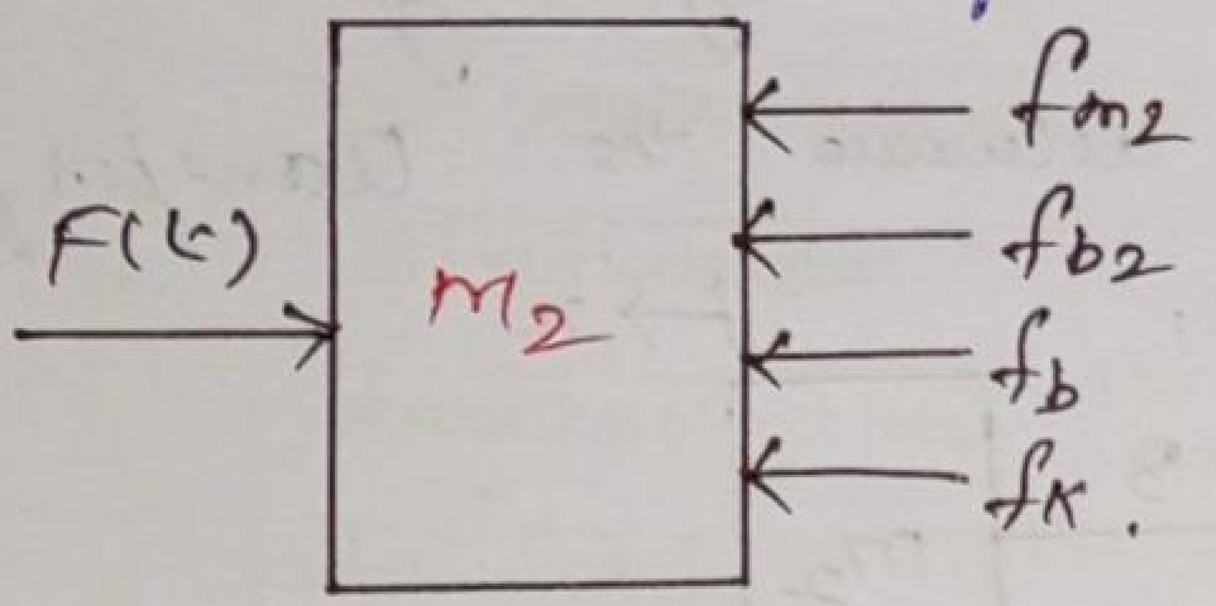
——— 3 marks

Force balance equation of M_2.

Free body diagram of M_2.



$$F(t) = f_{m_2} + f_{b_2} + f_b + f_k \quad \text{— (6)}$$

$$F(t) = m_2 \frac{d^2 x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + k(x - x_1) \quad \text{— (7)}$$

Taking laplace on both sides,

$$F(s) = M_2 s^2 X(s) + B_2 s X(s) + Bs\left[X(s) - X_1(s)\right] + K\left[X(s) - X_1(s)\right] \quad \text{— (8)}$$

$$F(s) = X(s)\left[M_2 s^2 + B_2 s + Bs + K\right] - X_1(s)\left[Bs + K\right] \quad \text{— (9)}$$

From equation (5),

$$X_1(s) = \frac{X(s)\left[Bs + K\right]}{M_1 s^2 + B_1 s + Bs + K_1 + K} \quad \text{— (10)}$$

——— 3 marks

Substitute eqn (10) in eq (9),

$$\therefore F(s) = X(s)\left[M_2 s^2 + B_2 s + Bs + K\right] - \frac{X(s)\left[Bs + K\right]}{M_1 s^2 + B_1 s + Bs + K_1 + K} \cdot \left[Bs + K\right] \quad \text{— (11)}$$

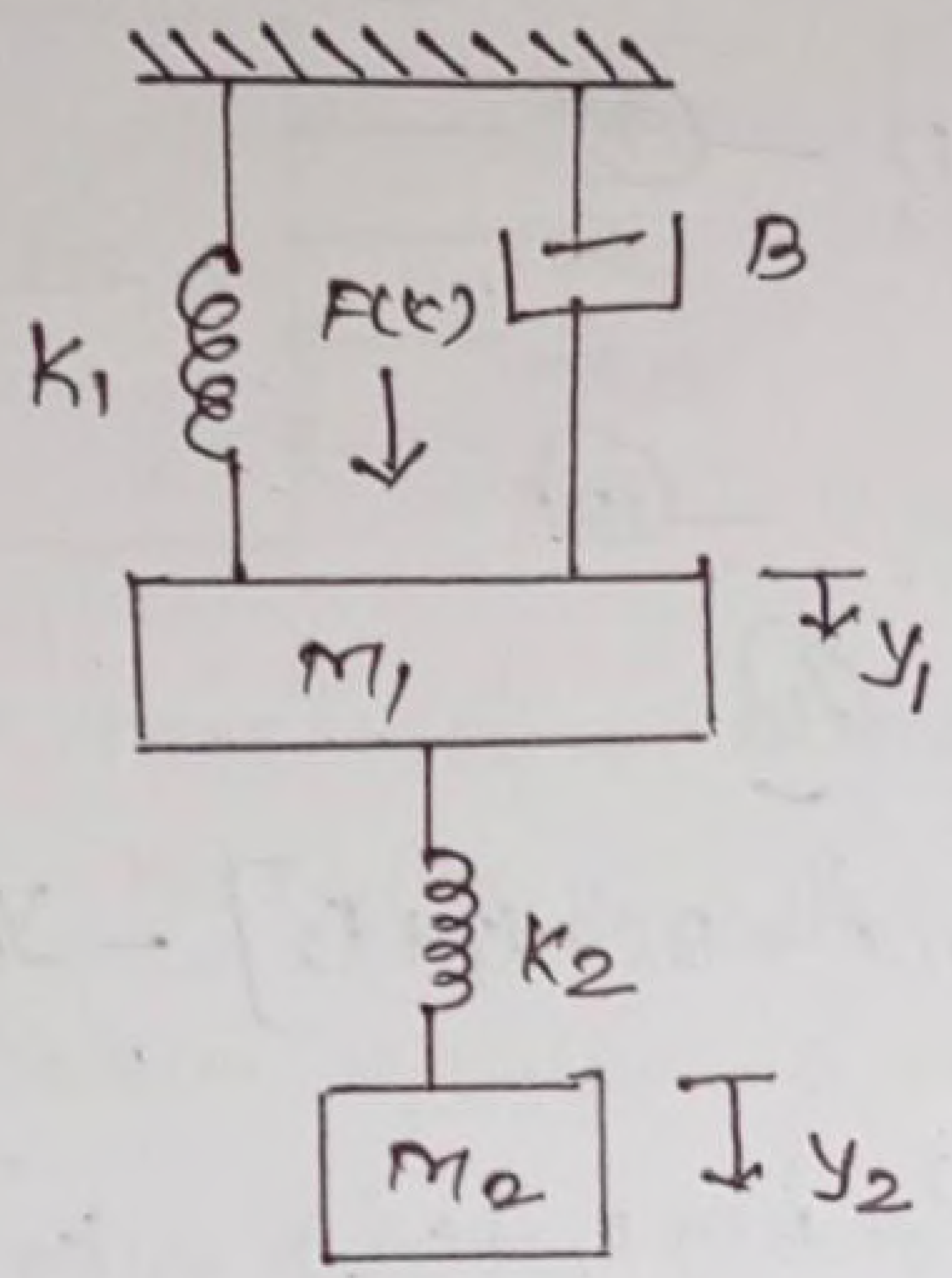
$$F(s) = X(s)\left[M_2 s^2 + B_2 s + Bs + K - \frac{(Bs + K)^2}{M_1 s^2 + B_1 s + Bs + K_1 + K}\right] \quad \text{— (12)}$$

$$F(s) = X(s)\left[\frac{(M_2 s^2 + B_2 s + Bs + K)(M_1 s^2 + B_1 s + Bs + K_1 + K) - (Bs + K)^2}{M_1 s^2 + B_1 s + Bs + K_1 + K}\right]$$

$$\Rightarrow \frac{X(s)}{F(s)} = \frac{M_1 s^2 + B_1 s + Bs + K_1 + K}{(M_2 s^2 + B_2 s + Bs + K)(M_1 s^2 + B_1 s + Bs + K_1 + K) - (Bs + K)^2}$$

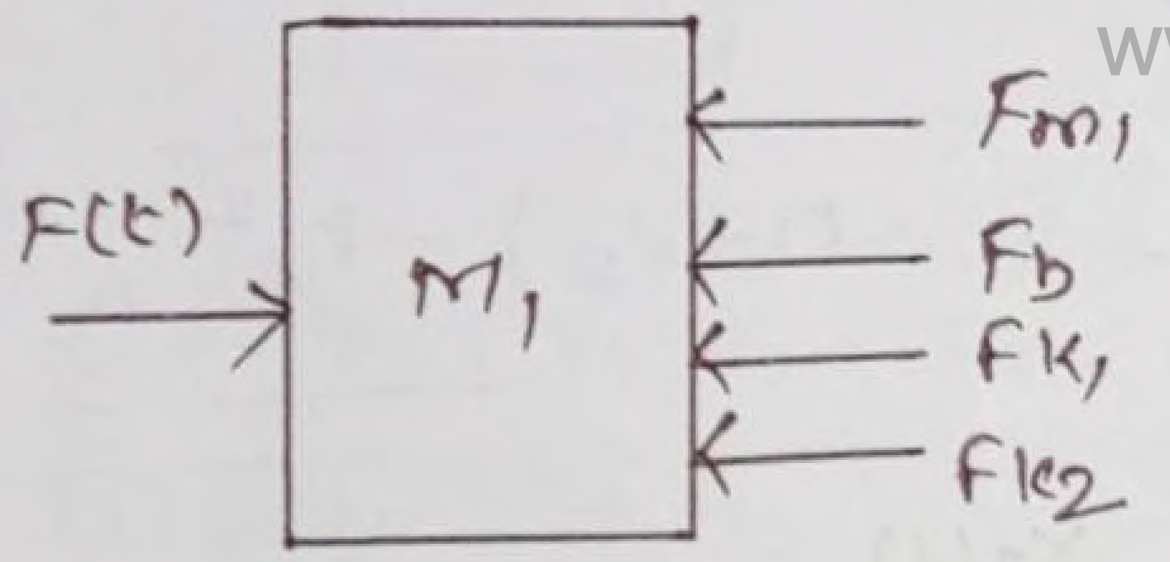
——— 2 marks

Determine the transfer function $\dfrac{Y_2(s)}{F(s)}$.



Force balance equation of M_1

Free body diagram of M_1



$$F(t) = F_{m_1} + F_b + F_{k_1} + F_{k_2} \quad \text{--- (1)}$$

$$F(t) = M_1 \frac{d^2 y_1}{dt^2} + B\frac{dy_1}{dt} + K_1 y_1 + K_2(y_1 - y_2) \quad \text{--- (2)}$$

Taking laplace on both sides

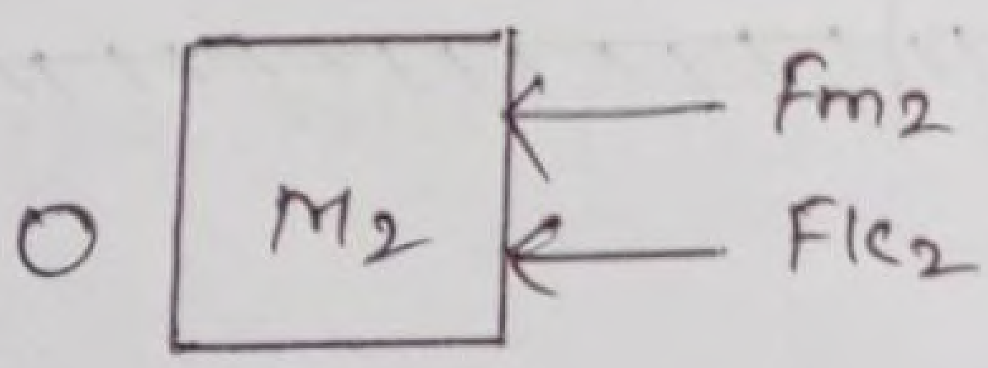
$$F(s) = M_1 s^2 Y_1(s) + BsY_1(s) + K_1 Y_1(s) + K_2\left[Y_1(s) - Y_2(s)\right] \quad \text{--- (3)}$$

$$F(s) = Y_1(s)\left[M_1 s^2 + Bs + K_1 + K_2\right] - Y_2(s) K_2 \quad \text{--- (4)}$$

— 3 marks.

Force balance equation of M_2

Free body diagram of M_2.



$$0 = F_{m_2} + F_{k_2} \quad \text{--- (5)}$$

$$0 = M_2 \frac{d^2 y_2}{dt^2} + K_2(y_2 - y_1) \quad \text{--- (6)}$$

Taking laplace on both sides,

$$0 = M_2 s^2 Y_2(s) + K_2[Y_2(s) - Y_1(s)] \quad \text{--- (7)}$$

$$0 = Y_2(s)[M_2 s^2 + K_2] - K_2 Y_1(s) \quad \text{--- (8)}$$

$$Y_2(s)[M_2 s^2 + K_2] = K_2 Y_1(s) \quad \text{--- (9)}$$

From eqn (9),

——— 3 marks.

$$Y_1(s) = Y_2(s)\left[\frac{M_2 s^2 + K_2}{K_2}\right] \quad \text{--- (10)}$$

Substitute eqn (10) in eqn (2)

$$F(s) = Y_2(s)\left[\frac{M_2 s^2 + K_2}{K_2}\right]\left[M_1 s^2 + Bs + K_1 + K_2\right] - Y_2(s) K_2 \quad \text{--- (11)}$$

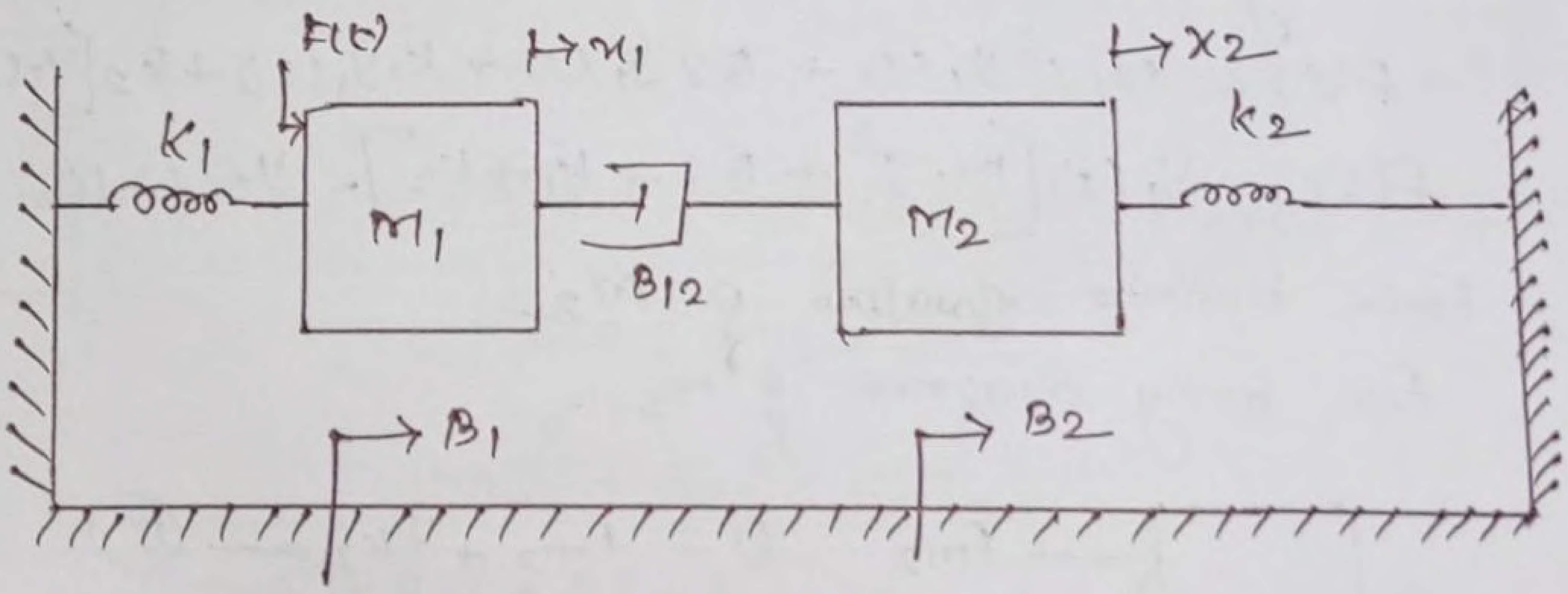
$$F(s) = Y_2(s)\left[\frac{M_2 s^2 + K_2(M_1 s^2 + Bs + K_1 + K_2)}{K_2} - K_2\right] \quad \text{--- (12)}$$

$$F(s) = Y_2(s)\left[\frac{M_2 s^2 + K_2(M_1 s^2 + Bs + K_1 + K_2) - K_2^2}{K_2}\right] \quad \text{--- (13)}$$

$$\therefore \frac{Y_2(s)}{F(s)} = \frac{K_2}{M_2 s^2 + K_2(M_1 s^2 + Bs + K_1 + K_2) - K_2^2}$$

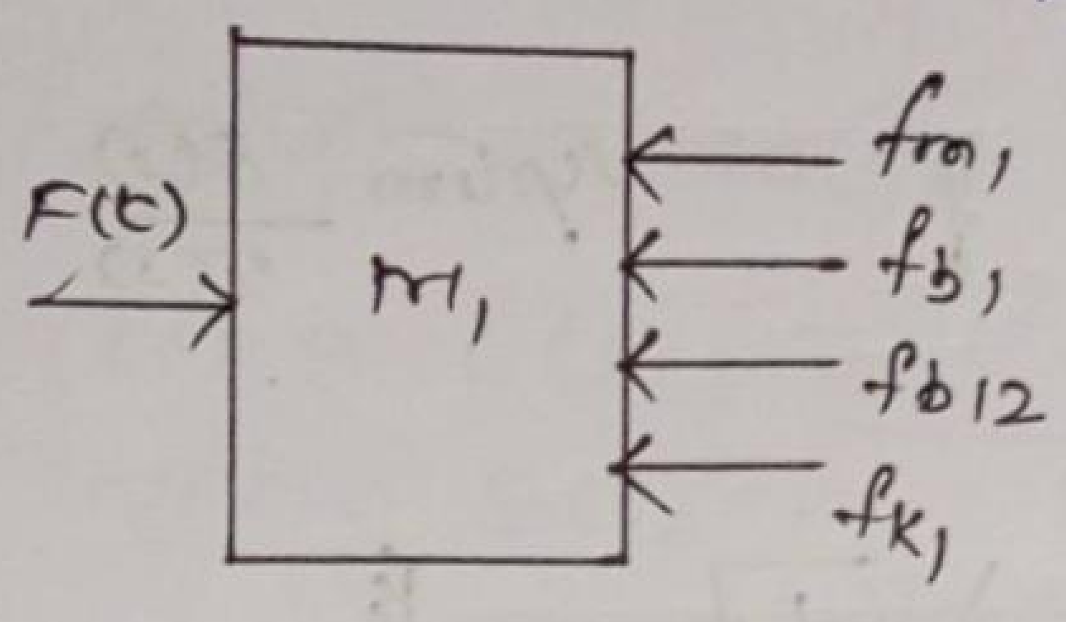
——— 2 marks.

Determine the transfer function $\frac{X_2(s)}{F(s)}$.



Force balance equation of m_1

Free body diagram of m_1



$$F(t) = f_{m1} + f_{b1} + f_{b12} + f_{k1} \quad \text{---} (1)$$

$$F(t) = m_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x_2) + k_1 x_1 \quad \text{---} (2)$$

Taking laplace on both sides,

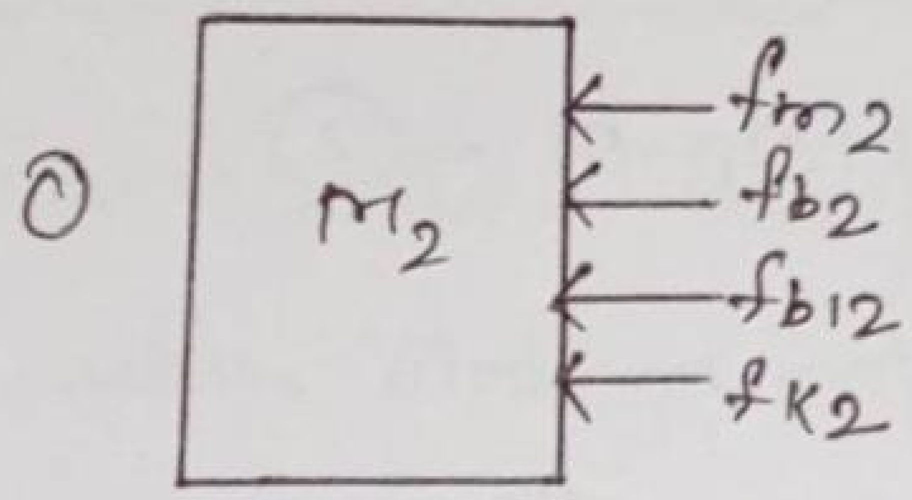
$$F(s) = m_1 s^2 x_1(s) + B_1 s x_1(s) + B_{12} s (x_1(s) - x_2(s)) + k_1 x_1(s) \quad \text{---} (3)$$

$$F(s) = x_1(s)(m_1 s^2 + B_1 s + B_{12} s + k_1) - x_2(s)(B_{12} s) \quad \text{---} (4)$$

— 3 marks.

Force balance equation of m_2.

Free body diagram of m_2.



$$0 = f_{m2} + f_{b2} + f_{b12} + f_{k2} \quad \text{---} (5)$$

$$0 = m_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + K_2 x_2 \quad \text{---} (6)$$

Taking laplace on both sides,

$$0 = m_2 s^2 x_2(s) + B_2 s x_2(s) + B_{12} s (x_2(s) - x_1(s)) + k_2 x_2(s) \quad \text{---} (7)$$

$$0 = (m_2 s^2 + B_2 s + B_{12} s + k_2) x_2(s) - x_1(s) B_{12} s \quad \text{---} (8)$$

$$x_2(s)[m_2 s^2 + B_2 s + B_{12} s + k_2] = x_1(s)[B_{12} s] \quad \text{---} (9)$$

$$\therefore \quad x_1(s) = x_2(s)\left[\frac{m_2 s^2 + B_2 s + B_{12} s + k_2}{B_{12} s}\right] \quad \text{---} (10)$$

— 3 marks.

Substitute equ (10) in equ (4)

$$F(s) = x_2(s)\left[\frac{m_2 s^2 + B_2 s + B_{12} s + k_2}{B_{12} s}\right]\left[m_1 s^2 + B_1 s + B_{12} s + k\right] - x_2(s)(B_{12} s)$$

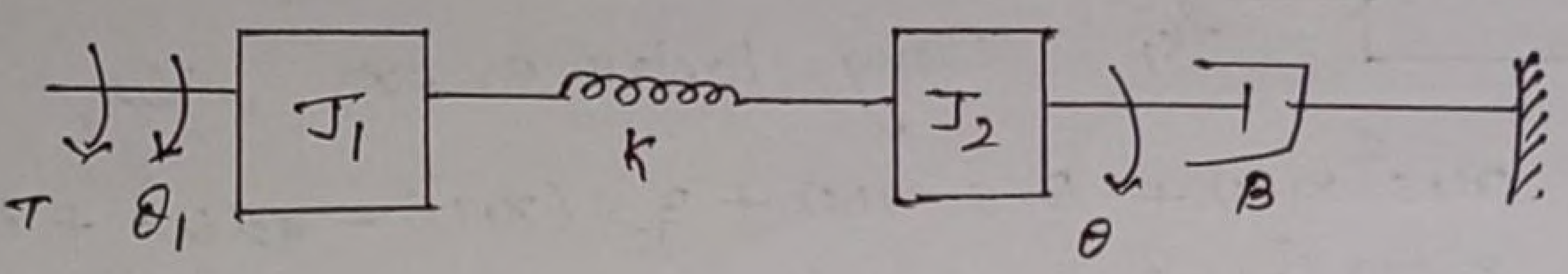
$$F(s) = x_2(s)\left[\frac{(m_2 s^2 + B_2 s + B_{12} s + k_2)(m_1 s^2 + B_1 s + B_{12} s + k)}{B_{12} s}\right] - x_2(s)(B_{12} s)$$

$$F(s) = x_2(s)\left[\frac{(m_2 s^2 + B_2 s + B_{12} s + k_2)(m_1 s^2 + B_1 s + B_{12} s + k) - (B_{12} s)^2}{B_{12} s}\right]$$

— 2 marks.

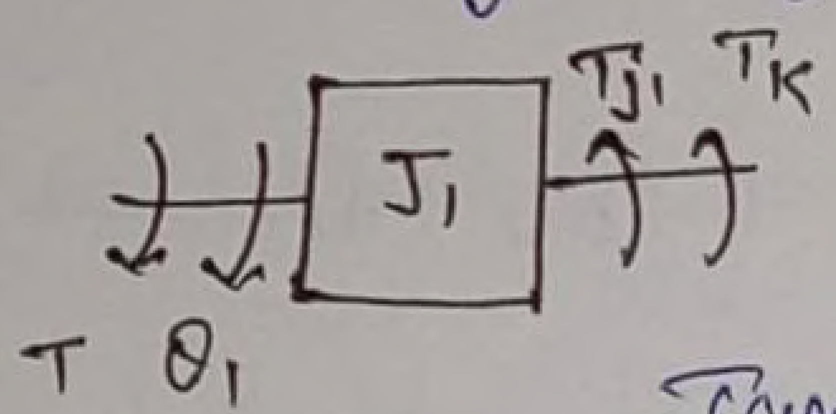
$$\Rightarrow \frac{X_2(s)}{F(s)} = \frac{B_{12}s}{(M_2s^2 + B_2s + B_{12}s + K_2)(M_1s^2 + B_1s + B_{12}s + K_1 - (B_{12}s)}$$

Evaluate the transfer function of a system $\frac{\theta(s)}{T(s)}$.



Torque balance equation of J_1

Free body diagram of J_1



$$T = T_{J_1} + T_K \quad \text{—} ①$$

$$T = J_1\frac{d^2\theta}{dt^2} + K(\theta_1 - \theta) \quad \text{—} ②$$

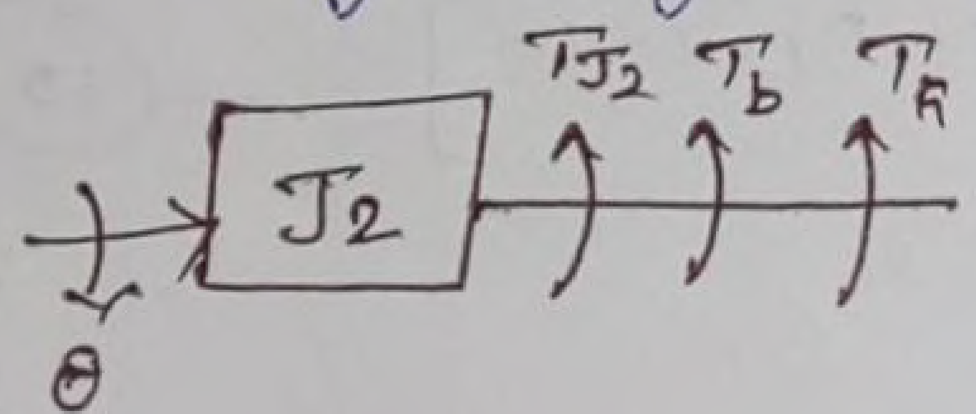
Taking laplace transform on both sides,

$$T(s) = J_1s^2\theta_1(s) + K[\theta_1(s) - \theta(s)] \quad \text{—} ③$$

$$T(s) = \theta_1(s)[J_1s^2 + K] - \theta(s)K \quad \text{—} ④$$

Torque balance equation of J_2. — 3 marks

Free body diagram of J_2.



$$0 = T_{j2} + T_b + T_K \quad \text{—} ⑤$$

$$0 = J_2\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) \quad \text{—} ⑥$$

Taking laplace transform on both sides.

$$0 = J_2s^2\theta(s) + Bs\theta(s) + K[\theta(s) - \theta_1(s)] \quad \text{—} ⑦$$

$$0 = [J_2s^2 + Bs + K]\theta(s) - \theta_1(s)K \quad \text{—} ⑧$$

$$\theta(s)[J_2s^2 + Bs + K] = \theta_1(s)K \quad \text{—} ⑨$$

— 3 marks.

From equation (9),

$$\theta_1(s) = \theta(s)\left[\frac{J_2 s^2 + Bs + K}{K}\right] \quad \text{— (10)}$$

Substitute eqn (10) in eqn (4),

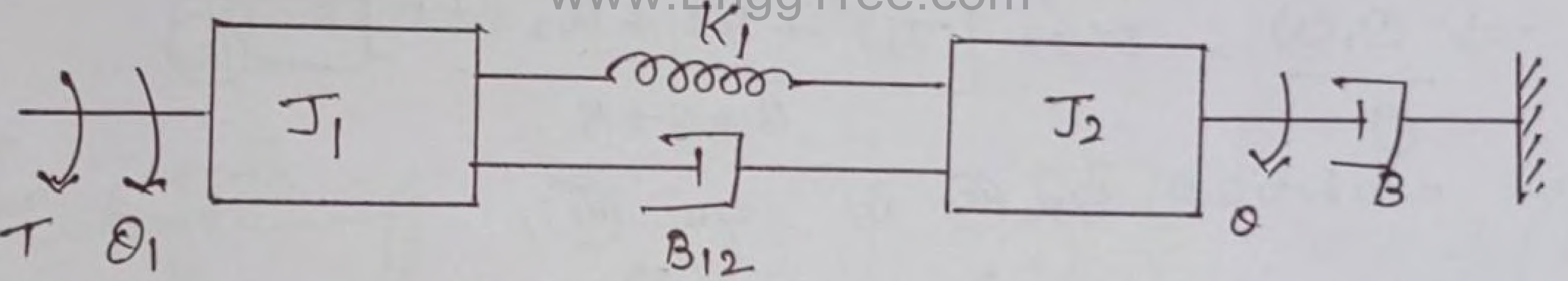
$$T(s) = \theta(s)\left[\frac{J_2 s^2 + Bs + K}{K}\right]\left[J_1 s^2 + K\right] - \theta(s)K$$

$$T(s) = \theta(s)\left[\frac{(J_2 s^2 + Bs + K)(J_1 s^2 + K)}{K} - K\right]$$

$$T(s) = \theta(s)\left[\frac{(J_2 s^2 + Bs + K)(J_1 s^2 + K) - K^2}{K}\right]$$

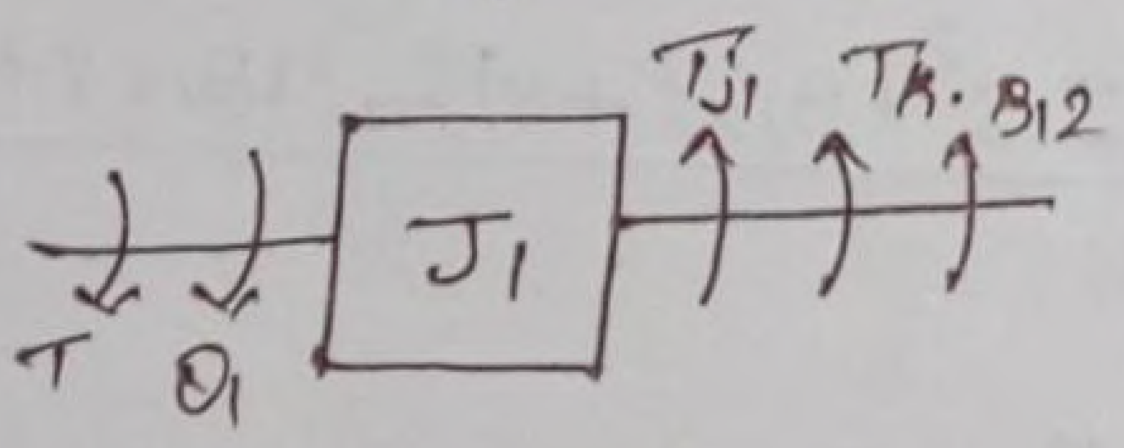
$$\therefore \frac{\theta(s)}{T(s)} = \frac{K}{(J_2 s^2 + Bs + K)(J_1 s^2 + K - K^2)}$$ — 2 marks.

Determine the transfer function $\frac{\theta(s)}{T(s)}$.



Torque balance equation of J_1

Free body diagram of J_1.



$$T = T_{J_1} + T_{B_{12}} + T_K \quad \text{— (1)}$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + B_{12}\frac{d(\theta_1 - \theta)}{dt} + K(\theta_1 - \theta) \quad \text{— (2)}$$

Taking laplace on both sides,

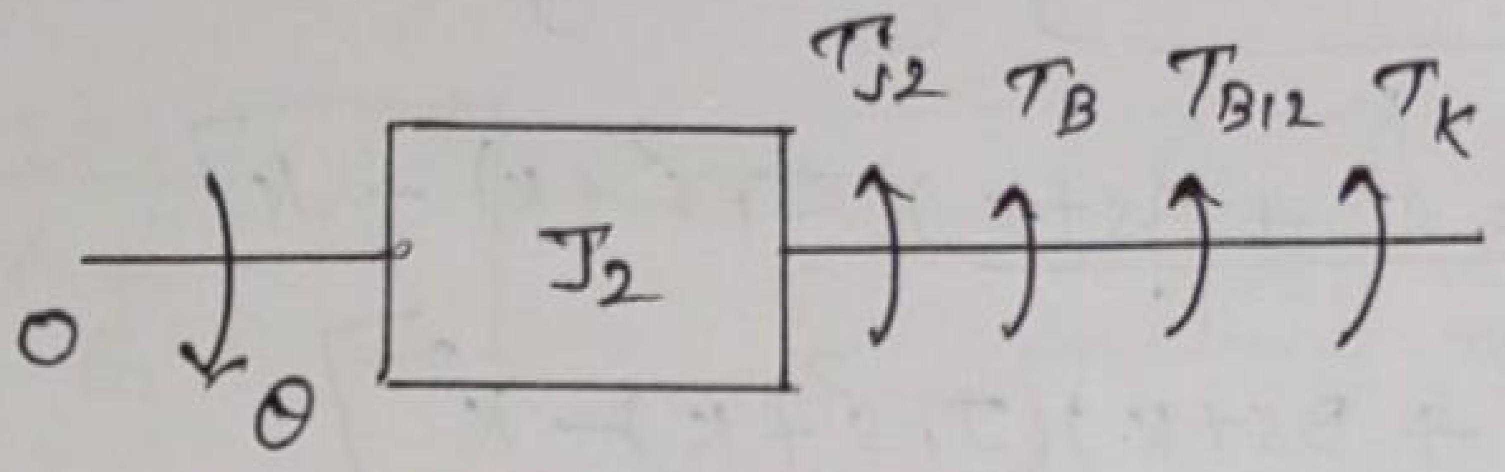
$$T(s) = J_1 s^2 \theta_1(s) + B_{12} s(\theta_1(s) - \theta(s)) + K(\theta_1(s) - \theta(s)) \quad \text{— (3)}$$

$$T(s) = \theta_1(s)\left(J_1 s^2 + B_{12} s + K\right) - \theta(s)\left(B_{12} s + K\right) \quad \text{— (4)}$$

— 3 marks

Torque balance equation of J_2.

Free body diagram of J_2.



$$0 = T_{J_2} + T_B + T_{B12} + T_K \quad \text{— (5)}$$

$$0 = J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + B_{12}\frac{d}{dt}(\theta - \theta_1) + K(\theta - \theta_1) \quad \text{— (6)}$$

Taking laplace on both sides,

$$0 = J_2 s^2 \theta(s) + Bs\,\theta(s) + B_{12} s\left[\theta(s) - \theta_1(s)\right] + K\left[\theta(s) - \theta_1(s)\right] \quad \text{— (7)}$$

$$0 = \theta(s)\left[J_2 s^2 + Bs + B_{12}s + K\right] - \theta_1(s)\left[B_{12}s + K\right] \quad \text{— (8)}$$

$$\therefore \theta(s)\left(J_2 s^2 + Bs + B_{12}s + K\right) = \theta_1(s)\left(B_{12}s + K\right) \quad \text{— (9)}$$

from eqn (9),

$$\Rightarrow \theta_1(s) = \theta(s)\left[\frac{J_2 s^2 + Bs + B_{12}s + K}{B_{12}s + K}\right] \quad \text{— (10)}$$

Substitute eqn (10) in eqn (4),

— 3 marks.

$$T(s) = \theta(s)\left[\frac{J_2 s^2 + Bs + B_{12}s + K}{B_{12}s + K}\right]\left[J_1 s^2 + B_{12}s + K\right] - \theta(s)\left(B_{12}s + K\right)$$

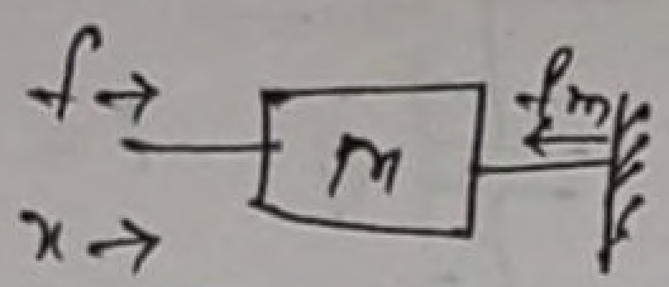
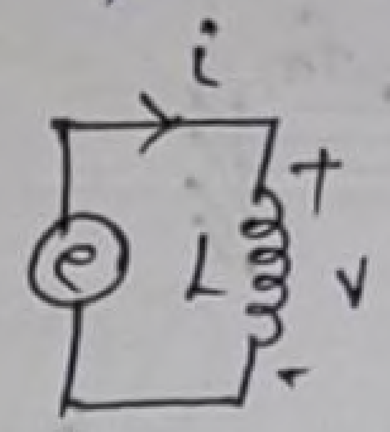
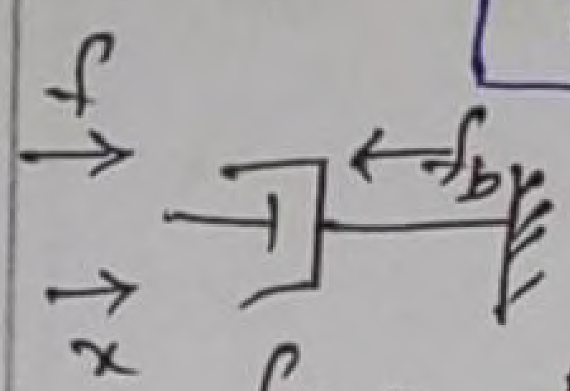
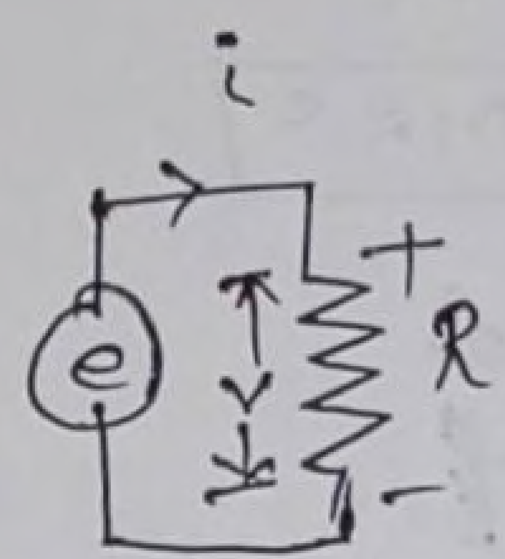
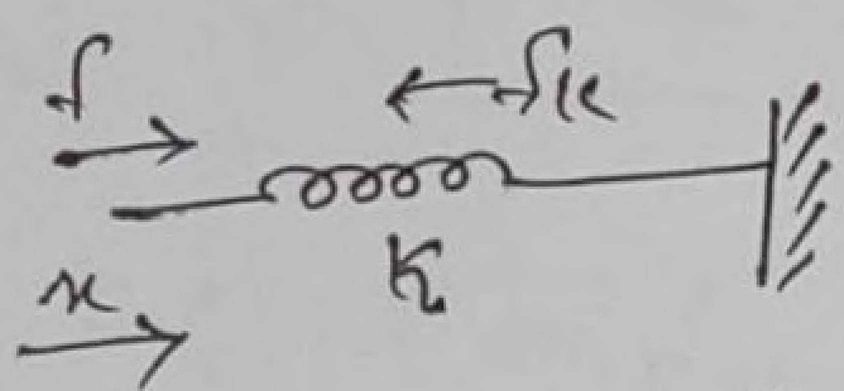
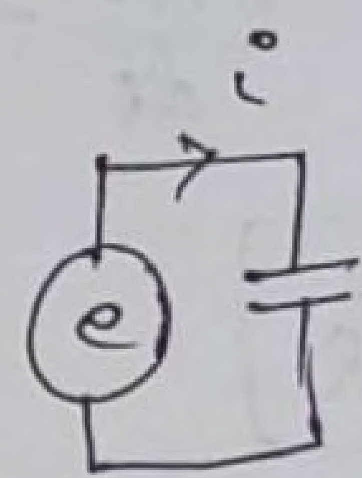
$$T(s) = \theta(s)\left[\frac{(J_2 s^2 + Bs + B_{12}s + K)(J_1 s^2 + B_{12}s + K)}{B_{12}s + K} - (B_{12}s + K)\right]$$

$$T(s) = \theta(s)\left[\frac{(J_2 s^2 + Bs + B_{12}s + K)(J_1 s^2 + B_{12}s + K) - (B_{12}s + K)^2}{B_{12}s + K}\right]$$

$$\Rightarrow \frac{\theta(s)}{T(s)} = \left[\frac{B_{12}s + K}{(J_2 s^2 + Bs + B_{12}s + K)(J_1 s^2 + B_{12}s + K) - (B_{12}s + K)^2}\right]$$

— 2 marks

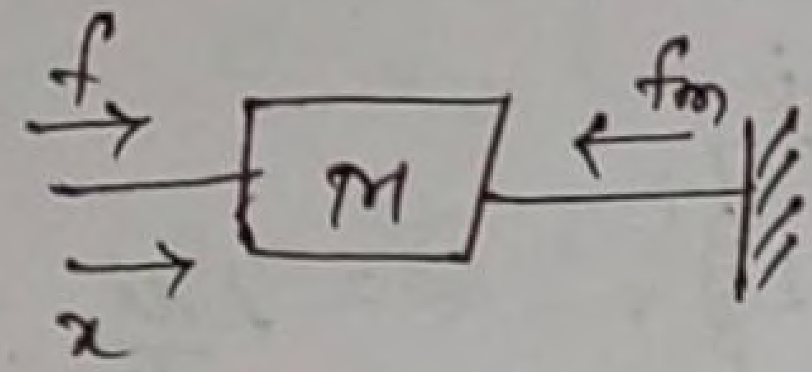
Force Voltage Analogy.

Mechanical system	Electrical system.
Input : Force	Input : Voltage.
Output : Velocity	Output : current.
	
$f_m = M\frac{d^2x}{dt} = M\frac{dv}{dt}$ $\boxed{M = L}$	$e = L\frac{di}{dt}$.
	
$f_b = B\frac{dx}{dt} = BV$ $\boxed{B = R}$	$e = iR$.
	
$f_K = Kx = x\int v\,dt \Rightarrow \boxed{K = \frac{1}{C}}$	$e = \frac{1}{C}\int i\,dt$.
$\frac{d^2x}{dt^2}$	$\frac{dv}{dt}$
$\frac{dx}{dt}$	V
x	$\int v\,dt$.

— 4 marks

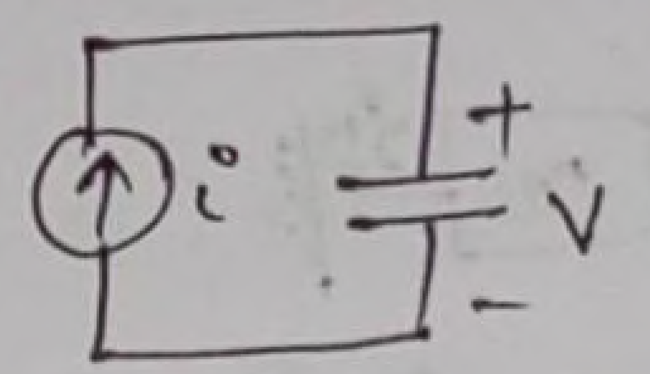
Force Current Analogy.

Mechanical System	Electrical System.
Input : Force	Input : Current
Output : Velocity	Output : Voltage.

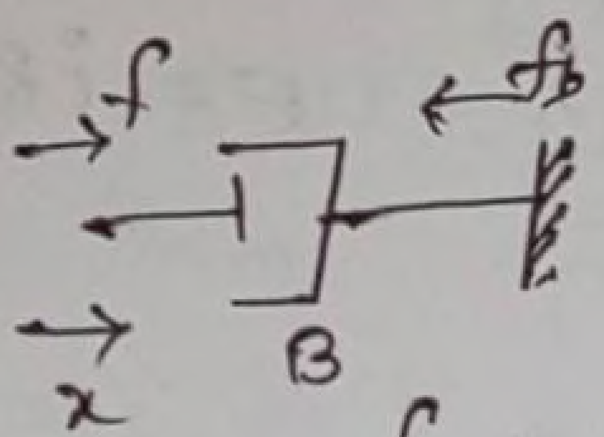


$$f_m = \frac{M d^2x}{dt^2} = M \cdot \frac{dv}{dt}$$

$$\boxed{M = C}$$

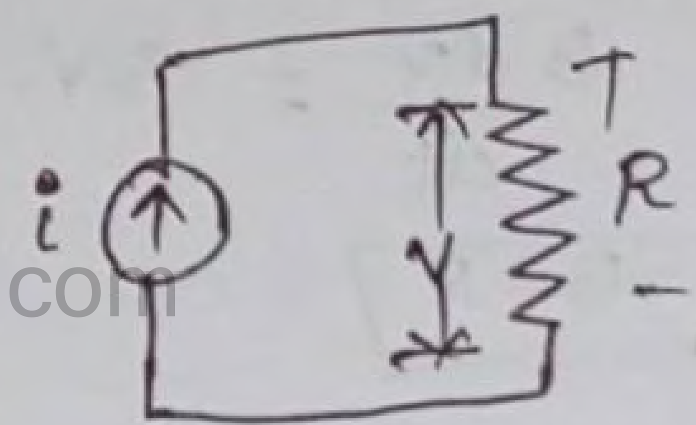


$$i = C \frac{dv}{dt}$$



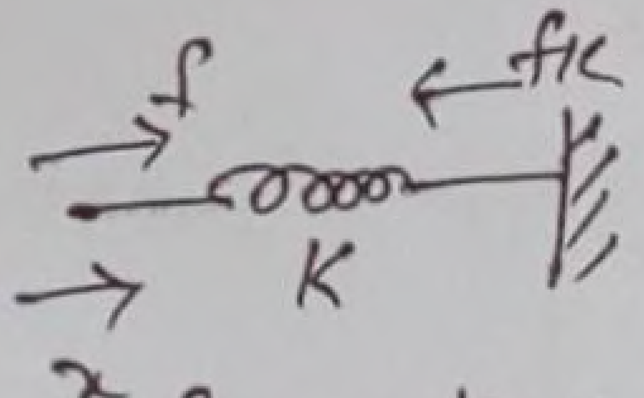
$$f_b = \frac{B\,dx}{dt} = BV$$

$$\boxed{B = \frac{1}{R}}$$

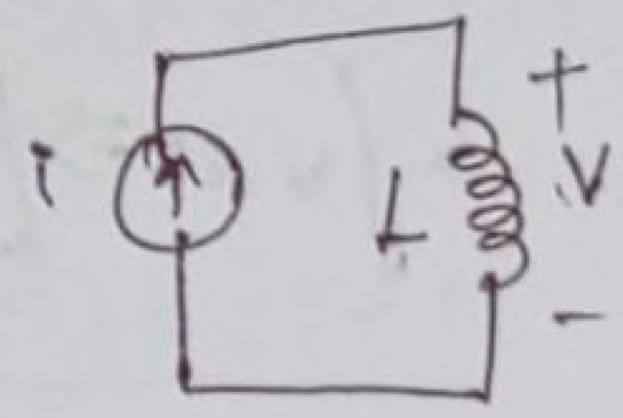


$$i = \frac{V}{R}$$

$$V \propto \frac{1}{R}$$



$$f_K = Kx = K\int v\,dt.$$

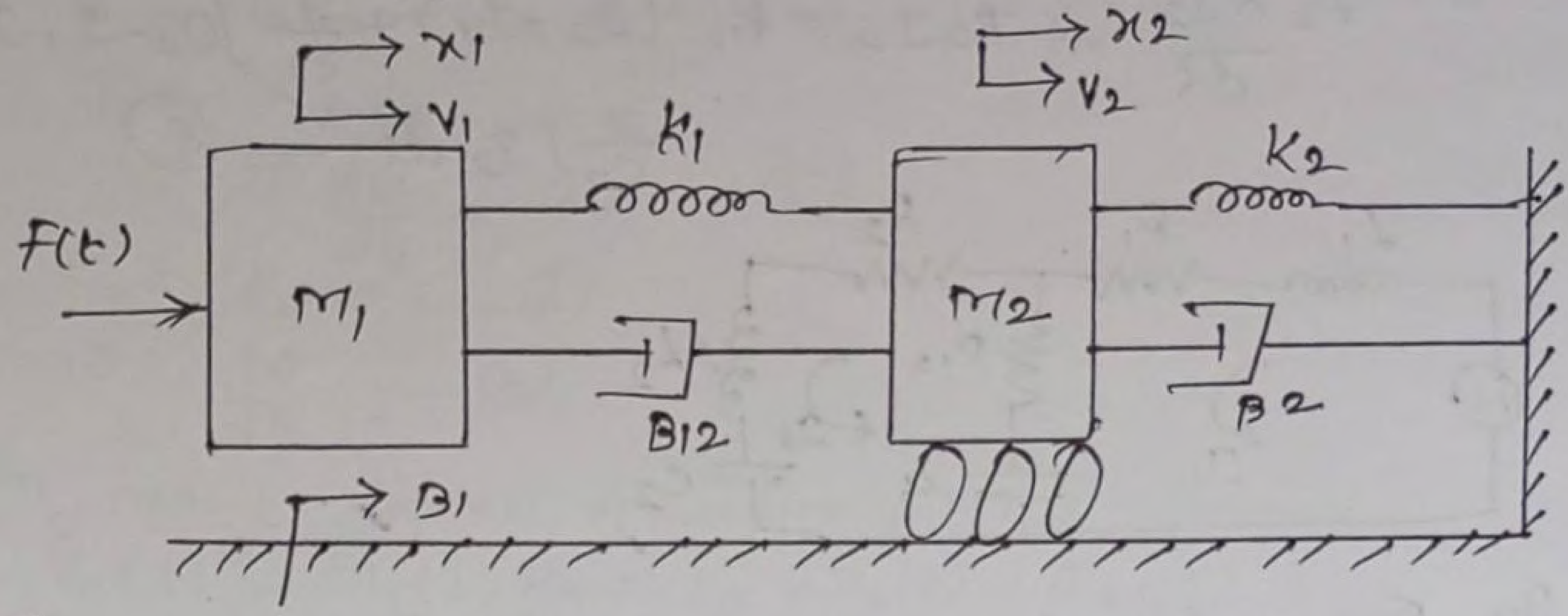


$$i = \frac{1}{L}\int v\,dt$$

← 4 marks

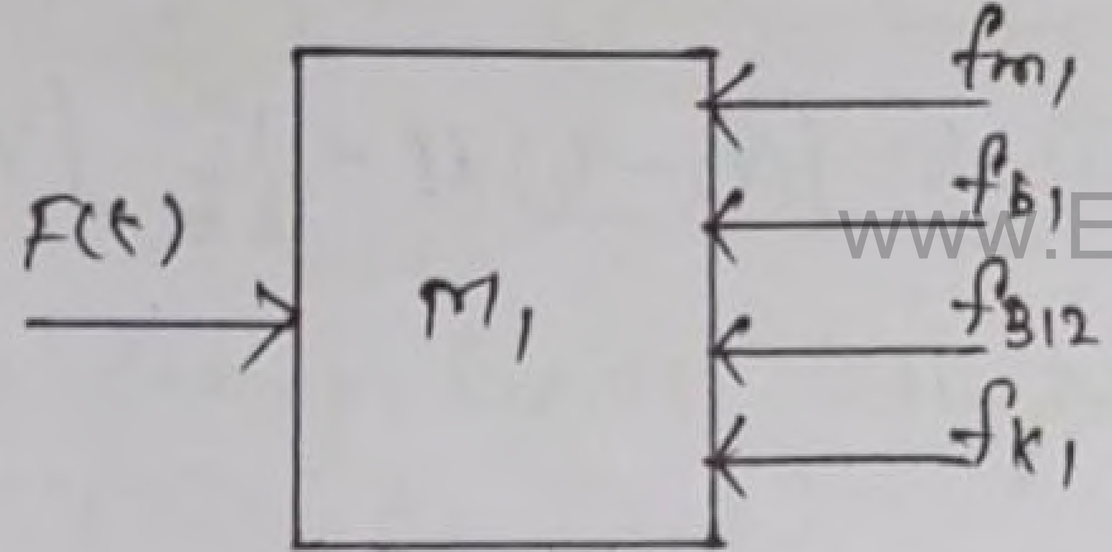
6

Evaluate the differential equation governing the mechanical system shown. Construct the force voltage and force current electrical analogy circuit.



Force balance equation of M_1

Free body diagram of M_1



$$F(t) = f_{m1} + f_{b1} + f_{b12} + f_{k1} \quad (1)$$

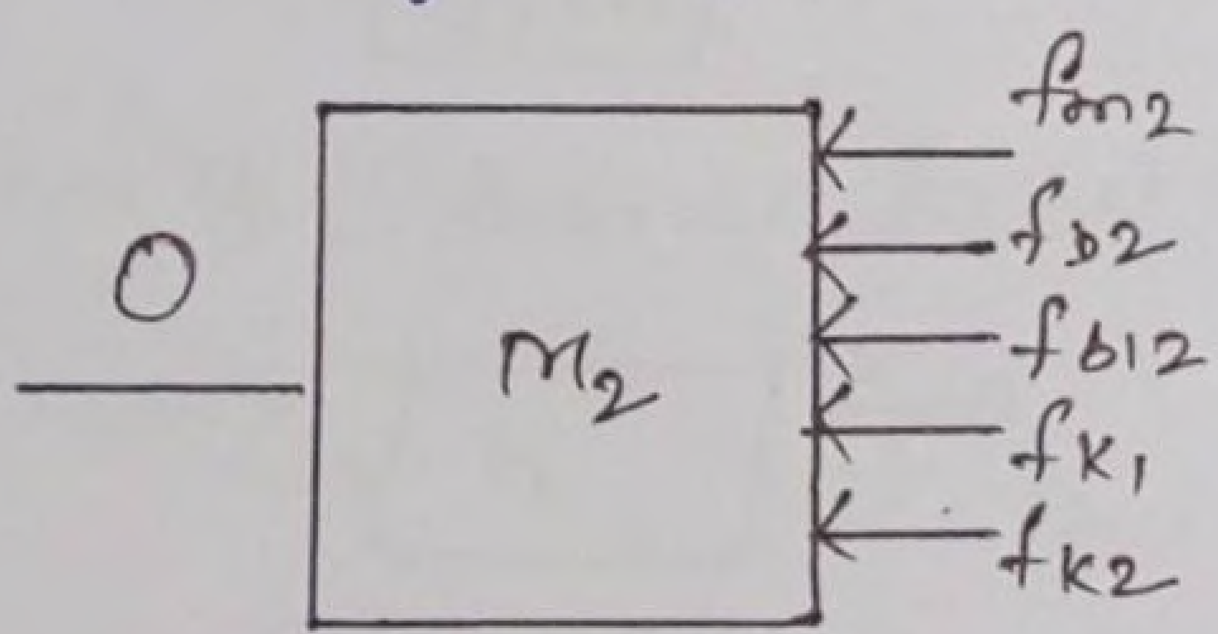
$$F(t) = M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B_{12}\frac{d}{dt}(x_1-x_2) + K_1(x_1-x_2) \quad (2)$$

$$F(t) = M_1\frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1-v_2) + K_1\int(v_1-v_2)\,dt \quad (3)$$

— 2 marks

Force balance equation of M_2

Free body diagram of M_2



$$0 = f_{m2} + f_{b2} + f_{b12} + f_{k1} + f_{k2} \quad (4)$$

$$0 = M_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + B_{12}(x_2-x_1) + K_1(x_2-x_1) + K_2x_2 \quad (5)$$

$$0 = M_2\frac{dv_2}{dt} + B_2v_2 + B(v_2-v_1) + K_1\int(v_2-v_1)\,dt + K_2\int v_2\,dt \quad (6)$$

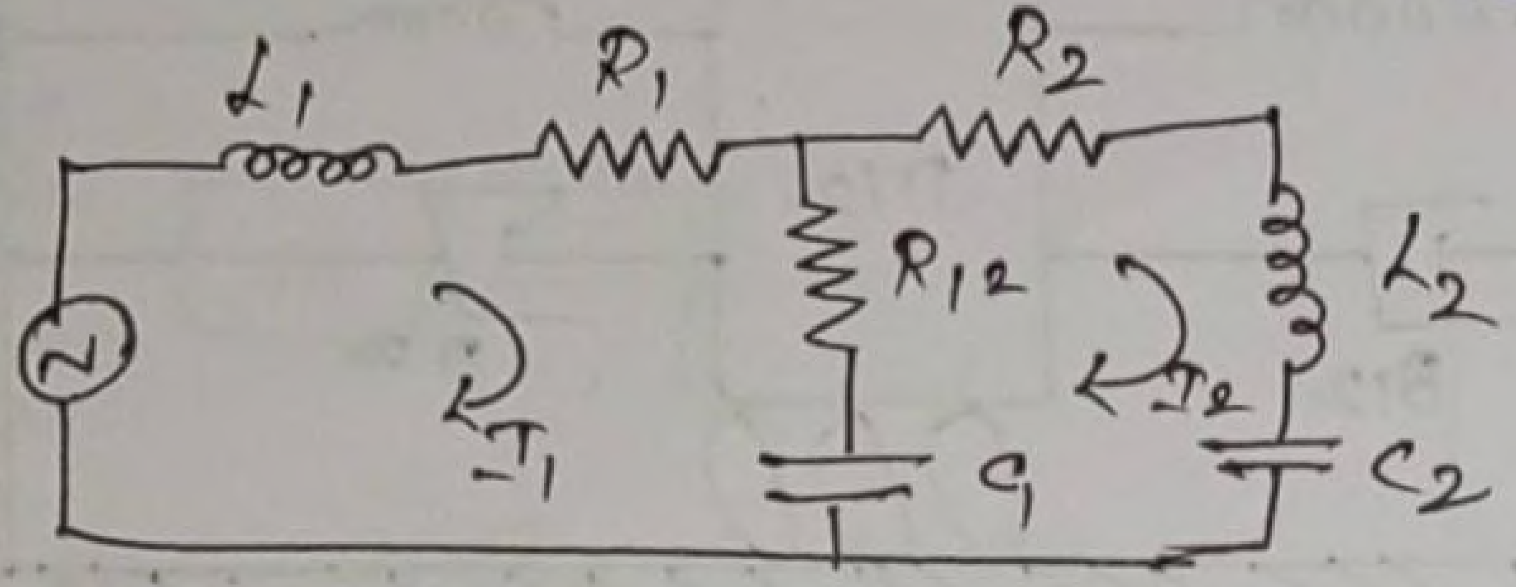
— 2 marks.

By Force - Voltage analogy, eqn ③ becomes,

$$V(t) = L_1 \frac{dI_1}{dt} + R_1 I_1 + R_{12}(I_1 - I_2) + \frac{1}{C_1}\int (I_1 - I_2)\,dt \quad \text{--- (7)}$$

III^ly eqn ⑥,

$$0 = L_2 \frac{dI_2}{dt} + R_2 I_2 + R_{12}(I_2 - I_1) + \frac{1}{C_1}\int (I_2 - I_1)\,dt + \frac{1}{C_2}\int I_2\,dt \quad \text{--- (8)}$$

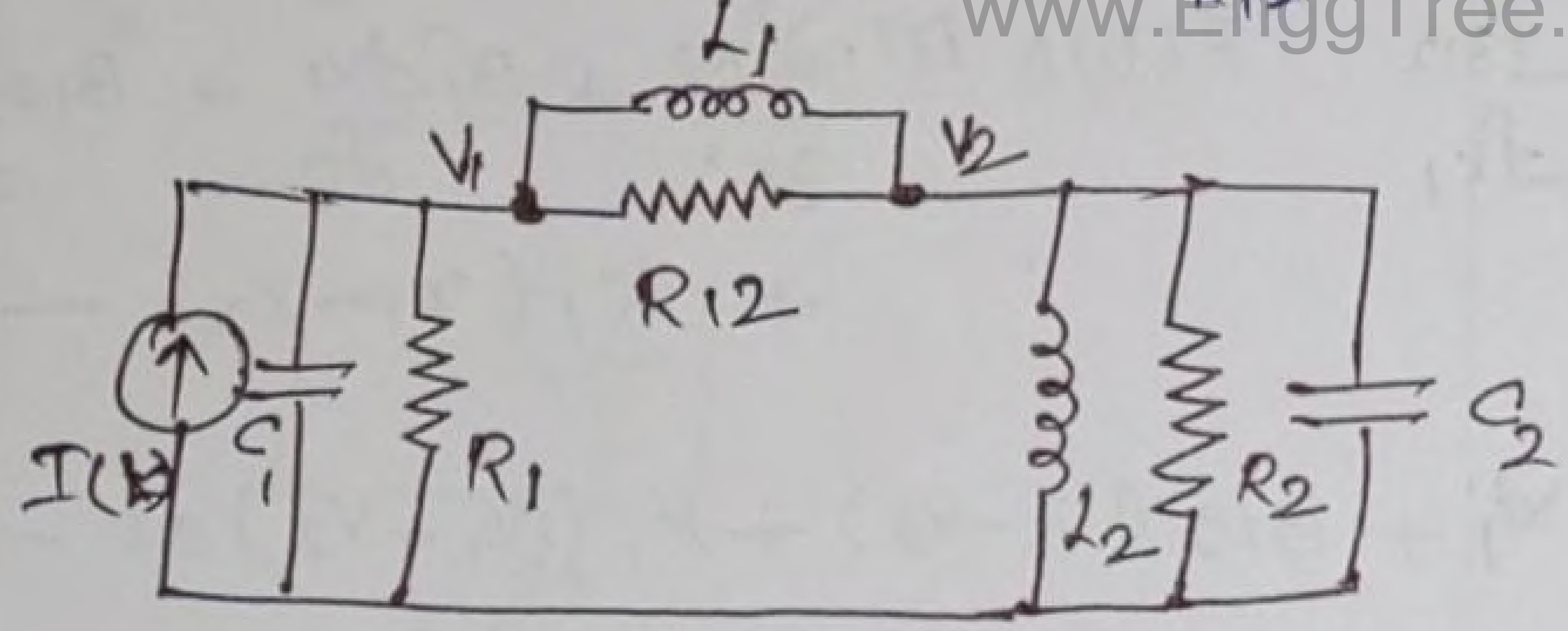


— 2 marks.

By Force - current analogy, eqn ⑤ becomes,

$$I(t) = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_{12}} + \frac{1}{L_1}\int (V_1 - V_2)\,dt \quad \text{--- (9)}$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{V_2}{R_2} + \frac{V_2 - V_1}{R_{12}} + \frac{1}{L_1}\int (V_2 - V_1)\,dt + \frac{1}{L_2}\int V_2\,dt \quad \text{--- (10)}$$



— 2 marks.

BLOCK DIAGRAM AND SIGNAL FLOW GRAPH REPRESENTATION

Construct the block diagram of armature controlled dc motor.

The differential equations governing the armature controlled dc motor are

$$V_a = i_a R_a + L_a \frac{di_a}{dt} + e_b \quad \text{--- (1)}$$

$$T = K_t i_a \quad \text{--- (2)}$$

$$T = J \frac{d\omega}{dt} + B\omega \quad \text{--- (3)}$$

$$e_b = K_b \omega \quad \text{--- (4)}$$

$$\omega = \frac{d\theta}{dt} \quad \text{--- (5)}.$$

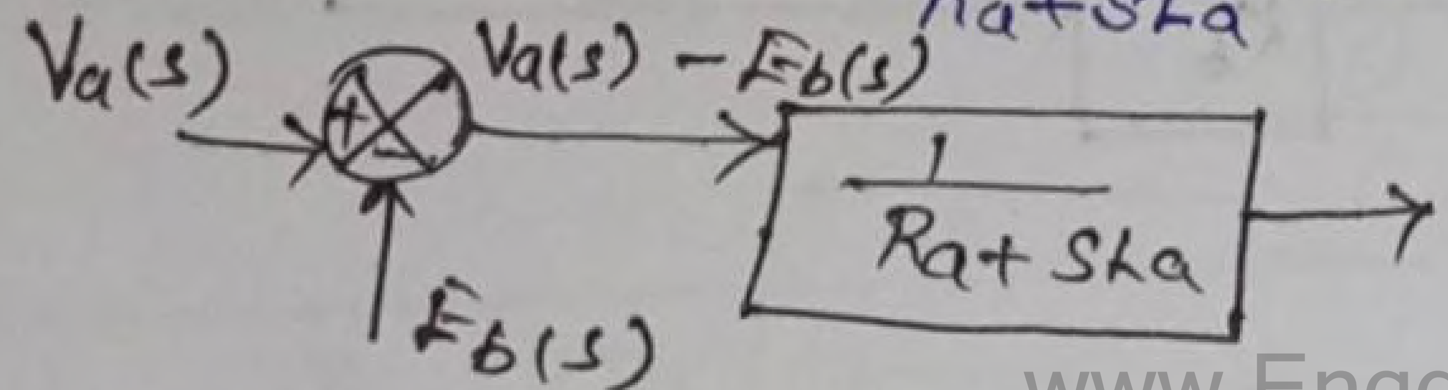
On taking Laplace transform, eqn (1) becomes

$$V_a(s) = I_a(s) R_a + L_a s I_a(s) + E_b(s)$$

$$V_a(s) - E_b(s) = I_a(s) [R_a + s L_a]$$

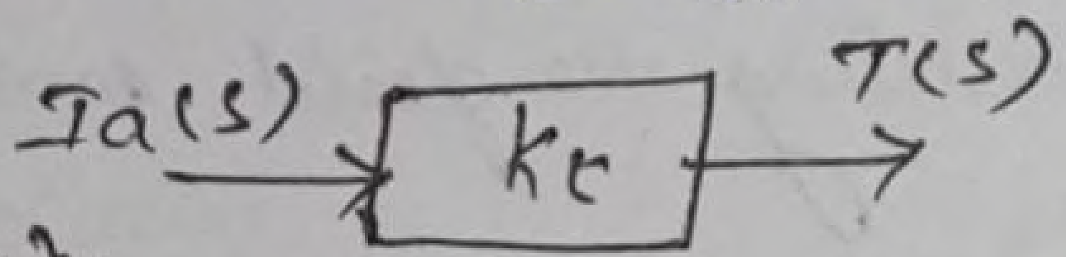
$$\therefore I_a(s) = \frac{1}{R_a + sL_a} [V_a(s) - E_b(s)]$$

— 3 marks



On taking laplace transform, eqn (2) becomes

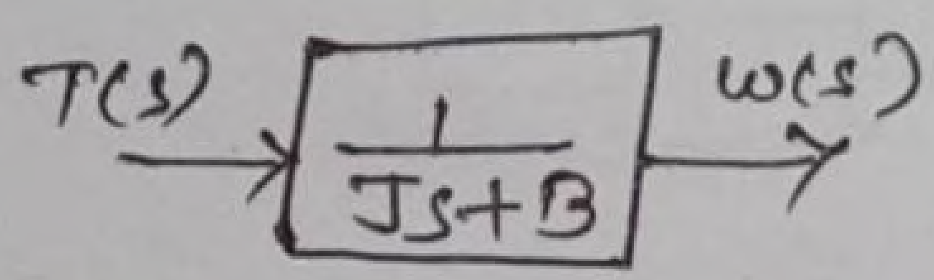
$$T(s) = K_t I_a(s).$$



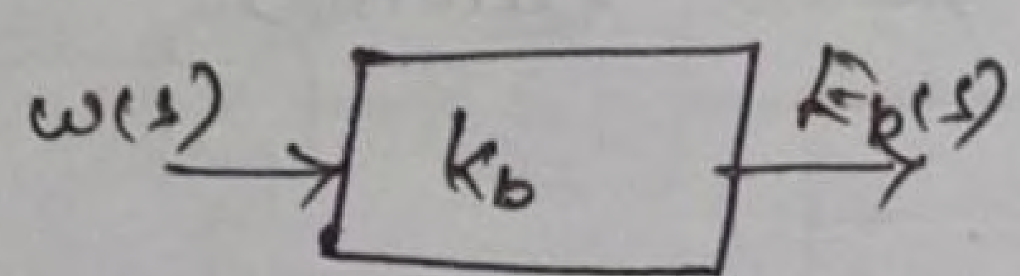
Il^{ly} eqn (3) becomes, $T(s) = Js\,\omega(s) + B\omega(s)$

$$= (Js + B)\,\omega(s)$$

$$\Rightarrow \omega(s) = \frac{1}{Js+B} \cdot T(s).$$



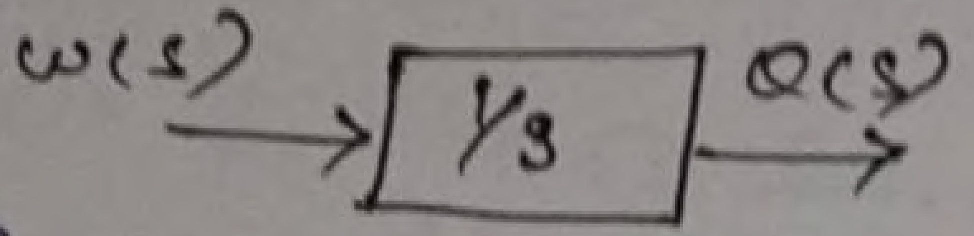
eqn (4), becomes $E_b(s) = K_b\,\omega(s)$



eqn ⑤ becomes,

$$\omega(s) = s\,\theta(s).$$

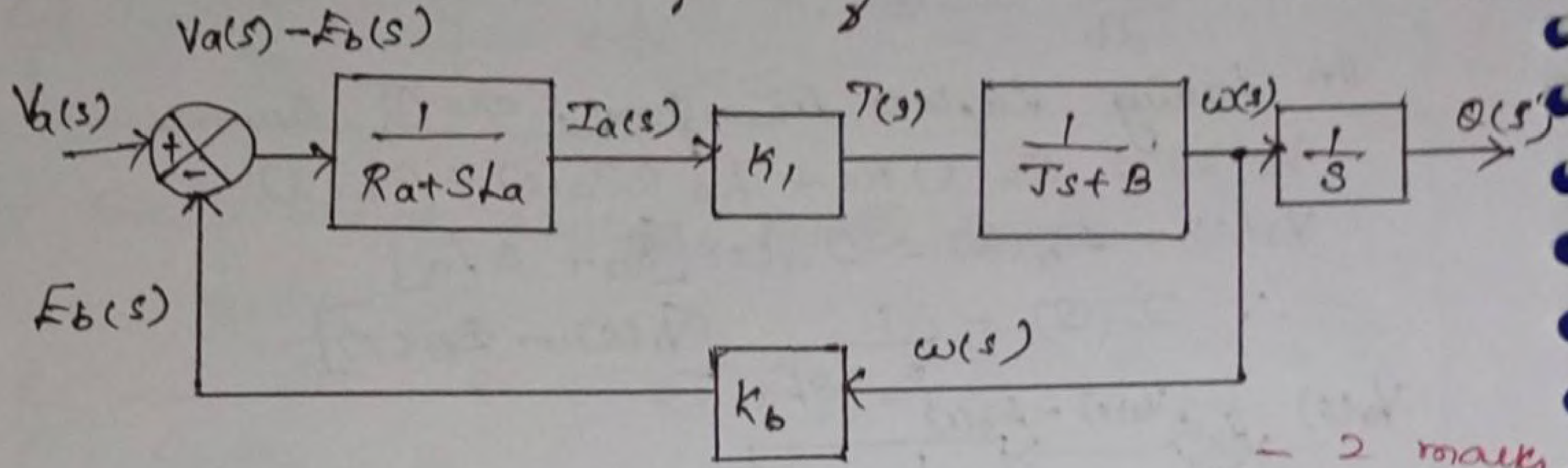
$$\theta(s) = \frac{1}{s}(\omega(s))$$



— 3 marks

The overall block diagram of armature controlled dc motor is obtained by connecting the various section,

Block diagram of armature controlled dc motor.



— 2 marks

Construct the block diagram of field controlled dc motor.

The differential equations governing the field controlled dc motor are

$$V_f = R_f\, i_f + L_f \frac{di_f}{dt} \quad \text{— ①}$$

$$T = K_{tf} \cdot i_f \quad \text{— ②}$$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \quad \text{— ③}.$$

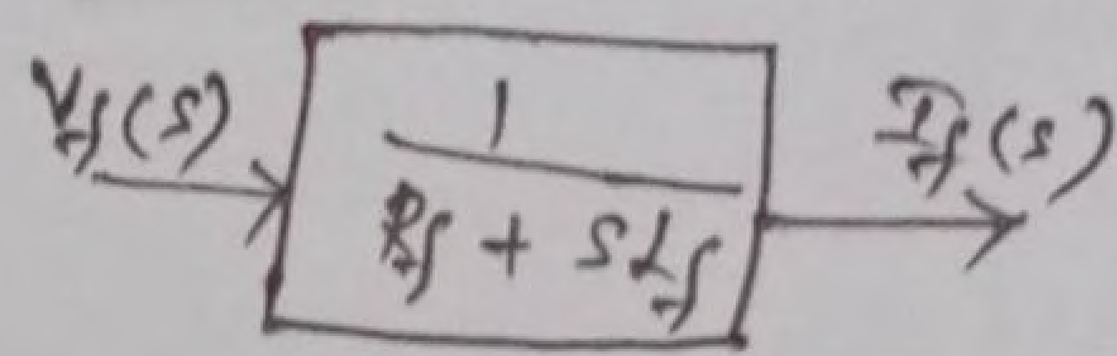
— 2 marks

On taking Laplace transform of eqn ① becomes

$$V_f(s) = R_f\, I_f(s) + L_f\, s \cdot I_f(s).$$

$$V_f(s) = I_f(s)\left[R_f + L_f \cdot s\right]$$

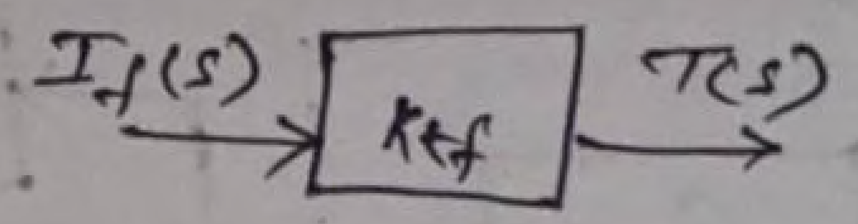
$$\Rightarrow I_f(s) = \frac{1}{R_f + sL_f} \cdot V_f(s).$$



— 2 marks

equ (5) becomes,

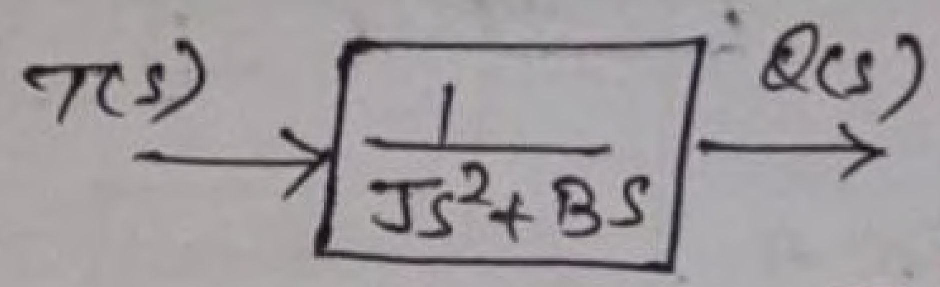
$$T(s) = K_{tf}\, I_f(s)$$



equ (3) becomes,

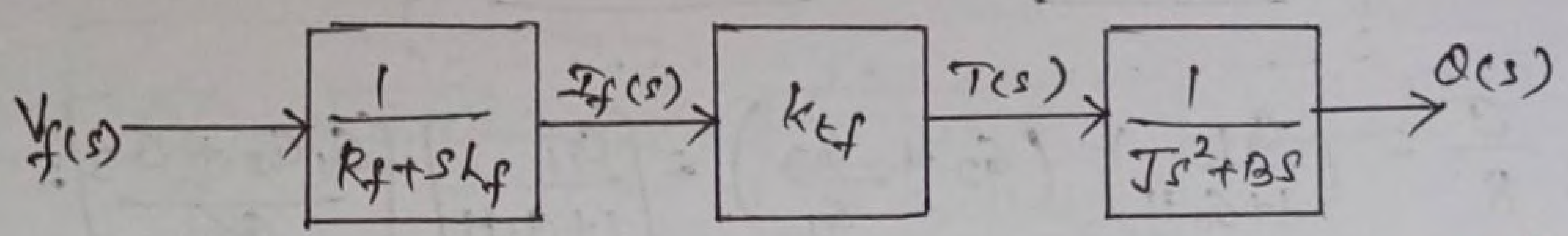
$$T(s) = J s^2 \theta(s) + B s \theta(s).$$

$$\therefore \theta(s) = \frac{1}{Js^2 + Bs} \cdot T(s).$$



— 2 marks

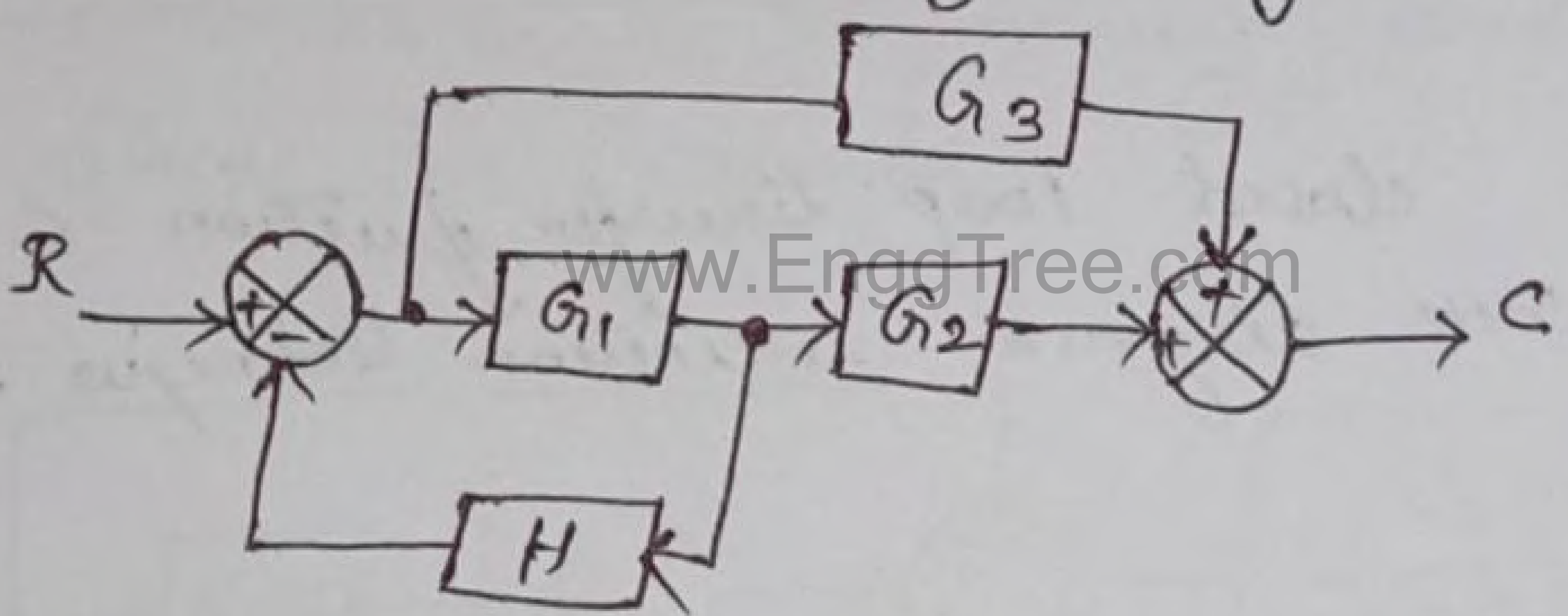
Overall block diagram of field controlled dc motor.



— 2 marks

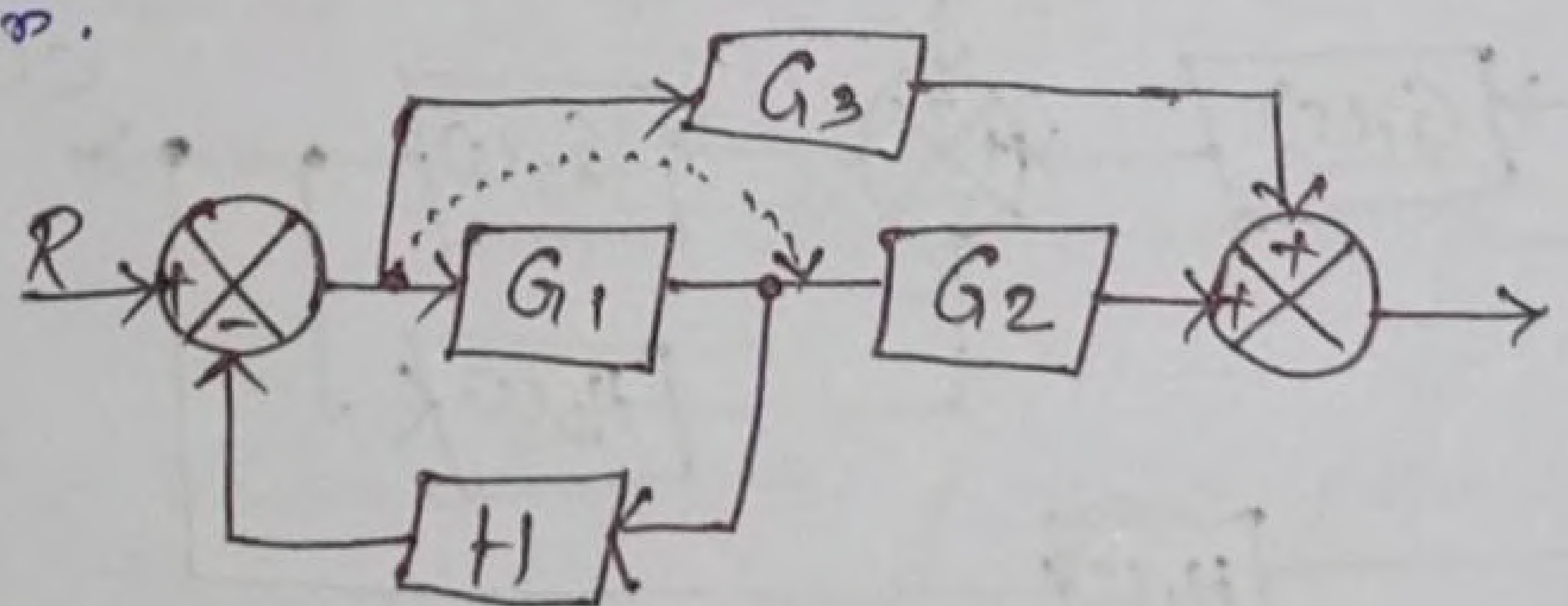
REDUCTION OF BLOCK DIAGRAM.

Reduce the block diagram given and find C/R.



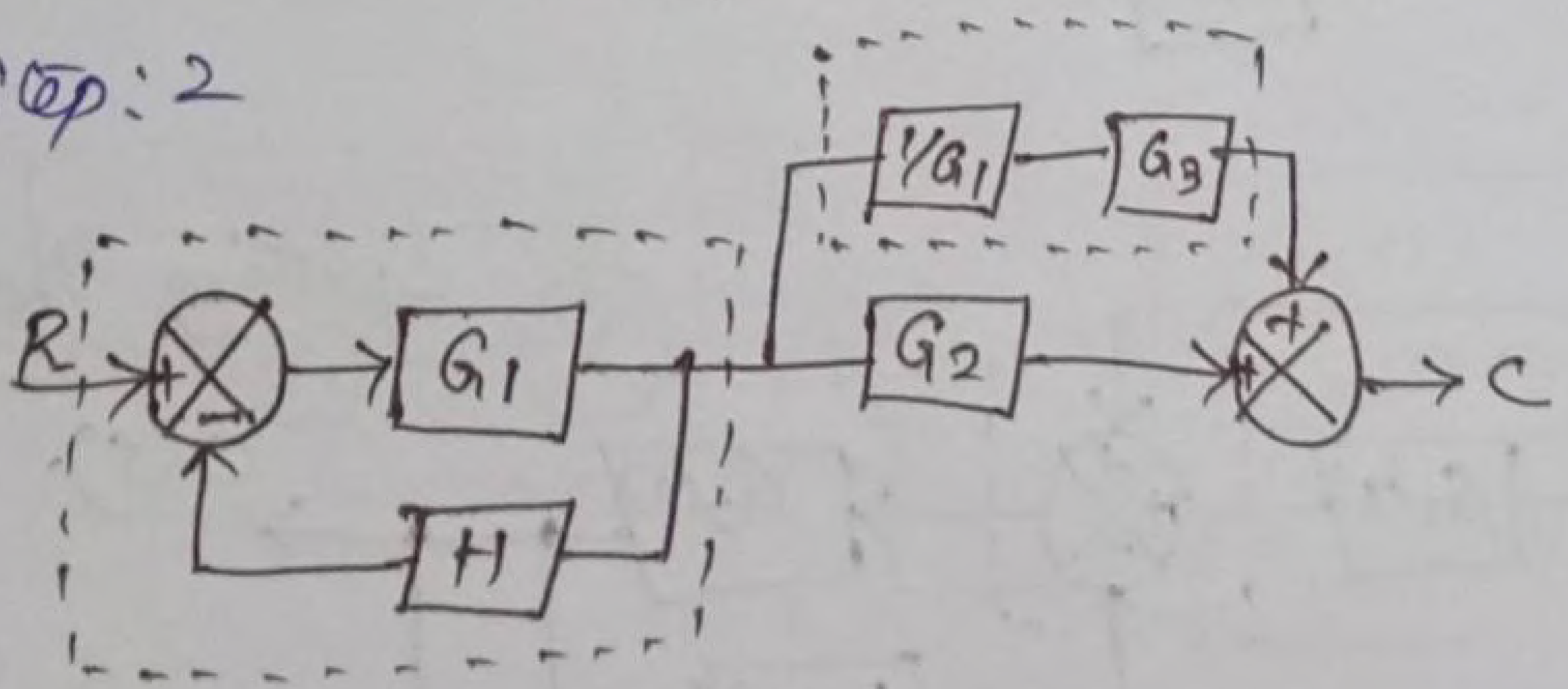
Solution.

Step: 1



moving branch point after the block.

Step: 2

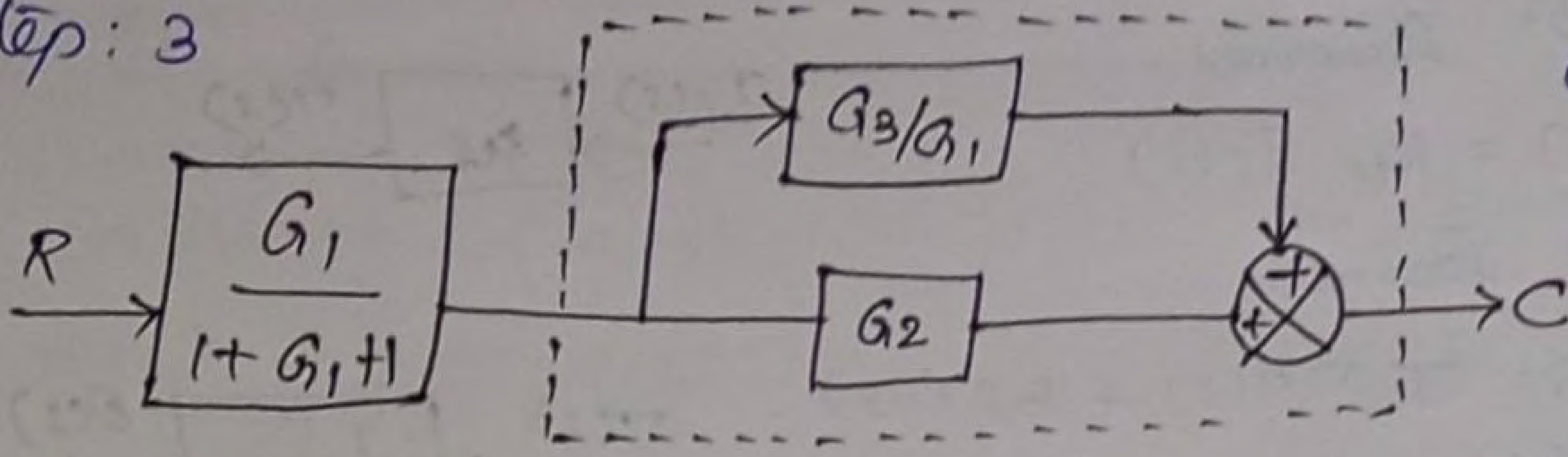


Eliminating the feedback path and combining blocks in cascade

— 4 marks.

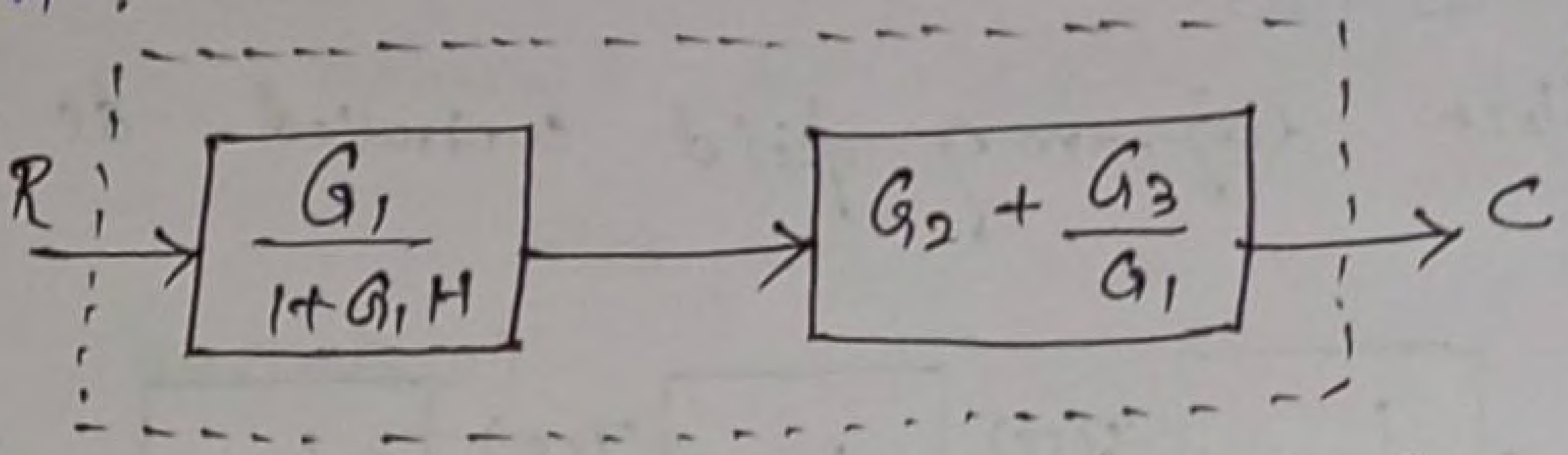
Step: 3

Combining parallel blocks.



Step: 4

Combining blocks in Cascade.

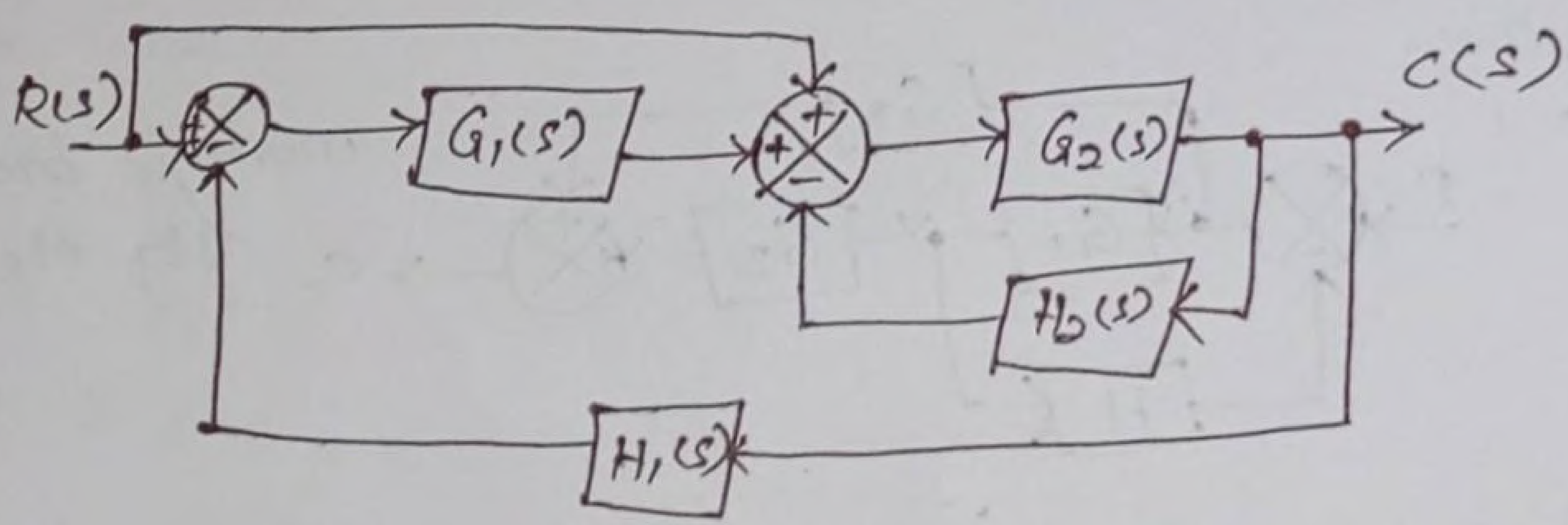


$$\frac{C}{R} = \left(\frac{G_1}{1+G_1H}\right)\left(G_2 + \frac{G_3}{G_1}\right) = \left[\frac{\cancel{G_1}}{1+G_1H}\right]\left[\frac{G_1G_2+G_3}{\cancel{G_1}}\right] = \frac{G_1G_2+G_3}{1+G_1H}$$

$\therefore$ The overall transfer function of the system, $\frac{C}{R} = \frac{G_1G_2+G_3}{1+G_1H}$.

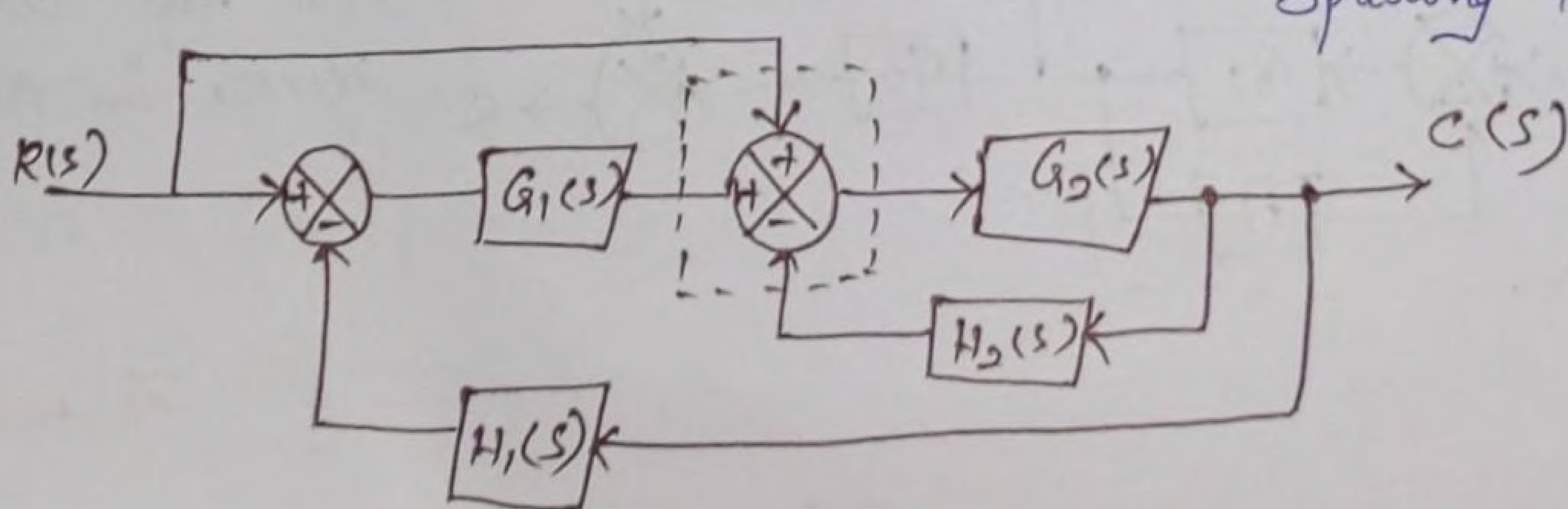
4 marks.

Determine the closed loop transfer function $C(s)/R(s)$, by using the block diagram reduction technique.



Step: 1

Splitting the Summing Point.



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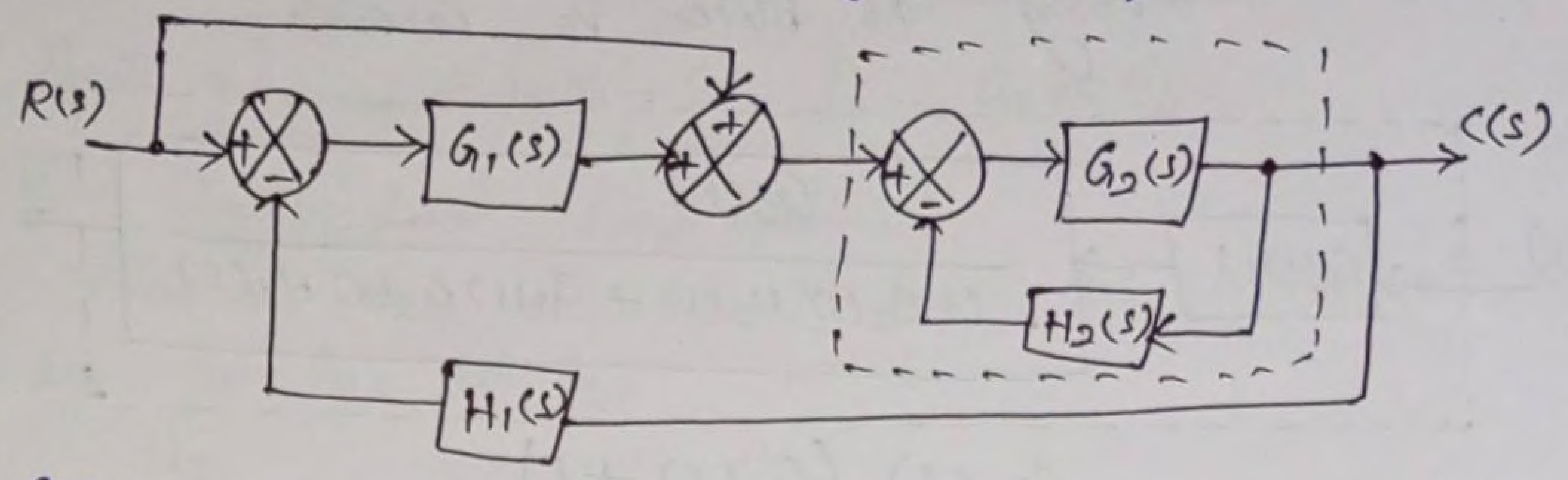
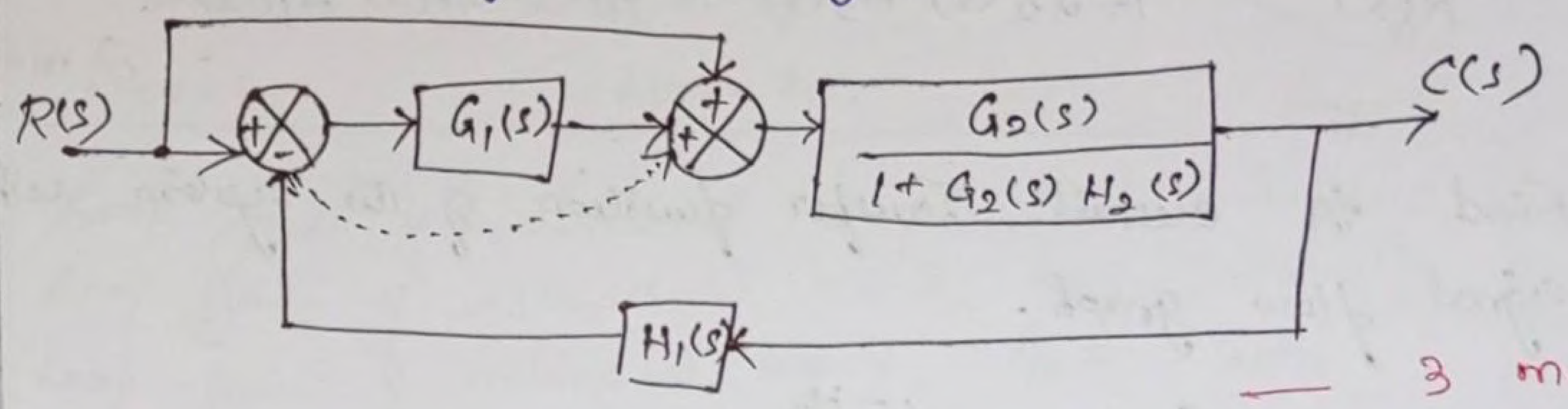
Step: 2. Eliminating the feedback path.

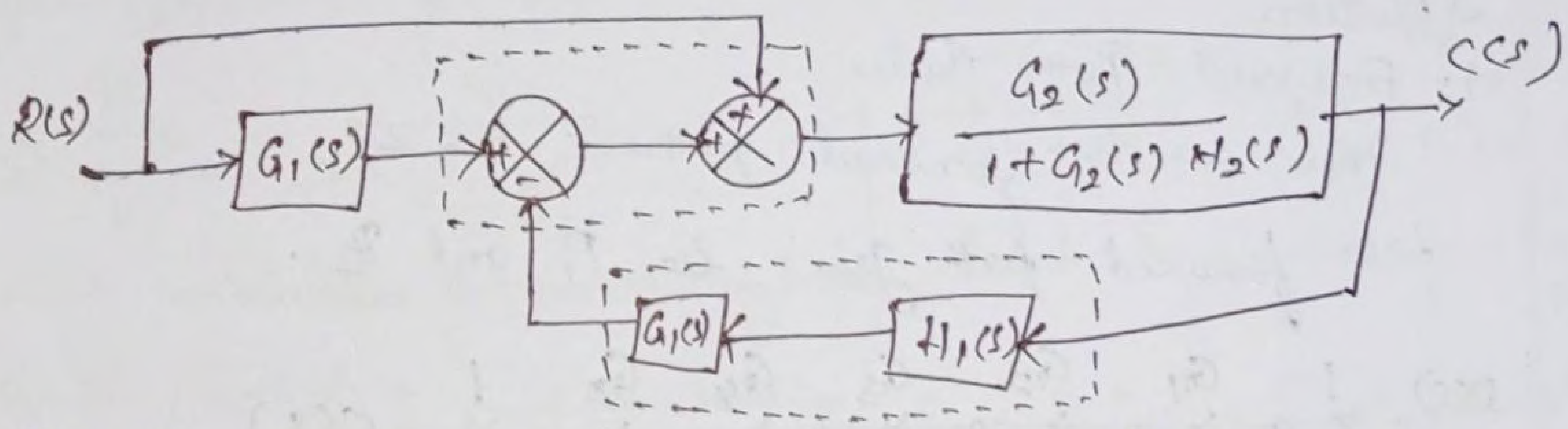


Step: 3. Moving the summing point after the block.

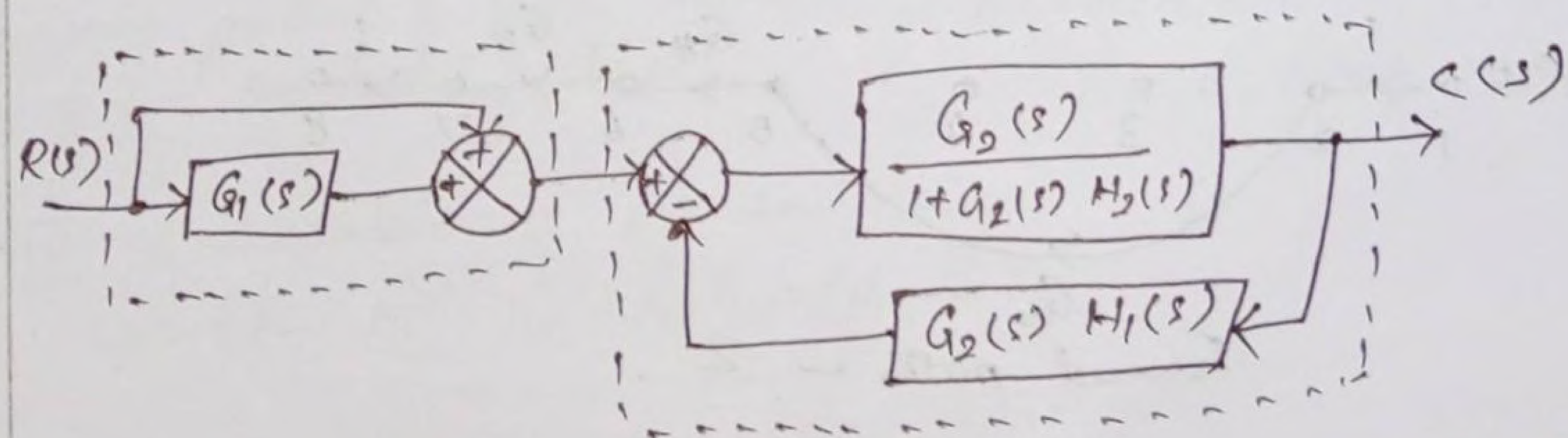


— 3 marks

Step: 4 Interchanging the summing points and combining the blocks in cascade.



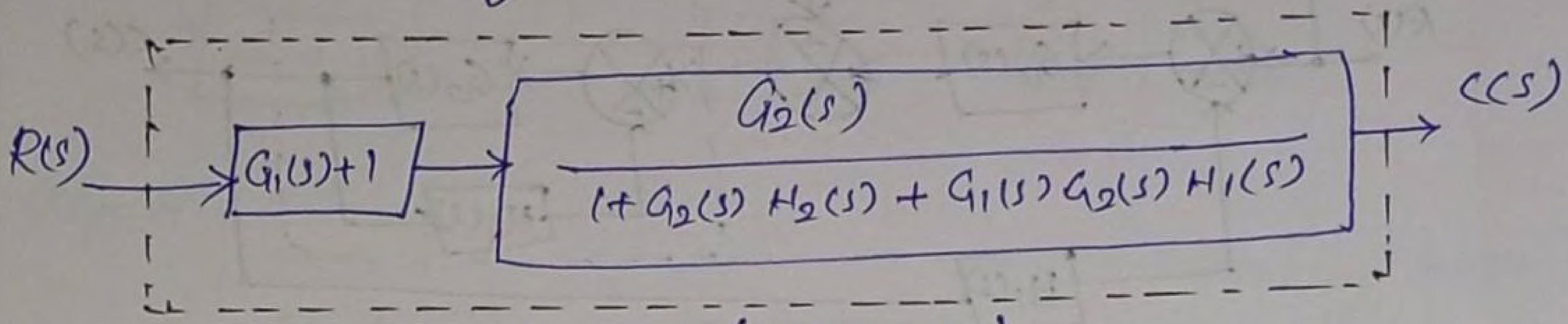
Step: 5 Eliminating the feedback path and feed forward path.



— 3 marks

REDUCTION OF SIGNAL FLOW GRAPH.

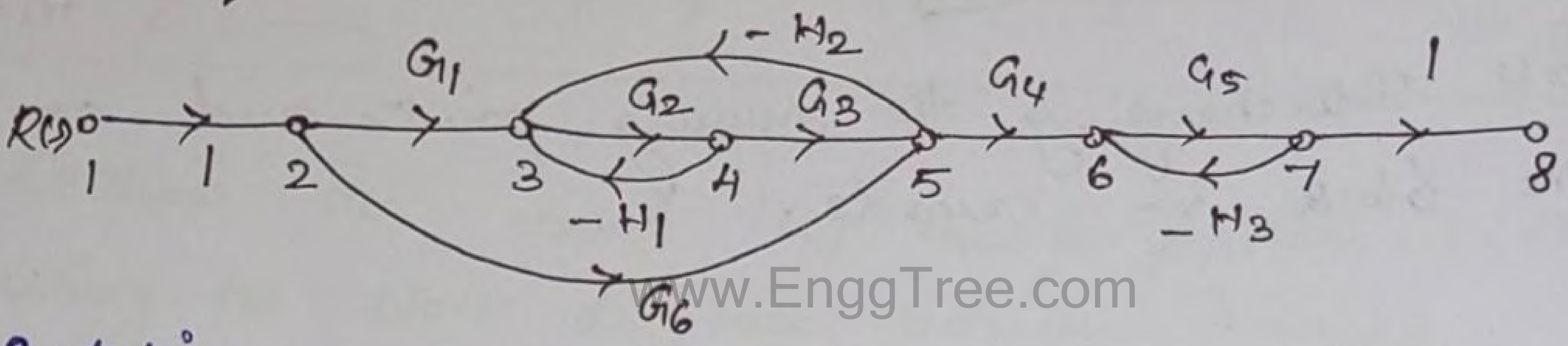
Step: 6 Combining the blocks in cascade,



$$\therefore \frac{C(s)}{R(s)} = \frac{G_2(s)\,(G_1(s)+1)}{1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)}.$$

— 2 marks.

Find the overall transfer function of the system whose signal flow graph.

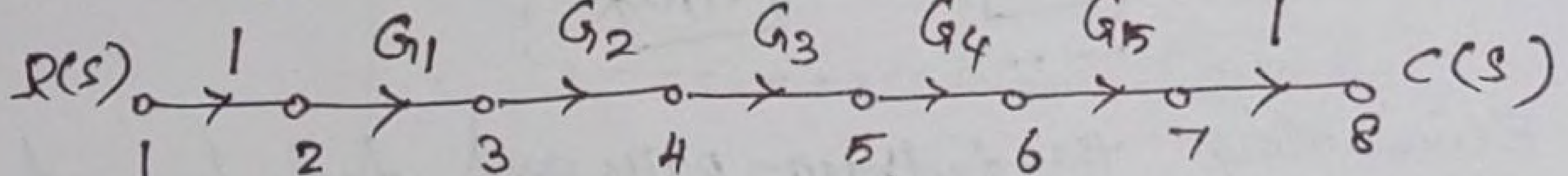


Solution

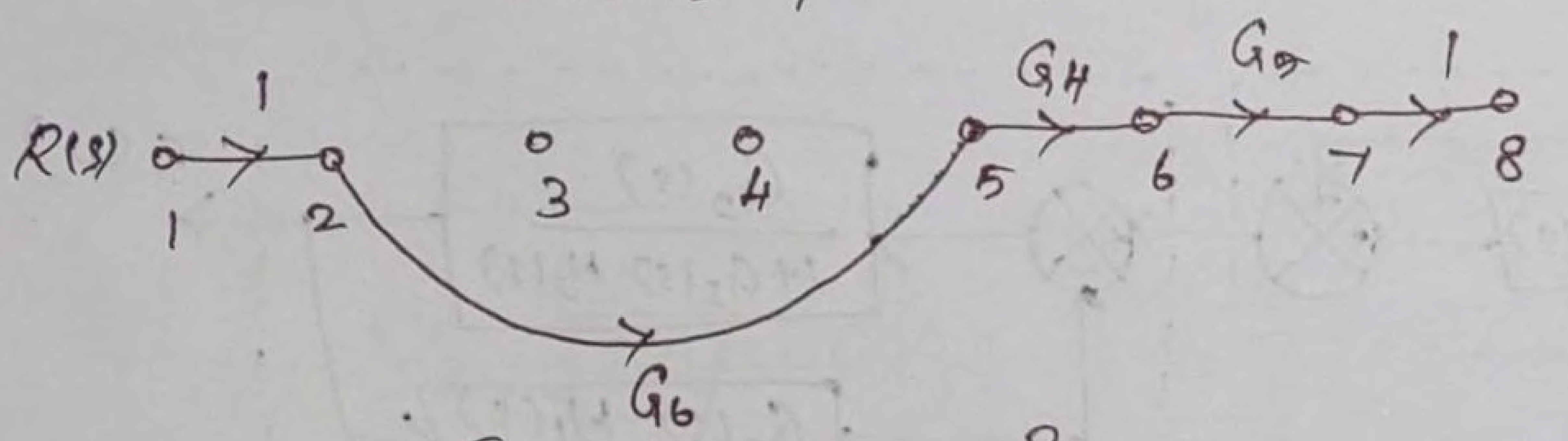
1. Forward Path Gain

There are two forward paths $\therefore K = 2$.

Let forward path gains be P_1 and P_2.



Forward path – 1



Forward path – 2.

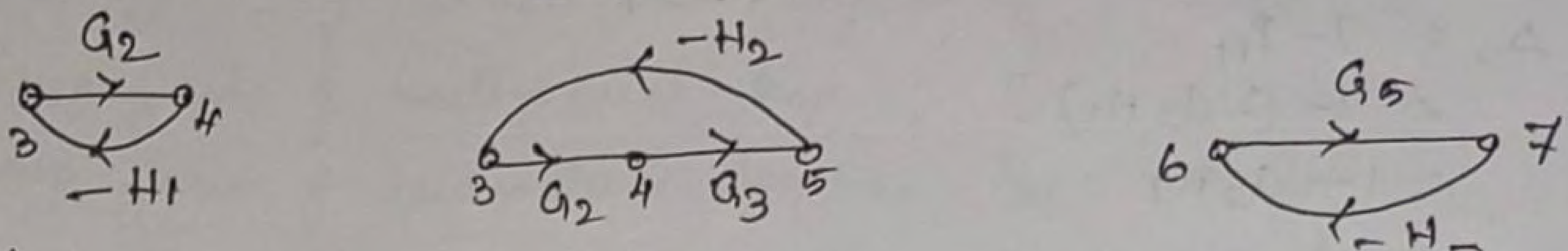
Gain of forward path-1, $P_1 = G_1 G_2 G_3 G_4 G_5$.

Gain of forward path-2, $P_2 = G_4 G_5 G_6$.

— 1 mark

II. Individual Loop Gain.

There are three individual loops. Let individual loop gains be P_{11}, P_{21} and P_{31}.



Loop-1 Loop-2 Loop-3.

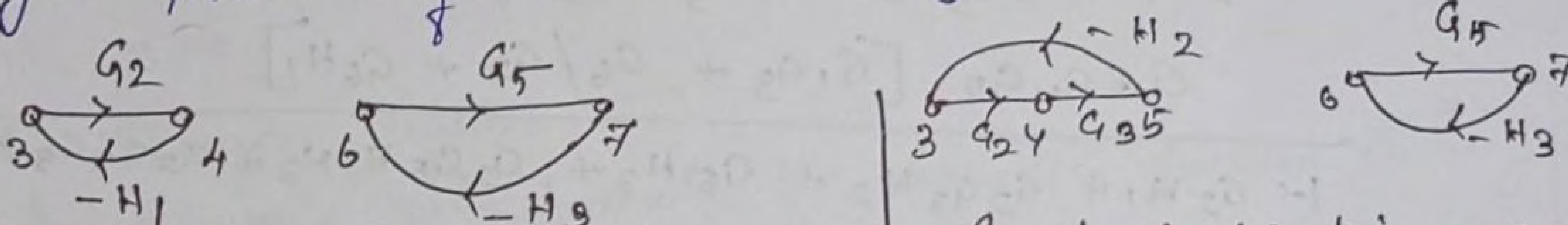
Loop gain of individual loop-1, $P_{11} = -G_2H_1$

Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Loop gain of individual loop-3, $P_{31} = -G_5H_3$ — 2 marks

III - Gain Products of Two Non-touching Loops.

There are two combinations of two non-touching loops. Let the gain products of two non-touching loops be P_{12} and P_{22}.



First combination of 2-non-touching loops. Second combination of 2-non-touching loops.

Gain product of first combination of two non touching loops $\Big\} = P_{12} = P_{11} P_{31} = (-G_2H_1)(-G_5H_3)$
$= G_2G_5H_1H_3$.

Gain product of second combination of two non touching loops $\Big\} = P_{22} = P_{21} P_{31} = (-G_2G_3H_2)(-G_5H_3)$
$= G_2G_3G_5H_2H_3$.

— 2 marks.

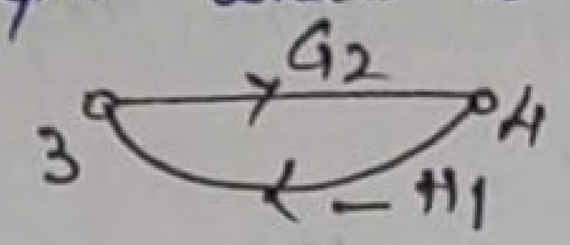
IV Calculation of Δ and Δ_g.

$\Delta = (1 - P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$.

$$\Delta = 1-(-G_2H_1 - G_2G_3H_2 - G_5H_2) + (G_2G_5H_1H_2 + G_2G_3G_5H_2H_3)$$
$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3.$$

$\Delta_1 = 1$, Since there is no part of graph which is not touching with first forward path.

The part of the graph which is not touching with second forward path is



$$\Delta_2 = 1 - P_{11}$$
$$= 1-(-G_2H_1)$$
$$= 1 + G_2H_1$$

V. Transfer function, T.

By Mason's gain formula the transfer function, T is given by

$$T = \frac{1}{\Delta}\sum_K P_K \Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

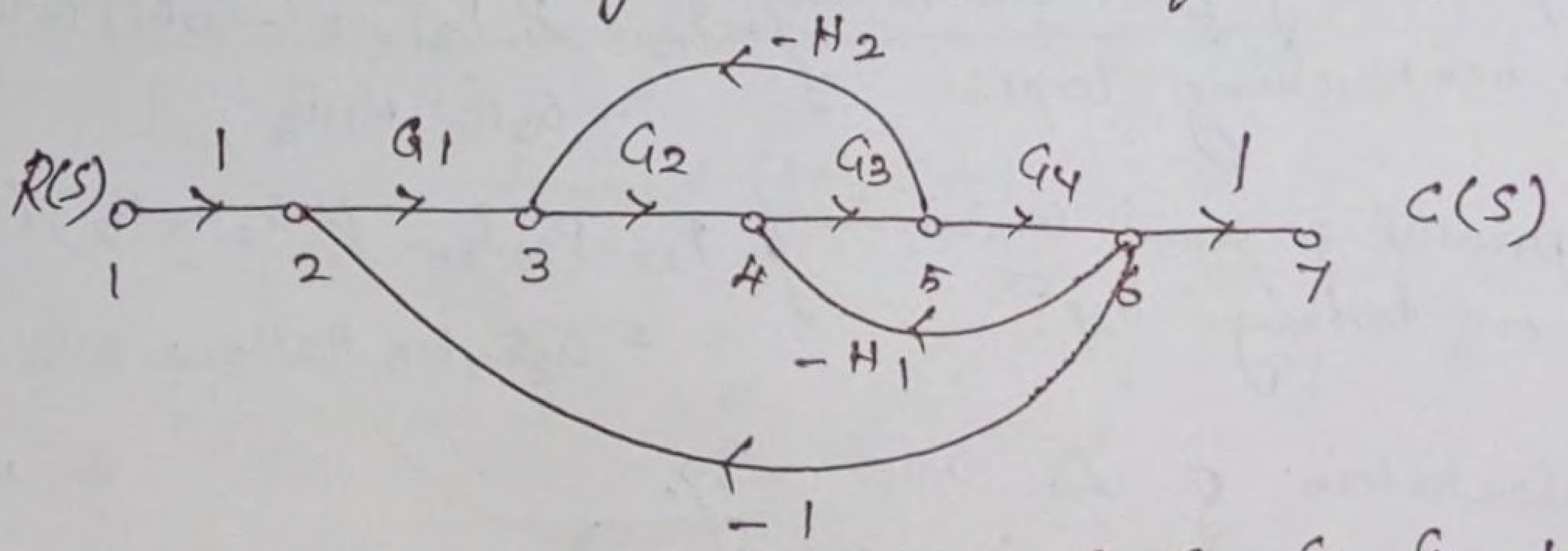
$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1+G_2H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

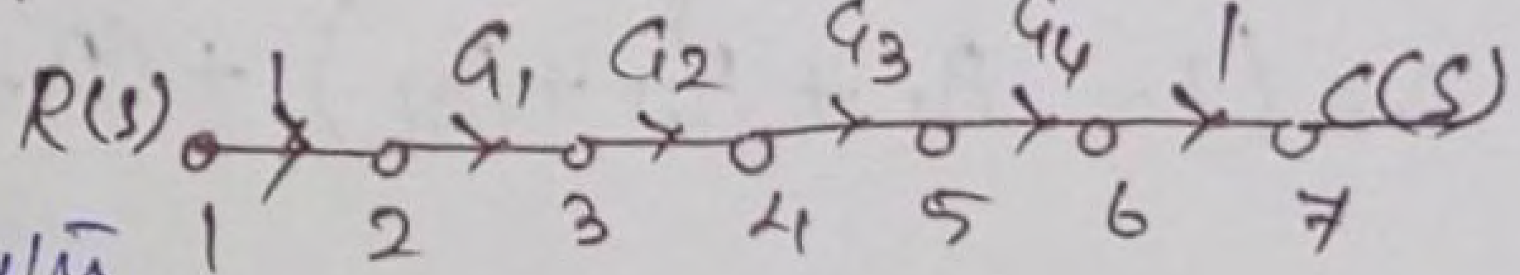
$$= \frac{G_2G_4G_5\left[G_1G_3 + G_6/G_2 + G_6H_1\right]}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}.$$

— 3 marks

Find the overall gain $C(s)/R(s)$ for the signal flow graph.



Forward Path Gain.



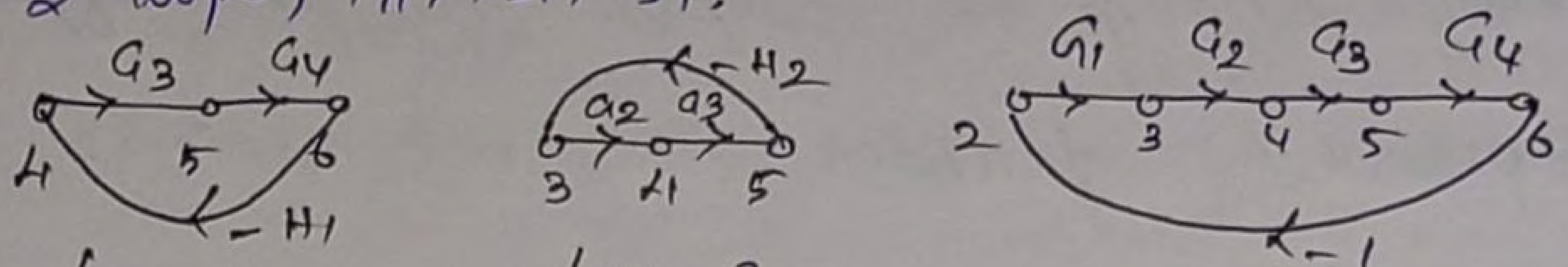
There is only one forward path

$\therefore K = 1$. $\therefore$ $P_1 = G_1G_2G_3G_4$.

— 1 mark

II. Individual loop gain.

3 loops, P_{11}, P_{21}, P_{31}.



Loop-1 Loop-2 Loop-3.

Loop gain of individual loop-1, $P_{11} = -G_3G_4H_1$

Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Loop gain of individual loop-3, $P_{31} = -G_1G_2G_3G_4$. — 2 marks

III. Gain Products of Two Non-touching Loops.

There are no possible combinations of two non-touching loops, three non-touching loops, etc. — 1 mark

IV Calculation of Δ and Δ_K.

$$\Delta = 1 - (P_{11} + P_{21} + P_{31})$$
$$= 1 - (-G_3G_4H_1 - G_2G_3H_2 - G_1G_2G_3G_4)$$
$$= 1 + G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4$$

Since no part of the graph is non-touching with forward path-1

$\Delta_1 = 1$. — 2 marks

V Transfer function, T.

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_K P_K \Delta_K = \frac{1}{\Delta} P_1 \Delta_1$$

$$= \frac{G_1G_2G_3G_4}{1 + G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4}$$

— 2 marks.

OPEN LOOP & CLOSED LOOP SYSTEMS.

OPEN LOOP SYSTEMS.

* Any change in output has no effect on the input (ie). feedback does not exist.
* Output measurement is not required for operation of system.
* Feedback element is absent.
* Error detector is absent.
* It is inaccurate and unreliable.
* Highly sensitive to the disturbances
* Highly sensitive to the environmental factors

* Bandwidth is small
* Simple to construct and cheap.
* Generally are stable in nature
* Highly affected by non linearities.
* Control system in which the output has an affect upon the input quantity in order to maintain the desired output values.

CLOSED LOOP SYSTEMS.

* Changes in output, affects the input which is possible by use of feedback.
* Output measurement is necessary.
* Feedback element is present.
* Error detection is necessary.
* Highly accurate and reliable.
* Less sensitive to the disturbances.
* Less sensitive to the environmental factors.
* Bandwidth is large.
* Complication to design and costly.
* Stability is the major consideration while designing.
* Reduced effect of non linearities.
* Closed loop system is also called automatic control system.

INTRODUCTION TO PHYSIOLOGICAL CONTROL SYSTEMS.

* A physiological control system explores some control problem related to biological environment and provides solution to the control researchers.

* The involvement of control theory makes the biomedical application more efficient and natural.

* Physiological control systems for LVADs should be designed to respond to changes in hemodynamic across a variety of clinical scenarios and patients by automatically adjusting the heart pump speed.

* The set of control systems and the set of CPSs have got a (non-empty) intersection.

* For this intersection the term control system defines the application perspective and the term CPSs defines the infrastructure perspective.

* Physiological regulation can be defined as the integrated neural control mechanism, underlying somatic, autonomic and neuroendocri activity.

* The science of CPS has the capability to impact technology in a wide variety of industri and Organizations and CPS allow us to imagin create, develop, refine and perpetuate smart syste in fields that result in the betterment of industr communities and individuals.

* Although there are several types of contro systems, there are two main types: open-loop a closed-loop control systems.

* A manual control system is also considered an Open-loop system.

* The main physiological systems are

1. Cardiovascular system
2. Digestive system
3. Endocrine system
4. Immune system
5. Muscular system.

* The four main physiological processes are

1. Transduction
2. Transmission
3. Modulation
4. Perception.

* The human organism consists of eleven organ systems.

1. Integumentary system
2. Skeletal system
3. Muscular system
4. Nervous system
5. Endocrine system

6. Cardiovascular System
7. Lymphatic System
8. Respiratory System
9. Digestive System
10. Urinary System
11. Reproductive System.

* Physiology is the study of how the human body works.

* Physiology is an experimental scientific discipline and is of central importance in medicine and related health sciences.

* It provides a thorough understanding of normal body function, enabling more effective treatment of abnormal or disease states.

* A feedback control system consists of five basic components.

1. Input
2. Process being controlled
3. Output
4. Sensing elements
5. Controller and actuating devices.

LINEAR MODELS OF PHYSIOLOGICAL SYSTEMS.

* Physiological modeling defines the development of mathematical models that replicate the multi-physics behavior of biomedical organs and systems.

* The various types of physiological models are

1. Manikin
2. Simulators
3. Heat Stress
4. Thermal Manikin

* Physiologically based pharmacokinetic (PBPK) modeling is a computational process that simulates the absorption, distribution, metabolism and excretion of a substances in the body of an organism based on the interrelationships among key physiological, biochemical and ~~(physiologi)~~ physiochemical factors using mathematical equations.

* Each system is composed of many subsystems.

* For examples, the physiological system includes such commonly known subsystems as skeletal, muscular, nervous, digestive, excretory, respiratory, circulatory, metabolic and many others.

* The three types of modeling in physchology are live, Verbal and symbolic.

* The four types of system models are System analysis, Hard systems modelling or Operational research modeling. Soft system modeling. Process based system modelling.

* The different ways of modeling physical systems are

1. Translating Mechanical systems (motion back and forth along a straight line)
2. Rotating Mechanical Systems
3. Electrical systems
4. Electromechanical Systems
5. Thermal Systems
6. Fluid system
7. Biological System.

DIFFERENCE BETWEEN ENGINEERING AND PHYSIOLOGICAL CONTROL SYSTEMS.

Engineering control systems	Physiological control systems.
* Engineering control systems are finely designed to accomplish optimally to a defined tasks.	* Physiological control systems are built for versatility and may be capable of serving several different functions.
* An Engineering control systems is synthesized by the designer, the characteristics of it various components are generally known.	* The physiological control system usually consists components that are unknown and difficult to analyse.
* Engineering control systems can be linear (or) non-linear	* Physiological systems are generally non-linear.
* The Engineering designer prefers the use of linear system components since they have properties that are well-behaved and easy to predict.	* Physiological control system is general are adaptive. The system may be able to offset any change in output not only through feedback but also by allowing the controller.

* An engineering system is a which desired effect is achieved by operating on the various inputs to the plant until the output, which measure desired effect falls within an acceptable range of values.	* There is an extensive degree to cross-coupling or interaction among different physiological control system.
* There is extensive cross coupling or interaction among the physiological control system, their governing parameters hence be disturbed extensively.	* Need to apply system identification techniques to determine how these various subsystem behave before we are able to proceed to analyzing the overall control system.
* For instance, although a primary purpose of the respiratory system is provide gas exchange a secondary.	* The proper functioning of the cardiovascular system for instance, is to a large extent dependent on interactions with the respiratory, renal endocrine and other organ system

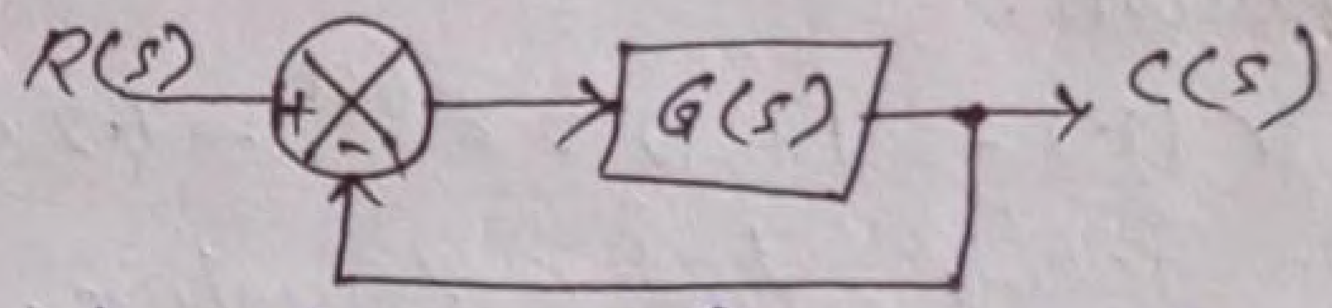
Unit - 2 . TIME RESPONSE ANALYSIS

STEP RESPONSE OF FIRST ORDER SYSTEMS

Obtain the response of unity feedback system whose open loop transfer function is $G(s) = \frac{4}{s(s+5)}$ and when the input is unit step.

Solution.

The closed loop system.



The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$.

$$\frac{C(s)}{R(s)} = \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s(s+5)}} = \frac{\frac{4}{\cancel{s(s+5)}}}{\frac{s(s+5)+4}{\cancel{s(s+5)}}} = \frac{4}{s(s+5)+4} = \frac{4}{s^2+5s+4}$$

$$\frac{C(s)}{R(s)} = \frac{4}{(s+4)(s+1)}$$

∴ The response in s- domain, $C(s) = R(s) \cdot \frac{4}{(s+1)(s+4)}$.

For unit step input, $R(s) = \frac{1}{s}$

$\Rightarrow C(s) = \frac{4}{s(s+1)(s+4)}$ —— 4 marks.

By partial fraction,

$$C(s) = \frac{4}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}.$$

$$A = C(s) \times s|_{s=0} = \frac{4}{(s+1)(s+4)} = \frac{4}{1\times 4} = 1.$$

$$B = C(s) \times s+1|_{s=-1} = \frac{4}{s(s+4)} = \frac{4}{-1(-1+4)} = \frac{-4}{3}.$$

$$C = C(s) \times s+4|_{s=-4} = \frac{4}{s(s+1)} = \frac{4}{-4(-4+1)} = \frac{4}{12} = 1/3.$$

The time domain response $c(t)$ is obtained by taking inverse laplace transform of $C(s)$.

Response in time domain, $c(t) = \mathcal{L}^{-1}[C(s)] = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s+4}\right]$

$$= 1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t} = 1 - \frac{1}{3}\left[4e^{-t} - e^{-4t}\right]$$ —— 4 marks.

STEP RESPONSE OF SECOND ORDER SYSTEMS

The response of a servomechanism is, $c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$ when subject to a unit step input. Obtain an expression for closed loop transfer function. Determine the undamped natural frequency and damping ratio.

Solution:

$$c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}.$$

On taking Laplace transform,

$$C(s) = \frac{1}{s} + 0.2\frac{1}{(s+60)} - 1.2\frac{1}{(s+10)} = \frac{(s+60)(s+10) + 0.2s(s+10) - 1.2s(s+60)}{s(s+60)(s+10)}.$$

$$= \frac{\cancel{s^2} + \cancel{70s} + 600 + \cancel{0.2s^2} + \cancel{2s} - \cancel{1.2s^2} - \cancel{72s}}{s(s+60)(s+10)}.$$

$$= \frac{600}{s(s+60)(s+10)}$$

$$= \frac{1}{s} \cdot \frac{600}{(s+60)(s+10)}$$

——— 3 marks

For unit step input, $R(s) = 1/s$.

$$\therefore C(s) = R(s) \cdot \frac{600}{(s+60)(s+10)} = R(s)\frac{600}{s^2+70s+600}$$

Closed loop transfer function of the system, $\frac{C(s)}{R(s)} = \frac{600}{s^2+70s+600}$

By comparing the system transfer function with standard form of second order transfer functions

——— 2 marks.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{600}{s^2+70s+600}$$

$\Rightarrow \omega_n^2 = 600$

$\omega_n = \sqrt{600}$

$= 24.49$ rad/sec.

$2\zeta\omega_n = 70$

$\therefore \zeta = \frac{70}{2\omega_n} = \frac{70}{2 \times 24.49}$

$\zeta = 1.43$

$$\therefore \frac{C(s)}{R(s)} = \frac{600}{s^2+70s+600}.$$

Natural frequency, $\omega_n = 24.49$ rad/sec

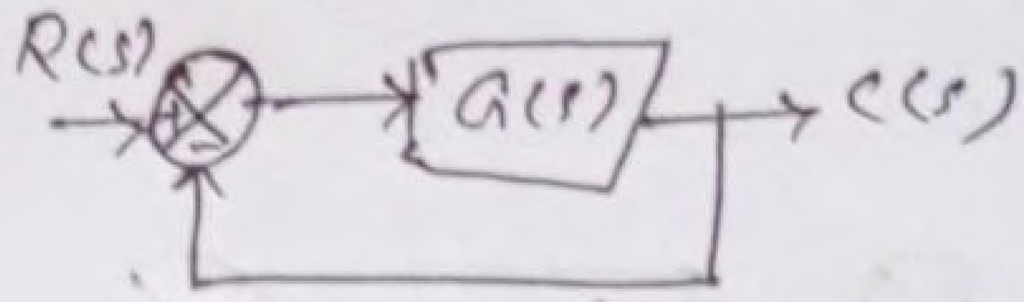
Damping ratio, $\zeta = 1.43$.

——— 3 marks.

The unity feedback system is characterized by an open loop transfer function $G(s) = K/s(s+10)$. Determine the gain K, so that the system will have a damping ratio of 0.5 for this value of K. Determine settling time, peak overshoot and time at peak overshoot for a unit step input.

Solution:

unity feed back system



The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$.

Given, $G(s) = K/s(s+10)$.

$$\therefore \frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+10)}}{1+\frac{K}{s(s+10)}} = \frac{\frac{K}{\cancel{s(s+10)}}}{\frac{s(s+10)+K}{\cancel{s(s+10)}}} = \frac{K}{s^2+10s+K}.$$

—— 3 marks

Comparing the s/m transfer function with standard form of 2nd order T.F,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{K}{s^2+10s+K}.$$

$\Rightarrow \omega_n^2 = K$

$\omega_n = \sqrt{K}$.

$2\zeta\omega_n = 10$

Put $\zeta = 0.5$ & $\omega_n = \sqrt{K}$

$2 \times 0.5 \times \sqrt{K} = 10$

$\sqrt{K} = 10$.

$\therefore K = \omega_n^2$

$\Rightarrow K = 100$

$\omega_n = 10$ rad/sec.

—— 2 marks

Percentage peak overshoot, $\% M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100$

$= e^{-0.5\pi/\sqrt{1-(0.5)^2}} \times 100$

$= 0.163 \times 100$

$= 16.3\%$

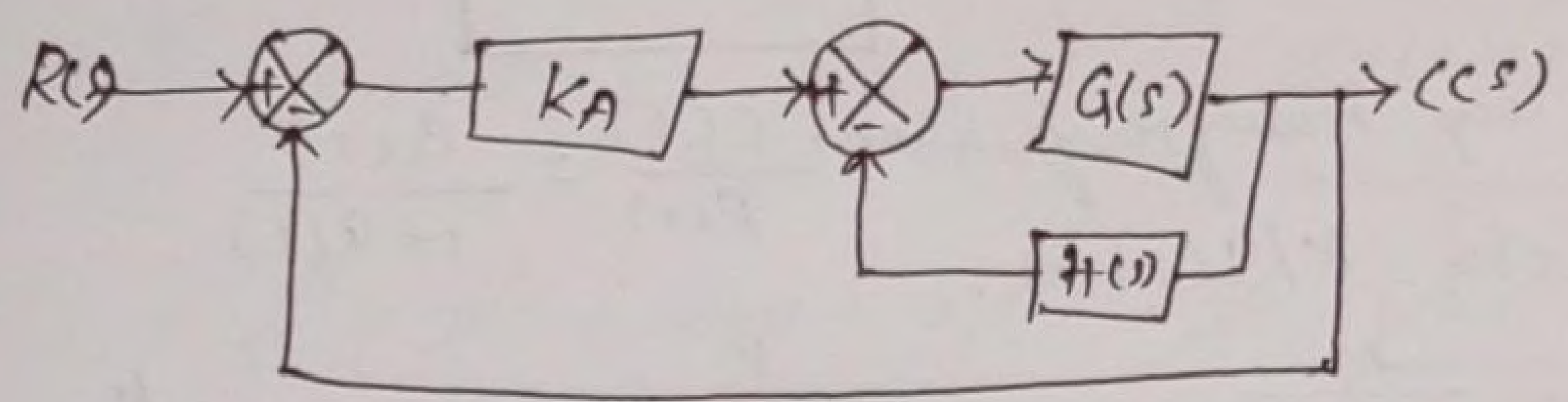
—— 2 marks

Peak time, $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{10\sqrt{1-(0.5)^2}} = 0.363$ sec.

—— 1 mark.

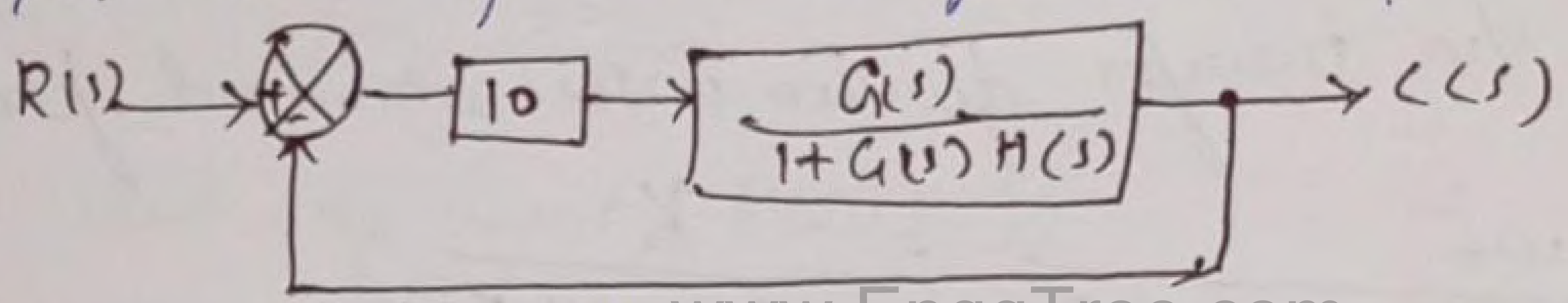
A unity feedback control system has an amplifier with gain $K_A = 10$ and gain ratio, $G(s) = 1/s(s+2)$ in the feed forward path. A derivative feedback, $H(s) = sK_0$ is introduced as a minor loop around $G(s)$. Determine the derivative feedback constant, K_0 so that the system damping factor is 0

Solution.



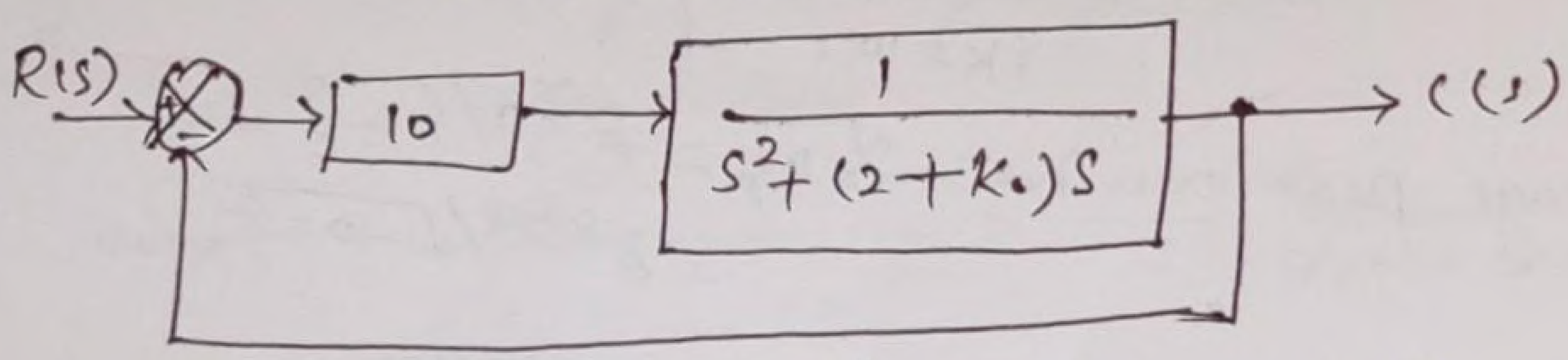
Here, $K_A = 10$; $G(s) = \frac{1}{s(s+2)}$ and $H(s) = sK_0$.

Step:1 Reducing the inner feedback loop.

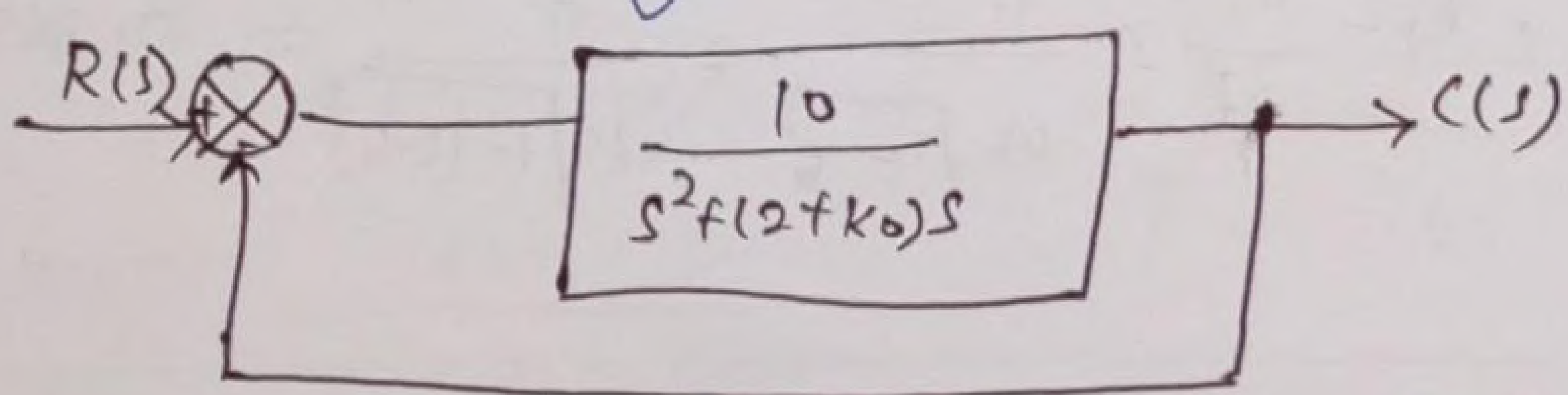


$$\frac{G(s)}{1+G(s)\times H(s)} = \frac{\frac{1}{s(s+2)}}{1+\frac{1}{s(s+2)}sK_0} = \frac{1}{s(s+2)+sK_0} = \frac{1}{s^2+2s+sK_0}$$

$$= \frac{1}{s^2+(2+K_0)s}$$

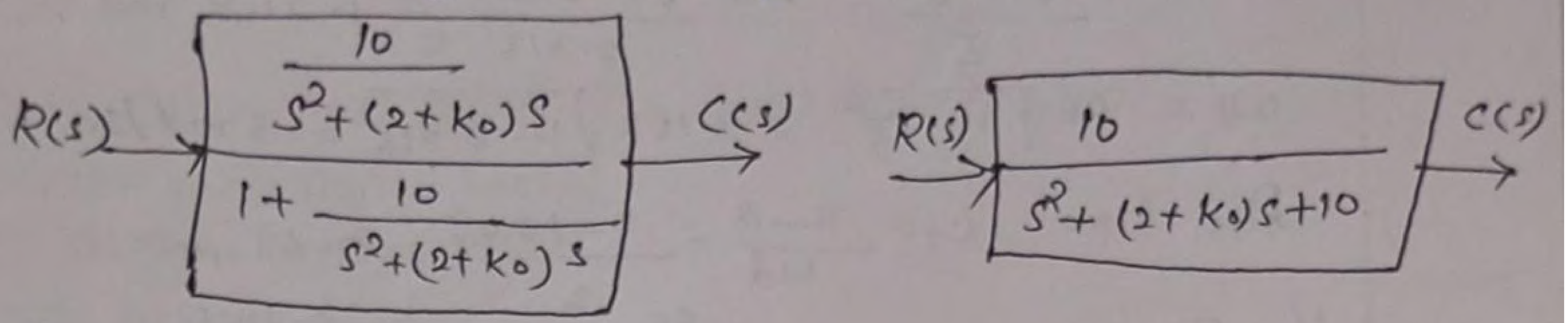


Step: 2: Combining blocks in cascade.



— 4 marks.

Step 3: Reducing the unity feedback path.



Closed loop T.F, $\frac{C(s)}{R(s)} = \frac{10}{s^2+(2+K_0)s+10}$

Standard form of 2nd order T.F, $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

On comparing,

$\omega_n^2 = 10$

$\omega_n = \sqrt{10}$

$= 3.162$ rad/sec

$2+K_0 = 2\zeta\omega_n$

$K_0 = 2\zeta\omega_n - 2$

$= 2\times 0.6\times 3.162 - 2$

$= 1.7944$.

∴ The value of constant, $K_0 = 1.7944$. — 4 marks.

A unity feedback control system has an open loop transfer function, $G(s) = 10/s(s+2)$. Find the rise time, % overshoot, peak time and settling time for a step input of 12 units.

Closed loop T.F $= \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$.

$G(s) = \frac{10}{s(s+2)}$.

$$\therefore \frac{C(s)}{R(s)} = \frac{\frac{10}{s(s+2)}}{1+\frac{10}{s(s+2)}} = \frac{10}{s(s+2)+10} = \frac{10}{s^2+2s+10}.$$

By comparing,

$\omega_n^2 = 10$

$\omega_n = \sqrt{10}$

$= 3.162$ rad/sec

$2\zeta\omega_n = 2$

$\therefore \zeta = \frac{2}{2\omega_n} = \frac{1}{3.162}$

$\zeta = 0.316$.

— 3 marks

$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} - \tan^{-1}\frac{\sqrt{1-0.316^2}}{0.316} = 1.249$ rad.

$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.162\sqrt{1-0.316^2} = 3$ rad/sec.

Rise time, $t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-1.249}{3} = 0.63$ sec.

% Overshoot, $\% M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100 = e^{-\frac{0.316\pi}{\sqrt{1-0.316^2}}} \times 100$

$= 0.3512 \times 100 = 35.12\%$

Peak Overshoot $= \frac{35.12}{100} \times 12$ units

$= 4.2144$ units.

Peak time, $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3} = 1.047$ sec. — 3 marks

Time constant, $T = \frac{1}{\xi\omega_n} = \frac{1}{0.316 \times 3.162} = 1$ sec.

∴ For 5% error,

settling time, $t_s = 3T = 3$ sec.

For 2% error,

settling time, $t_s = 4T = 4$ sec.

— 2 marks

STEADY STATE ERROR CONSTANTS.

Consider a unity feedback system with a closed loop transfer function $\frac{C(s)}{R(s)} = \frac{ks+b}{s^2+as+b}$. Determine open loop transfer function $G(s)$. Show that steady state error with unit ramp input is given by $\left(\frac{a-k}{b}\right)$.

Solution.

For unity feedback system, $H(s) = 1$.

The closed loop transfer function, $M(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$.

4

$\therefore \quad M(s) = \dfrac{G(s)}{1+G(s)}$

$\Rightarrow G(s) = M(s)[1+G(s)] = M(s) + M(s)G(s).$

$\therefore G(s) - M(s)G(s) = M(s).$

$G(s)[1-M(s)] = M(s)$

$\Rightarrow M(s) = \dfrac{ks+b}{s^2+as+b}.$ ——— 3 marks

Open loop transfer function, $G(s) = \dfrac{M(s)}{1-M(s)} = \dfrac{\frac{ks+b}{s^2+as+b}}{1-\frac{ks+b}{s^2+as+b}}.$

$$= \frac{ks+b}{s^2+as+b-(ks+b)}$$

$$= \frac{ks+b}{s^2+as+\cancel{b}-ks-\cancel{b}}$$

$$= \frac{ks+b}{s^2+(a-k)s.}$$

$$G(s) = \frac{ks+b}{s[s+(a-k)]}$$ ——— 3 marks

Velocity error constant, $K_v = \lim_{s\to 0} s\,G(s)H(s) = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{s\,k...}{s(s...}$

$$\boxed{K_v = \frac{b}{a-k}}.$$

with Velocity input, Steady state error, $e_{ss} = \frac{1}{K_v} = \frac{a-k}{b}.$

——— 2 ma...

A unity feedback system has the forward transfer f...

$G(s) = \dfrac{K_1(2s+1)}{s(5s+1)(1+s)^2}.$ When the input $r(t) = 1+6t$, dete... the minimum value of K_1 so that the steady error is le... than 0.1.

Solution.

Input $[r(t)] = 1+6t.$

Taking laplace transform,

$\therefore R(s) = \mathcal{L}[r(t)] = \mathcal{L}[1+6t] = \frac{1}{s} + \frac{6}{s^2}$.

The error signal in S-domain E(s) is given by,

$$E(s) = \frac{R(s)}{1+G(s)H(s)} = \frac{\frac{1}{s} + \frac{6}{s^2}}{1 + \frac{K_1(2s+1)}{s(5s+1)(1+s)^2}} = \frac{\frac{1}{s} + \frac{6}{s^2}}{\frac{s(5s+1)(1+s)^2 + K_1(2s+1)}{s(5s+1)(1+s)^2}}$$

$$= \frac{1}{s}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right] + \frac{6}{s^2}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right]$$

← 4 marks

The steady state error e_{ss} can be obtained from final value theorem,

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s\left\{\frac{1}{s}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right] + \frac{6}{s^2}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right]\right\}$$

$$= \lim_{s\to 0}\left\{\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)} + \frac{6(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right\}$$

$$= 0 + \frac{6}{K_1} = \frac{6}{K_1}$$

Given that, $e_{ss} < 0.1$,

$$\therefore 0.1 = \frac{6}{K_1}$$

(or) $K_1 = \frac{6}{0.1} = 60$.

Result: ← 4 marks.

For steady state error, $e_{ss} < 0.1$, the value of K_1 should be greater than 60.

IMPULSE RESPONSE OF FIRST ORDER SYSTEMS

* The standard form of closed loop transfer function of second order system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

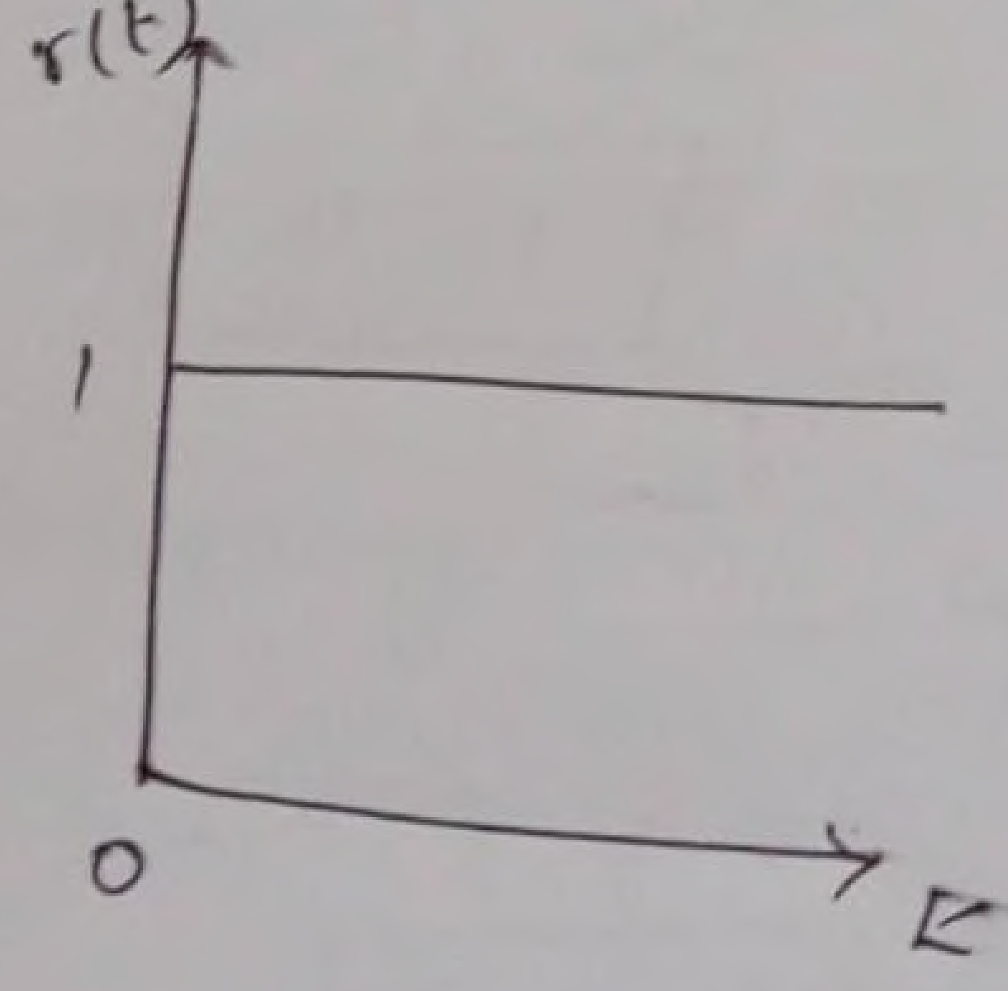
where,

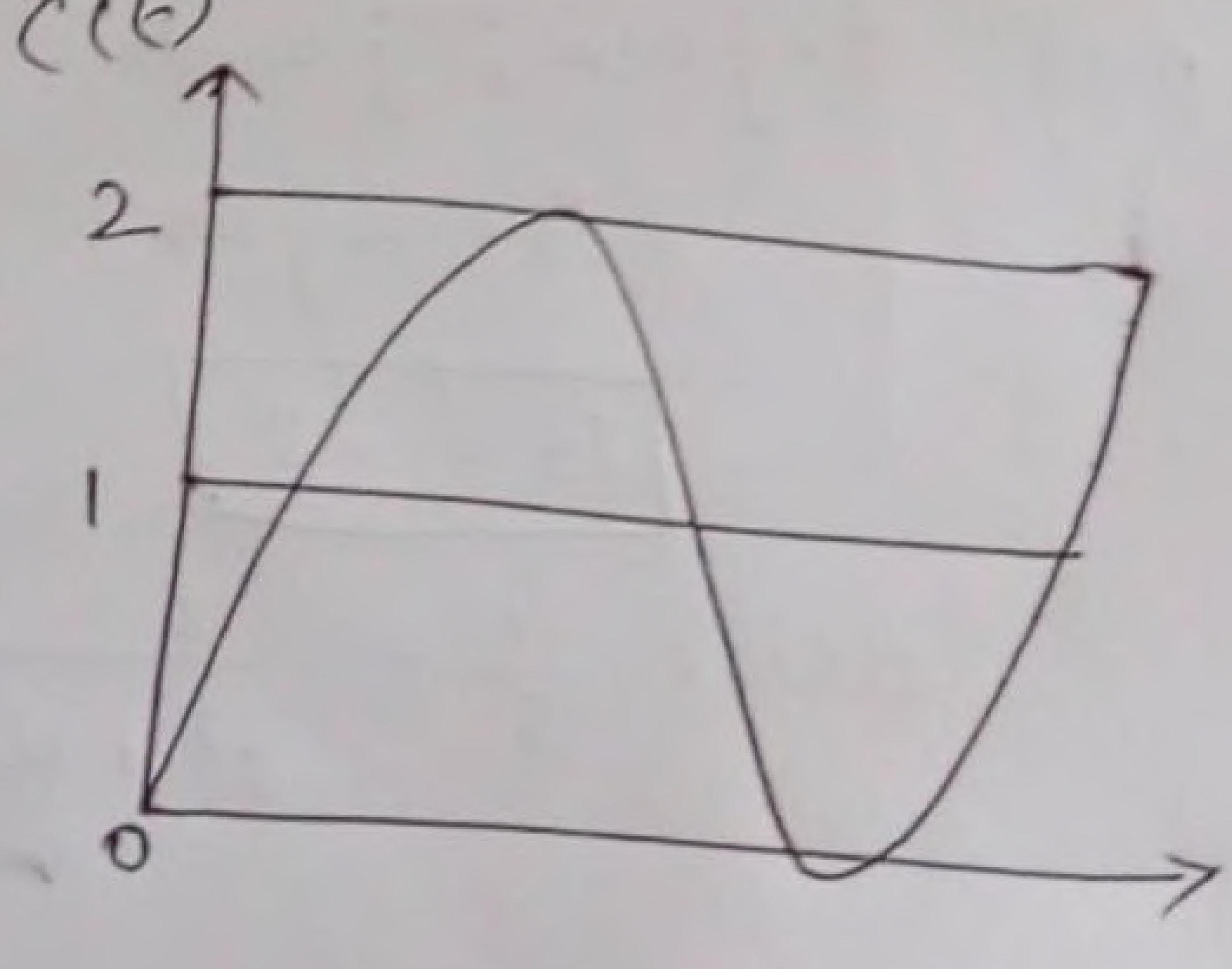
ω_n = undamped natural frequency rad/sec.

ξ_n = damping ratio.

* Depending on the value of ξ, the second order system is classified into 4 types.

1. Undamped system, $\xi = 0$
2. Underdamped system, $0 < \xi < 1$
3. Critically damped system ; $\xi = 1$
4. Overdamped system ; $\xi > 1$





* $$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

put $\zeta = 0$

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}.$$

$\Rightarrow r(t) = 1$ and $R(s) = \frac{1}{s}$.

$\therefore$ The response is s-domain,

* $$C(s) = R(s) \cdot \frac{\omega_n^2}{s^2 + \omega_n^2}.$$

$$= \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + \omega_n^2}.$$

* By partial fraction expansion

$$C(s) = \frac{\omega_n^2}{s(s^2 + \omega_n^2)} = \frac{A}{s} + \frac{B}{s^2 + \omega_n^2}$$

* $$A(s^2 + \omega_n^2) + Bs = \omega_n^2.$$

put $s = 0$; $\omega_n^2 A = \omega_n^2$

$$\boxed{A = 1}.$$

put $s = j\omega_n$; $j\omega_n B = \omega_n^2$

$$B = -j\omega_n = -s$$

$$\boxed{B = -s}$$

$$\therefore C(s) = \frac{1}{s} - \frac{s}{s^2 + \omega_n^2}$$

* Time response, $c(t) = L^{-1}[C(s)]$

$$= L^{-1}\left[\frac{1}{s} - \frac{s}{s^2+\omega_n^2}\right]$$

$$\boxed{c(t) = 1 - \cos\omega_n t.}$$

IMPULSE RESPONSE OF SECOND ORDER SYSTEMS

* $C(s) = R(s) \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$$= \frac{1}{s} \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

* By Partial fraction expansion.

$$C(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$A(s^2+2\zeta\omega_n s+\omega_n^2) + Bs^2 + Cs = \omega_n^2$$

* Comparing constant terms,

$$A\omega_n^2 = \omega_n^2$$

$$\boxed{A = 1}$$

* Comparing the coefficient of s^2,

$$A + B = 0$$

$$\boxed{B = -1}$$

* Comparing the coefficient of S

$$s(2\zeta\omega_n) + C = 0$$

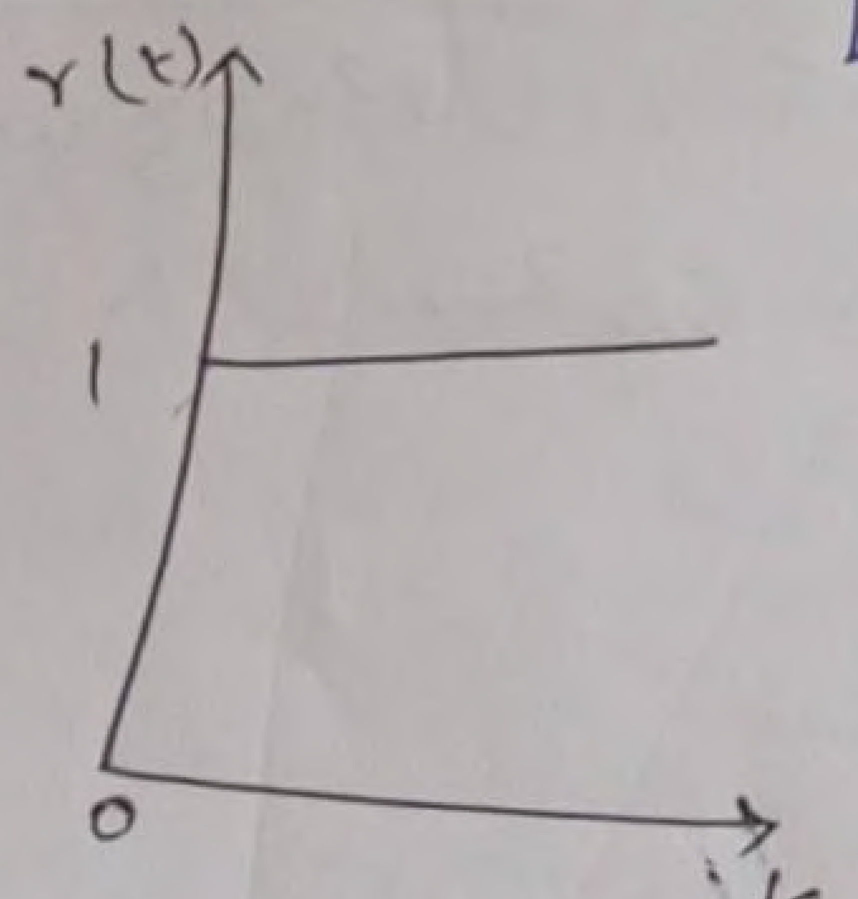
$$C = -2\zeta\omega_n .$$

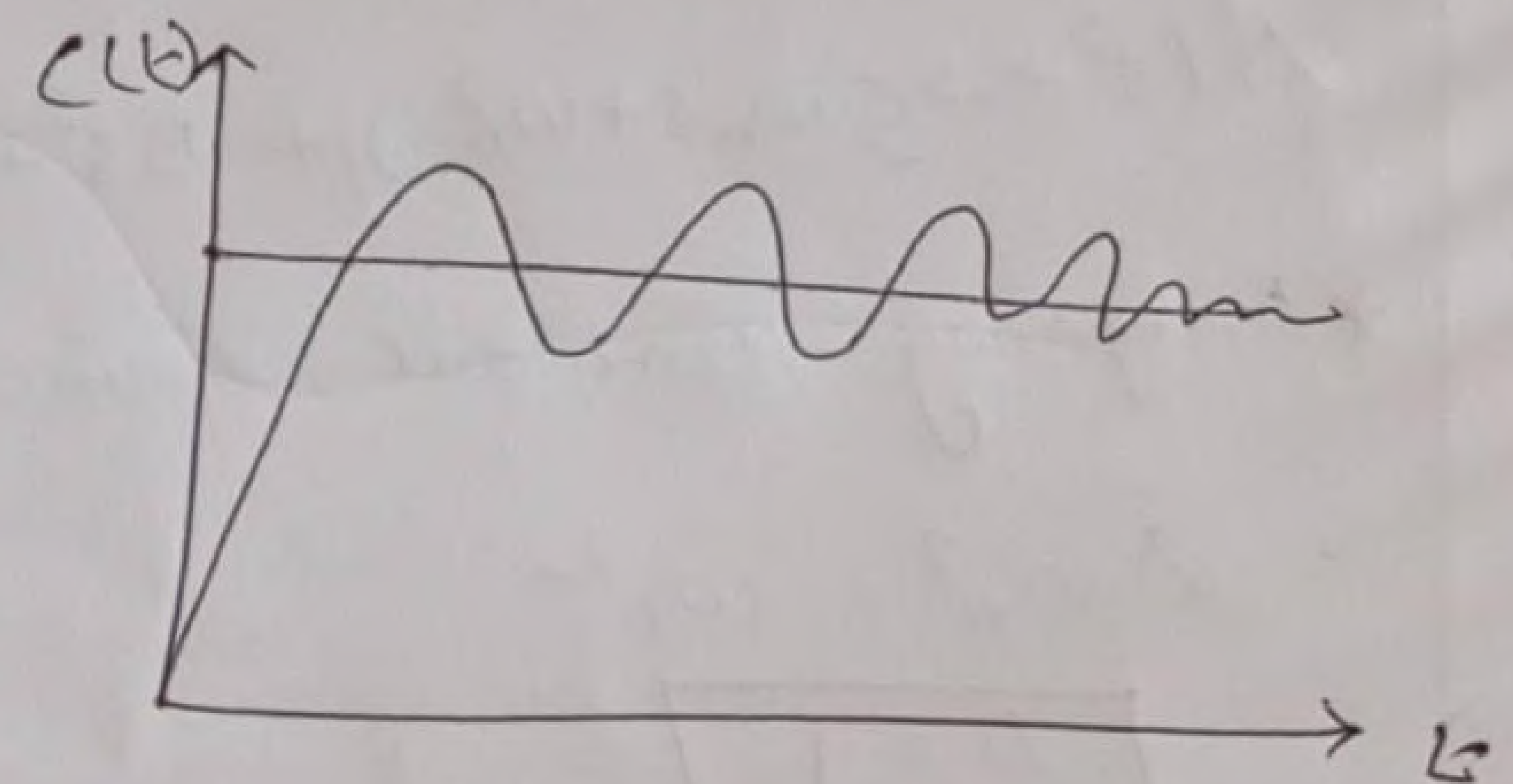
$$\therefore C(s) = \frac{1}{S} - \frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2}$$

* $$C(s) = \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n \quad \omega_d}{\omega_d(s+\zeta\omega_n^2)+\omega_d^2}$$

* $$c(t) = 1 - \frac{e^{-\zeta\omega_n}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)$$

* where $\theta = \tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$





* The response of underdamped second order system for unit step input oscillates before settling to a final value.

TIME DOMAIN SPECIFICATIONS OF FIRST ORDER SYSTEMS

* The transient response characteristics of a control system to a unit step input is specified in terms of the following time domain specifications.

1. Delay time (t_d)
2. Rise time (t_r)
3. Peak time (t_p)
4. Maximum overshoot (M_p)
5. Settling time (t_s)
6. steady state error (e_{ss})

* Delay time (t_d) is the time required to reach at 50% of its final value by a time response signal during its first cycle of oscillation.

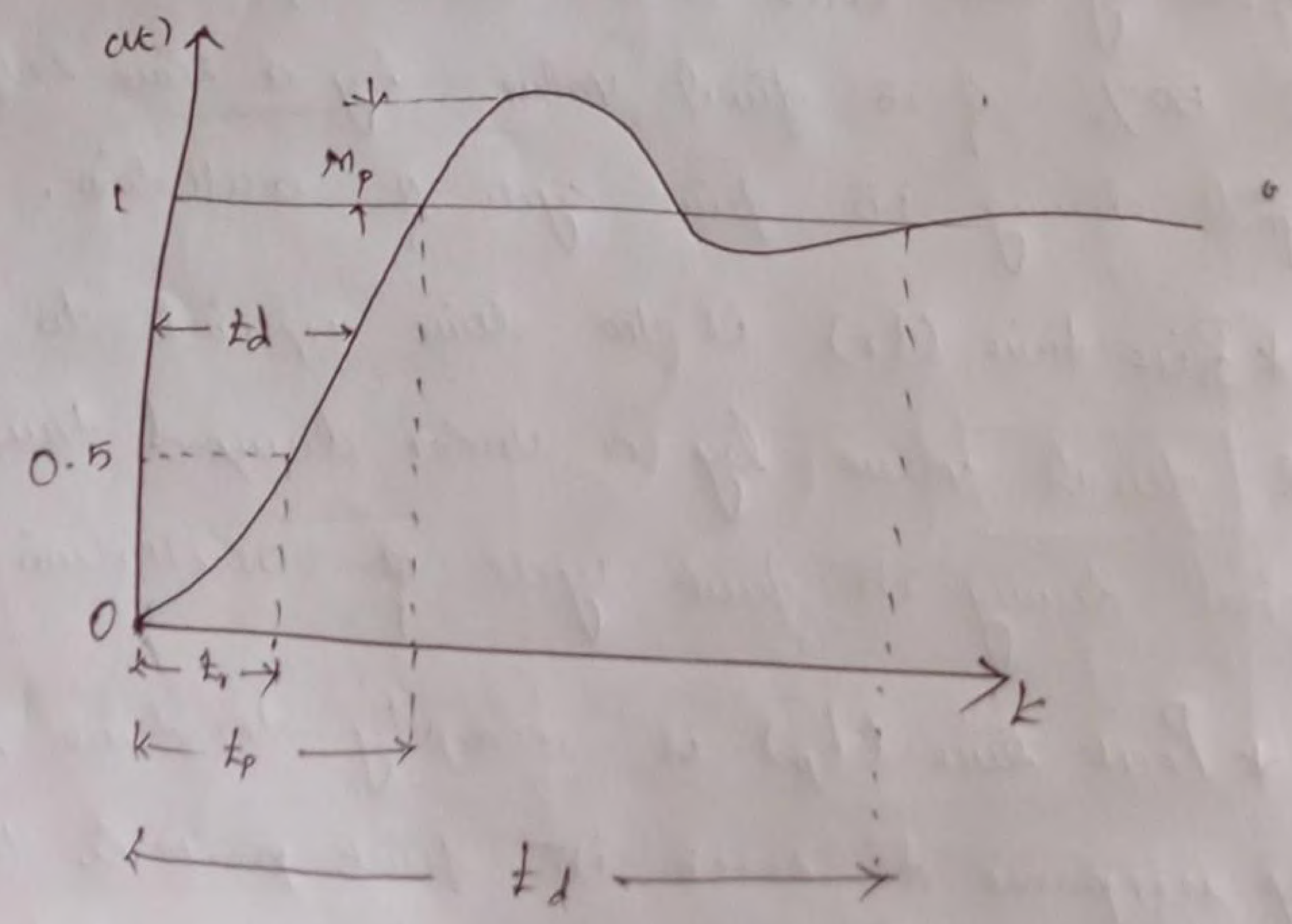
* Rise time (t_r) is the time required to reach at final value by a under damped time response signal during its first cycle of oscillation.

* Peak time (t_p) is simply the time required by response to reach its first peak i.e. the peak of first cycle of oscillation or first overshoot.

*Maximum overshoot (M_p) is straight way difference between the magnitude of the highest peak of time response and magnitude of its steady state.

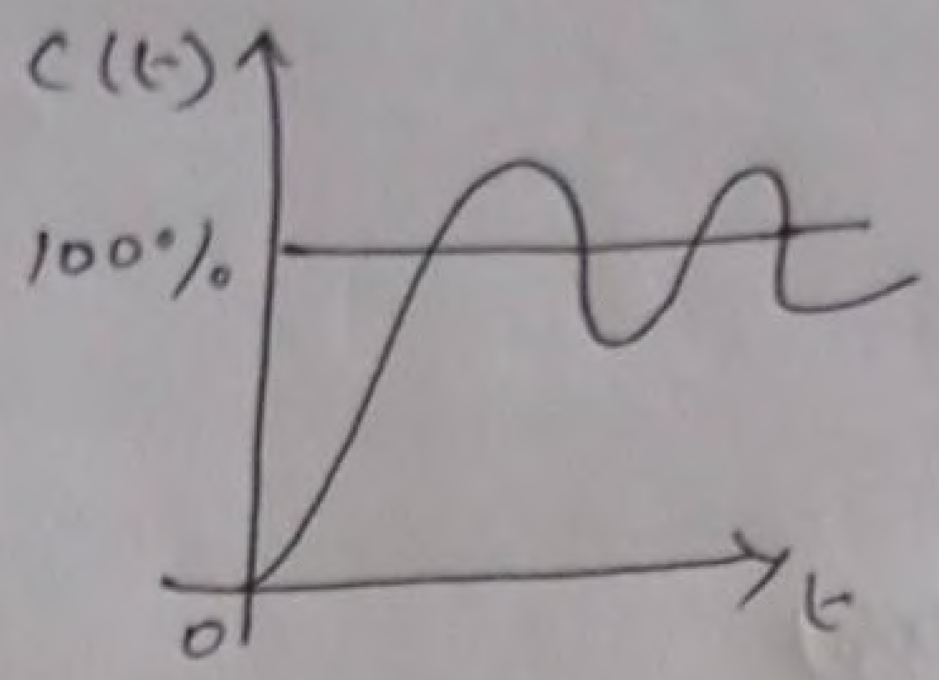
*Settling time (t_s) Time required for a response to become steady. It is defined as the time required by the response to reach and steady within specified range of 2% to 5% of its final value.

*Steady state error (e_{ss}) is the difference between actual output and desired output at the infinite range of time.



TIME DOMAIN SPECIFICATIONS OF SECOND ORDER SYSTEMS

Rise time t_r.



$$c(t) = 1 - \frac{e^{-\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)$$

At rise time $c(t) = 1$

$$\Rightarrow 1 = 1 - \frac{e^{-\zeta\omega_n t_r}}{\sqrt{1-\zeta^2}} \cdot \sin(\omega_d t_r + \theta)$$

$$-\frac{e^{-\zeta\omega_n t_r}}{\sqrt{1-\zeta^2}} \cdot \sin(\omega_d t_r + \theta) = 0 .$$

Equation will get satisfied if $\sin(\omega_d t_r + \theta) = 0$;

$$\Rightarrow (\omega_d t_r + \theta) = n\pi \quad \text{where } n = 1, 2$$

Let $n = 1$

$$\omega_d t_r + \theta = \pi$$

$$\boxed{\therefore t_r = \frac{\pi - \theta}{\omega_d}}$$

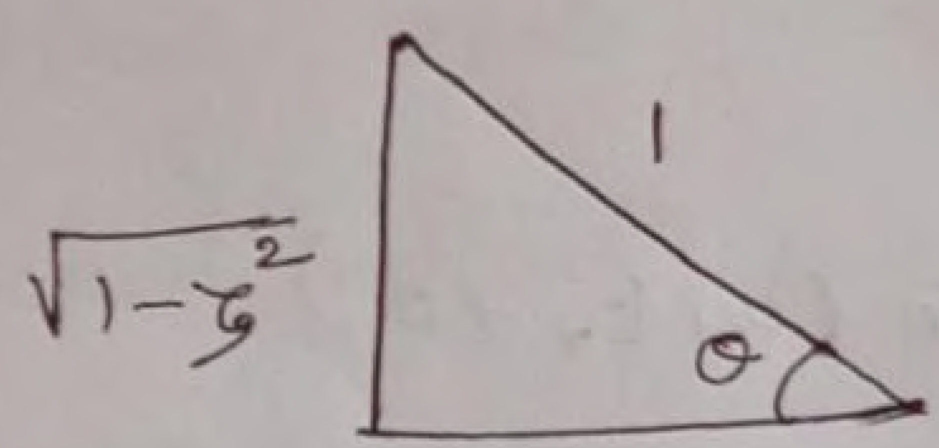
Peak time t_p.

* $t_p = \frac{\pi}{\omega_d}$

* $t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$

Maximum peak overshoot (% M_p)

* $M_p = e^{\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$.



Setting time (t_s)

* $c(t)$ at $(t = t_s) = 0.98$

$c(t) = 1 - e^{\zeta\omega_n t_s}$

$0.98 = 1 - e^{\zeta\omega_n t_s}$

* $e^{\zeta\omega_n t_s} = 0.02$

$t_s = \frac{3.912}{\zeta\omega_n}$.

* $t_s = \frac{2.995}{\zeta\omega_n} = \frac{3}{\zeta\omega_n} = 3T$.

UNIT - 3 STABILITY ANALYSIS.

ROUTH - HURWITZ CRITERIA OF STABILITY.

Using Routh criterion, determine the stability of the system represented by the characteristic equation, $s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$.

The characteristic equation of the system is, $s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$

$$
\begin{array}{lccc}
s^4 : & 1 & 18 & 5 \\
s^3 : & 8 & 16 & \\
s^4 : & 1 & 18 & 5 \\
s^3 : & 1 & 2 & \\
s^2 : & 16 & 5 & \\
s^1 : & 1.7 & & \\
s^0 : & 5 & &
\end{array}
$$

↑ column-1. —— 3 marks

$s^2 : \dfrac{1\times18 - 2\times1}{1} \quad \dfrac{1\times5 - 0\times1}{1}$

$s^2 : 16 \quad 5$

$s^1 : \dfrac{16\times2 - 5\times1}{16}$

$s^1 : 1.6875 = 1.7$

$s^0 : \dfrac{1.7\times5 - 0\times16}{1.7}$

$s^0 = 5$. —— 3 marks

The elements in the first column of routh array are +ve, there is no sign change. Hence all the roots are lying on the left half of s-plane and the system is stable.

—— 2 marks

By routh stability criterion determine the stability of the system represented by the characteristic equation, $9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$.

The characteristic polynomial of the system is, $9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$.

$$
\begin{array}{lccc}
s^5 : & 9 & 10 & -9 \\
s^4 : & -20 & -1 & -10 \\
s^3 : & 9.55 & -13.5 & \\
s^2 : & -29.3 & -10 & \\
s^1 : & -16.8 & & \\
s^0 : & -10 & &
\end{array}
$$

↑ column-1. —— 3 marks

s^3: $\dfrac{-20\times 10-(-1\times 9)}{-20}$ $\dfrac{-20\times(-9)-(-10)\times 9}{-20}$

s^3: 9.55 -13.5

s^2: $\dfrac{9.55\times(-1)-(-13.5)\times(-20)}{9.55}$ $\dfrac{9.55(-10)}{9.55}$

s^2: -29.3 -10.

s^1: $\dfrac{-29.3\times(-13.5)-(-10)\times 9.55}{-29.3}$

s^1: -16.8

$s^0 = \dfrac{-16.8\times(-10)}{-16.8}$

s^0: -10. — 3 marks

By examining the elements of 1st coloumn of routh array it is observed that there are three sign changes and so three roots are lying on the right half of s-plane and the remaining two roots are lying on the left half of s-plane. The system is unstable. — 2 marks

The characteristic polynomial of a system is $s^7+9s^6+24s^5+24s^4+24s^3+24s^2+23s+15=0$. Determine the location of roots on s-plane and hence the stability of the system.

The characteristic equation is $s^7+9s^6+24s^5+24s^4+24s^3+24s^2+23s+15=0$.

s^7: 1 24 24 23

s^6: 9 24 24 15

$\div$ s^6 by 9,

s^7 :	1	24	24	23	
s^6 :	3	8	8	5	
s^5 :	1	1	1		
s^4 :	1	1	1		
s^3 :	0	0			
s^3 :	2	1			
s^2 :	0.5	1			
s^1 :	-3				
s^0 :	1				

Column – 1.

s^5: $\frac{3\times24-8\times1}{3} = 21.33$

$\frac{3\times24-8\times1}{3} = 21.33$

$\frac{3\times23-5\times1}{3} = 21.33$

$\therefore s^5 = 1 : 1 \quad 1$

$s^4 = \frac{1\times8-1\times3}{1} \quad \frac{1\times8-1\times3}{1} \quad \frac{1\times5}{1}$

s^4: 5 5 5

s^4: 1 1 1

s^3 : $\frac{1\times1-1\times1}{1} \quad \frac{1\times1-1\times1}{1}$

s^3 : 0 0

— 3 marks

The auxilliary polynomial is,

$A = s^4 + s^2 + 1$

$\therefore \frac{dA}{ds} = 4s^3 + 2s.$

s^3 : 4 2

s^3 : 2 1

s^2 : $\frac{2\times1-1\times1}{2} \quad \frac{2\times1-0\times1}{2}$

s^2 : 0.5 1

s^1 : $\frac{0.5\times1-1\times2}{0.5}$

s^1 : -3

s^0 : $\frac{-3\times1}{-3}$

s^0 : 1

— 2 marks

Put $s^2 = x$ in the auxilliary equation.

$s^4 + s^2 + 1 = x^2 + x + 1 = 0$

$\therefore$ The roots of quadratic eq,

$x = -\frac{1}{2} \pm j\sqrt{3}/2$

$= 1\angle 120^\circ$ or $1\angle -120^\circ$.

$s^2 = x \; \therefore \; s = \pm\sqrt{x} = \pm\sqrt{1\angle 120^\circ}$ or $\pm\sqrt{1\angle -120^\circ}$ —— 2 mark

$\therefore$ The system is unstable.

Two roots are lying on right half of s-plane and five roots are lying on left half of s-plane. —— 1 mark.

Use the routh stability criterion to determine the location of roots on the s-plane and hence the stability of the s/m represented by the characteristic equation $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$.

The characteristic equation of the system is,

$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0.$

s^5 :	1	8	7
s^4 :	4	8	4
$\div 4$	1	2	1
s^3 :	1	1	
s^2 :	1	1	
s^1 :	ϵ		
s^0 :	1		

↑ Column – 1. —— 3 marks

s^3 : $\frac{1\times 8-2\times 1}{1}$ $\frac{1\times 7-1\times 1}{1}$

s^3 : 6 6

$\div$ by 6

s^3 : 1 1

s^2 : $\frac{1\times 2-1\times 1}{1}$ $\frac{1\times 1-0\times 1}{1}$

s^2 : 1 1

s^1 : $\frac{1\times 1-1\times 1}{1}$

s^1 : 0

Let $0 \rightarrow \epsilon$.

s^1 : ϵ

s^0 : $\frac{\epsilon\times 1-0\times 1}{\epsilon}$

s^0 : 1.

The auxilliary polynomial is $s^2+1=0$.

$\therefore s^2=-1$

$s=\pm\sqrt{-1}$

$=\pm j1$

——— 3 marks

The roots of auxilliary polynomial are $+j1$ and $-j1$, lying on imaginary axis.

The system is limitedly or marginally stable.

Two roots are lying on imaginary axis and three roots are lying on left half of s-plane.

——— 2 marks

Determine the range of K for stability of unity feedback system whose open loop transfer function is $G(s) = \frac{K}{s(s+1)(s+2)}$

The closed loop T.F, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+1)(s+2)}}{1+\frac{K}{s(s+1)(s+2)}} = \frac{K}{s(s+1)(s+2)+K}$

The characteristic equation is, $s(s+1)(s+2)+K=0$.

$s(s^2+3s+2)+K=0 \Rightarrow s^3+3s^2+2s+K=0$.

s^3 :	1	2
s^2 :	3	K
s^1 :	$\frac{6-K}{3}$	
s^0 :	K	

↑ Column-1

$s^1 : \frac{3\times 2 - K\times 1}{3}$

$: \frac{6-K}{3}$

$s^0 : \frac{\frac{6-K}{3}\times K - 0\times 3}{\frac{6-K}{3}}$

$: K$. — 3 marks

— 3 marks

From s^0 row, for the system to be stable,

$K > 0$

From s^1 row, for the system to be stable,

$\frac{6-K}{3} > 0$.

For $\frac{6-K}{3} > 0$, the value of K should be less than 6.

∴ The range of K for the s/m to be stable is $0 < K < 6$

— 2 marks

A feedback system has open loop transfer function of $G(s) = \frac{Ke^{-s}}{s(s^2+5s+9)}$. Determine the maximum value of K for stability of closed loop system.

For low frequency ranges, the term e^{-s} can be replaced by $1-s$, (ie $e^{-sT} \approx 1-sT$.)

$$\therefore \quad G(s) = \frac{Ke^{-s}}{s(s^2+5s+9)} = \frac{K(1-s)}{s(s^2+5s+9)}$$

The closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K(1-s)}{s(s^2+5s+9)}}{1+\frac{K(1-s)}{s(s^2+5s+9)}} = \frac{K(1-s)}{s(s^2+5s+9)+K(1-s)}$$

The characteristic equation is, $s(s^2+5s+9)+K(1-s)=0$.

$$s^3+5s^2+(9-K)s+K=0.$$

Routh array.

s^3 :	1	$(9-K)$
s^2 :	5	K
s^1 :	$9-1.2K$	
s^0 :	K	

column-1.

— 3 marks

$$s^1 = \frac{5\times(9-K) - K\times 1}{5} = \frac{45-5K-K}{5} = \frac{45-6K}{5} = 9-1.2K$$

$$s^0 : \frac{(9-1.2)K \times K}{(9-1.2K)}$$

s^0 : K. — 3 marks

From s^1 row, for stability of the system, $(9-1.2K)>0$.

If $(9-1.2K)>0$ then $1.2K<9$; $\therefore K< \frac{9}{1.2} = 7.5$.

From s^0 row, $K>0$.

$\Rightarrow$ For stability of the system K should be in the range of $0 < K < 7.5$. — 2 marks.

DEFINITION OF STABILITY.

* A linear relaxed system is said to have BIBO stability if every bounded input result in a bounded output.

* Relative stability is degree of closeness of the system, it is an indication of strength or degree of stability.

* If the root of characteristic equation lies on imaginary axis the nature of impulse response is oscillatory.

STUDY OF STABILITY.

* If the coefficients of characteristics polynomial are negative or zero, then some of the roots lies on the negative half of the s plane.

* Hence the system is unstable.

* If the coefficients of the characteristic polynomial are positive and if no coefficient is zero, then there is possibility of the system to be stable provided all the roots are lying on the left half of the s-plane.

ROOT LOCUS TECHNIQUES.

In control theory and stability theory, root locus analysis is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly a gain within a feedback system.

Sketch the root locus of the system whose open loop transfer function is $G(s) = \frac{K}{s(s+2)(s+4)}$. Find the value of K, so that the damping ratio of the closed loop system is 0.5.

* Poles are lying at $s = 0, -2, -4$.

Let us denote poles $P_1 = 0$, $P_2 = -2$, $P_3 = -4$.

* The root locus starts from pole $P_1 = 0$ and terminates at $P_2 = -2$ and it form the part of root locus and the root locus starts from P_3 & terminates at open loop zero at infinity.

* Angle of asymptotes $= \frac{\pm 180°(2q+1)}{n-m}$

$q = 0, 1, 2, 3 \ldots n-m$

Here $n = 3$, $m = 0$, $\therefore q = 0, 1, 2, 3$.

* when $q=0$, $\phi_A = \frac{\pm 180^\circ}{3} = \pm 60^\circ$

* when $q=1$, $\phi_A = \frac{\pm 180^\circ \times 3}{3} = \pm 180^\circ$

* when $q=2$, $\phi_A = \frac{\pm 180^\circ \times 5}{3} = \pm 300^\circ$

* Centroid $= \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$

* $\sigma_A = \frac{0-2-4-0}{3}$

$= -2$

* $\frac{C(s)}{R(s)} = \frac{K}{s(s+2)(s+4)+K}$.

* The characteristic equation is given by

$s(s+2)(s+4)+K=0$.

$s(s^2+6s+8)+K=0$.

$s^3+6s^2+8s+K=0$.

$K = -[s^3+6s^2+8s]$

* $\frac{dK}{ds} = -[3s^2+12s+8]$

put $\frac{dK}{ds} = 0$

$\Rightarrow 3s^2+12s+8=0$.

* $S = \dfrac{-12 \pm \sqrt{12^2 - 4 \times 3 \times 8}}{2 \times 3}$

$= -0.845$ or -3.154.

* when $S = -0.845$, the K is given by

* $K = -[(-0.845)^3 + 6(0.845)^2 + 8(0.845)]$
$= 3.08$.

* Since K is +ve and real for $s = -0.845$, this point is actual break away point.

* when $S = -3.154$, $K = -3.08$.

* Since K is -ve for $S = -3.154$, this is not a actual breakaway point.

CONSTRUCTION OF ROOT LOCUS.

Step: 1 To locate poles and zeros

Step: 2 To find the root locus on the real axis.

Step: 3 To find asymptotes and centroid.

$$\text{angle of asymptotes} = \frac{\pm 180^\circ (2q+1)}{n-m}$$

where $q = 0, 1, 2, 3 \ldots n-m$.

$$\text{centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}.$$

Step: 4 To find the break away and break in points

step: 5 To find angle of departure.

step: 6 To find the crossing point on imaginary axis

The Open loop transfer of a unity feedback control system is given by $G(s) = \frac{k}{s^2(s^2+4s+13)}$. Sketch the root locus.

Step: 1

* Poles,

$$s = 0, \frac{-4 \pm \sqrt{4^2 - 4\times 13}}{2}$$

$$= 0, -2+j3, -2-j3.$$

Let $P_1 = 0$, $P_2 = -2+j3$, $P_3 = -2-j3$.

Zeros = NIL.

Step: 2

* There is only one pole at origin

* Hence the entire −ve real axis will be a part of root locus.

Step 3

* Angle of asymptote $= \phi_A = \frac{\pm 180^\circ (2q+1)}{n-m}$.

* Here $n = 3$, $q = 0, 1, 2, 3$

* when $q = 0$, $\phi_A = \pm 60^\circ$

* when $q = 1$, $\phi_A = \pm 180^\circ$

* when $q=2$, $\phi_A = \pm 300^\circ = \pm 60^\circ$
* when $q=3$, $\phi_A = \pm 420^\circ = \pm 60^\circ$.
* centroid $\sigma_A = \dfrac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$

$$= \frac{-4}{3}$$
$$= -1.33.$$

Step: 7

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$= \frac{K}{s(s^2+4s+13)+K}$$

* The characteristic equation is $s(s^2+4s+13)+K=0$.

$$s(s^2+4s+13)+K=0.$$

$$K = -(s^3+4s^2+13s).$$

$$\frac{dK}{ds} = -[3s^2+8s+13]$$

$$\frac{dK}{ds} = 0$$

$$\Rightarrow 3s^2+8s+13=0.$$

$$\therefore s = \frac{-8 \pm \sqrt{8^2-4\times3\times13}}{2\times3}$$

$$= -1.33 \pm j1.6$$

* Check for K.
  * when $s = -1.33 + j1.6$ the value of K is given by

* $K = -(s^3 + 4s^2 + 13s)$

$$= -\left[(-1.33 + j1.6)^3 + 4(-1.33 + j1.6)^2 + 13(-1.33 + j1.6)\right]$$

$\neq$ real tue.

* Similarly when $s = -1.33 - j1.6$, the value of K is not positive and real.

* Therefore, the root locus has either break away nor breakin point.

Step: 4

* Here $\theta_1 = 180^\circ - \tan^{-1} 3/2$

$= 123.7$

$\theta_2 = 90^\circ$.

Step: 5

* Angle of departure from the complex poles P_2

$= 180^\circ - (\theta_1 + \theta_2)$

$= 180^\circ - (123.7 + 90^\circ)$

$\phi_d = + 33.7^\circ$

Step: 6

* The characteristic equation is given by

$$s^3 + 4s^2 + 13s + K = 0.$$

* put $s = j\omega$

$$(j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + K = 0.$$

$$-j\omega^3 + 4j^2\omega^2 + j13\omega + K = 0.$$

$$-j\omega^3 - 4j\omega^2 + j13\omega + K = 0.$$

$\omega^2 = 13$

* $\omega = \pm\sqrt{13}$

Unit-4 FREQUENCY RESPONSE ANALYSIS.

BODE PLOT.

Construct Bode plot for the following transfer function and determine the system gain K for the gain cross over freque[ncy] to be 5 rad/sec.

$$G(s) = \frac{ks^2}{(1+0.2s)(1+0.02s)}.$$

Solution.

Put $s = j\omega$,

$$\therefore G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

Let $K=1$, $\therefore G(j\omega) = \frac{(j\omega)^2}{(1+j0.2\omega)(1+j0.02\omega)}$

magnitude plot.

The corner frequencies, $\omega c_1 = \frac{1}{0.2} = 5$ rad/sec.

$\omega c_2 = \frac{1}{0.02} = 50$ rad/sec.

Term	corner frequency rad/sec.	slope db/dec.	change in slope db/dec.
$(j\omega)^2$	–	+40	
$\frac{1}{(1+j0.2\omega)}$	$\omega c_1 = \frac{1}{0.2} = 5$	-20	$40-20 = 20$
$\frac{1}{(1+j0.02\omega)}$	$\omega c_2 = \frac{1}{0.02} = 50$	-20	$20-20 = 0$,

Low frequency, $\omega_l < \omega c_1$

High frequency, $\omega_h > \omega c_2$.

$\therefore \omega_l = 0.5$ rad/sec

$\omega_h = 100$ rad/sec.

—— 4 marks

$A = |G(j\omega)|$ in db.

$A = 20 \log |(j\omega)^2| = 20 \log (\omega)^2 = 20 \log (0.5)^2 = -12$ db.

$A = 20 \log |(j\omega)^2| = 20 \log (\omega)^2 = 20 \log (5)^2 = 28$ db.

$A = \left[\text{Slope from } \omega c_1 \text{ to } \omega c_2 \times \log \frac{\omega c_2}{\omega c_1}\right] + A_{at\ \omega = \omega c_1}$

$= 20 \times \log \frac{50}{5} + 28$

$= 48$ db.

$A = \left[\text{Slope from } \omega c_2 \text{ to } \omega_h \times \log \frac{\omega_h}{\omega c_2}\right] + A_{at\ \omega = \omega c_2}$

$= 0 \times \log \frac{100}{50} + 48$

$= 48$ db.

Phase Plot.

$\phi = \angle G(j\omega) = 180^\circ - \tan^{-1} 0.2\omega - \tan^{-1} 0.02\omega.$

ω rad/sec.	$\tan^{-1} 0.2\omega$ deg	$\tan^{-1} 0.02\omega$ deg	$\phi = \angle G(j\omega)$ deg
0.5	5.7	0.6	174
1	11.3	1.1	168
5	45	5.7	130
10	63.4	11.3	106
50	84.3	45	50
100	87.1	63.4	30.

Calculation of K.

$20 \log K = -28$ db

$\log K = \frac{-28}{20} = 10^{-\frac{28}{20}} = 0.0398.$

—— 4 marks.

Given, $G(s) = \frac{Ke^{-0.2s}}{s(s+2)(s+8)}$. Find K so that the system is stable with (a) gain margin equal to 2db and (b) phase margin equal to 45°.

Assume, K=1,

$G(s) = \frac{e^{-0.2s}}{s(s+2)(s+8)} = \frac{e^{-0.2s}}{s\times 2(1+\frac{s}{2})\times 8(1+s/8)} = \frac{0.0625\,e^{-0.2s}}{s(1+0.5s)(1+0.125s)}$

Put, $s=j\omega$.

$\therefore G(j\omega) = \frac{0.0625\,e^{-j0.2\omega}}{j\omega(1+j0.5\omega)(1+j0.125\omega)}$

$\omega_{c1} = \frac{1}{0.5} = 2$ rad/sec

$\omega_{c2} = \frac{1}{0.125} = 8$ rad/sec.

Term	Corner frequency rad/sec	slope db/dec	change in slope db/dec.
$\frac{0.0625}{j\omega}$	–	-20	
$\frac{1}{1+j0.5\omega}$	$\omega_{c1} = 2$	-20	$-20-20=-40$
$\frac{1}{1+j0.125\omega}$	$\omega_{c2} = 8$	-20	$-40-20=-60$

$\therefore \omega_l = 0.5$ rad/sec

$\omega_h = 50$ rad/sec. —— 4 marks

$\therefore A = |G(j\omega)|$ at $\omega_l, \omega_{c1}, \omega_{c2}$ & ω_h.

At $\omega=\omega_l$, $A = 20\log\left|\frac{0.0625}{j\omega}\right| = 20\log\frac{0.0625}{0.5} = -18$ db.

At $\omega=\omega_{c1}$, $A = 20\log\left|\frac{0.0625}{j\omega}\right| = 20\log\frac{0.0625}{2} = -30$ db

At $\omega = \omega_{c_2} \Rightarrow A = \left[\text{Slope from } \omega_{c_1} \text{ to } \omega_{c_2} \times \log \frac{\omega_{c_2}}{\omega_{c_1}}\right] + A_{\text{at } \omega = \omega_{c_1}}$

$= -40 \times \log \frac{8}{2} + (-30)$

$= -54 \text{ db}.$

At $\omega = \omega_h \Rightarrow A = \left[\text{Slope from } \omega_{c_2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c_2}}\right] + A_{\text{at } \omega = \omega_{c_2}}$

$= -60 \times \log \frac{50}{8} + (-54)$

$= -102 \text{ db}.$

Phase angle,

$\phi = 0.2\omega \times \frac{180}{\pi} - 90^\circ - \tan^{-1} 0.5\omega - \tan^{-1} 0.125\omega.$

$\phi = \angle G(j\omega).$

ω rad/sec	ϕ_{deg}.
0.01	-90°
0.1	-94
0.5	-114
1	-134
2	-172
3	-202
4	-226.

Phase Margin,

$\gamma = 180^\circ + \phi_{gc}.$

$\gamma = 45, \quad \phi_{gc} = \gamma - 180^\circ = 45 - 180^\circ = -135^\circ.$

$\therefore 20 \log K = 24 \quad ; \quad K = 10^{24/20} \quad ; \quad K = 15.84.$

$\therefore 20 \log K = 30 \quad ; \quad K = 10^{30/20} \quad ; \quad K = 31.62.$

—— 4 marks

Plot the Bode diagram for the following transfer function and obtain the gain and phase cross-over frequencies.

$$G(s) = \frac{10}{s(1+0.4s)(1+0.1s)}$$

Put $s=j\omega$,

$$\therefore G(j\omega) = \frac{10}{j\omega(1+j0.4\omega)(1+j0.1\omega)}$$

Magnitude plot,

$\omega_{c1} = \frac{1}{0.4} = 2.5$ rad/sec,

$\omega_{c2} = \frac{1}{0.1} = 10$ rad/sec.

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec.
$\frac{10}{j\omega}$	–	-20	
$\frac{1}{1+j0.4\omega}$	$\omega_{c1} = 2.5$	-20	$-20-20 = -40$
$\frac{1}{1+j0.1\omega}$	$\omega_{c2} = 10$	-20	$-40-20 = -60$

$\omega_l = 0.1$ rad/sec

$\omega_h = 50$ rad/sec, ——— 4 marks

$A = |G(j\omega)|$ in db,

At $\omega = \omega_l$, $A = 20\log\left|\frac{10}{j\omega}\right| = 20\log\frac{10}{0.1} = 40$ db.

At $\omega = \omega_{c1}$, $A = 20\log\left|\frac{10}{j\omega}\right| = 20\log\frac{10}{2.5} = 12$ db.

At $\omega = \omega_{c_2}$, $A = \left[\text{Slope from } \omega_{c_1} \text{ to } \omega_{c_2} \times \log \frac{\omega_{c_2}}{\omega_{c_1}}\right] + A \text{ at } \omega = \omega_{c_1}$

$= -40 \times \log \frac{10}{2.5} + 12$

$= -12 \text{ db}.$

At $\omega = \omega_h$, $A = \left[\text{Slope from } \omega_{c_2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c_2}}\right] + A \text{ at } \omega = \omega_{c_2}$

$= -60 \times \log \frac{50}{10} + (-12)$

$= -54 \text{ db}.$

Phase Plot,

$\phi = -90^\circ - \tan^{-1} 0.4\omega - \tan^{-1} 0.1\omega$.

ω rad/sec.	$\phi = \angle G(j\omega)$ deg.
0.1	−92
1	−118
2.5	−150
4	−170
10	−210
20	− 236.

Gain cross-over frequency: 5 rad/sec

Phase cross-over frequency: 5 rad/sec. —— 4 marks.

POLAR PLOTS.

The Open loop transfer function of a unity feedback system is given by $G(s) = 1/s(1+s)(1+2s)$. Sketch the polar plot and determine the gain margin and phase margin.

$$G(s) = \frac{1}{s(1+s)(1+2s)}$$

Put $s = j\omega$.

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}.$$

$\omega_{c1} = \frac{1}{2} = 0.5$ rad/sec.

$\omega_{c2} = \frac{1}{1} = 1$ rad/sec.

$$G(j\omega) = \frac{1}{(j\omega)(1+j\omega)(1+j2\omega)} = \frac{1}{\omega\angle 90^\circ \sqrt{1+\omega^2}\angle \tan^{-1}\omega \sqrt{1+4\omega^2}\angle \tan^{-1}2\omega}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

← 4 marks

$$\therefore |G(j\omega)| = \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}}$$

$$= \frac{1}{\omega\sqrt{1+4\omega^2+\omega^2+4\omega^4}}$$

$$= \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}.$$

Gain margin $K_g = 1.4286$

Phase margin $\gamma = +12^\circ$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega.$$

magnitude and Phase of $G(j\omega)$ at Various frequencies.

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$\lvert G(j\omega)\rvert$	2.2	1.8	1.5	1.2	0.9	0.7	0.3
$\angle G(j\omega)$	-144	-150	-156	-162	-171	-180	-198.

← 4 marks

The Open loop transfer function of a unity feedback system is given by $G(s) = 1/s^2(1+s)(1+2s)$. Sketch the polar plot and determine the gain margin and phase margin.

Given that, $G(s) = \dfrac{1}{s^2(1+s)(1+2s)}$

Put $s = j\omega$,

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+j2\omega)}.$$

$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

$$= \frac{1}{\omega^2\sqrt{1+5\omega^2+4\omega^4}}$$

$\angle G(j\omega) = -180° - \tan^{-1}\omega - \tan^{-1}2\omega.$ —— 4 marks

Magnitude and phase plot of $G(j\omega)$ at various frequencies.

ω rad/sec.	0.415	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$\|G(j\omega)\|$	3.3	2.5	1.9	1.5	1.2	1	0.8	0.3
$\angle G(j\omega)$ deg.	−246	−251	−256	−261	−265	−269	−273	−288

Gain margin $K_g = 0$,

Phase margin, $\gamma = -90°$. —— 4 marks

The Open loop transfer function of a unity feedback system is given by $G(s) = \dfrac{(1+0.2s)(1+0.025s)}{s^3(1+0.005s)(1+0.001s)}$. Sketch the polar plot and determine the phase margin.

$$G(s) = \frac{(1+0.2s)(1+0.025s)}{s^3(1+0.005s)(1+0.001s)}$$

$$\therefore G(j\omega) = \frac{(1+j0.2\omega)(1+j0.025\omega)}{(j\omega)^3(1+j0.005\omega)(1+j0.001\omega)}$$

$$|G(j\omega)| = \frac{\sqrt{1+(0.2\omega)^2}\sqrt{(1+0.025\omega)^2}}{\omega^3\sqrt{(1+0.005\omega)^2}\sqrt{(1+0.001\omega)^2}}$$

$$\angle G(j\omega) = \tan^{-1}0.2\omega + \tan^{-1}0.0025\omega - 270^\circ - \tan^{-1}0.005\omega - \tan^{-1}0.001\omega$$

— 4 marks

Magnitude and phase of $G(j\omega)$

ω, rad/sec	0.9	0.95	1.0	1.1	1.2	1.4	1.7
$\lvert G(j\omega)\rvert$	1.4	1.2	1.0	0.8	0.6	0.4	0.2
$\angle G(j\omega)$, deg.	−259	−258	−257	−256	−255	−253	−249

Phase margin, $\gamma = -77^\circ$.

Real and imaginary part of $G(j\omega)$.

ω, rad/sec	0.9	0.95	1.0	1.1	1.2	1.4	1.7
$G_R(j\omega)$	−0.27	−0.25	−0.22	−0.19	−0.16	−0.12	−0.07
$G_I(j\omega)$	1.37	1.17	0.97	0.78	0.58	0.38	0.19

Phase margin, $\gamma = -77^\circ$. — 4 marks

Consider a unity feedback system having an open loop transfer function $G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$. Sketch the polar plot and determine the value of K so that i) Gain margin is 18 db & ii) phase margin is 60°.

$$G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$$

Put K=1, $s = j\omega \Rightarrow G(j\omega) = \frac{1}{j\omega(1+j0.2\omega)(1+j0.05\omega)}$

$\omega_{c1} = \frac{1}{0.2} = 5$ rad/sec; $\omega_{c2} = \frac{1}{0.05} = 20$ rad/sec.

$$|G(j\omega)| = \frac{1}{\omega\sqrt{(1+(0.2\omega)^2}\sqrt{(1+(0.5)^{\omega})^2}}$$

$\angle G(j\omega) = -90° - \tan^{-1} 0.2\omega - \tan^{-1} 0.05\omega$. —— 4 marks

Magnitude & phase of $G(j\omega)$ at various frequencies.

ω rad/sec	0.6	1	3	5	7	11	14
$\lvert G(j\omega)\rvert$	1.65	1.0	0.3	0.14	0.07	0.03	0.02
$\angle G(j\omega)$ deg.	−98	−104	−129.4	−149	−164	−184	−195

K=1

Gain margin, $K_g = \frac{1}{0.04} = 25$

Gain margin in db = 20 log 25 = 28 db.

Phase margin, $\gamma = 76°$. —— 4 marks

NICHOL'S CHART.

Consider a unity feedback system having an open loop transfer function $G(s) = \frac{K(1+10s)}{s^2(1+s)(1+2s)}$. Sketch the Nichols plot and determine the value of K. so that (i) Gain margin is 10 db ii) phase margin is 10°.

$$G(s) = \frac{K(1+10s)}{s^2(1+s)(1+2s)}$$

$$K=1;\ s=j\omega,\quad G(j\omega) = \frac{(1+j10\omega)}{(j\omega)^2(1+j\omega)(1+j2\omega)}$$

$$|G(j\omega)| = \frac{\sqrt{1+100\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

$$\therefore |G(j\omega)| \text{ in db} = 20\log\left[\frac{\sqrt{1+100\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}\right]$$

$$\angle G(j\omega) = \tan^{-1}\omega - 180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega.$$

Magnitude of Phase of $G(j\omega)$. —— 4 marks

ω rad/sec	0.2	0.4	0.6	0.8	1.0	1.5	2.0	4.0
$\lvert G(j\omega)\rvert$ db	34.1	25.4	19.3	14.3	10	1.4	−5.3	−22.5
$\angle G(j\omega)$ deg.	−150	−164	−181	−194	−204	−222	−232	−250

Gain margin = −19.5 db

Phase margin = −45°

$$\therefore 20\log K_1 = -29.5\text{ db} \Rightarrow \log K_1 = \frac{-29.5}{10} \Rightarrow K_1 = 10^{\frac{-29.5}{20}}$$

$$= 0.0335.$$

$\phi_{gc2} = \gamma_2 - 180° = 10° - 180° = -170°$

$\therefore 20 \log k_2 = -23 \Rightarrow \log k_2 = -23/20 \Rightarrow k_2 = 10^{\frac{-23}{20}} = 0.07$

Gain Margin = −19.5 db

Phase margin = −45°

For a gain of 10db, $K = K_1 = 0.0335$

For a phase margin of 10°, $K = K_2 = 0.07$. — 4 marks

A unity feedback system has open loop transfer function, $G(s) = \frac{20}{s(s+2)(s+5)}$. Using Nichol's chart determine the closed loop frequency response and estimate M_r, ω_r and ω_b.

$$G(s) = \frac{20}{s(s+2)(s+5)}$$

The transfer function G(s) is converted to time constant or bode form.

$$G(s) = \frac{20}{s \times 2\left(\frac{s}{2}+1\right) \times 5\left(\frac{s}{5}+1\right)} = \frac{20/2\times5}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)} = \frac{2}{s(1+0.5s)(1+0.2s)}$$

Put $s = j\omega$,

$$G(j\omega) = \frac{2}{(j\omega)(1+j0.5\omega)(1+j0.2\omega)}$$

$$|G(j\omega)| = \frac{2}{\omega\sqrt{1+0.25\omega^2}\sqrt{1+0.04\omega^2}}$$ — 4 marks

$$|G(j\omega)| \text{ in db} = 20 \log\left[\frac{2}{\omega\sqrt{1+0.25\omega^2}\sqrt{1+0.04\omega^2}}\right]$$

$$\angle G(j\omega) = -90° - \tan^{-1} 0.5\omega - \tan^{-1} 0.2\omega.$$

Resonant peak M_r = 4 db

Resonant frequency ω_r = 1.6 rad/sec.

Bandwidth, ω_b = 2.5 rad/sec. — 4 marks

NYQUIST PLOT.

A unity feedback control system has $G(s) = \frac{10}{s(s+1)(s+2)}$. Draw the Nyquist plot and determine the closed loop stability.

Solution.

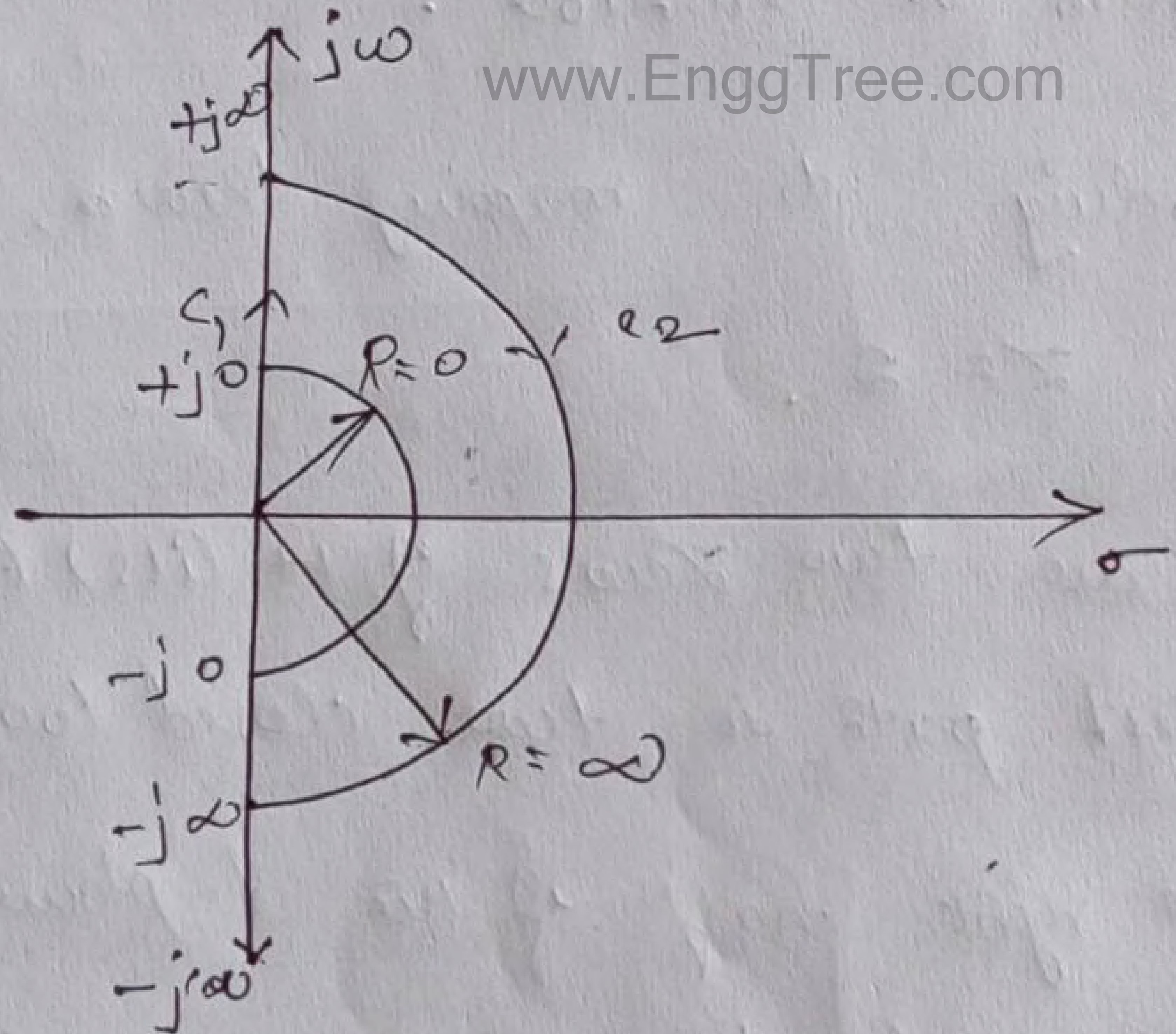
$$G(s).H(s) = \frac{10}{s(s+1)(s+2)}$$

as $H(s) = 1$.

No of poles in the right half of the s-plane $P = 0$.

For stability no of encirclements, $N = -P = 0$.

The Nyquist plot should not encircle $(-1+j0)$ points for absolute stability of this system.



$$G(j\omega)\, H(j\omega) = \frac{10}{(j\omega)(1+j\omega)(2+j\omega)}$$

$$M = |G(j\omega) H(j\omega)| = \frac{10}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}.$$

* $\phi = \angle G(j\omega)H(j\omega) = -90° - \tan^{-1}\omega - \tan^{-1}\omega/2$.

*

ω	M	ϕ
0	∞	$-90°$
∞	0	$-270°$.

* $G(j\omega)H(j\omega) = \frac{-30 \times 2}{(4+2)(4+2)} = \frac{-60}{36}$

$= -1.667$.

* The number of encirclement of $(-1+j0)$ are $N = +2$. However for stability, $N=0$, the given system is unstable.

* According to mapping theorem, $N = Z - P$.

$Z = 2$.

* There are two zeros of $1 + G(s)H(s)$ encircled by Nyquist path ie two closed loop poles are there in the right half of the s-plane due to which the closed loop system is unstable.

NYQUIST STABILITY CRITERION.

* If the nyquist plot of the open loop transfer function G(s) corresponding to the nyquist contour in the s plane encircle the critical point $-1+j0$ in the contour in anti-clockwise direction as many times as the number of right half of s plane poles of (G(s), the closed loop system is stable.

* If the roots of characteristic equation lies on imaginary axis the nature of impulse response is oscillatory.

* The contour that enclosed entire right half of s plane is called nyquist contour.

* The two segments of Nyquist contour are

1. An infinite line segment C_1 along the imaginary axis.
2. An arc C_2 of infinite radius.

CLOSED LOOP STABILITY.

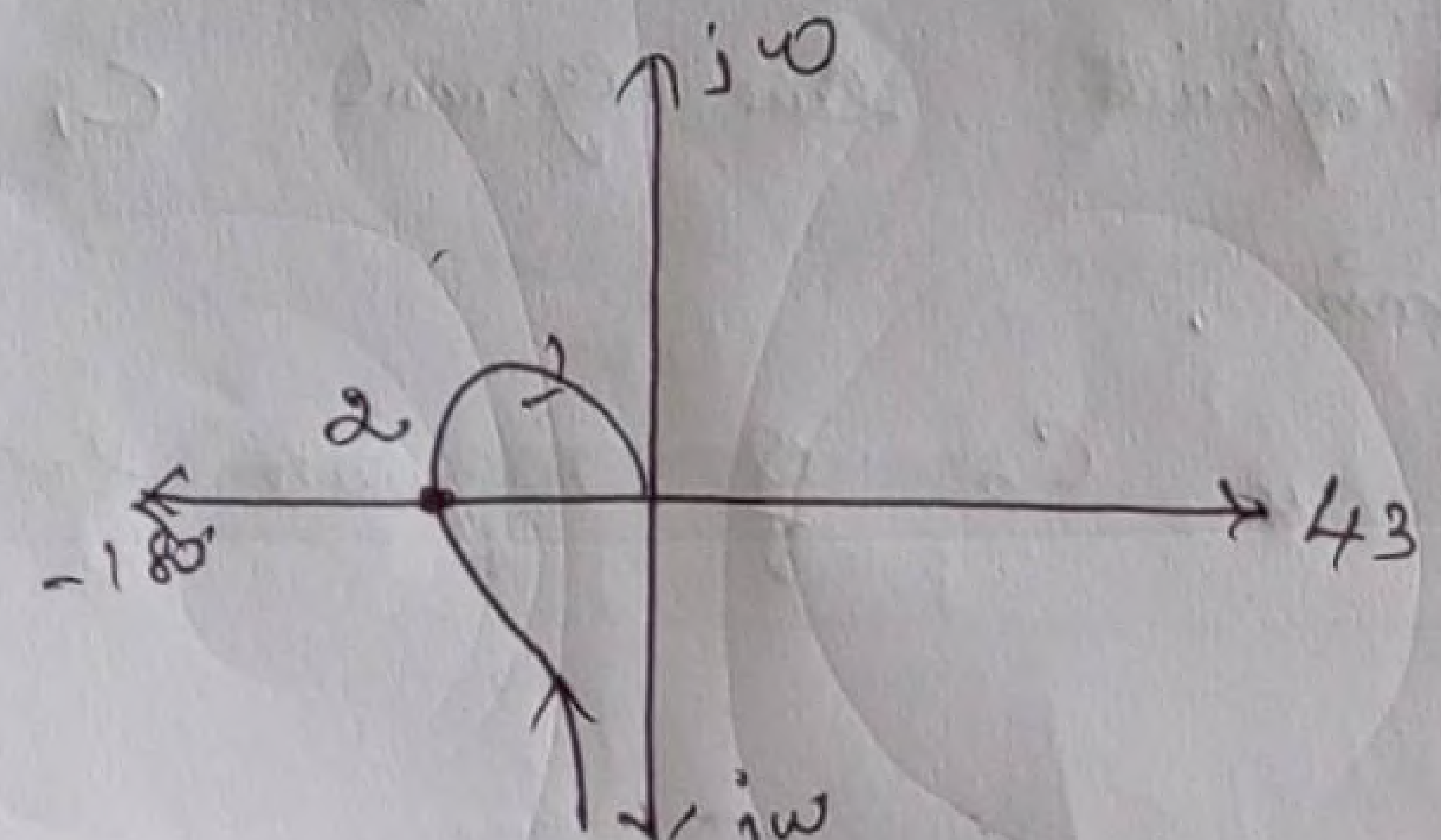
Draw the Nyquist plot for the system whose OLTF is $G(s)H(s) = \frac{K}{s(s+2)(s+10)}$. Determine the range of K for which the closed loop system is stable.

Solution.

1. No of poles in the right of half of s-plane $P = 0$.
2. For stability, $N = -P = 0$.
3. As there is one pole at origin, the Nyquist contour is chosen, C_1, C_2, C_3 & C_4.
4. $G(j\omega) H(j\omega) = \frac{K}{(j\omega)(j\omega+2)(j\omega+10)}$

$$M = |G(j\omega) H(j\omega)| = \frac{K}{\omega\sqrt{\omega^2+4}\sqrt{\omega^2+100}}$$

ω	M	ϕ
0	∞	$-90°$
∞	0	$-270°$



5. $G(s)H(s) = \frac{K}{s^3}$

$G(s)H(s) = 0\, e^{-j3\pi/2}$

$G(s)H(s) = 0\, e^{+j3\pi/2}$

6. The range of value of K for stability is

$0 < K < 240.$

CONSTANT M AND N CIRCLES.

Prove that the loci of the constant magnitude of closed loop transfer function is a circle.

Constant M circles

* $T(j\omega) = \frac{C(j\omega)}{R(j\omega)}$

$= \frac{G(j\omega)}{1+G(j\omega)}$

$= \frac{x+jy}{1+x+jy}$

* $|T(j\omega)|^2 = \frac{x^2+y^2}{(1+x)^2+y^2}$

* Let $|T(j\omega)| = M$

$\Rightarrow M^2 = \frac{x^2+y^2}{(1+x)^2+y^2}$

$$M^2(1+x)^2 + M^2y^2 = x^2+y^2$$

$$x^2(M^2-1) + 2xM^2 + y^2(M^2-1) = -M^2$$

$$x^2 + \frac{2M^2}{M^2-1}x + y^2 = \frac{-M^2}{M^2-1}$$

Making a perfect square of the term, we have

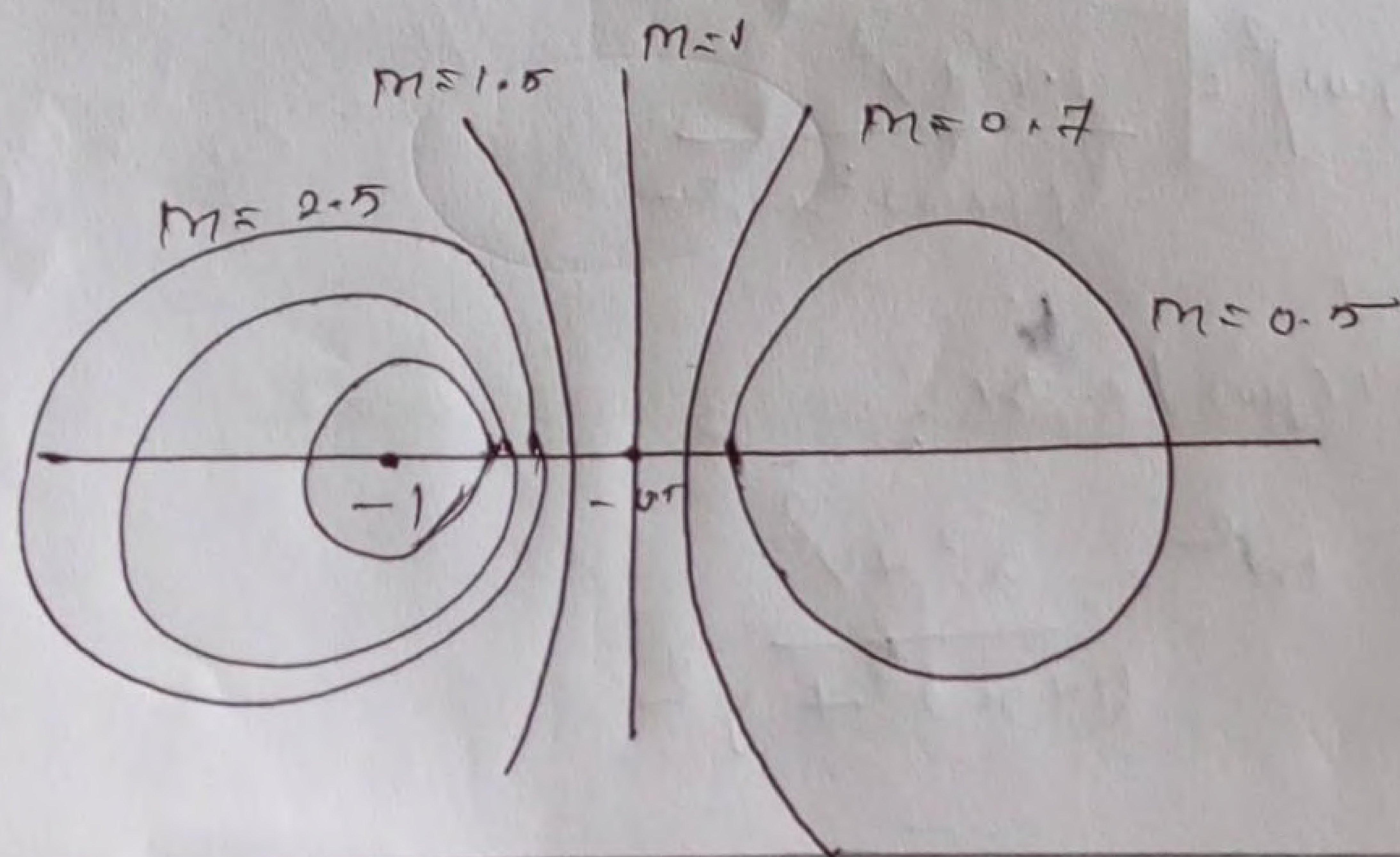
$$\left(x + \frac{M^2}{M^2-1}\right)^2 + y^2 = \frac{-M^2}{M^2-1} + \frac{M^4}{(M^2-1)^2}$$

$$= \frac{-M^2(M^2-1)+M^4}{(M^2-1)^2}$$

$$= \left(\frac{M}{M^2-1}\right)^2$$

* Represent a circle with a radius of $\frac{M}{M^2-1}$ and centre at $\left(\frac{M^2}{M^2-1}, 0\right)$

* These circles are called constant M circles.



FREQUENCY DOMAIN SPECIFICATIONS -

* The frequency domain specifications are resonant peak, resonant frequency and bandwidth. Substitute $s = j\omega$, in the above equation.

* A time domain graph shows how a signal changes with time, whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies.

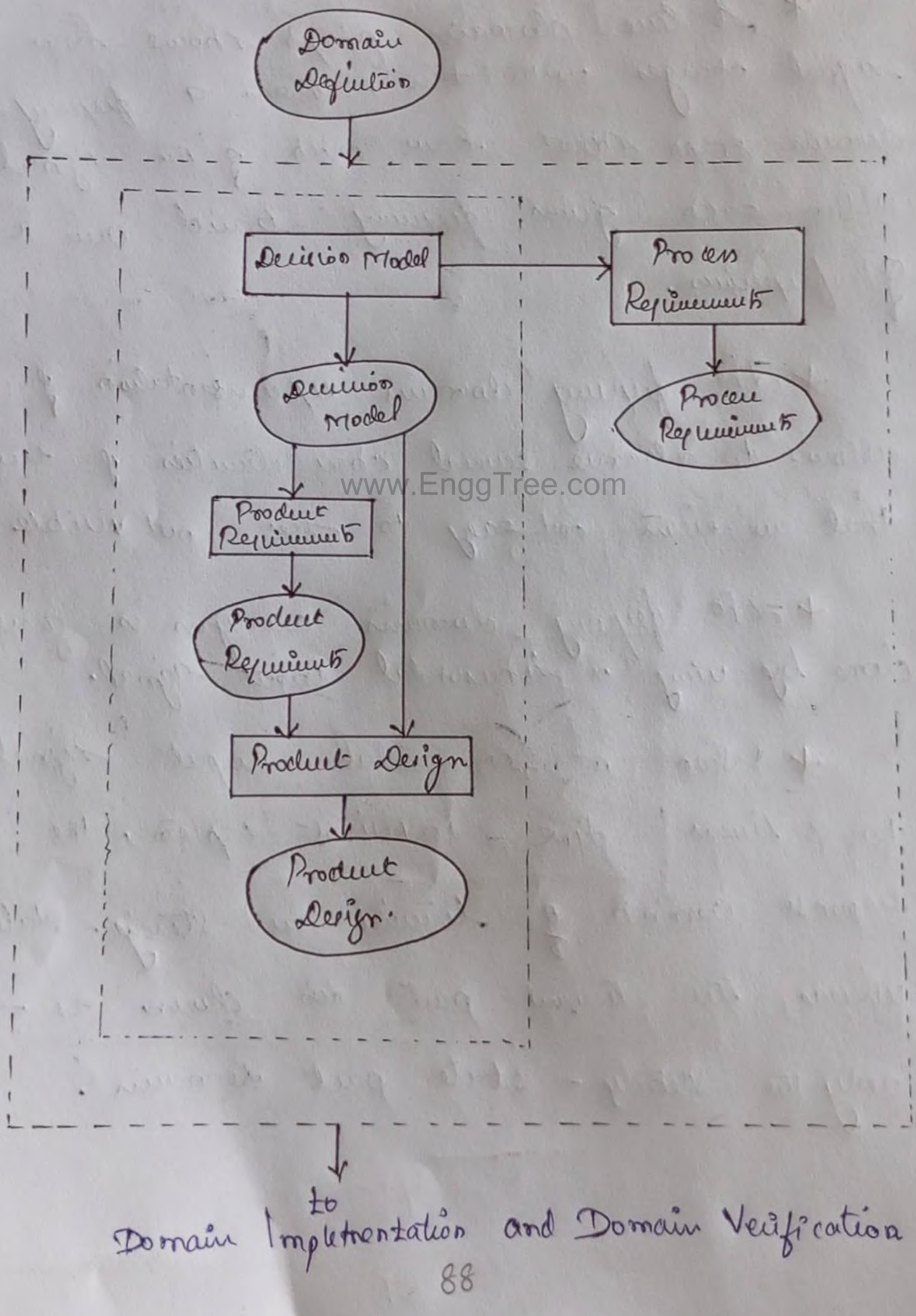
* The frequency domain representation of a signal allows to observe several characteristics of the signal that are either not easy to see, not visible at all.

* The frequency domain analysis is generally done by using a sinusoidal input signal.

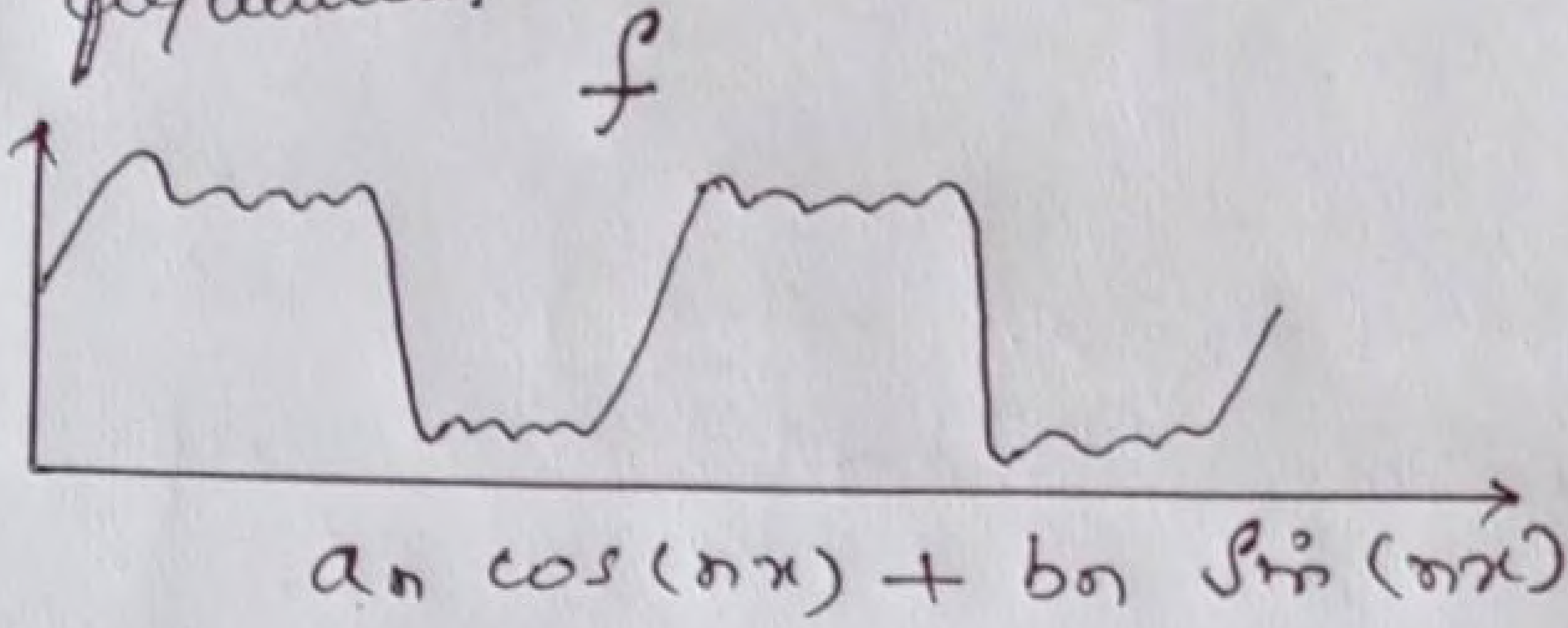
* When a sinusoidal input signal is given to a linear time - invariant system, the output response consists of transient and steady - state parts, whereas the transient part dies down as $\to \infty$, only the steady - state part remain.

* In Engineering and statistics, frequency domain is a term used to describe the analysis of mathematical functions or signals with respect to frequency, rather than time.

Domain Specification Process.

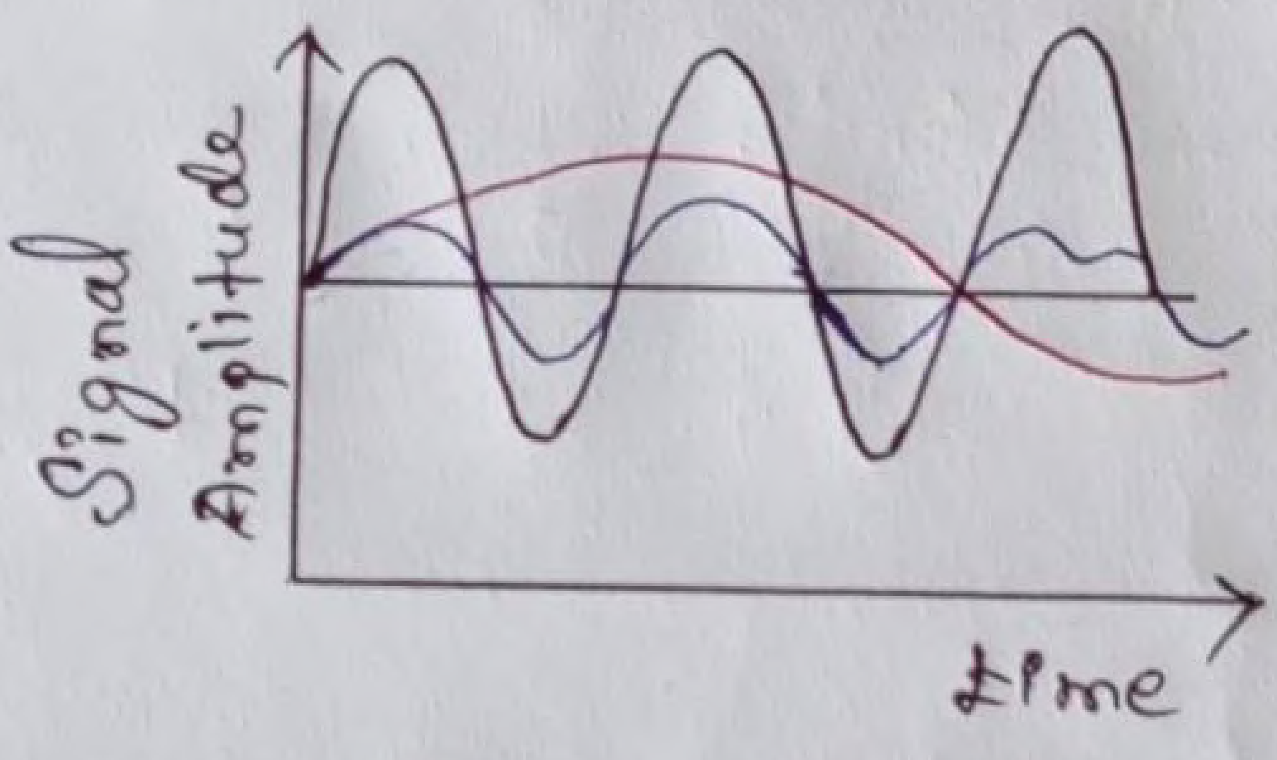


* A time - domain graph shows how a signal changes from over time, whereas a frequency - domain graph shows how much of the signal lies within each given frequency band over a range of frequencies.

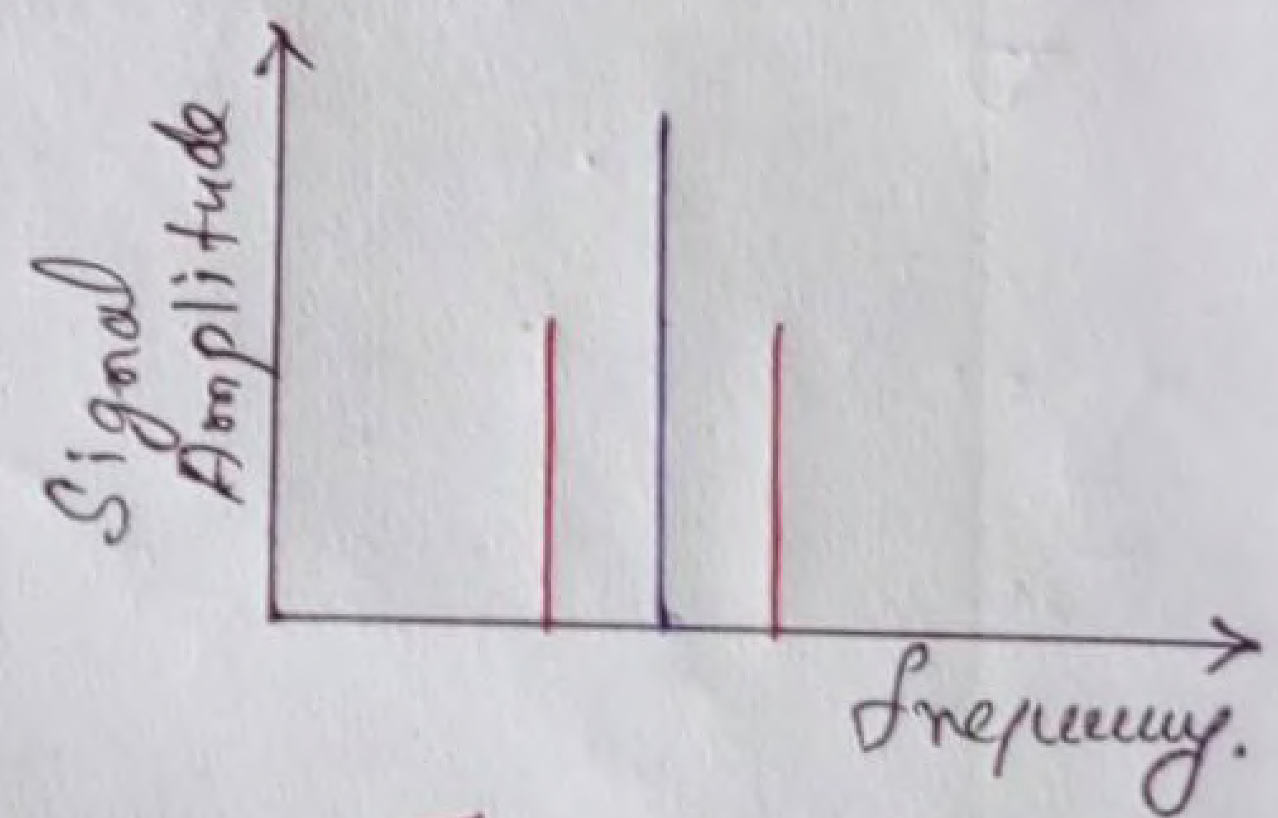


* Advantages of frequency domain are System performance and stability can be tuned and optimized efficiently, requires its frequency - response function.

* Its ability to describe "transfer functions", which aid in the analysis of individual components of complex system.



Time domain



Frequency domain.

* Disadvantages of frequency domain analysis are
→ Requires transfer function of plant be known.
→ Difficult to infer all performance value.
→ Hard to extract steady - state response.

The state equation and initial condition vector of an linear time-invariant system are given below. Determine the solution of state equation.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; \quad x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}; \quad sI - A = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$|sI - A| = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix} = (s-1)^2 - 0 = (s-1)^2.$$

$$(sI - A)^{-1} = \frac{1}{(s-1)^2} \begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

$$e^{At} = \phi(t) = L^{-1}[\phi(s)]$$

$$= L^{-1}[(sI - A)^{-1}]$$

$$= \begin{bmatrix} e^t & 0 \\ t e^t & e^t \end{bmatrix}$$

$\therefore$ The solution of the state equation is,

$$x(t) = e^{At} x_0 = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} e^t \\ te^t \end{bmatrix}.$$

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Given that, $A_1 = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}$; $A_2 = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}$; $A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$ Compute e^{At}

Here, $A = A_1 + A_2$

$$e^{At} = e^{(A_1 + A_2)t}$$
$$= e^{A_1 t} \cdot e^{A_2 t}$$

$$SI - A_1 = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix} = \begin{bmatrix} s-\sigma & 0 \\ 0 & s-\sigma \end{bmatrix}$$

$$\Delta_1 = |SI - A_1| = \begin{vmatrix} s-\sigma & 0 \\ 0 & s-\sigma \end{vmatrix} = (s-\sigma)^2$$

$$(SI - A_1)^{-1} = \frac{1}{\Delta_1}\begin{bmatrix} s-\sigma & 0 \\ 0 & s-\sigma \end{bmatrix} = \frac{1}{(s-\sigma)^2}\begin{bmatrix} s-\sigma & 0 \\ 0 & s-\sigma \end{bmatrix} = \begin{bmatrix} \frac{1}{s-\sigma} & 0 \\ 0 & \frac{1}{s-\sigma} \end{bmatrix}$$

$$e^{A_1 t} = L^{-1}[(SI - A_1)^{-1}] = \begin{bmatrix} e^{-\sigma t} & 0 \\ 0 & e^{-\sigma t} \end{bmatrix}$$

$$SI - A_2 = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} = \begin{bmatrix} s & -\omega \\ \omega & s \end{bmatrix}$$

$$\Delta_2 = |SI - A_2| = \begin{vmatrix} s & -\omega \\ \omega & s \end{vmatrix} = s^2 + \omega^2$$

$$[SI - A_2]^{-1} = \frac{1}{\Delta_2}\begin{bmatrix} s & \omega \\ \omega & s \end{bmatrix} = \frac{1}{s^2+\omega^2}\begin{bmatrix} s & \omega \\ -\omega & s \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2+\omega^2} & \frac{\omega}{s^2+\omega^2} \\ \frac{-\omega}{s^2+\omega^2} & \frac{s}{s^2+\omega^2} \end{bmatrix}$$

$$e^{A_2 t} = L^{-1}[(SI - A_2)^{-1}] = \begin{bmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{bmatrix}$$

$$e^{At} = e^{A_1 t} e^{A_2 t} = \begin{bmatrix} e^{-\sigma t} & 0 \\ 0 & e^{-\sigma t} \end{bmatrix}\begin{bmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{bmatrix}$$

$$\therefore e^{At} = \begin{bmatrix} e^{-\sigma t}\cos\omega t & e^{-\sigma t}\sin\omega t \\ -e^{-\sigma t}\sin\omega t & e^{-\sigma t}\cos\omega t \end{bmatrix}$$

The output equation is $y = 3x_1 - 4x_2 + x_3$

The state model in matrix form is given below.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

$$y = \begin{bmatrix} 3 & -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Consider the matrix A. Compute e^{At} by two methods.

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$e^{At} = \left[I + At + \frac{1}{2!} A^2 t^2 + \frac{1}{3!} A^3 t^3 + \dots \right]$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 6 & 7 \\ -14 & -15 \end{bmatrix}$$

$$A^4 = A^3 \cdot A = \begin{bmatrix} -14 & -15 \\ 30 & 31 \end{bmatrix}$$

$$\therefore e^{At} = I + At + \frac{1}{2!} A^2 t^2 + \frac{1}{3!} A^3 t^3 + \frac{1}{4!} A^4 t^4 + \dots$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} t + \frac{1}{2} \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix} t^2 + \frac{1}{6} \begin{bmatrix} 6 & 7 \\ -14 & -15 \end{bmatrix} t^3 + \frac{1}{24} \begin{bmatrix} -14 & -15 \\ 30 & 31 \end{bmatrix} t^4$$

$$= \begin{bmatrix} 1 - t^2 + t^3 - \frac{7}{12} t^4 + \dots & 1 - \frac{3}{2} t^2 + \frac{7}{6} t^3 - \frac{5}{8} t^4 + \dots \\ -2t + 3t^2 - \frac{7}{3} t^3 + \frac{5}{4} t^4 + \dots & 1 - 3t + \frac{7}{2} t^2 - \frac{5}{2} t^3 + \frac{31}{24} t^4 + \dots \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

Determine the canonical state model of the system, whose transfer function is $T(s) = 2(s+5)/[(s+2)(s+3)(s+4)]$.

By partial fraction expansion,

$$\frac{Y(s)}{U(s)} = \frac{2(s+5)}{(s+2)(s+3)(s+4)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+4}.$$

$$A = \left.\frac{2(s+5)}{(s+3)(s+4)}\right|_{s=-2} = \frac{2(-2+5)}{(-2+3)(-2+4)} = \frac{2\times 3}{1\times 2} = 3.$$

$$B = \left.\frac{2(s+5)}{(s+2)(s+4)}\right|_{s=-3} = \frac{2(-3+5)}{(-3+2)(-3+4)} = \frac{2\times 2}{-1\times 1} = -4.$$

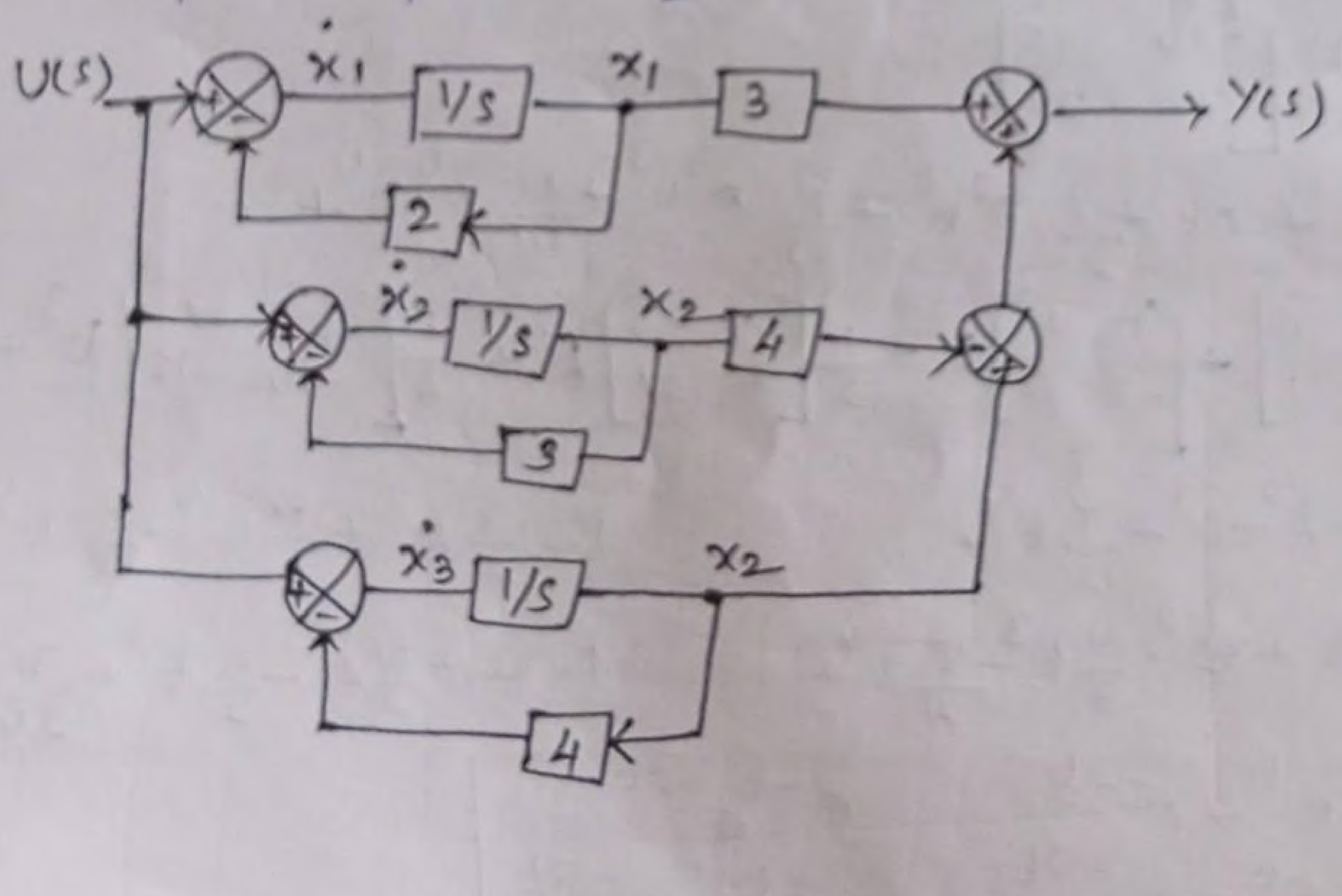
$$C = \left.\frac{2(s+5)}{(s+2)(s+3)}\right|_{s=-4} = \frac{2(-4+5)}{(-4+2)(-4+3)} = \frac{2\times 1}{-2\times -1} = 1.$$

$$\therefore \frac{Y(s)}{U(s)} = \frac{3}{s(1+2/s)} - \frac{4}{s(1+3/s)} + \frac{1}{s(1+4/s)}$$

$$\Rightarrow Y(s) = \left[(3)\cdot\frac{\frac{1}{s}}{1+\frac{1}{s}\cdot 2}\right]U(s) - \left[\frac{\frac{1}{s}}{1+\frac{1}{s}\times 3}\times(4)\right]U(s) + \left[\frac{\frac{1}{s}}{1+\frac{1}{s}\times 4}\right]U(s)$$

The state equations are

$$\dot{x}_1 = -2x_1 + u \;;\; \dot{x}_2 = -3x_2 + u \;;\; \dot{x}_3 = -4x_3 + u.$$



Obtain the state model of the system whose transfer function is given as, $\frac{Y(s)}{U(s)} = \frac{10}{s^3+4s^2+2s+1}$

Given that $\frac{Y(s)}{U(s)} = \frac{10}{s^3+4s^2+2s+1}$ — ①

$$Y(s)\left[s^3+4s^2+2s+1\right] = 10\,U(s)$$

$$s^3 Y(s) + 4s^2 Y(s) + 2s Y(s) + Y(s) = 10\,U(s). \quad \text{— ②}$$

Taking inverse laplace transform of equ ②,

$$\dddot{y} + 4\ddot{y} + 2\dot{y} + y = 10U \quad \text{— ③}$$

State variables are,

$$x_1 = y$$
$$x_2 = \dot{y}$$
$$x_3 = \ddot{y}$$

Put $\dddot{y} = \dot{x}_3$

$\ddot{y} = x_3$

$\dot{y} = x_2$

$y = x_1$ in equ — ③

$$\Rightarrow \dot{x}_3 + 4x_3 + 2x_2 + x_1 = 10U.$$

$$\dot{x}_3 = -x_1 - 2x_2 - 4x_3 + 10U.$$

The state equations are

$$\dot{x}_1 = x_2 \;;\; \dot{x}_2 = x_3 \;;\; \dot{x}_3 = -x_1 - 2x_2 - 4x_3 + 10U.$$

The output equa is $y = x_1$

State model in the matrix form is,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & 2 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 10\end{bmatrix}\begin{bmatrix}U\end{bmatrix} \quad \& \quad y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Put $y = x_1$, $\frac{dy}{dt} = x_2$ and $\frac{d^2y}{dt^2} = x_3$ and $\frac{d^3y}{dt^3} = \dot{x}_3$ in the given equation,

$$\therefore \quad \dot{x}_3 + 6x_3 + 11x_2 + 6x_1 + u = 0.$$

$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 - u.$$

The state equations are

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 - u.$$

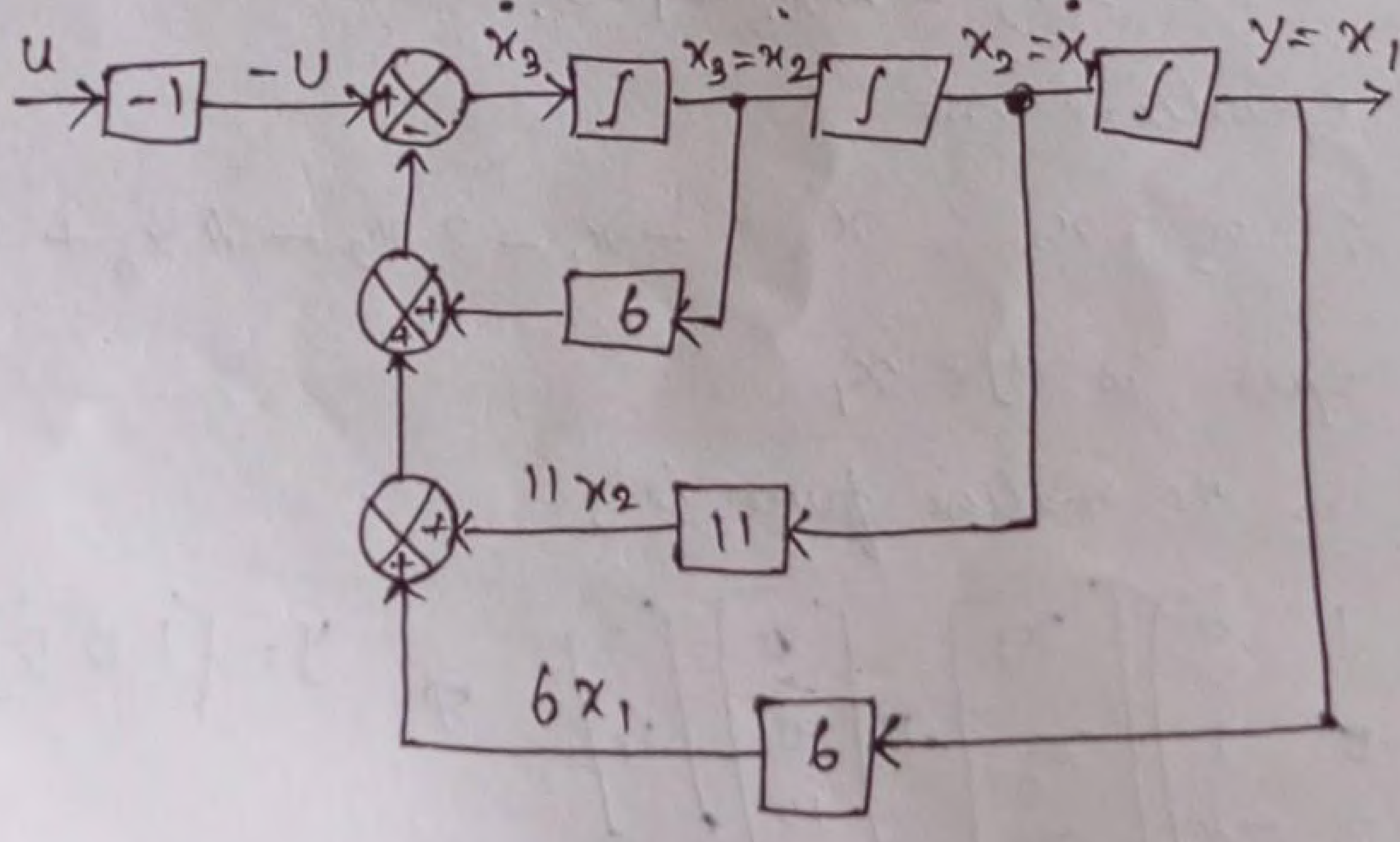
On arranging the state equations in the matrix form we get,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

Here y = output, $y = x_1$.

$\therefore$ The o/p equation is $y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Block diagram form of State diagram.



$$C\dot{x}_2 = \frac{x_1}{R} - \frac{x_2}{R} - \frac{x_2}{R} + \frac{U}{R}$$

$$\dot{x}_2 = \frac{1}{RC}x_1 - \frac{2}{RC}x_2 + \frac{1}{RC}U \quad \text{--- (6)}$$

∴ State equations of the s/m are

$$\dot{x}_1 = -\frac{1}{RC}x_1 + \frac{1}{RC}x_2$$

$$\dot{x}_2 = \frac{1}{RC}x_1 - \frac{2}{RC}x_2 + \frac{1}{RC}U.$$

State equations in the matrix form,

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-\frac{1}{RC} & \frac{1}{RC} \\ \frac{1}{RC} & -\frac{2}{RC}\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{RC}\end{bmatrix}\begin{bmatrix}U\end{bmatrix} \quad \text{--- (7)}$$

The Output, $y = V_1(t) = x_1$

∴ The output equation is $y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix}$

Construct a state model for a system characterized by the differential equation.

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y + U = 0.$$

Give the block diagram representation of the state model.

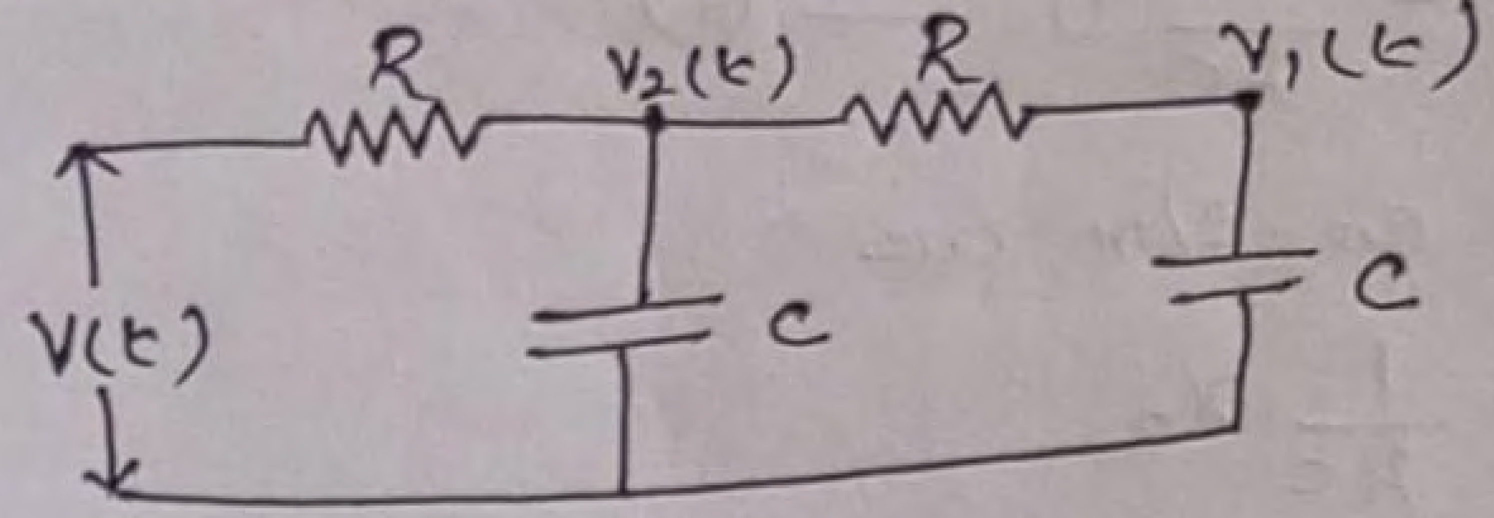
The state variables x_1, x_2 and x_3 are related to phase variables,

$$x_1 = y$$

$$x_2 = \frac{dy}{dt} = \dot{x}_1$$

$$x_3 = \frac{d^2y}{dt^2} = \dot{x}_2$$

Obtain the state model of the electrical networks by choosing $v_1(t)$ and $v_2(t)$ as state variables.



At node : 1, by KCL,

$$\frac{V_1 - V_2}{R} + C\frac{dV_1}{dt} = 0 \quad — (1)$$

At node: 2, by KCL,

$$\frac{V_2 - V_1}{R} + \frac{V_2}{R} + C\frac{dV_2}{dt} = \frac{v(t)}{R} \quad — (2)$$

Let state variables be x_1 and x_2.

$$V_1 = x_1 \quad \text{and} \quad V_2 = x_2$$

Let $v(t) = u =$ input.

On substituting in eq (1) & (2),

$$\frac{x_1 - x_2}{R} + C\frac{dx_1}{dt} = 0 \quad — (3)$$

$$\frac{x_2 - x_1}{R} + \frac{x_2}{R} + C\frac{dx_2}{dt} = \frac{u}{R} \quad — (4)$$

$$\frac{x_1}{R} - \frac{x_2}{R} + C\dot{x}_1 = 0.$$

from eq (3)

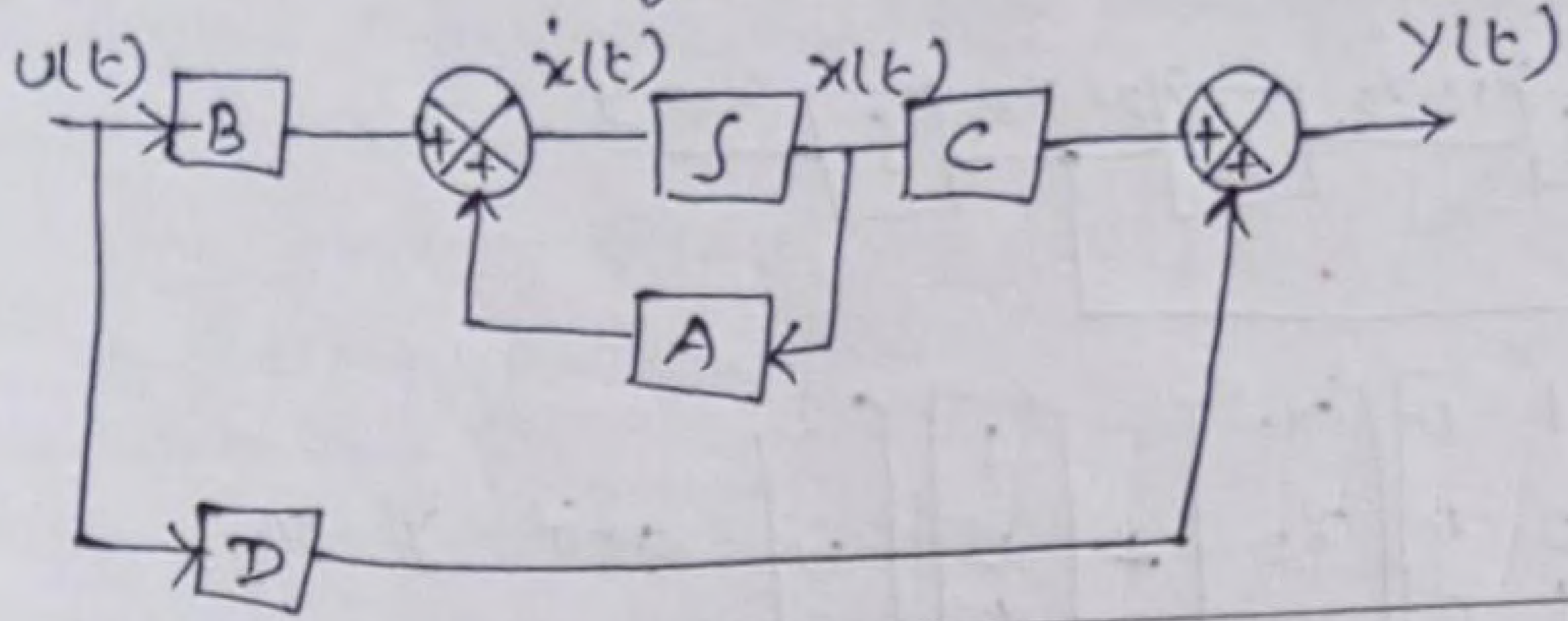
$$C\dot{x}_1 = \frac{-x_1}{R} + \frac{x_2}{R}$$

$$\dot{x}_1 = \frac{-x_1}{RC} + \frac{1}{RC}\cdot x_2 \quad — (5)$$

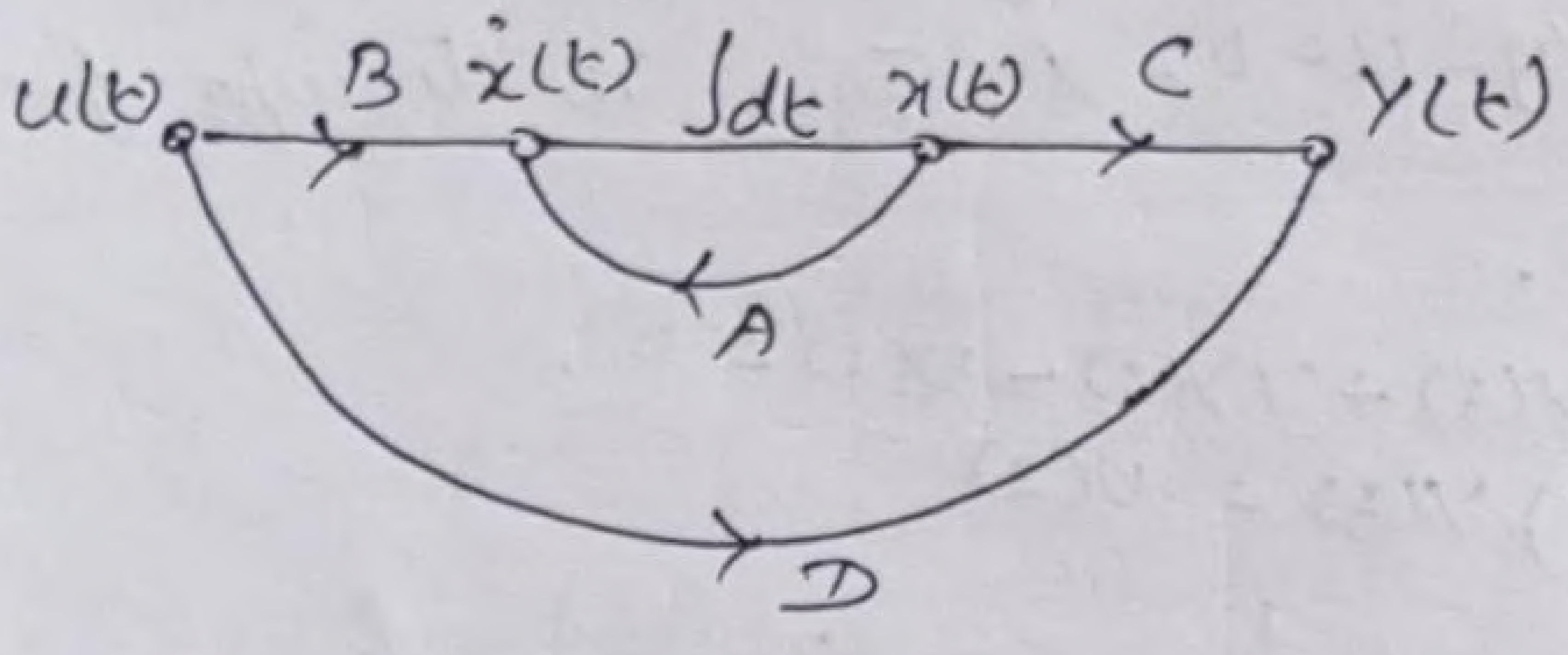
from eq (4)

$$\frac{x_2}{R} - \frac{x_1}{R} + \frac{x_2}{R} + C\dot{x}_2 = \frac{u}{R}$$

31. Draw the block diagram representation of state model?



32. Draw the signal flow graph representation of state model?



33. Illustrate the basic elements used to construct the state diagram.

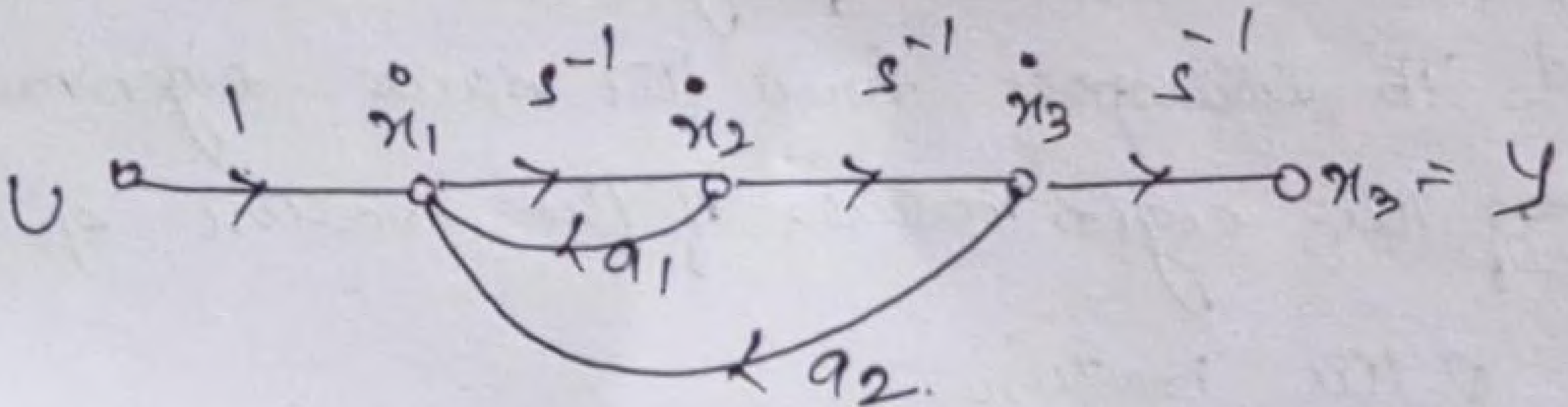
The basic elements used to construct the state diagram are scalar, Adder and Integrator.

34. Construct the signal flow graph of the system described by the state model.

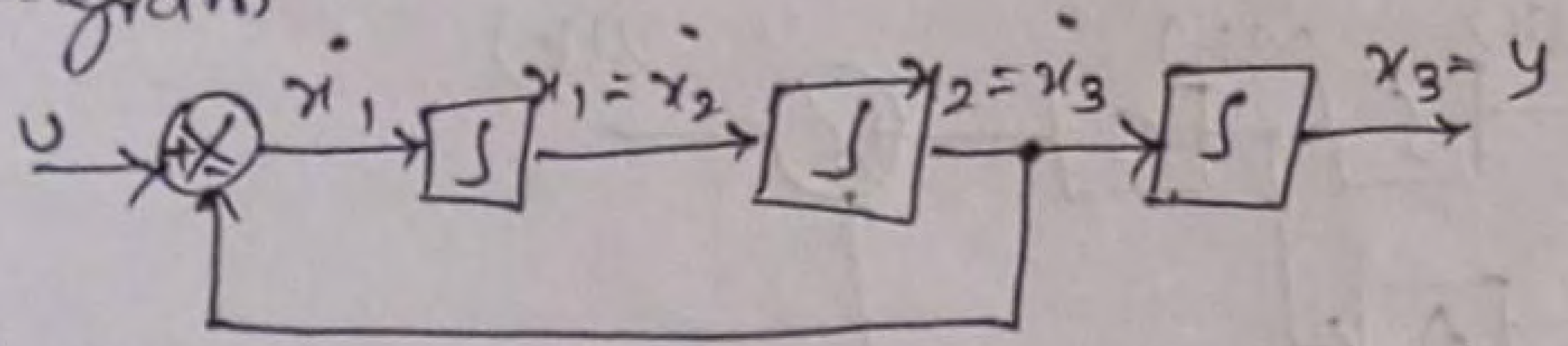
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \text{ and } y = x_3.$$

$\dot{x}_2 = x_1 + x_3$

$\dot{x}_3 = x_2$.



35. Determine the state model of the system represented by the block diagram:



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \text{ and } y = x_3 .$$

36. A system is characterized by the differential equation.
$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 7y - u = 0$. Determine its transfer function.

Laplace transform,

$$s^2 Y(s) + 10 s Y(s) + 7 Y(s) - U(s) = 0 .$$
$$(s^2 + 10 s + 7) Y(s) = U(s).$$
$$\therefore \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 10s + 7} .$$

37. The transfer function of a system is given by $\frac{Y(s)}{U(s)} = \frac{10}{4s^2 + 2s + 1}$.
Determine differential equation governing the system.

$$\frac{Y(s)}{U(s)} = \frac{10}{4s^2 + 2s + 1} .$$
$$(4s^2 + 2s + 1) Y(s) = 10 U(s) .$$
$$4s^2 Y(s) + 2s Y(s) + Y(s) = 10 U(s) .$$

Taking inverse laplace transform,

$$4\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 10u$$
$$4\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y - 10u = 0.$$

38. Illustrate any two properties of eigenvalues.

* A matrix and its transpose have the same eigenvalues.
* The product of the eigenvalues of the matrix equals the determinant of the matrix.

39. Explain resolvant matrix?

* The laplace transform of state transition matrix is called resolvant matrix.
* Resolvant matrix, $\phi(s) = L[\phi(t)] = L[e^{At}]$.
  Also, $\phi(s) = [SI - A]^{-1}$.

40. Explore similarity transformation.

* The process of transforming a square matrix A to another similar matrix B by the transformation $P^{-1}AP = B$ is called similarity transformation.
* The matrix P is called transformation matrix.

41. Illustrate the advantage and disadvantage in canonical form of state model.

* The advantage of canonical form is that the state equation are independent of each other.
* The disadvantage is that the canonical variables are not physical variables and so they are not available for measurement and control.

42. Evaluate the properties of state transition matrix.

1. $\phi(0) = e^{A \times 0} = I$ (Unit matrix)
2. $\phi(t) = e^{At} = (e^{-At})^{-1} = [\phi(-t)]^{-1}$
3. $\phi(t_1+t_2) = e^{A(t_1+t_2)}$
   $= e^{At_1} \cdot e^{At_2}$
   $= \phi(t_1)\,\phi(t_2)$
   $= \phi(t_2) \cdot \phi(t_1)$.

43. Define the characteristic equation of a matrix.

The characteristic equation of a nxn matrix A is the n^{th} degree polynomial of equation, $[\lambda I - A] = 0$, where I is the unit matrix.

44. Explain about diagonalization.
* The process of converting the system matrix A into a diagonal matrix by a similarity transformation using the modal matrix M is called diagonalization.

45. Define modal matrix.
* The modal matrix is a matrix used to diagonalize the system matrix.
* It is also called diagonalization matrix.
* If A = system matrix
M = Modal matrix
M^{-1} = Inverse of modal matrix, then $M^{-1}AM$ will be a diagonalized system matrix.

46. Create the transformed canonical state model of a system.
The transformed state model of a system is

$$\dot{Z} = \Lambda Z + \tilde{B} U$$
$$Y = \tilde{C} Z + DU$$

where, Z = transformed state vector ; $\tilde{B} = M^{-1}B$
M = Model matrix ; $\tilde{C} = CM$.
$\Lambda = M^{-1}AM$.

47. Infer when modal matrix is called vander monde matrix.
When the system matrix A is in the companion or bush form then the modal matrix is given by a special matrix called vander monde matrix, V.

$$M = V = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_n^2 \\ \vdots & \vdots & & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_n^{n-1} \end{bmatrix}$$

48. Describe Jordan canonical form.

* When the eigenvalues have multiplicity the system matrix cannot be diagonalized.

* But the transformation, $X = MZ$ will transform the system matrix to a form called Jordan matrix (J), where J is $M^{-1}AM$.

* The transformed state model in this case is called Jordan canonical form.

49. Discuss the need for controllability test.

* A system is said to be completely state controllable, if it is possible to transfer the system state from any initial state $x(t_0)$ at any other desired state $x(t)$, in specified finite time by a control vector $u(t)$.

50. Express the condition for controllability by Kalman's method.

For state equation, $\dot{x} = AX + BU$, form a composite matrix, Q_c

where, $Q_c = [B \quad AB \quad A^2B \ \ldots\ldots \ A^{n-1}.B]$

27. Illustrate the frequency domain specifications.
→ Resonant peak, M_r
→ Resonant frequency, ω_r
→ Bandwidth, ω_b
→ cut-off rate
→ Gain margin, K_g
→ Phase margin, γ.

28. Explain Resonant peak.
The maximum value of the magnitude of closed loop transfer function is called Resonant peak.

29. Create the expression for resonant peak and resonant frequency.
Resonant Peak, $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

Resonant frequency, $\omega_r = \omega_n\sqrt{1-2\xi^2}$

30. The damping ratio and natural frequency of oscillation of a second order system is 0.5 and 8 rad/sec respectively. Calculate the resonant peak and resonant frequency.
Resonant peak, $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times0.5\sqrt{1-(0.5)^2}} = 1.154$.

Resonant frequency, $\omega_r = \omega_n\sqrt{1-2\xi^2} = 8\sqrt{1-2(0.5)^2} = 5.657$ rad/s

31. Discuss Resonant frequency.
The frequency at which the resonant peak occurs is called resonant frequency. The resonant peak is the max value of the magnitude of closed loop transfer function.

32. Discuss Bandwidth.

The Bandwidth is the range of frequency for which the system gain is more than -3 db.

33. Explain cut-off rate.

The slope of the log-magnitude curve near the cut-off frequency is called cut-off rate.

34. Illustrate the expression of gain margin.

Gain margin, $K_g = \dfrac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}$

$$K_g \text{ in db} = 20 \log \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}$$

$$= -20 \log |G(j\omega)|_{\omega=\omega_{pc}}.$$

35. Create the expression for phase margin.

Phase margin, $\gamma = 180° + \phi_{gc}$

where $\phi_{gc} = \angle G(j\omega)\Big|_{\omega=\omega_{gc}}$.

36. Explain Corner frequency.

The magnitude plot can be approximated by asymptotic straight lines. The frequencies corresponding to the meeting point of asymptotes are called corner frequency. The slope of the magnitude plot changes at every corner frequency.

37. Identify the value of error in the approximate magnitude plot of a quadratic factor with $\zeta = 1$ at the corner frequency

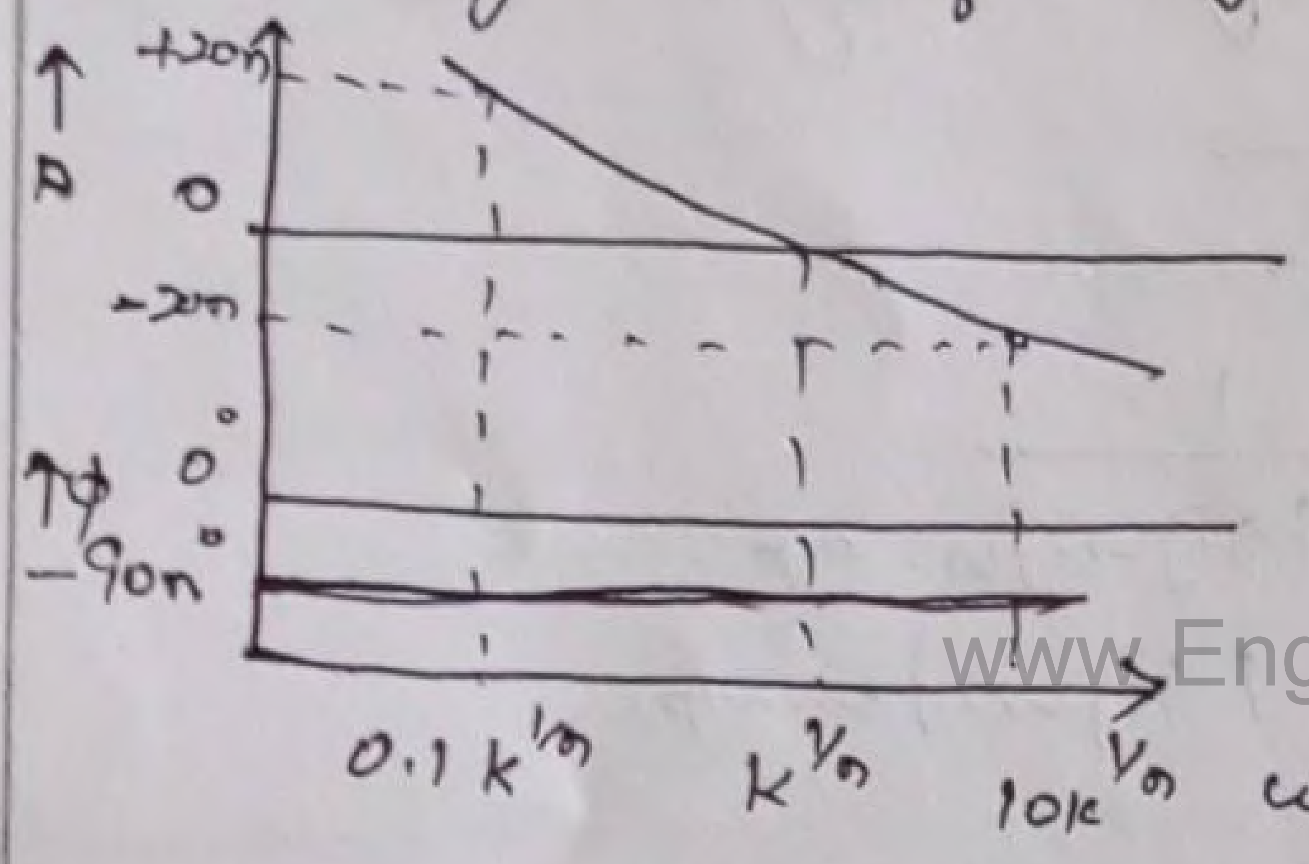
* The error is ± 6 db, for the quadratic factor with $\zeta = 1$.
* Positive error for numerator factor
* Negative error for denominator factor.

38. Draw the bode plot of, $G(s) = \frac{K}{s^n}$.

Let $s = j\omega$,

$\Rightarrow G(j\omega) = \frac{K}{(j\omega)^n}$.

The magnitude of $G(j\omega)$ is unity when $\omega = K^{1/n}$.



39. Illustrate the advantages of Bode plot.

→ The frequency domain specifications can be easily determined.

→ The bode plot can be used to analyse both open loop and closed loop system.

→ The magnitude are expressed in db and so, a simple procedure is available to add magnitude of each term one by one.

→ The approximate bode plot can be quickly sketched and the corrections can be made at corner frequency to get the exact plot.

40. Discuss minimum phase system.

* The minimum phase system are systems with minimum phase transfer function.

* In minimum phase transfer function, all poles and zeros will lie on the left half of s-plane.

41. Explain All-Pass systems.

→ The all-pass system are systems with all pass transfer function.

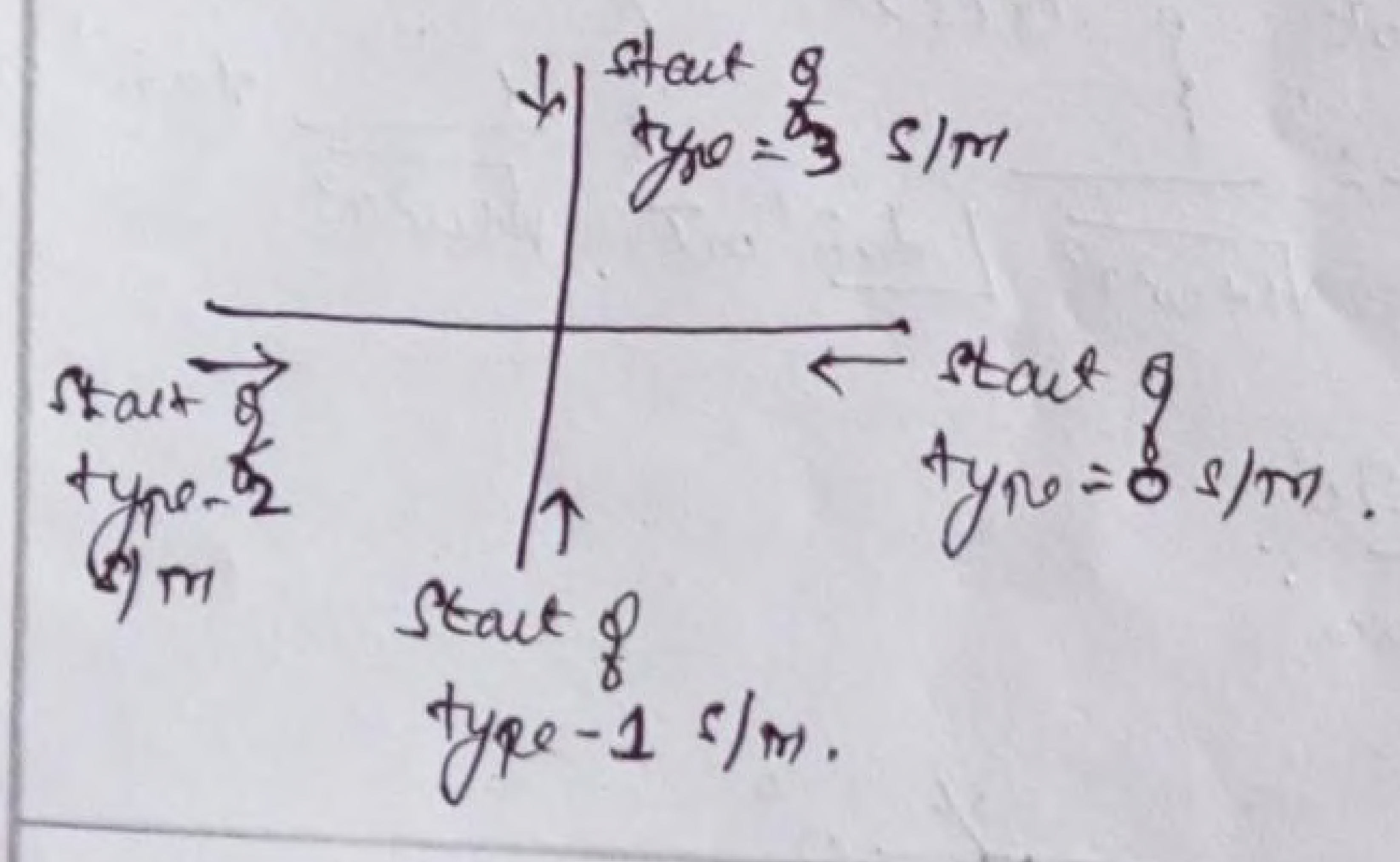
→ In all-pass transfer functions, the magnitude is unity at all frequencies and the transfer-function will have anti-symmetric pole zero pattern.

42. Explain non-minimum phase transfer function?

* A transfer function which has one or more zeros in the right half s-plane is known as non-minimum phase transfer function.

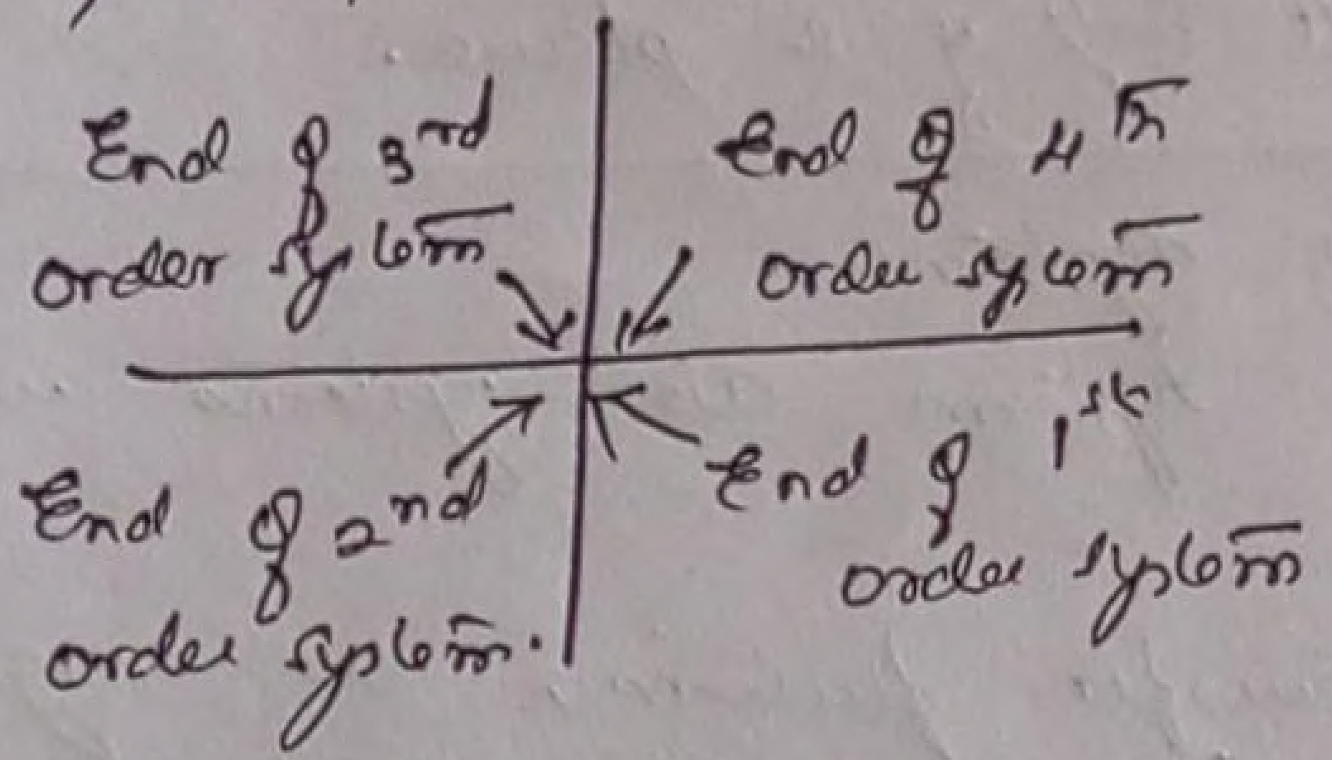
43. In minimum phase system, how the start of polar plot are identified?

→ The type of the system determines the quadrant in which the polar plot starts.



44. In minimum phase system, how the end of polar plot identified?

→ The order of a s/m determines the quadrant in which the polar plot ends.

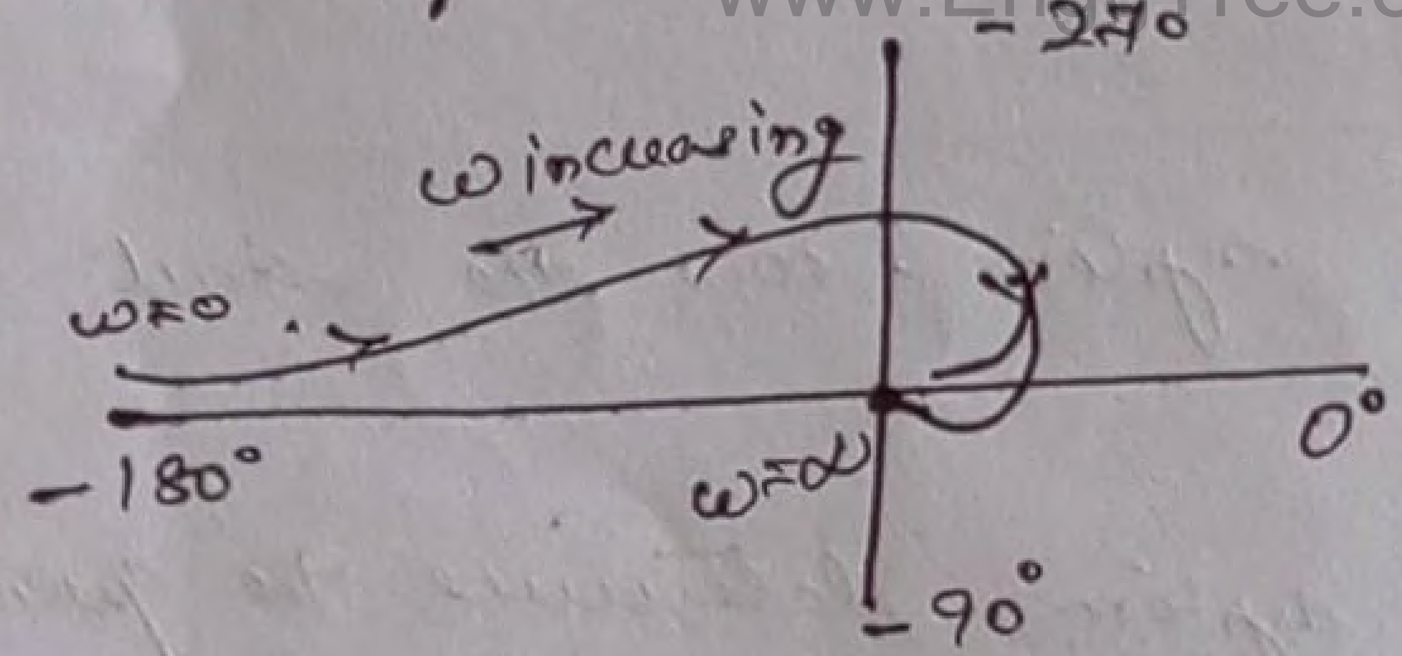


45. Construct the polar plot of $G(s) = \frac{1}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$

→ The given system is all pole minimum phase system.

→ The type number of the system is 2 and order is 5

→ Hence the polar plot starts in second quadrant and ends in fifth quadrant.

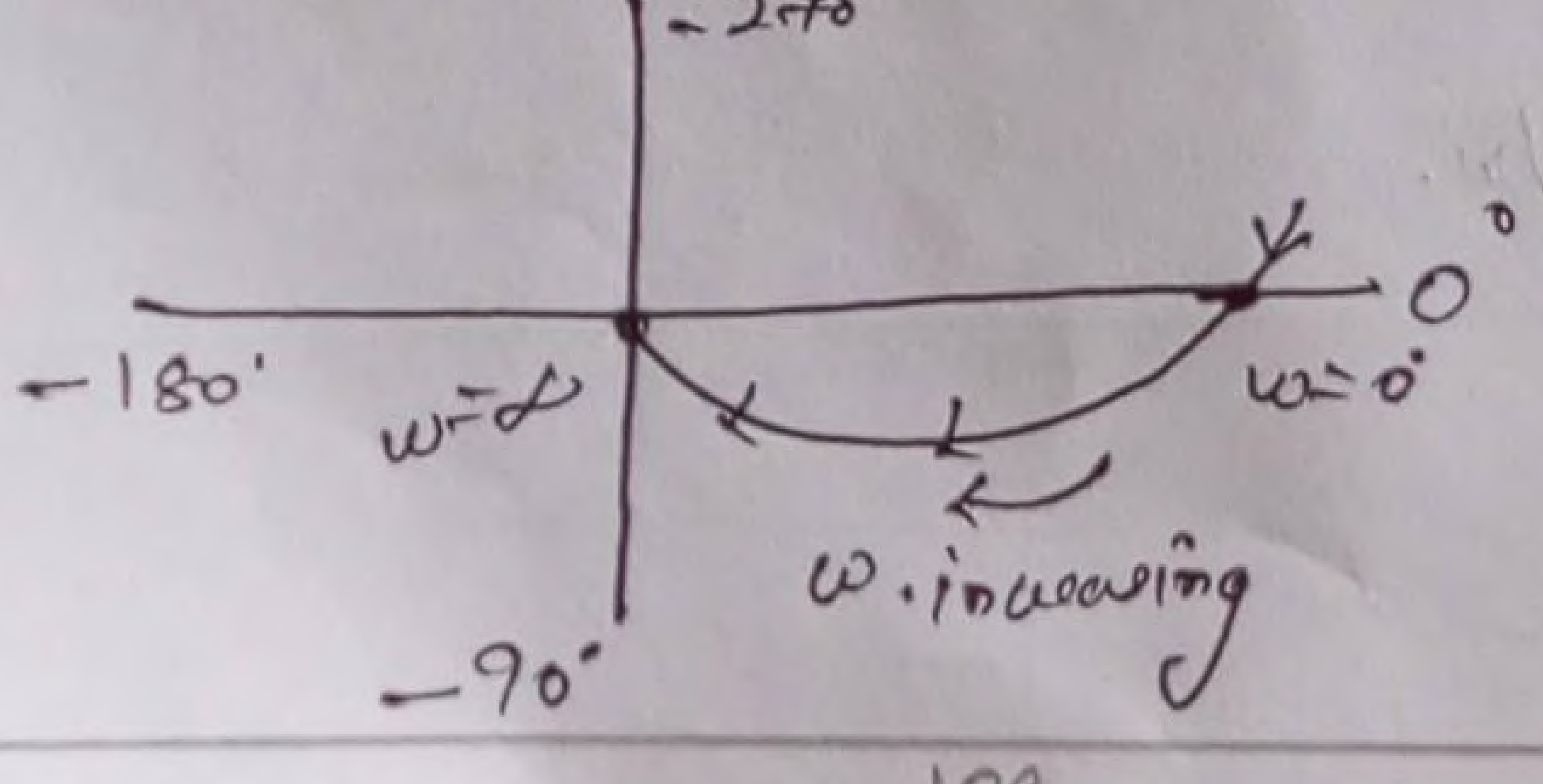


46. Construct the polar plot of $G(s) = 1/(1+sT)$.

Let $s=j\omega$, $G(j\omega) = \frac{1}{1+j\omega T} = \frac{1}{\sqrt{1+\omega^2T^2}} \angle \tan^{-1}\omega T$. $= \frac{1}{\sqrt{1+\omega^2T^2}} \tan^{-1}\omega T$.

As $\omega \to 0$, $G(j\omega) \to 1\angle 0^\circ$

As $\omega \to \infty$, $G(j\omega) \to 0 \angle -90^\circ$.



47. Discuss Nichol's plot.

→ The Nichols plot is a frequency response plot of the open loop transfer function of a system.

→ It is a graph between magnitude of $G(j\omega)$ in db and the phase of $G(j\omega)$ in degree, plotted on a ordinary graph sheet.

48. How closed loop frequency response is determined from open loop frequency response using M and N circles.

→ The $G(j\omega)$ locus or polar plot of open loop system is sketched on the standard M and N circles chart.

→ The meeting point of M circle with $G(j\omega)$ locus gives the magnitude of closed loop system.

→ The meeting point of N circle with $G(j\omega)$ gives the value of phase of closed loop system.

49. Illustrate the advantages of Nichols chart.

→ It is used to find closed loop frequency response from open loop frequency response.

→ The frequency domain specifications can be determined from Nichols chart.

50. How closed loop frequency response is determined from the open loop frequency response using Nichols chart?

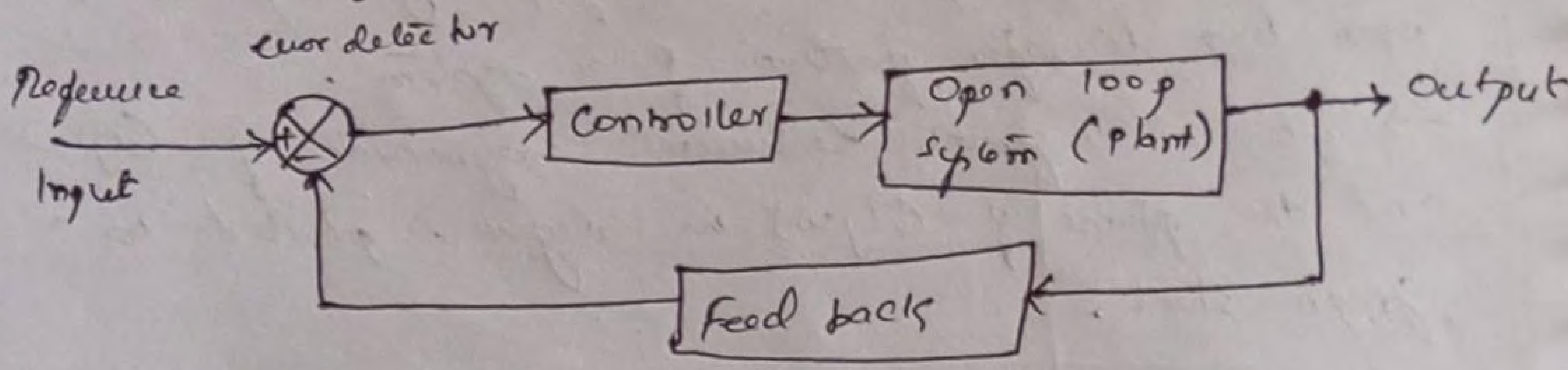
→ The $G(j\omega)$ locus or the Nichols chart is sketched on the standard Nichols chart.

→ The meeting point of M-countour with $G(j\omega)$ gives the magnitude

→ The meeting point of N-circle gives the phase / argument of the closed loop system.

35. Illustrate the components of feedback control systems.

→ Plant, feedback path elements, error detector and controller



36. Illustrate the characteristics of negative feedback.

→ accuracy in tracking steady state value.

→ rejection of disturbance signals

→ low sensitivity to parameter variations

→ reduction in gain at the expense of better stability.

37. Explain the effect of positive feedback on stability.

→ The positive feedback increases the error signal and drives the output to instability.

→ Used in minor loops in control system to amplify certain internal signals or parameters.

38. Explore why negative feedback is invariably preferred in a closed loop system?

→ The negative feedback results in better stability in steady state and rejects any disturbance signals.

→ It also has low sensitivity to parameter variations

39. Elucidate the two major type of control system.

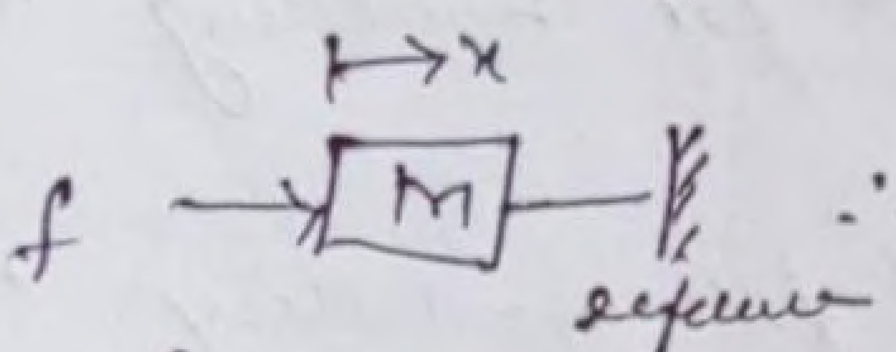
The two major types of control systems are

1. Open loop system and
2. closed loop systems.

40. Identify the basic elements used for modelling mechanical translational system.

* The model of mechanical translational system can be obtained by using three basic elements mass, spring and dashpot.

41. Compute the force balance equation of ideal mass element.

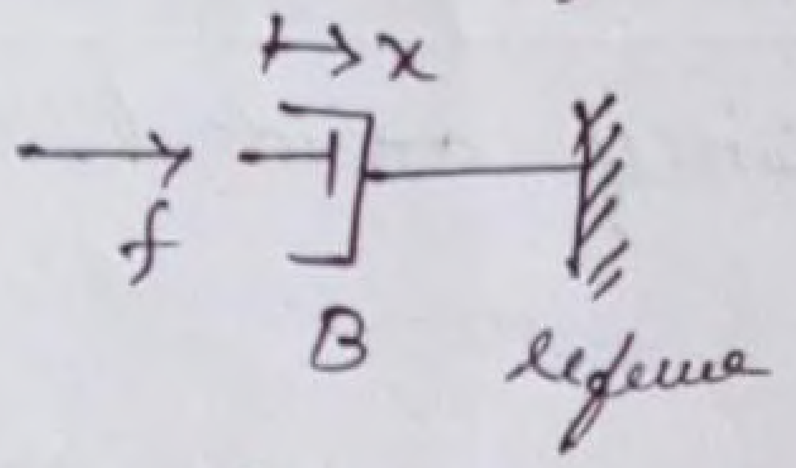


$$\therefore f = f_m = M\frac{d^2x}{dt^2}.$$

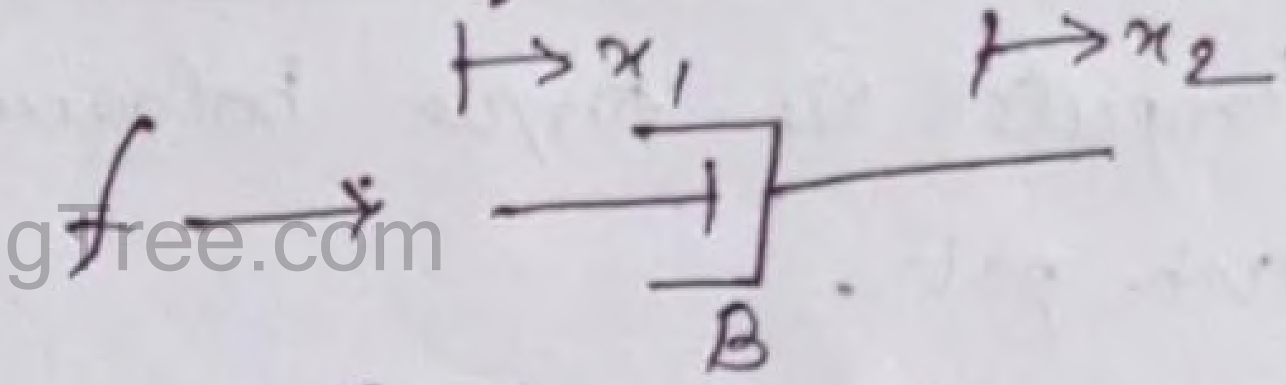
$f \rightarrow$ force applied to an ideal mass M.

$f_m \rightarrow$ opposing force.

42. Compute the force balance equation of ideal dashpot.

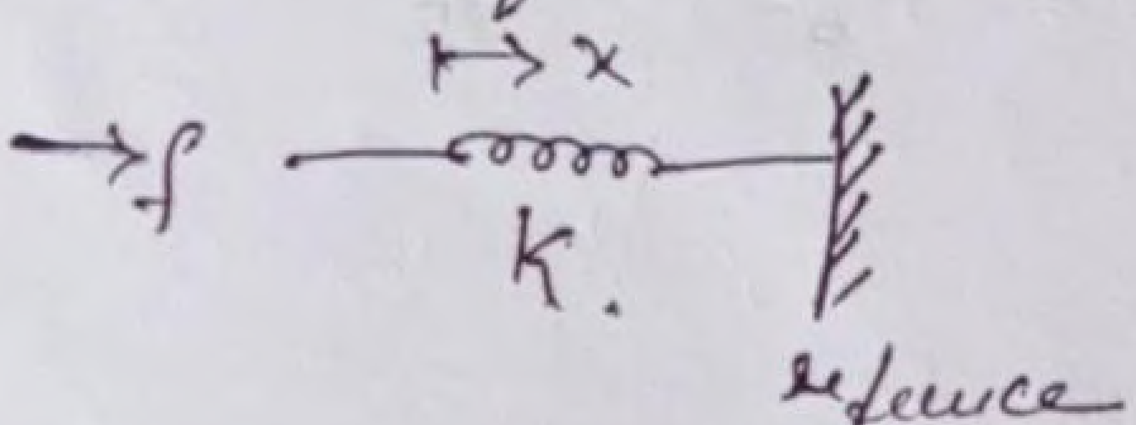


$$\therefore f = f_b = B\frac{dx}{dt}.$$

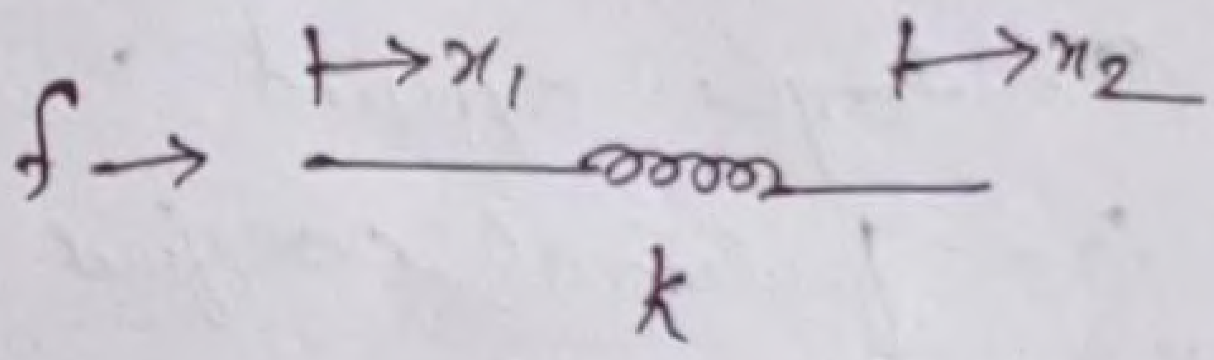


$$\therefore f = f_b = B \cdot \frac{d}{dt}(x_1 - x_2).$$

43. Compute the force balance equation of ideal spring.



$$\therefore f = f_K = Kx.$$

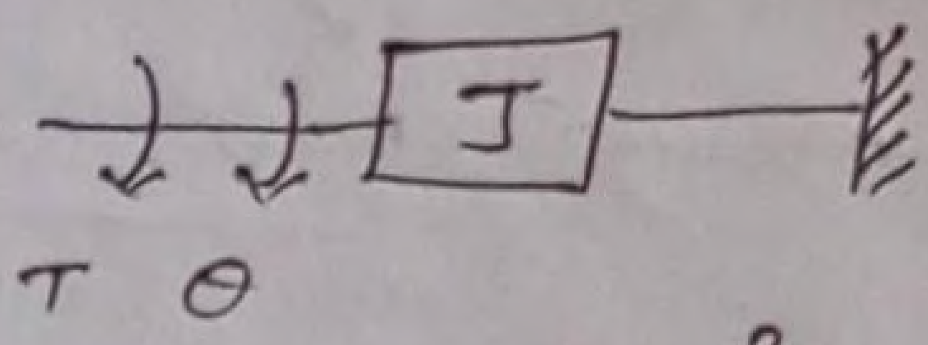


$$\therefore f = f_K = K(x_1 - x_2).$$

44. Identify the basis for framing the rules of block diagram reduction technique.

$\rightarrow$ The rules for block diagram reduction technique are framed such that any modification made on the diagram does not alter the input output relation.

45. Compute the torque balance equation of an ideal rotational mass element.



$T \rightarrow$ torque applied to an ideal mass with moment of inertia . J.

$T_J \rightarrow$ Opposing torque $\propto$ angular acceleration.

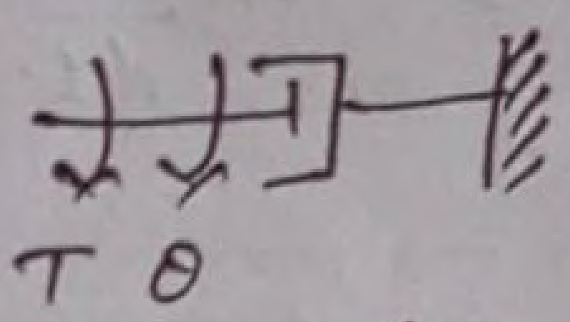
$$T = T_J = J\frac{d^2\theta}{dt^2}$$

46. Illustrate the basic elements used for modelling mechanical rotational system.

→ The three basic elements used for modelling mechanical rotational system are

* moment of inertia , J
* dash-pot with rotational frictional coefficient, B
* torsional spring with stiffness, K.

47. Compute the torque balance equation of an ideal rotational dash-pot.

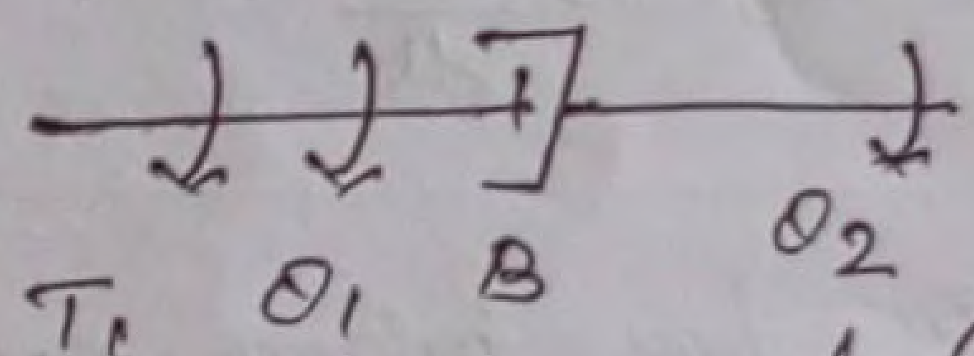


$$T = T_b = B \cdot \frac{d\theta}{dt}$$

T - torque applied .

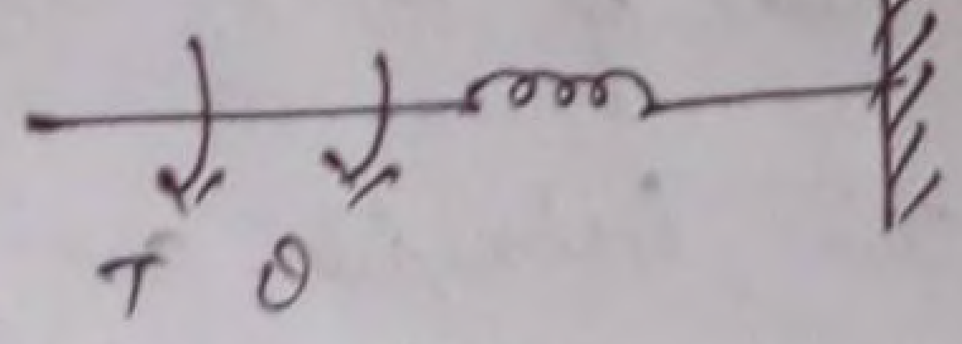
B - frictional coefficient

T_b - opposing torque .

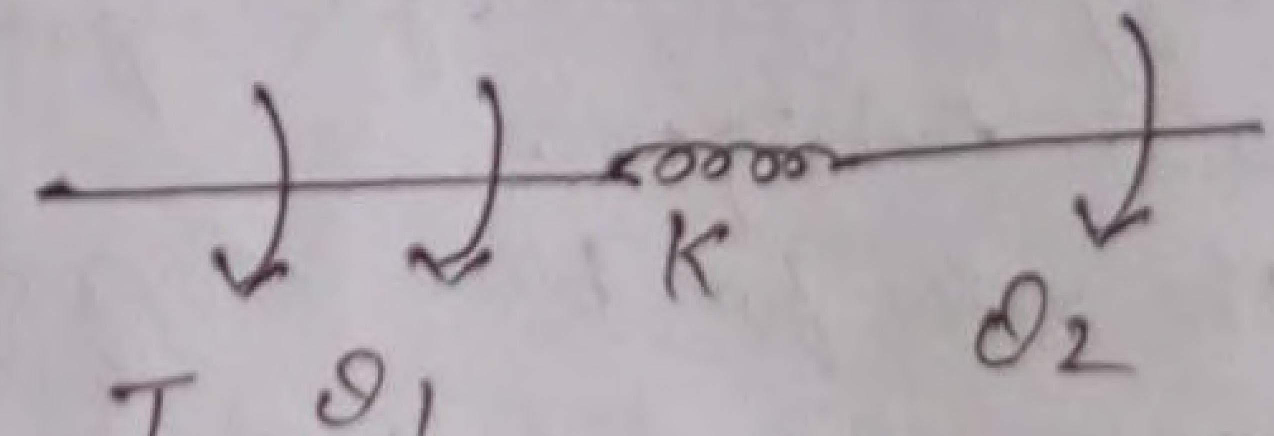


$$T = T_b = B\frac{d}{dt}(\theta_1 - \theta_2).$$

48. Compute the torque balance equation of ideal rotational spring



$$\therefore T_K = K\theta$$



$$\therefore T_K = K(\theta_1 - \theta_2).$$

49. Identify the two types of electrical analogous for mechanical system.

→ The two types of analogies for the mechanical systems are

1) force - voltage analogy
2) force - current analogy.

50. Compute the analogous electrical elements in force-voltage analogy for the elements of mechanical translational system

Force, f → Voltage, e
Velocity, v → current, i
Displacement, x → charge q
Mass, M → Inductance, L
Frictional coefficient, B → Resistance, R
Stiffness, K → Inverse of capacitance, $1/C$
Newton's second law, $\Sigma f = 0$ → Kirchhoff's voltage law, $\Sigma v =$

38. Explain transient and steady state response.

→ The transient response is the response of the system when the input changes from one state to another.

→ The response of the system as $t \to \infty$ is called steady state response.

39. Classify the test signals used in control system.

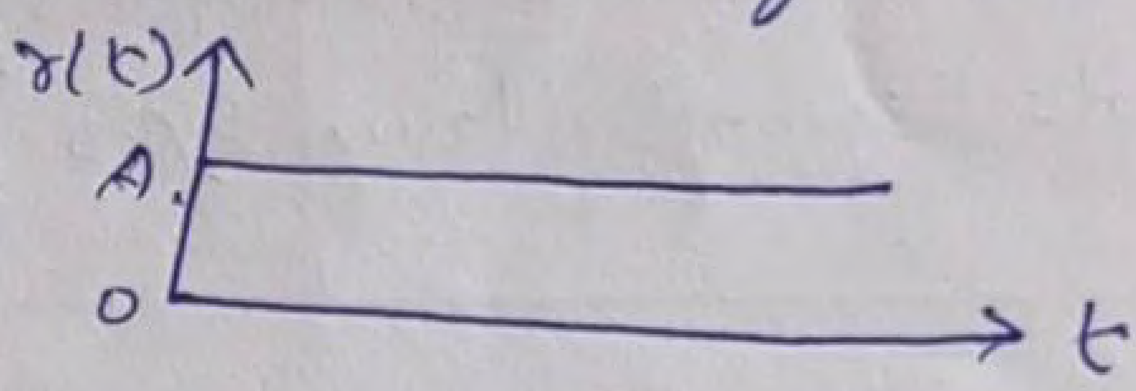
→ The commonly used test input signals in control system are

* Impulse
* Step
* Ramp
* Acceleration and
* Sinusoidal signals.

40. Explain step signal.

→ The step signal is a signal whose value changes from 0 to A and remains constant at A for $t > 0$.

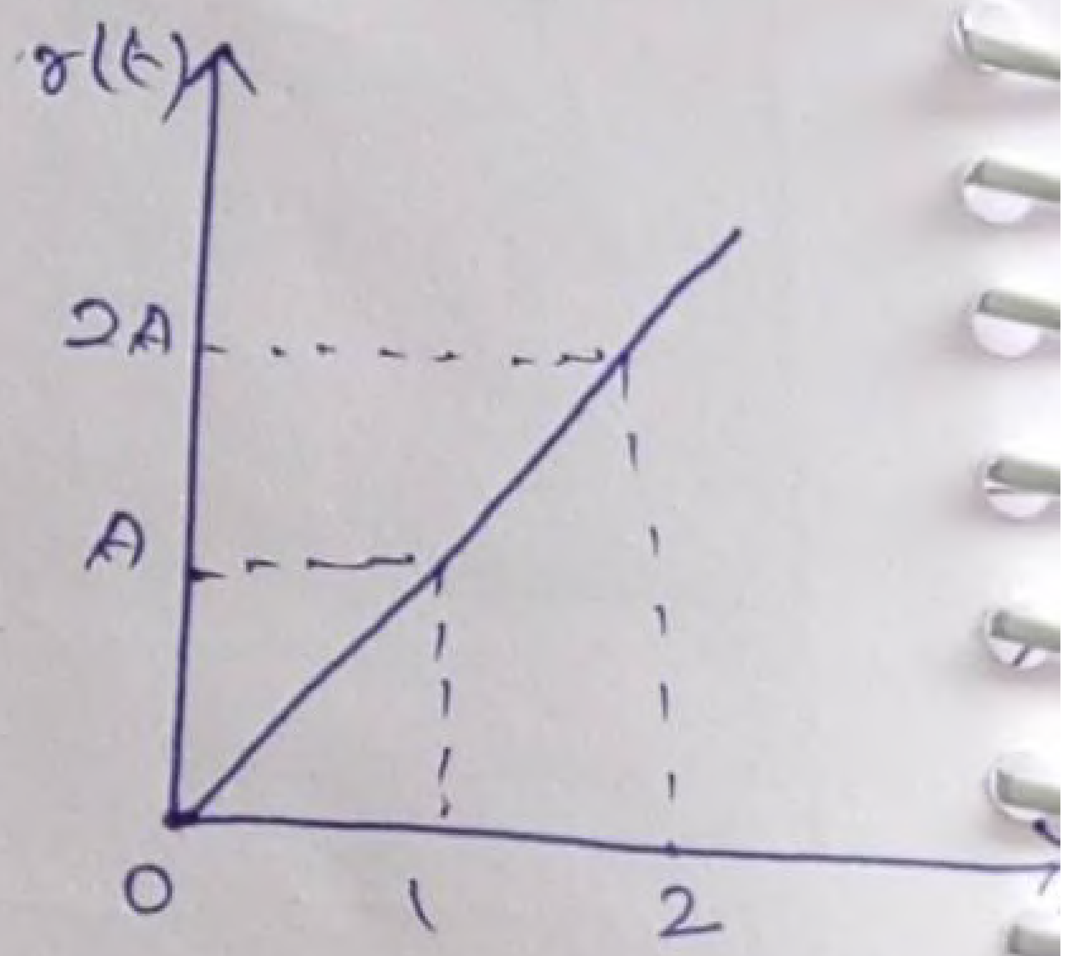
→ $r(t) = A, \quad t \geq 0$
$\quad\;\; = 0, \quad t < 0.$



41. Define ramp signal.

→ A ramp signal is a signal whose values varies as square of the time from an initial value of zero at $t=0$.

→ $r(t) = \frac{At^2}{2}, \quad t \geq 0$
$\quad\;\; = 0, \quad t < 0.$



42. Discuss weighting function.
→ The impulse response of system is called weighting function.
→ It is given by inverse laplace transform of system transfer function.

43. Explain pole.
→ The pole of a function, F(s) is the value at which the function F(s) becomes infinite, where F(s) = function of complex variable. s.

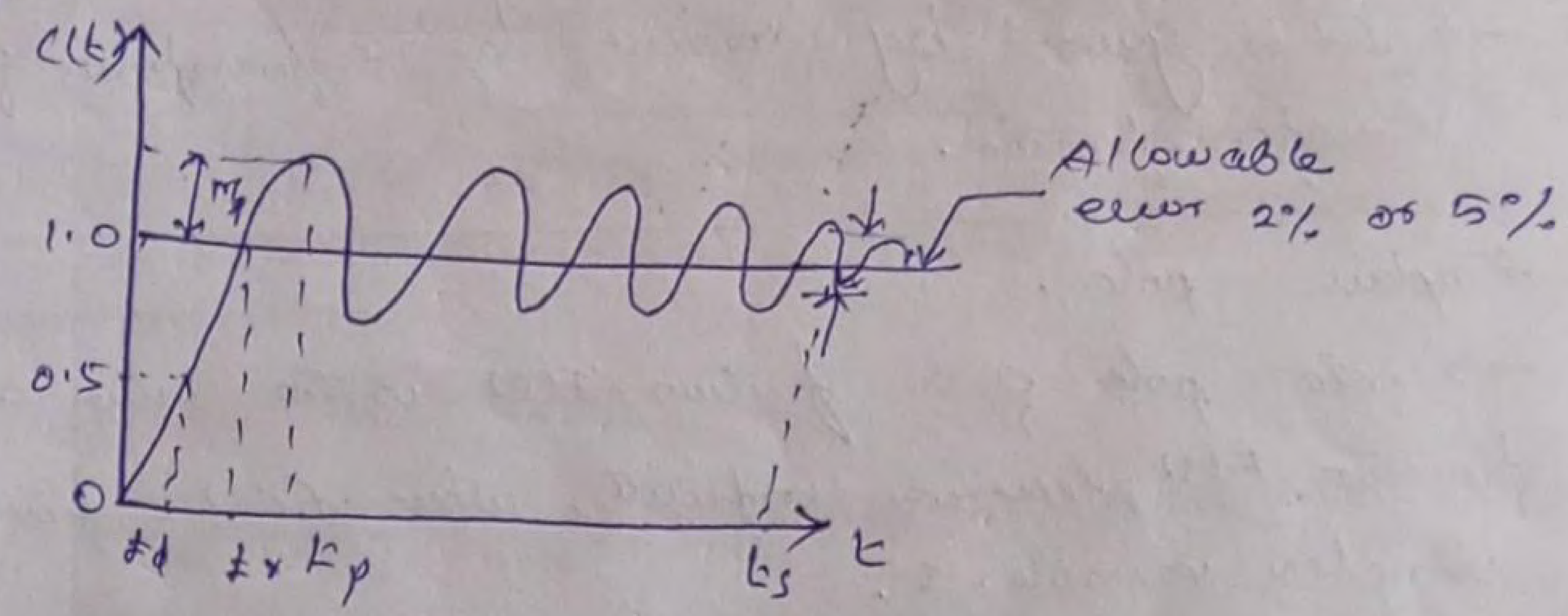
44. Define damping ratio.
→ The damping ratio is defined as the ratio of actual damping to critical damping.
→ damping ratio = $\frac{\text{actual damping}}{\text{critical damping}}$.

45. Explain zero.
→ The zero of a function, F(s) is the value at which the function F(s) become zero,
where F(s) = function of complex variable. s.

46. How the system is classified depending on the value of damping.
→ Undamped system, $\zeta = 0$
→ Underdamped system, $0 < \zeta < 1$
→ critically damped system, $\zeta = 1$
→ Overdamped system, $\zeta > 1$.

✓

47. Construct the response of a second order under damped system.



48. A second order system has a damping ratio of 0.6 and natural frequency of oscillation is 10 rad/sec. Determine the damped frequency of oscillation.

Damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\xi^2} = 10\sqrt{1-(0.6)^2} = 10 \times 0.8$

$\therefore \omega_d = 8$ rad/sec.

49. Classify the time domain specifications.

→ Delay time
→ Rise time
→ Peak time
→ Maximum Overshoot
→ Settling time.

50. Why derivative controller is not used in control system?

→ The derivative controller produces a control action based on rate of change of error signal and it does not produce corrective measures for any constant error.

Hence derivative controller is not used in control systems.

UNIT - V BIOLOGICAL CONTROL SYSTEM ANALYSIS.

SIMPLE MODELS OF MUSCLE STRETCH REFLEX ACTION.

* The pathway starts when the muscle spindle is stretched.

* The stretch reflex is also referred to as the deep tendon reflex or myotatic reflex.

* It is a simple pre-programmed response by the human body in response to the muscle being passively stretched.

* The stretch reflex or myotatic reflex refers to the contraction of a muscle in response to its passive stretching by increasing its contractility as long as the stretch is within physiological limits.

* The monosynaptic stretch reflex, or sometimes also referred as the muscle stretch reflex, deep tendon reflex, is a reflex arc that provides direct communication between sensory and motor neurons innervating the muscle.

* The knee-jerk reflex is a great example of the stretch reflex.

* When the doctor taps your patellar tendon just below your knee, it stretches your patellar tendon, quadriceps tendon and quadriceps muscles.

* The four different stretches for different situations are

1. Active stretching
2. Passive stretching
3. Dynamic stretching
4. PNF stretching.

* The examples of simple reflexes are the contraction of a muscle in response to stretching, the blink of the eye when the cornea is touched, and salivation at the sight of food.

* Reflexes of these types are usually involved in maintaining homeostasis.

STEADY STATE ANALYSIS OF MUSCLE STRETCH REFLEX ACTION.

* It is a reflex contraction of a muscle when it is passively stretched.

* The different pathway are as follows

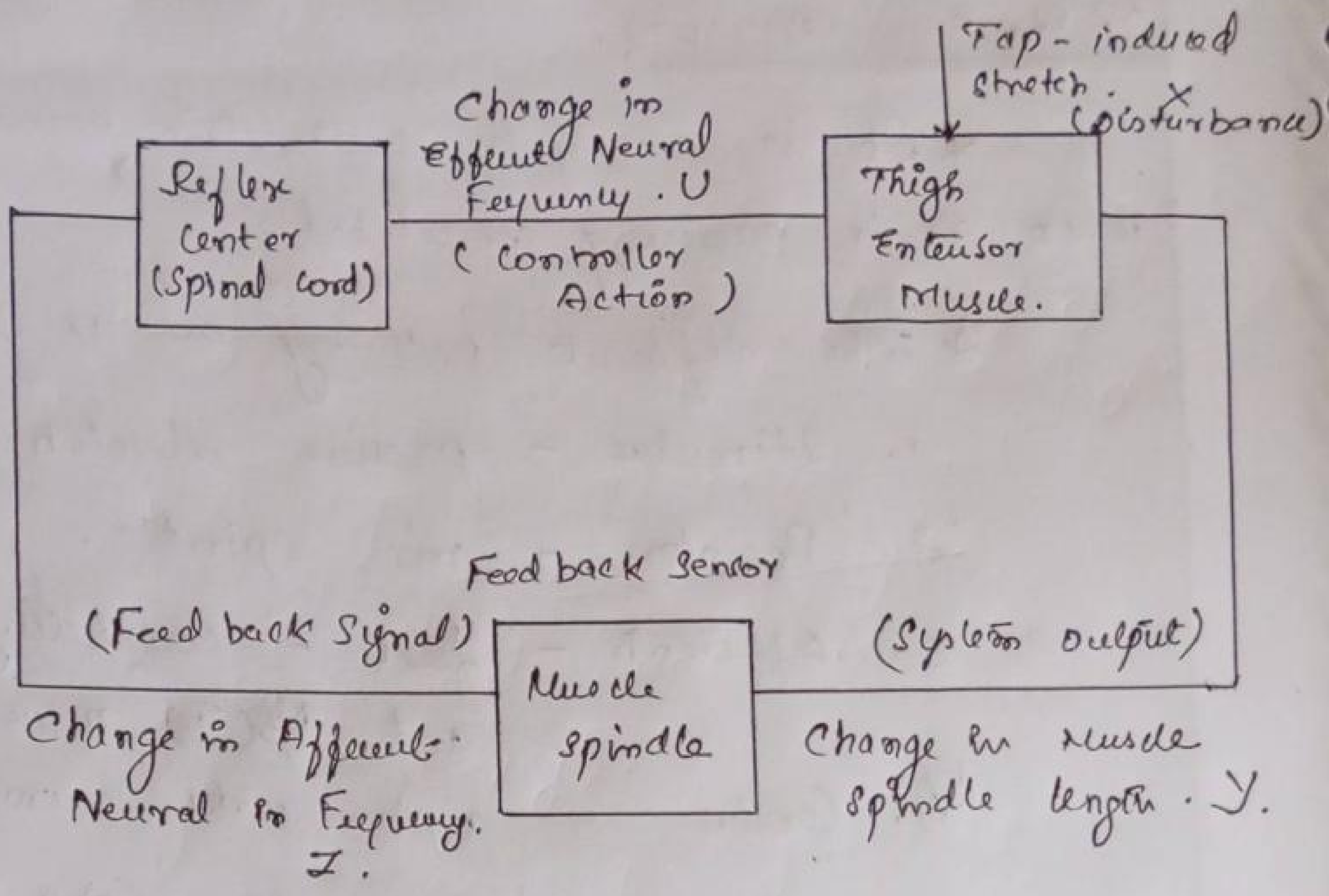
1. Stimulus - passive stretch.
2. Receptors - ms spindle.
3. Afferents - fast - conducting Aα (Ia) and Aβ(II) nerve fibers.
4. Centers - alpha motor neurons in AHCs.
5. Efferents - axons of alpha motor neuron
6. Effector Organ - Extrafusal ms fibers
7. Response - ms contraction.

* The stretch reflex or myotatic reflex refers to the contraction of a muscle in response to its passive stretching by increasing its contractility as long as the stretch is within physiological limits.

* The SSC is described as a rapid cyclical muscle action whereby the muscle undergoes a eccentric contraction, followed by a transitional period prior to the concentric contraction.

STRETCH.

BLOCK DIAGRAM REPRESENTATION OF THE MUSCLE REFLEX



* The spindles translate this mechanical quantity into an increase in afferent neural traffic (Z) sent back to the reflex center in the spinal cord which corresponds to our controller.

* In turn, the controller action is an increase in efferent neural traffic (U) directed back to the thigh muscle, which subsequently contracts in order to offset the initial stretch.

* Although this closed-loop control system differs in some details from the canonical structure. It is indeed a negative feedback system, leads to a controller action.

TRANSIENT RESPONSE OF NEUROMUSCULAR REFLEX MODEL ACTION.

* Stretching of the muscle activates nerve impulses which travel to the spinal cord.

* Here the incoming impulses activate motor neurons, which travel back to the muscle and result in muscle contraction.

* This reflex arc is primarily a spinal reflex, but is influenced by other pathways to and from the brain.

* Neuromuscular changes in the aging population often include muscle weakness, reduced joint proprioception and decreased power.

* These declines can significantly affect dynamic postural control and balance, altering gait mechanics, causing faulty perception and increasing injury risk.

* Examples of neuromuscular disorder include Amyotrophic lateral sclerosis, Muscular dystrophy, Myasthenia gravis.

* The neuromuscular system includes all the muscles in the body and the nerves serving th

* Every movement the body makes requires communication between the brain and the muscles.

* The nervous system provides the link between thoughts and actions by relaying messages from the brain to other parts of the body.

* As age goes on, brain and nervous system go through natural changes.

* Brain and Spinal cord lose nerve cells and weight (atrophy).

* Nerve cells may begin to pass messages more slowly than in the past.

* Waste products or other chemicals such as beta amyloid can collect in the brain tissue as nerve cells break down.

* Muscle fibers reduce in number and shrink in size.

* Muscle tissue is replaced more slowly and lost muscle tissue is replaced with a tough, fibrous tissue.

* Changes in the nervous system cause muscles to have reduced tone and ability to contract.

FREQUENCY RESPONSE OF CIRCULATORY CONTROL MODEL

* A frequency response describes the steady-state response of a system to sinusoidal inputs of varying frequencies.

* Frequency response describes the range of frequencies or musical tones a component can reproduce.

* Frequency response measures if and how well a particular audio component reproduces all of these audible frequencies and if it makes any changes to the signal on the way through.

* Formula for frequency response is as follows.

$$H(\omega) = \frac{e^{j\omega n}(b_1 + b_2 e^{-j\omega} + b_3 e^{-2j\omega})}{e^{j\omega n}(a_1 + a_2 e^{-j\omega} + a_3 e^{-2j\omega})}$$

* This form of the frequency response can be generalized to LTI difference equations with an arbitary number of terms.

* A second order system has a natural angular frequency of 2.0 rad/sec and a damped frequency of 1.8 rad/sec.

* Frequency response function of a linear mechanical system is defined as the Fourier transform of the

time domain response divided by the Fourier transform of the time domain input.

* Systems respond differently to inputs of different frequencies.

* Some systems may amplify components of certain frequencies and attenuate components of other frequencies.

* The way that the system output is related to the system input for different frequencies is called the frequency response of the system.

* The frequency response is a steady state response of the system to a sinusoidal input signal.

* If a system has sinusoidal input, the output will also be sinusoidal.

* The changes can occur in magnitude and the phase shift.

* It is the transfer function in the time-constant form.

STABILITY ANALYSIS OF PUPILLARY LIGHT REFLEX

* Eyes allow visualization of the world by receiving and processing the energy of light as it enters the eye.

* This light interacts with the structures and nerves of the eyes to create images.

* Adjustments via the muscles connected to the lens, ciliary bodies and muscles that make up the iris are stimulated by several nerves. This is known as pupillary light reflex.

* The pupillary light reflex constricts the pupil in response to light and pupillary constriction is achieved through the innervation of the iris sphincter muscle.

* Light travels through the cornea, anterior chamber, pupil, lens and the posterior chamber, eventually reaching the retina.

* Photoreceptor cells in the outer layers of retina which are called rods and cones, convert light stimuli into neuronal impulses.

* The optic nerve sends impulses to the brain for further processing and image recognition.

* This entire pathway is being tested when a light is shined in the eyes.

* The room light should be dim, barely allowing to see the iris and pupil.

* Encourage patients to relax and alleviate their anxiety, if any.

* Gently point the focal light into one eye, this is known as direct pupillary light.

* Then, withdraw the light for few seconds, followed by stimulating the same eye again, but this time observe the indirect or consensual, PLR in the opposite eye.

* Pupillary escape is a phenomenon that can occur in the setting of a diseased optic nerve or retina.

* When light is shown on the affected pupil, there will be a transient pupillary constriction and then a slow dilation to the original size.

* Nursing, Allied Health and Interprofessional Team monitoring.

- Vital signs
- Clarity of vision
- Discharge the patient, only if they are fully recovered from the effect of mydriatics.

C. 6/8/23.