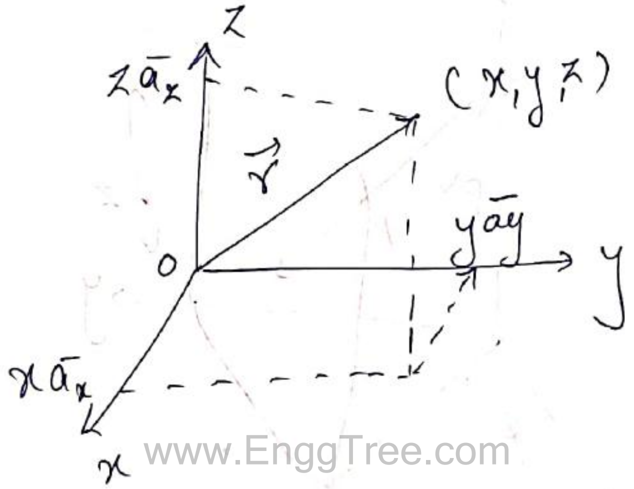
UNIT I

ELECTROSTATICS – I

1.) Cordinate Systems :-

→ Simplex way of representing Vectors.

1.1 The Cartesian Co-ordinate Systems:-



$$r = \bar{a}_x x + \bar{a}_y y + \bar{a}_z z$$

where a_x, a_y, a_z are unit-Vectors.

Unit Vector → in a given direction is a Vector in that direction divided by magnitude, It is given by

$$a_r = \frac{x\bar{a}_x + y\bar{a}_y + z\bar{a}_z}{\sqrt{x^2+y^2+z^2}}$$

→ The differential length dl from P to Q is a diagonal is given by,

$$dl = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

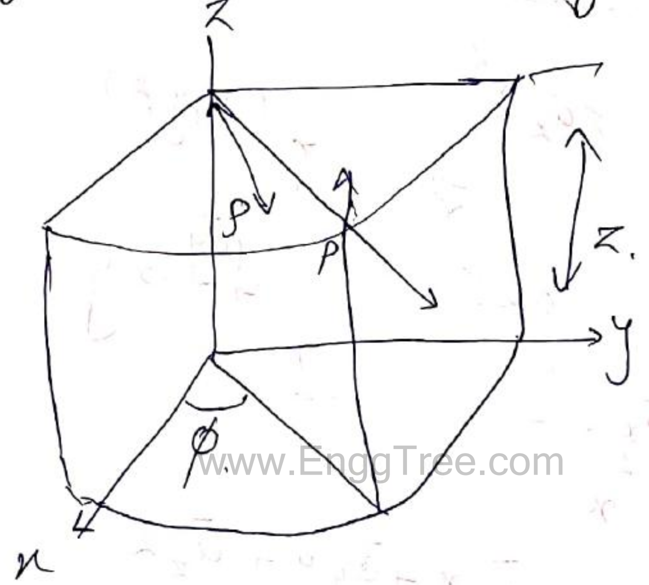
The differential area $ds = dx\,dy$

$= dy\,dz$

$= dz\,dx$.

differential Volume $dv = dx\,dy\,dz$.

1.2. Cylindrical Co-ordinate System :-



→ Consider any point as the intersection of three mutually perpendicular surfaces.

→ a Cylinder (ρ), a plane (ϕ) and another plane (z)

→ for a parallelopiped has the sides $d\rho$, $\rho d\phi$ and dz.

$$dl = \sqrt{d\rho^2 + (\rho\, d\phi)^2 + (dz)^2}$$

$$ds = \rho\, d\rho\, d\phi = d\rho\, dz = \rho\, d\phi\, dz.$$

1.3 Spherical Co-ordinate System:-

→ In this System Consider any point of intersection of the Spherical surface (radius r), Conical Surface (θ, angle b/w r & z) & a plane Surface (ϕ = constant), The Co-ordinate are r, θ, ϕ.

differential length $dl = \sqrt{dr^2 + (rd\theta)^2 + (r\sin\theta d\phi)^2}$

$ds = dr . rd\theta = rdrd\theta$

$= dr\, r\sin\theta\, d\phi = r\sin\theta\, d\phi\, dr.$

$= rd\theta . r\sin\theta\, d\phi.$

$= r^2 \sin^2\theta\, d\theta\, d\phi.$

Conversion

(i) Cartesian to Cylindrical

x	$\rho = \sqrt{x^2 + y^2}$
y	$\phi = \tan^{-1}(y/x)$
z	$z = z.$

(ii) Cylindrical to Cartesian.

ρ	$x = \rho\cos\theta$
ϕ	$y = \rho\sin\phi.$
z	$z = z.$

(iii) Cartesian to Spherical

$x \qquad r = \sqrt{x^2+y^2+z^2}$

$y \qquad \theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$z \qquad \phi = \tan^{-1} y/x$.

1.4. Vector fields :-

→ The Quantity that contains direction.

→ for Vector multiplication take

$|A \times B| = AB \sin\theta$.

$$A \times B = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

→ Co-efficients of Vectors in each direction.

(i) Gradient :-

$$\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z$$

This operation is called gradient

$$\boxed{\nabla V = \text{grad } V}$$

(ii) Divergence :-

$$\nabla \cdot A = \underset{v \to 0}{Lt} \frac{1}{v} \oint A \cdot \hat{n}\, ds.$$

Can be expressed as

$$\nabla \cdot A = \left[\bar{a}_x \frac{\partial}{\partial x} + \bar{a}_y \frac{\partial}{\partial y} + \bar{a}_z \frac{\partial}{\partial z}\right]$$

$$\left[a_x A_x + a_y A_y + \bar{a}_z A_z\right].$$

So ∴ $\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$

$$\boxed{\nabla \cdot A = div\, A}$$

(iii) Curl :-

$$curl\, A = \nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

1.5. Theorms & Applications :-

1) Divergence theorm :-

→ The Volume integral of the divergence of a Vector field over a Volume is equal to the Surface integral of the normal Component of this Surface bounding the Volume.

$$\iiint_V \nabla \cdot A\, dV = \oiint_S A \cdot ds.$$

→ for the proof ref. EMT (by Dhanajayan) page 1.17.

2) Stokes theorm:-

→ The line integral of a vector around a closedpath is equal to the surface integral of the normal component of its curl over any closed surface.

$$\oint H \cdot dl = \iint_S \nabla \times H\, ds.$$

Note:- proof ref text EMT page no 1.19.

Applications:-

Exp: Using divergence theorm, evaluate $\iint_S \vec{F} \cdot n\, ds$ where $\vec{F} = 2xy\,\hat{i} + y^2\,\hat{j} + 4yz\,\hat{k}$ and S is the surface of the cube bounded by $x=0, x=1, y=0, y=1$ and $z=0, z=1$.

Hint:- find using divergence theorm.

Exp 2. Check the validity of the divergence theorm Considering the field

$$D = 2xy\,\hat{a}_x + x^2\,\hat{a}_y \ C/m^2$$ and

rectangular parallelopiped formed by the planes $x=0, x=1; y=0, y=2$ and $z=0, z=3$.

Hint :- use Divergence theorm and Solve of both sides.

Exp 3. If $F = x^2\hat{a}_x + y^2\hat{a}_y + z^2 a_z$, then

find divergence of F & Curl of F.

Hint: $\text{Div}\,F = \nabla \cdot F$

$\text{Curl}\,F = \nabla \times F$.

Exp 4: Solve that $\vec{H} = x^2 a_x + y^2 a_y + z^2 a_z$ is a Conservative field.

Hint :- if $\vec{H}$ is Conservative field if $\text{Curl}\,\vec{F} = 0$.

1.6 Coulomb's Law:-

→ force b/w two small object seperated by a distance which is large compared to their size is proportional to the charge on each & inversely proportional to the square of distance b/w them.

$$F \propto \frac{Q_1 Q_2}{r^2}$$

$$F = k \frac{Q_1 Q_2}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 9\times10^9.$$

ϵ_0 → Absolute permittivity in free space

→ 8.85×10^{-12} F/m

ϵ → permittivity of medium

ϵ_r → relative permittivity

$$\boxed{\epsilon_0 = \frac{\epsilon}{\epsilon_r}}$$

1.7 Continuous Charge Distribution:-

(i) Linear charge density:-

→ total charge distributed over a line or curve.

$$\rho_l = \frac{Q}{l}. \text{ Coulomb}/m.$$

(ii) Surface charge density:-

→ Total charge distributed over a surface.

$$\rho_s = \frac{Q}{S} = \frac{Q}{A}. \rightarrow C/m^2$$

(iii) Volume charge density:-

→ Total charge distributed over a Volume.

$$\rho_v = \frac{Q}{V} \rightarrow C/m^3.$$

1.8. Electric field Intensity:-

→ Electric force per unit charge

$$E = F/q.$$

According to Coulomb's law

$$F = \frac{Qq}{4\pi\epsilon_0 r^2}$$

$$E = F/q = \frac{Q}{4\pi\epsilon_0 r^2} \quad N/Coulomb.$$

1.9. Gauss Law & its Applications :-

Gauss law :-

→ Electric flux passing through any closed Surface is equal to the total charge enclosed by the Surface.

→ Consider a small element of area ds in a plane Surface having a charge Q & P be a point in the element — electric flux density D will have a value Ds. — makes an angle θ.

$$d\psi = D_s \, normal \cdot ds$$
$$= D_s \cos\theta \cdot ds.$$

$$d\psi = D_s \cdot ds \quad \text{(dot product)}.$$

→ total flux passing through the closed Surface is given by

$$\Psi = \int d\Psi = \oint D_s\, ds.$$

$$\Psi = Q.$$

In terms of volume charge density

$$\Psi = \int_V \rho_v\, dv = Q.$$

Note: - proof - refer EMT page 2.2.

Electric flux :- (Ψ)

→ Electric flux is equal to charge itself.

$$\Psi = Q.$$

Electric flux density (D)

→ electric flux per unit area.

$$D = Q/A.$$

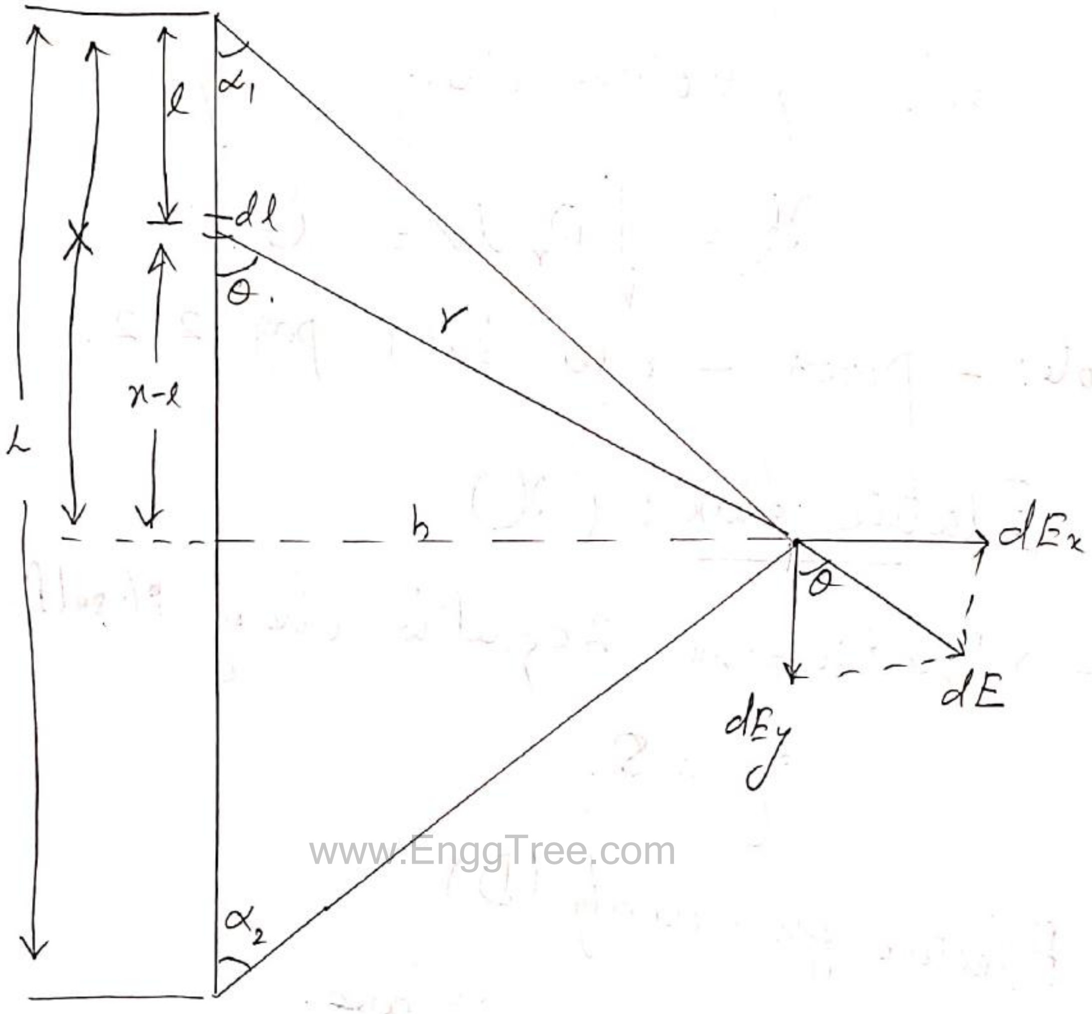
$$A = 4\pi r^2.$$

$$D = \frac{Q}{4\pi r^2}$$

$$\text{but } E = \frac{Q}{4\pi \epsilon_0 r^2}$$

$$\boxed{D = \epsilon E}.$$

1.10. Field line of charge :-



→ Consider a uniform charged line of length 'L' whose linear charge density is ρ_l C/m.

→ Consider a small element 'dl' at a distance 'l' from one end of line.

→ point 'P' is taken from distance 'r' from 'dl'.

The electric field at point P due to charged element $\rho\, dl$ is

$$dE = \frac{\rho_l\, dl}{4\pi\varepsilon r^2}$$

→ The x & y component of electric field dE is given by

$$dE_x = dE \sin\theta$$
$$dE_y = dE \cos\theta.$$

Then $$dE_x = \frac{\rho_e dl \sin\theta}{4\pi\epsilon r^2}$$

From the figure
$$x - l = h \cot\theta$$
$$-dl = -h \operatorname{cosec}^2\theta \, d\theta.$$

and $h/r = \sin\theta$, $r = h \operatorname{cosec}\theta$

$$dE_x = \frac{\rho_e dl \sin\theta}{4\pi\epsilon h.}$$

The electric field E_x due to entire length of line of charge is given by

$$E_x = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_e \sin\theta \, d\theta}{4\pi\epsilon h}$$

$$= \frac{\rho_l}{4\pi\epsilon h} \int_{\alpha_1}^{\pi-\alpha_2} \sin\theta \, d\theta.$$

$$= \frac{\rho_l}{4\pi\epsilon h} \Big[-\cos\theta\Big]_{\alpha_1}^{\pi-\alpha_2} = \frac{\rho_l}{4\pi\epsilon h}\left[\cos\alpha_1 + \cos\alpha_2\right]$$

IIIly for E_y we get

$$E_y = \frac{\rho_l}{4\pi\epsilon h}\left[\sin\alpha_2 - \sin\alpha_1\right]$$

Case 1: if the point P is at bisector of a line

then $\alpha_1 = \alpha_2 = \alpha$

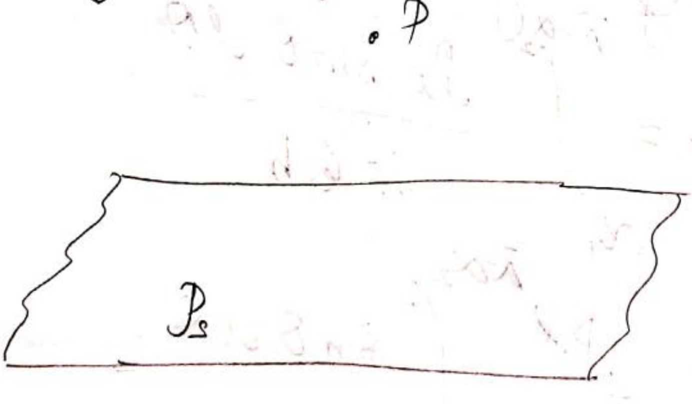
then $E_y = 0$, So $E = \dfrac{\rho_l}{2\pi\epsilon h}\cos\alpha$.

Case 2: if the line infinitely long $\alpha = 0$

$E_y = 0$ E becomes E_x

$$\boxed{E = \frac{\rho_l}{2\pi\epsilon h}}$$

1.11. Electric field Intensity at a point due to charged Circular Disc: ~~/~~ infinite plane sheet of charge



→ Consider an infinite plane sheet which is uniformly charged with charged density of ρ_s C/m^2

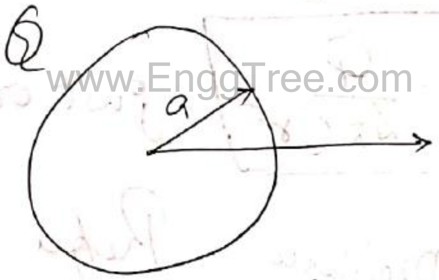
→ The field Intensity at any point 'P' due to plane sheet of charge is

$$E = \frac{\rho_s}{2\varepsilon}(1-\cos\alpha)$$

$$E = \frac{\rho_s}{2\varepsilon} \quad [\because \alpha = 90]$$

1.12 Electric field Intensity due to shell of charge:-

→ Let positive charge is uniformly distributed over a spherical surface of radius as shown in figure.



→ by Applying Gauss law inside the shell, the integral of flux density D over a spherical Surface is Zero as no charge enclosed by the Surface.

$$\oint D \cdot ds = 0 \qquad r < a.$$

$$\varepsilon \oint E \cdot ds = 0$$

$$E = 0 \qquad r < a. \text{ (Inside the shell.)}$$

→ Applying Gauss law just outside the shell, the ~~&~~ Integral of flux density D over a spherical surface is the charge of shell.

$$\oint_s D \cdot ds = Q$$

$$\varepsilon \oint_s E \cdot ds = Q$$

$$\oint_s E \cdot ds = \frac{Q}{\varepsilon}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$ just outside the shell.

Note: for problems Refer EMT by Dananjayan — page – 1.20 to 1.35.

2.1 Electric potential:-

→ Consider a uniform Electric field 'E' & a unit positive 'q'.

→ force with which unit test charge is moved from one point to another due to electric field

$$F = qE$$

→ The work done in moving unit positive charge from one point to another in an electric field is called potential Difference (V)

$$W = -\int_{r_1}^{r_2} qE \cdot dr.$$

$$W = -q\int_{r_1}^{r_2} E \cdot dr$$

but

$$V = \frac{W}{q} = -\int_{r_1}^{r_2} E \cdot dr \quad J/C.$$

But $E = \dfrac{Q}{4\pi\varepsilon r^2}$, So $V = -\dfrac{Q}{4\pi\varepsilon_0}\displaystyle\int_{r_1}^{r_2} \frac{1}{r^2}\, dr$

$$\therefore V = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_{r_1}^{r_2}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \text{ volts. or}$$

Electric potential is defined as work done in moving unit positive charge from infinity a given point in Electric field.

$$\boxed{V = \frac{Q}{4\pi\varepsilon r}} \text{ Volts.}$$

→ In a Conservative field

$$\oint E.dr = 0.$$

Relation b/w Electric field & potential:-

→ In two points, work done will be

$$dw = dv = -E dr.$$

→ Since Scalar potential V is a function of x, y, z, above equation can be written as

$$\frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} dz = -E.dr.$$

This can be re-written as,

$$\left[\vec{a}_x \frac{\partial v}{\partial x} + \vec{a}_y \frac{\partial v}{\partial y} + \vec{a}_z \frac{\partial v}{\partial z}\right] . \left[\vec{a}_x dx + a_y dy + a_z dz\right]$$

$$= -E.dr.$$

$$\nabla V.\cancel{dr} = -E.\cancel{dr}$$

$$\boxed{E = -\nabla V}$$

2.2. Conductors & Dielectrics:-

→ Conduction properties can be explained on the basis of electrons having energies in the Valance & Conduction bands.

→ If the Valance band merges smoothly into a Conduction band — resulting in electron flow — called a Solid mettalic Conductor — no forbidden energy gap.

→ The forbidden energy gap b/w Valance and Conduction band of material is high — requires large energy to conduct — called dielectric.

→ In electromagnetics Conductors & dielectrics are defined on basis of ratio of Conduction to displacement current. Ratio of Conduction current to displacement current is $\frac{\sigma}{\omega \epsilon}$.

* If $\frac{\sigma}{\omega\epsilon} = 1$ — b/w Conductors & dielectrics.

$\frac{\sigma}{\omega\epsilon} > 1$ — good Conductors.

$\frac{\sigma}{\omega\epsilon} < 1$ — dielectrics.

2.3. Dielectric polarization :-

→ dielectric material is divided into two types

(i) polar

(ii) non-polar molecules.

→ molecules whose centre of positive & negative charges coincide — nonpolar

→ molecules whose centre is displaced from eachother is polar.

→ when non-polar dielectric material is placed in a electric field — displacement of negative & positive charges takes place in opposite directions & produce a dipole.

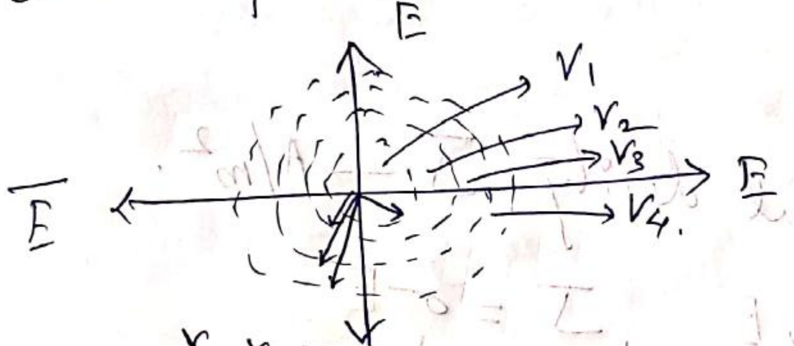
→ polarization is defined as dipole moment per unit volume.

$$P = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_{i=1}^{n \Delta V} P_i \quad C/m^2.$$

→ If the dipoles are aligned in random orientation — polarization has a

2.4. Equipotential Surface:-

→ It is an imaginary Surface in a electric field of a given charge distribution in which all the Surface points are having same electric potential.



r_1, r_2, r_3, r_4.

$$V_1 = V_2 = V_3 = V_4 = V = \frac{Q}{4\pi\epsilon_0 r}$$

$$r_1 = r_2 = r_3 = r_4 = r$$

$$V \propto \frac{1}{r}.$$

2.5. uniform & Non uniform field:-

$\vec{E}$ at Equipotential Surface } right angle to each other.

workdone is Zero (uniform field).

In a non-uniform field workdone will not be zero.

2.6 Conduction Current & Displacement Current :-

→ when electric field is applied to an conductor — conduction current occurs due to drift motion of electron. when the electron move they encountered with forces called Resistance.

Current density $J \rightarrow A/m^2$.

$J \propto E$, $J = \sigma E$
↓ conductivity of material

Since $J = I/s$

$\sigma E = I/s$, $E = V/l$, $R = V/I = \frac{l}{\sigma s}$

$\sigma \frac{V}{l} = I/s$, $R = l/\sigma s$.

$$\boxed{R = \frac{\rho l}{s}}$$

→ Displacement current I_D is flowing through a capacitor when ac voltage is applied across the capacitor.

$$I_D = \frac{dQ}{dt}$$

$$= C \frac{dV}{dt}$$

for parallel plate capacitor $C = \frac{\epsilon_0 A}{d}$

$$I_D = \frac{\varepsilon_0 A}{d} \cdot \frac{dV}{dt}$$

$$= \varepsilon A \frac{dE}{dt} \quad [\because V = Ed]$$

$$= A \cdot \frac{\partial D}{\partial t} \quad [\because D = \varepsilon E.$$

Displacement current density

$$J_D = \frac{I_D}{A}$$

$$J_D = \frac{\partial D}{\partial t}.$$

2.7. Dielectric strength :-

→ It is the maximum electric field that a dielectric can tolerate or withstand without electrical breakdown.

2.8. Boundary Conditions :-

→ Conditions existing at the boundary of two medium when field passes from one medium to other.

→ Depending on the nature of medium, it can be classified into three types :-

(i) Boundary b/w dielectric & dielectric

(ii) Boundary b/w Conductor & dielectric

(iii) Boundary b/w Conductor & free space.

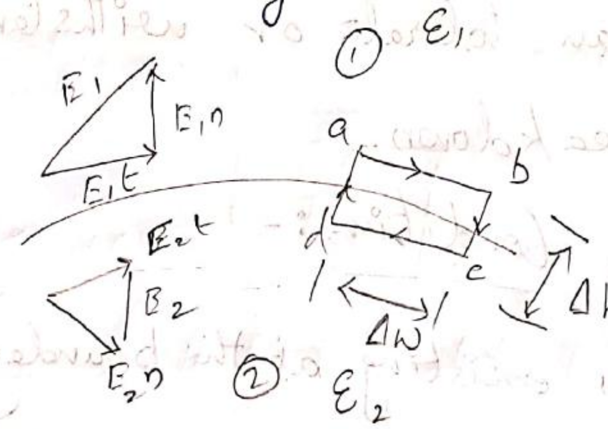
To determine the boundary conditions we need to use maxwell's equations

$$\oint E \cdot dl = 0$$

$$\oint_S D \cdot ds = Q_{enc.}$$

↓ free charge enclosed by the Surface.

1) Boundary b/w Dielectric & Dielectric:



Consider a field E existing in a region that consists of two diff dielectric by

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$$

$$\varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$$

$E = E_t + E_n$, $E_1 = E_{1t} + E_{1n}$,

$E_2 = E_{2t} + E_{2n}$

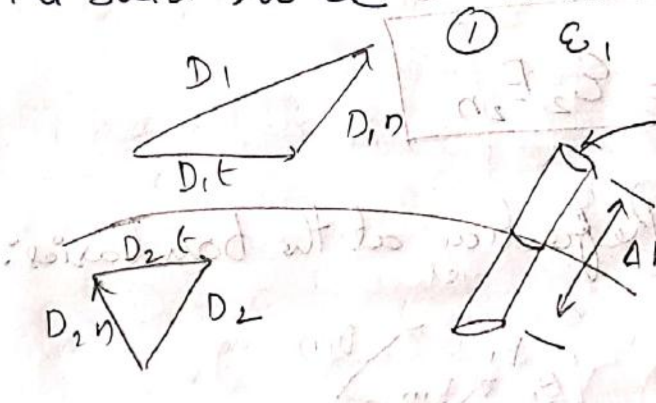
→ we apply Maxwell equ to closed loop abcda $\oint E \cdot dl = 0$

$$0 = E_{1t}\,\Delta w - E_{1n}\,\cancel{\Delta h/2} - E_{2n}\,\cancel{\Delta h/2}$$
$$+ E_{2t}\,\Delta w + E_{2n}\,\cancel{\Delta h/2} +$$
$$E_{1n}\,\cancel{\Delta h/2}$$

$$0 = \Delta w\,(E_{1t} - E_{2t})$$

$\Delta h = 0 \Rightarrow E_{1t} = E_{2t}$

E_t undergoes no change on the boundary & it said to be continuous across the boundary.



$D = \varepsilon E$, $\dfrac{D}{\varepsilon} = E$

$$E_{1t} = \frac{D_{1t}}{\varepsilon_1} \; ; \; E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

$$E_{1t} = E_{2t} \, , \; \frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

D_t undergoes some changes across the boundary and it is said to be discontinuous across the boundary.

$$\oint_S D \cdot ds = Q_{enc}.$$

Let us assume $Q_{enc} = 0$

$$\Delta h = 0.$$

$$\Delta Q = \rho_s \Delta s = D_{1n} \Delta s - D_{2n} \Delta s$$

$$\rho_s \Delta s = \Delta s (D_{1n} - D_{2n})$$

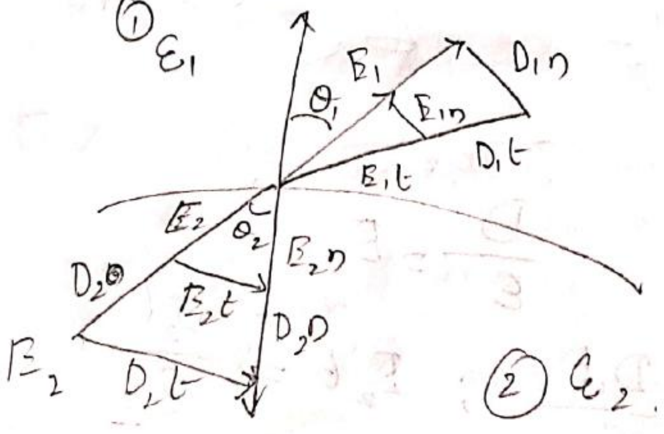
→ If no free charge is enclosed

$\rho_s = 0 \implies \boxed{D_{1n} = D_{2n}}$

D_n undergoes no change at the boundary

$$\boxed{\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}}$$

Reflection & Refraction at the boundaries:-



The normal components of D are given below

$$D_{n1} = D_1 \cos\theta_1$$

$$D_{2n} = D_2 \cos\theta_2$$

The tangential components of E are given below,

$$E_{t1} = E_1 \sin\theta_1$$

$$E_{t2} = E_2 \sin\theta_2 .$$

Apply the boundary condition

$$D_{n1} = D_{2n}$$

$$E_{1t} = E_{2t}$$

$$D_1 \cos\theta_1 = D_2 \cos\theta_2$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 .$$

Dividing $\dfrac{E_1}{D_1} \tan\theta_1 = \dfrac{E_2}{D_2} \tan\theta_2$

But $D_1 = \varepsilon_1 E_1$ and $D_2 = \varepsilon_2 E_2$

Then $\dfrac{E_1}{\varepsilon_1 E_1} \tan\theta_1 = \dfrac{E_2}{\varepsilon_2 E_2} \tan\theta_2$

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}, \quad \therefore \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}}$$

2.9. POISSON'S AND Laplace's Equations :-

According to Gauss' law point law form, divergence of electric flux density is equal to Volume charge density,

$$\nabla \cdot D = \rho_v$$

But $D = \varepsilon E$

$$\nabla \cdot (\varepsilon E) = \rho_v$$

$$\varepsilon \, \nabla \cdot E = \rho_v$$

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon}$$

But $E = -\nabla V$

$$\nabla \cdot (-\nabla \cdot V) = \frac{\rho_v}{\varepsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$ This is Poisson's Equation.

For Cartesian System

$$\nabla \cdot \nabla V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$$

This is poisson's equation in Cartesian form

If the Volume charge density (ρ_v) is Zero then.

$$\boxed{\nabla^2 V = 0}$$

This is Laplace equation. ∇^2 operator is Called as Laplacian operator.

2.10 Capacitance / Capacitor :-

→ Capacitor is a electricstatic device which Consists of two opposite Conductor charge density Saperated by a dielectric medium.

→ Capacitance is the property by which ratio of either charge to potential diffeence b/w Conductors.

$$C = Q/V. \longrightarrow C/V \Rightarrow \text{Farad}.$$

Assume uniform charge density D over the plates

$$D = Q/A. \text{, In terms of } E$$

$$D = \varepsilon E, \therefore \frac{Q}{A} = \varepsilon E, \quad Q = A\varepsilon E.$$

But electric field $E = V/d$,

$$Q = A\varepsilon \, V/d$$

$$\therefore C = \frac{A\varepsilon_0 \varepsilon_r}{d} \text{ Farad.}$$

2.11. Capacitance of parallel plate capacitor with two electric media.

$$V = V_1 + V_2$$

$$E_1 = \frac{D}{\varepsilon_{r_1}} = \frac{Q}{A\varepsilon_{r_1}\varepsilon_0}$$

$$E_2 = \frac{D}{\varepsilon_{r_2}} = \frac{Q}{A\varepsilon_{r_2}\varepsilon_0}.$$

The applied pottential $V = E_1 d_1 + E_2(d-d_1)$

$$V = \frac{Q}{A\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r_1}} + \frac{d-d_1}{\varepsilon_{r_2}}\right].$$

$$C = \frac{Q}{V}$$

$$\therefore C = \frac{A\varepsilon_0\varepsilon_{r1}\varepsilon_{r2}}{d_1\varepsilon_{r_2} + (d-d_1)\varepsilon_{r_1}}.$$

if medium 1 is air $\varepsilon_{r_1} = 1$, $\varepsilon_{r_2} = \varepsilon_r$

So $\therefore$ $C = \frac{A\varepsilon_0\varepsilon_r}{d_1\varepsilon_r + (d-d_1)}$

Three diffenent dielectrics $\varepsilon_{r_1}, \varepsilon_{r_2}, \varepsilon_{r_3}$.

$$C = \frac{A\varepsilon_0}{\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}} + \frac{d_3}{\varepsilon_{r_3}}}.$$

2.11.1 Capacitance of Isolated sphere:-

Consider a sphere of radius 'r'

The potential of sphere is by

$$V = -\int_{\infty}^{r} E.dr$$

But $E = \frac{Q}{4\pi\epsilon r^2}$

$$= -\int_{\infty}^{r} \frac{Q}{4\pi\epsilon r^2}.dr$$

$$= \frac{-Q}{4\pi\epsilon}\int_{\infty}^{r} \frac{1}{r^2}\,dr.$$

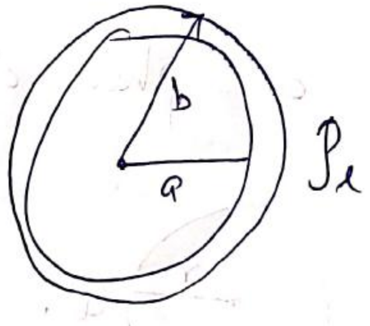
$V = \frac{Q}{4\pi\epsilon r}$, $C = \frac{Q}{V}$

$$\boxed{C = 4\pi\epsilon_0 r} \text{ Farad.}$$

Two Concentric spheres of inner radius 'a' & outer radius 'b'

$$C = 4\pi\epsilon\left[\frac{ab}{b-a}\right].$$

2.11.2. Capacitance of Co-axial Cable:-



Since $E = \dfrac{\rho_l}{2\pi \varepsilon r}$

Pd b/w two Co-axial Cable is

$$V = -\int_a^b E\,dr = \frac{\rho_l}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)$$

Capacitance of Co-axial Cable is

$$C = \rho_l / V$$

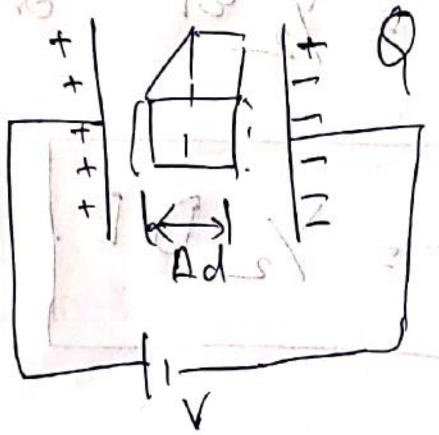
$$C = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln(b/a)} \text{ F/m.}$$

2.12. Energy density:-

The stored energy inside a Capacitance is in the form of Electrostatic form

$$W = \tfrac{1}{2} CV^2 = \tfrac{1}{2} QV \text{ Joules.}$$

Consider a Cube of side Δd parallel to the plates of a Capacitor



The Capacitance of Capacitor is

$$\Delta C = \frac{\varepsilon A}{\Delta d} = \frac{\varepsilon (\Delta d)^2}{\Delta d}$$

$$C = \varepsilon \Delta d$$

Energy stored $\Delta W = \frac{1}{2} \Delta C (\Delta V)^2$

Pd across the Cube is

$$\Delta V = E \cdot \Delta d$$

The stored energy

$$\Delta W = \frac{1}{2} (\varepsilon \Delta d)(E \cdot \Delta d)^2$$

$$= \frac{1}{2} \varepsilon E^2 (\Delta d)^3$$

$$= \frac{1}{2} \varepsilon E^2 \Delta v$$

$\Delta v = (\Delta d)^3$ is Volume.

Energy density is given by

$$\frac{\Delta W}{\Delta v} = \frac{1}{2}\varepsilon E^2 \text{ or}$$

$$\boxed{\frac{\Delta W}{\Delta v} = \frac{1}{2} D \cdot E} \quad \text{Joule}/m^3.$$

Note: problems refer text book

EMT by Dananjayan page no: 2.54 to

2.137. (Examples).

3.1 Magnetic flux & Magnetic flux density (13)

→ Magnetic flux (ϕ) is defined as magnetic lines of force passing unit Area.

$$\phi = \oint B \cdot ds \quad \text{weber.}$$

→ Magnetic flux density is defined as density of flux passing per unit Area.

$$B = \phi / A = Wb/m^2 \Rightarrow \text{Tesla.}$$

→ $B = \mu H$.

μ → permeability of medium

$\mu = \mu_o \mu_r$.

μ_o → permeability of free space

μ_r → Relative permeability.

$\mu_o = 4\pi \times 10^{-7}$.

H → magnetic field Intensity.

→ Since magnetic flux lines are closed & don't terminate.

→ Gauss law for magnetic field is

$$\oint_S B \cdot ds = 0.$$

By divergence theorem

$$\iint_S B \cdot ds = \nabla \cdot B$$

Then $\nabla \cdot B = 0$

→ total magnetic flux passing through any closed Surface is equal to zero.

3.2 Lorentz force:-

→ charged motion particle in a magnetic field experience a force.

→ force is proportional to product of magnitude of charge Q, its Velocity & flux density & sine of angle b/w v & B.

$$F = Q(\vec{V} \times \vec{B})$$

$$= BQv \sin\theta.$$

Electric force on a charged particle is

$$F = qE.$$

→ so Combine electric & Magnetic field will be

$$F = q\vec{E} + q\, V \times B$$

$$F = Q[\vec{E} + \vec{v} \times \vec{B}] \text{ Lorentz force.}$$

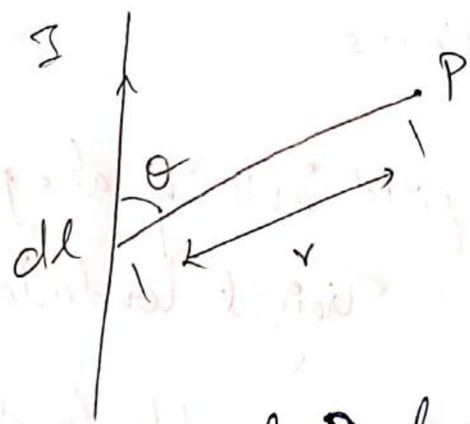
The magnitude of force is

$$F = BIl\sin\theta \quad \text{or}$$

$$F = (I \times B)l, \text{ force on a current element.}$$

3.3. Biot – Savart's Law :-

→ Law states that magnetic flux density produced by a current element at any point in a magnetic field is proportional to the current element & inversely proportional to square of the distance b/w them.



flux density at a point P due to current element Idl is given by

$$dB \propto \frac{Idl\sin\theta}{r^2}.$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}.$$

Magnetic field Intensity is given by

$$dH = \frac{I\,dl \sin\theta}{4\pi r^2}.$$

3.4. Ampre's Circuital Law

Law states that line integral of magnetic field Intensity H about any closed path is excatly equal to the direct current enclosed by that path.

$$\boxed{\oint H.dl = I.}$$

3.5. Applications

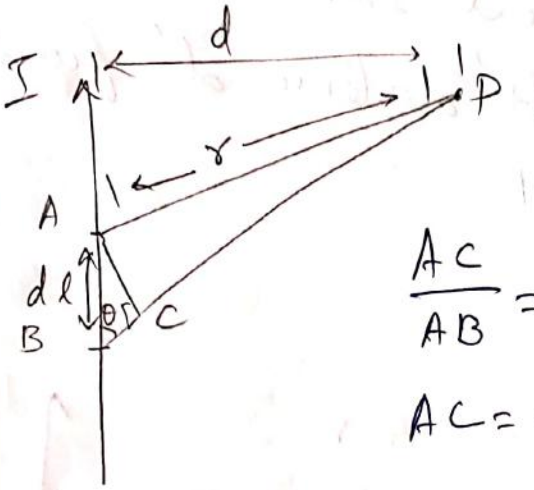
3.5.1. Magnetic field Intensity at a point due to Straight conductor.

Consider a long straight conductor Carrying Current I and also Consider a current element Idl. Let 'P' be a point at 'B' to measured.

According to Biot Savart's law,

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\theta}{r^2}$$

From ΔABC



$$\frac{AC}{AB} = \sin\theta$$

$$AC = dl \sin\theta$$

But arc $AC = r\,d\theta$.

$$dl \sin\theta = r\,d\theta$$

$$d\theta = \frac{dl \sin\theta}{r}$$

Sub this value in 'B' equation.

$$B = \frac{\mu_0 I}{4\pi} \int \frac{d\theta}{r} \quad [\text{from figure } d/r = \sin\theta]$$

$$\text{So } B = \frac{\mu_0 I}{4\pi d} \int_0^{\pi} \sin\theta\, d\theta.$$

$$= \frac{\mu_0 I}{4\pi d} \left[-\cos\theta\right]_0^{\pi}$$

$$B = \frac{\mu_0 I}{4\pi d} \times 2 = \frac{\mu_0 I}{2\pi d} \ \ wb/m^2$$

3.5.2 Magnetic flux density at any point along the axis of circular coil/loop:-

→ Consider a circular coil of radius 'a' carrying a current I and also consider current element Idl. Let 'P' be any point on 'B' distance 'd' from the centre of coil

→ Magnetic flux density 'B' at point 'P' due to current element dl is

$$dB = \frac{\mu_0 I\, dl}{4\pi r^2}$$

From ΔAOP $r^2 = a^2 + d^2$

$$dB = dB\cos\theta$$

$$= dB \cdot \frac{a}{\sqrt{a^2+d^2}}$$

$$= \frac{\mu_0 I\, dl}{4\pi r^2} \times \frac{a}{\sqrt{a^2+d^2}}$$

$$dB = \frac{\mu_0 I\, dl}{4\pi(a^2+d^2)} \times \frac{a}{\sqrt{a^2+d^2}}$$

$$dB = \frac{\mu_0 a I\, dl}{4\pi(a^2+d^2)^{3/2}}.$$

Magnetic flux density due to Circular Coil is given by

$$B = \frac{\mu_0 a I}{4\pi(a^2+d^2)^{3/2}} \int dl$$

$$= \frac{\mu_0 a I}{4\pi(a^2+d^2)^{3/2}} \times 2\pi a.$$

$$B = \frac{\mu_0 a^2 I}{2(a^2+d^2)^{3/2}} \quad Wb/m^2.$$

$$H = \frac{I a^2}{2(a^2+d^2)^{3/2}} \quad A/m.$$

If $d = 0$, field at Centre

$$B = \frac{\mu_0 a^2 I}{2a^3} = \frac{\mu_0 I}{2a}.$$

$$H = \frac{I}{2a} \quad A/m.$$

3.6 Magnetic potential :-

(i) Scalar magnetic potential :-

→ $\oint H \cdot dl = I$

If no current is enclosed ie $J = 0$

$\oint H \cdot dl = 0$

Magnetic field H can be expressed as negative gradient of a scalar function

$H = -\nabla V_m$

$V_m \rightarrow$ Scalar magnetic potential

$V_m = -\int H \cdot dl$

This also satisfies Laplace equation.

In free space $\nabla \cdot B = 0$

$\mu_0 \nabla \cdot H = 0$

But $H = -\nabla V_m$

$\mu_0 \nabla \cdot (-\nabla V_m) = 0$

$-\mu_0 \nabla^2 V_m = 0$

$\nabla^2 V_m = 0$

(ii) Vector magnetic potential

→ If current is enclosed, the potential depends upon current element.

→ Since divergence of a vector is a scalar, vector potential is expressed in curl

i.e, $\nabla \cdot B = 0$

$$B = \nabla \times A \quad \text{——} \ (1)$$

$A \longrightarrow$ magnetic vector potential.

Take curl on both sides in eq (1)

$$\nabla \times B = \nabla \times \nabla \times A$$

By the identity $\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$.

But $\nabla \times B = \mu J$

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu J$$

for the steady dc $(\nabla \cdot A) = 0$

then $-\nabla^2 A = \mu J$

Equating $\nabla^2 A_x = -\mu J_x$

$$\nabla^2 A_y = -\mu J_y.$$

$$\nabla^2 A_z = -\mu J_z.$$

→ in form of poissons equation.

Magnetic Vector pottential Canbe written as

$$A_x = \frac{\mu}{4\pi} \int_V \left(\frac{J_x}{r}\right) dv$$

$$A_y = \frac{\mu}{4\pi} \int_V \left(\frac{J_y}{r}\right) dv$$

$$A_z = \frac{\mu}{4\pi} \int_V \left(\frac{J_z}{r}\right) dv.$$

In general vector potential can be

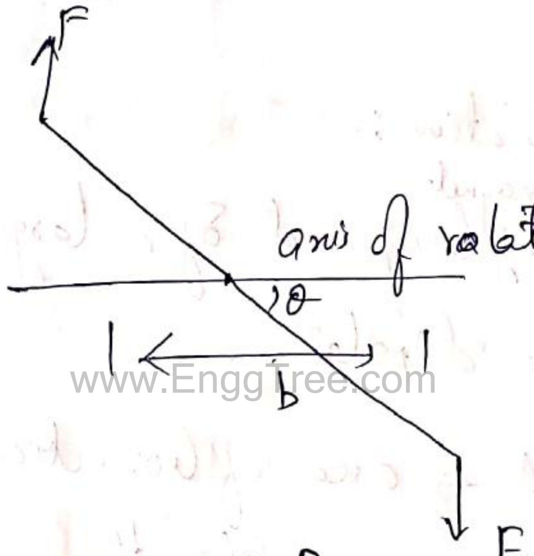
expressed as

$$A = \frac{\mu}{4\pi} \iiint \frac{J}{r} dv.$$

3.7 Magnetic Torque :-

→ when a current loop placed to a magnetic field, forces act on the loop; tend to rotate it.

→ Consider a rectangular loop of length 'l' & breadth 'b' carrying a current I in a uniform magnetic field B.



force $F = BIl\sin\theta$.

→ loop plane is parallel to the magnetic field

Total torque $T = 2 \times$ torque on each side

$= 2 \times$ force $\times$ distance

$= 2\, BIl\sin\theta\, b/2$

$= BIlb\sin\theta$

$T = BIA\sin\theta \quad (A = lb)$

magnetic moment is IA.

$m = IA$; $T = mB\sin\theta\,\hat{n}$

$T = m \times B$.

3.8 Magnetic dipole:-

→ polestrength Q_m & length l in a small bar magnet is treated as magnetic dipole

Since $\because m = IA$

$$\boxed{\therefore Q_m l = IA}$$

3.9. Magnetization:-

→ bar magnet composed of a large number of a magnetic dipoles.

$A \rightarrow$ area of crossection

$l \rightarrow$ axial length

$Al \rightarrow$ Volume.

$Q_m l \rightarrow$ dipole moment.

→ net dipole moment per unit Volume is Magnetization. (M)

$$M = \frac{Q_m l}{\text{Volume}} = \frac{Q_m}{A}.$$

Magnetic Susceptibility

$$\chi_m = M/H.$$

In free space flux density is given by

$$B = \mu_0 H.$$

for magnet, flux density is given by.

$$B = \mu_0 (H + M)$$

$$= \mu_0 H (1 + M/H)$$

$$= \mu_0 H \ (1 + \chi_m) \quad \text{—— (1)}.$$

Since $B = \mu_0 H$, eq (1) becomes.

$$\mu = \mu_0 (1 + \chi_m)$$

$$\boxed{\mu_r = 1 + \chi_m}$$

3.10. Magnetic Materials :-

→ Depending on the magnetic behaviour, Substances are classified into

(i) diamagnetic ($\mu_r \leq 1$)

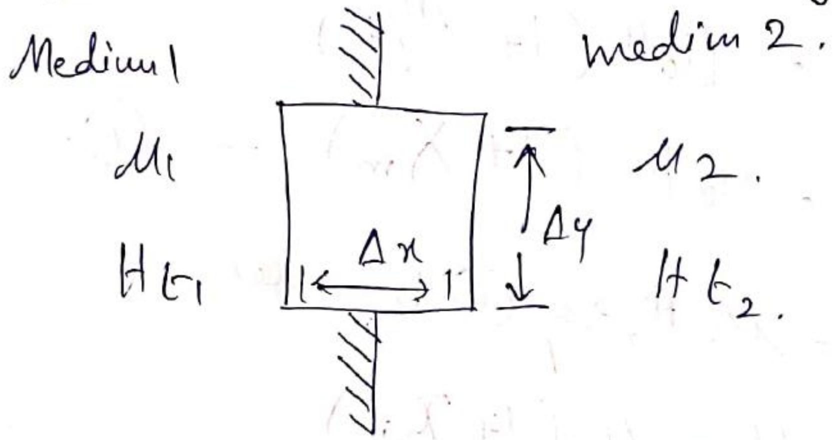
(ii) paramagnetic ($\mu_r \geq 1$)

(iii) ferro magnetic. ($\mu_r >> 1$)

3.11. Boundary Conditions:-

(1) Tangential Component of magnetic field Intensity is Continuous

2) Normal Component of B is Continuous across the boundary.



According to Ampere's law,

$$\oint H \cdot dl = I$$

If there is no current enclosed by the path

$$\oint H \cdot dl = 0$$

$$H_{t1} \Delta y - H_{t2} \Delta y = 0$$

$$H_{t1} = H_{t2}.$$

Let B_{n1} — normal Component in 1

B_{n2} — " in 2.

By Gauss law for magnetic field.

$$\iint_s B \cdot ds = 0$$

$$B_{n1}\, ds - B_{n2}\, ds = 0$$

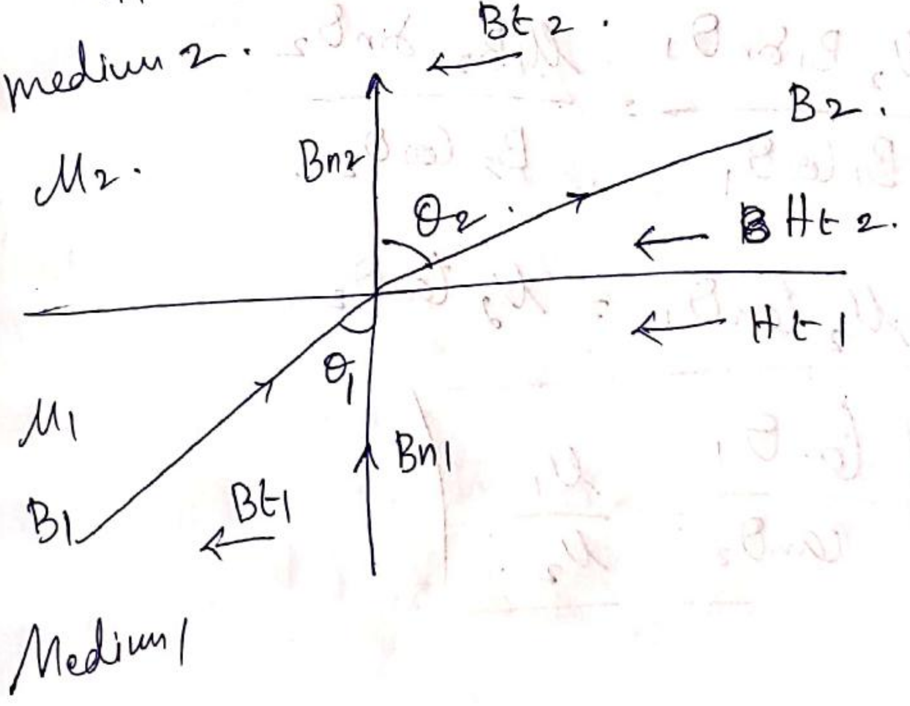
$$B_{n1} = B_{n2}.$$

$$B_{n1} = B_1 \cos\theta_1$$

$$B_{n2} = B_2 \cos\theta_2$$

But $B_{n1} = B_{n2}$

$$\cancel{B_{H1}}\; B_1 \cos\theta_1 = B_2 \cos\theta_2 \quad \text{——} \; (1)$$



$H_{t1} = B_{t1}/\mu_1$

$H_{t2} = B_{t2}/\mu_2$.

But $H_{t1} = H_{t2}$ (Boundary Condition.

$$\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}.$$

$\mu_2 B_{t1} = \mu_1 B_{t2}$.

when we Substitute

$\mu_2 B_1 \sin\theta_1 = \mu_1 B_2 \sin\theta_2$. —— (2).

Divide (2) by (1)

$$\frac{\mu_2 B_1 \sin\theta_1}{B_1 \cos\theta_1} = \frac{\mu_1 B_2 \sin\theta_2}{B_2 \cos\theta_2}.$$

$\mu_2 \tan\theta_1 = \mu_2 \tan\theta_2$.

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}.}$$

3.12. Inductance :- (L)

→ ratio of total flux linkage to the current produced in the flux is called inductance.

$$L = \frac{N\phi}{I}.$$

→ Inductance of Solenoid

Total flux linkage $= N\phi$

$\phi = BA.$

$= NBA.$

$B = \mu H$

∴ Total flux linkage $= N\mu HA.$

field Intensity is given by for solenoid

$$= N\mu \frac{NI}{l} A.$$

$$= \frac{\mu N^2 I A}{l}$$

$$H = \frac{NI}{l}$$

$$\text{Inductance} = \frac{\mu N^2 I A}{l I}$$

$$\boxed{L = \frac{\mu^2 N A}{l}}$$

→ Inductance of Torid :-

Total flux linkage $= N\phi$

$$\phi = BA.$$

$$= NBA$$

$$= N\mu HA.$$

$$H = \frac{NI}{2\pi R} = \frac{N\mu NIA}{2\pi R}.$$

$$\cancel{H} = \frac{\mu N^2 IA}{2\pi R}.$$

$$L = \frac{\mu N^2 \cancel{I} A}{2\pi R \cancel{I}}$$

$$L = \frac{\mu N^2 A}{2\pi R}$$

$$\boxed{L = \frac{N^2 R}{2}}$$

For Co-axial Cable

$$\boxed{L = \frac{\mu d \ln(b/a)}{2\pi}} \quad H.$$

Reluctance

$$R = \frac{mmf}{Total flux} = \frac{e_m}{\phi}.$$

$$R = \frac{l}{\mu s}.$$

Permeance $P = \frac{1}{R} = \frac{\phi}{e_m} = \frac{\mu s}{l}.$

$$mmf = \phi \times R.$$

$$mmf = H \times l.$$

3.13. Magnetic Energy:-

$$W = \frac{1}{2} L I^2.$$

$$W_m = \frac{1}{2} \Delta L (\Delta I)^2$$

$$dw = V i dt$$

Energy stored in magnetic field

$$W = \int_0^I V i dt.$$

$$= \int_0^I L \frac{di}{dt} i dt.$$

$$W = \frac{1}{2} L I^2.$$

Energy Density

$$W = \frac{1}{2} L I^2$$

Note: problems page - 3.27 to 3.48 & page - 4.25 to 4.46

Inductance of the Solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

Sub in above equation

$$W = \frac{1}{2} \frac{\mu_0 N^2 A}{l} I^2$$

$$= \frac{1}{2} \mu_0 \left(\frac{NI}{l}\right)^2 l A$$

$$W = \frac{1}{2} \mu_0 H^2 l A \qquad \left[\because H = \frac{NI}{l}\right]$$

Energy stored per unit Volume

$$w = \frac{1}{2} \mu_0 H^2$$

$$= \frac{1}{2} (\mu_0 H) H$$

$$w = \frac{1}{2} B H$$

The energy stored

$$W = \int_v w \, dv$$

$$W = \int \frac{1}{2} \mu H^2 \, dv$$

4.1 Magnetic Circuits

→ Magnetic flux are continuous & form closed paths — a single line flux called magnetic circuit.

→ Magnetic flux through magnetic circuit is defined as ratio of magneto motive force to Reluctance of the magnetic circuit.

$$\text{Magnetic flux} = \frac{\text{mmf}}{\text{Reluctance}}$$ (Magnetic ohm's law)

→ Reluctance is the opposition to the flux path in a magnetic circuit

→ It is also the Ratio of total mmf to the flux through it.

$$\text{Reluctance} = \frac{\text{mmf}}{\text{magnetic flux}}$$

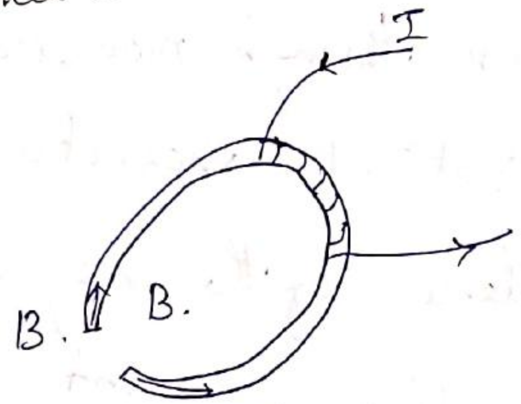
$$S = \frac{\oint H \cdot dl}{BA}$$

$$S = \frac{Hl}{BA} = \frac{Hl}{\mu HA} = \underline{\underline{\frac{l}{\mu A}}}\ H^{-1}$$

Reciprocal of Reluctance is Permeance

$$P = \frac{1}{S} = \frac{\mu A}{l} \text{ Henry.}$$

Magnetic circuit with air gap :-



→ thickness 't'

→ B — flux density in the gap is same as in the iron.

→ field Intensity in the iron is

$$H_i = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r}$$

and the field Intensity in the airgap is

$$H_g = \frac{B}{\mu_0}.$$

Note :- problems in Dhananjayan (Text) page - 4.25 to 4.43.

4.2 FARADAY's LAW And Its Implementation

Total electromagnetic force (emf) induced in a circuit is equal to the rate of decrease of total magnetic flux linking the circuit.

$$V = -\frac{d\phi}{dt}$$

→ 'N' number of turns, then total flux is $N\phi$, where ϕ is the magnetic flux

$$V = -N\frac{d\phi}{dt}$$

→ with reference to definition, the emf in a circuit is the line Integral of the electric field around the closed path

$$V = \oint E \cdot dl$$

E → Electric field.

→ According to Gauss law

$$\phi = \iint B \cdot ds.$$

Sub the value of ϕ in emf equation

$$V = -\frac{d}{dt}\iint B \cdot ds$$

$$V = -\iint \frac{\partial B}{\partial t} \cdot ds$$

But $V = \oint E \cdot dl$

$$\oint E \cdot dl = -\iint_S \frac{\partial B}{\partial t} \cdot ds$$

Applying strokes theorm

$$\oint E \cdot dl = \iint_S \nabla \times E \cdot ds.$$

Then

$$\iint_S \nabla \times E \cdot ds = -\iint \frac{\partial B}{\partial t} \cdot ds$$

$$\nabla \times E = -\frac{\partial B}{\partial t}.$$

This is refered to Maxwell equation.

4.2.1. Self Inductance & Mutual Inductance:-

→ changing current will produce an induced emf in the circuit to oppose the change in flux — this phenomenon is Self Inductance.

Consider a coil having 'N' number of turns — if AC is applied

Induced emf is proportional to rate of change of current

$$V \propto \frac{di}{dt}$$

$$V = L\frac{di}{dt}$$

$$L\frac{di}{dt} = N\frac{d\phi}{dt} \quad \left(\text{Since} \Rightarrow V = N\frac{d\phi}{dt}\right)$$

$$L = N\frac{d\phi}{di}$$

$$\therefore \boxed{L = \frac{N\phi}{i}}$$

Mutual Inductance is the ratio of induced magnetic flux linkage in one coil to the current through other coil.

$$M = N_1 \frac{\phi_{21}}{i_2}$$

&

$$M = N_2 \frac{\phi_{21}}{i_1}.$$

Co-efficient of coupling

$$K = \frac{M}{\sqrt{L_1 L_2}}$$ (K is always less than 1.)

4.3 Maxwell's Equations

→ Maxwell derived the concept of magnetic field produced by changing Electric field.

→ four electromagnetic Equations called Maxwell's Equations

→ It has both Integral & differential form

→ these equations are based on fundamental laws of Gauss, Faraday & Ampere.

→ Each differential equation has its integral counter part.

4.3.1 Maxwell equation - I.

From Ampres Circuital law:-

→ line Integral of the magnetic field Intensity H on any closed path is equal to Current enclosed by that path.

$$\oint_l H \cdot dl = I$$

$$= \iint_S J\, ds.$$

Total Current involves both Conduction Current and displacement Current.

→ Current through resistive element is called Conduction Current

→ Current through Capacitive element is called displacement Current.

Conduction Current density

$$I_c = \frac{V}{R}$$

$$\text{But } R = \frac{\rho l}{A} = \frac{l}{\sigma A}.$$

σ → Conductivity.

$$\therefore \frac{V\sigma A}{l} = I_c.$$

If E is the electric field, then

Voltage $V = E \cdot l$

$$I_c = \frac{E l \sigma A}{l}$$

$$I_c = E\sigma A.$$

$$\frac{I_c}{A} = E\sigma, \quad \boxed{J_c = \sigma E} \text{ --- (1)}$$

Displacement current density

$$I_D = \frac{dQ}{dt}.$$

$$Q = CV \Rightarrow I_D = C\frac{dV}{dt}.$$

Since $C = \dfrac{\varepsilon_0 A}{d}$

Sub in above equation

$$I_D = \frac{\varepsilon A}{d} \cdot \frac{dV}{dt}.$$

But $V = Ed$

$$I_D = \frac{\varepsilon A}{d} \cdot d \frac{dE}{dt}.$$

$$I_D = \varepsilon A \frac{dE}{dt}$$

$$\frac{I_D}{A} = \varepsilon \frac{\partial E}{\partial t}$$

$$= \frac{\partial D}{\partial t} \quad [\because D = \varepsilon E]$$

Displacement current density

$J_D = I/A$ is

$$\boxed{J_D = \frac{\partial D}{\partial t}} \quad \text{—} ②$$

Equation ① & ② are the Differential form.

Ampres law can be written as

$$\oint H \cdot dl = \iint_S (J_c + J_D)\, ds$$

then $$\oint H \cdot dl = \iint_S \left(\sigma E + \frac{\partial D}{\partial t}\right) ds$$

$$\oint H \cdot dl = \iint_S \left[\sigma E + \varepsilon \frac{\partial E}{\partial t}\right] ds$$

then $$\oint H \cdot dl = \iint \left(J + \frac{\partial D}{\partial t}\right) ds$$

This is the Maxwell's equation in integral form from Ampree law.

By Applying stokes theorm

$$\oint H \cdot dl = \iint_S \nabla \times H \; ds$$

Comparing above two equations

$$\iint \nabla \times H \, ds = \iint \left[J + \frac{\partial D}{\partial t} \right] ds$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\boxed{\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}}$$

Maxwell's equation in differtialform.

Statement :- mmf around a closed path is equal to the sum of the conduction current & displacement current enclosed by the path.

4.3.2 Maxwell's Equation - II

From Faraday's law :-

As faraday's law states,

$$V = -\frac{d\phi}{dt}$$

$$= \frac{d}{dt}\left(\iint_S B \cdot ds\right)$$

But

$$V = \oint E \cdot dl$$

$$\oint E \cdot dl = -\frac{d}{dt}\iint_S B \cdot ds.$$

$$\oint E \cdot dl = -\iint \frac{\partial B}{\partial t} \cdot ds.$$

Or $$\oint E \cdot dl = -\mu \iint \frac{\partial H}{\partial t} \cdot ds \text{ (Integral form)}$$

By Applying stokes theorm

$$\oint E \cdot dl = \iint_S \nabla \times E \, ds.$$

Comparing both equations

$$\iint_S \nabla \times E \, ds = -\iint \frac{\partial B}{\partial t} \cdot ds.$$

$$\nabla \times E = -\frac{\partial B}{\partial t}.$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}. \quad \text{(differential form)}.$$

Statement :- Emf around a closed path is equal to magnetic displacement (flux density) through that closed path.

4.3.3. Maxwell's Equation - III

From electric Gauss law,

$$\psi = Q.$$

$$\iint D \cdot ds = Q.$$

Or $$\iiint_V \rho_v \, dv = Q.$$

Then

$$\iint_S D \cdot ds = \iiint_V \rho_v \, dv. \quad \text{(Integral form)}.$$

By Applying Divergence theorem

$$\iint_S D \cdot ds = \iiint_V \nabla \cdot D \, dv$$

Comparing both equations.

$$\iiint_V \nabla \cdot D\,dv = \iiint_V \rho \cdot dv.$$

$$\boxed{\nabla \cdot D = \rho}$$ differential form.

statement :- Total electric displacement through the surface enclosing a volume is equal to the total charge within the volume.

4.3.4 Maxwell's Equation - IV

From magnetic Gauss law,

$\phi = 0$. (or)

$$\iint_S B \cdot ds = 0. \text{ (Integral form)}$$

Applying Divergence theorm

$$\iint_S B \cdot ds = \iiint_V \nabla \cdot B\,dv$$

Comparing both equations

$$\iiint_V \nabla\cdot B = 0.$$

$$\boxed{\nabla\cdot B = 0}$$ differential form.

Statement :- net magnetic flux emerging through any closed Surface is Zero.

4.3.5. Time Varying field :-

→ Electric & Magnetic fields are assumed to be time Varying fields.

→ So it is expressed in phasor Quantity.

$$E(x,t) = \text{Real part of}\left[E(x)\,e^{j\omega t}\right]$$

$$\frac{\partial E}{\partial t}(x,t) = \text{Real part of}\left(j\omega E(x)\,e^{j\omega t}\right)$$

Apply in Maxwell equation

$$\text{Real part of}\left[\nabla\times H\right] = \text{Real part of}\left[(\sigma E + j\omega\varepsilon E)\,e^{j\omega t}\right]$$

$$\nabla \times H = \sigma E + j\omega\varepsilon E$$

$$\boxed{\nabla \times H = (\sigma + j\omega\varepsilon)E} \longrightarrow (1).$$

For magnetic field

$$H(x,t) = Re\left[H(x)\, e^{j\omega t}\right]$$

$$\frac{\partial H}{\partial t}(x,t) = Re\left[j\omega H(x)\, e^{j\omega t}\right]$$

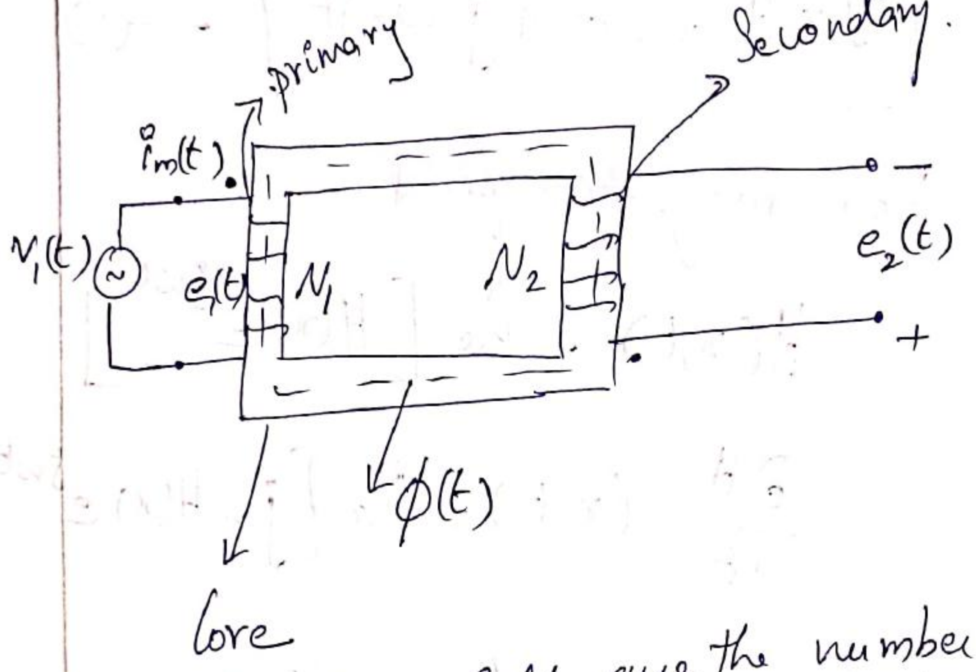
Apply in Maxwell equations

$$Re\left[\nabla \times E\right] = \left[-Re\left(j\omega\mu H e^{j\omega t}\right)\right]$$

$$\boxed{\nabla \times E = -j\omega\mu H} \longrightarrow (2).$$

∴ Note : Maxwell's equations will change incorporating the eq (1) & (2).

4.4. Transformer and Motional Emf.



→ If N_1 & N_2 are the number of turns in primary & Secondary windings respectively, the induced emf's are.

$$e_1 = N_1 \frac{d\phi}{dt}.$$

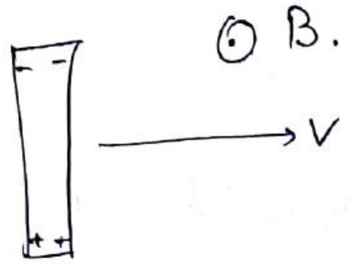
$$e_2 = N_2 \frac{d\phi}{dt}.$$

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} \quad (\text{emf ratio})$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a \quad (\text{Voltage ratio})$$

(Current ratio) $\dfrac{i_2}{i_1} = \dfrac{V_1}{V_2} = \dfrac{e_1}{e_2} = \dfrac{N_1}{N_2}$

→ Motional emf :-



Consider a conductor moving in a steady magnetic field.

→ if a charge Q moves in a Magnetic field $\vec{B}$ experiences a force.

$$\vec{F} = Q(\vec{V} \times \vec{B})$$

This force will cause the electrons in conductor to drift towards one end & leave other end positively charged.

when E & B forces reaches equilibrium, net force on the moving conductor is zero.

$$\frac{\vec{F}}{Q} = \vec{V} \times \vec{B} \text{ — Called}$$

motional emf. $\vec{E}_m = \vec{V} \times \vec{B}$.

The generated emf around the circuit is

$\oint_c \vec{V} \times \vec{B} \cdot dl$ → motional emf.

4.5 Relation b/w field theory & Circuit theory.

	Circuit theory	field theory
1.	analysis originated by itś own.	1. Evolved from Transmission theory.
2.	Applicable for portion of RF range.	Beyond RF range.
3.	dependent & independent parameter, I & V are directly obtained.	3. Not directly obtained, through E & H.
4.	parameters of medium are not involved	parameter are involved [ε & μ].
5.	Laplace transform is employed	5. Maxwell equation is employed.
6.	Z, Y, H parameters are used.	S parameter is used.
7	Low power Calculations	Relatively high power.
8.	Simple to understand	need Visualization ability
9.	Two dimensional	Three dimensional Analysis.
10.	Freq for reference	wave length for reference.
11.	Lumped Components used.	Distributed Components used

5.1 Electromagnetic Wave Equation:

→ application of Maxwell's equations — existance of electromagnetic wave.

→ wave equation can be derived from Maxwell's equation from Faraday's law in point form,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$= -\mu \frac{\partial H}{\partial t}.$$

Take curl on both sides,

$$\nabla \times \nabla \times E = \mu \, \nabla \times \frac{\partial H}{\partial t}. \quad \text{—— (1)}$$

But Maxwell's equation from Ampere's law is

$$\nabla \times H = J + \frac{\partial D}{\partial t}.$$

$$= \sigma E + \varepsilon \frac{\partial E}{\partial t}.$$

Differentiate

$$\nabla \times \frac{\partial H}{\partial t} = \frac{\partial \nabla \times H}{\partial t}.$$

$$= \frac{\partial}{\partial t}\left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla \times \frac{\partial H}{\partial t} = \sigma \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{— (2)}$$

Sub (2) in (1) we get.

$$\nabla \times \nabla \times E = -\mu \left[\sigma \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2} \right]$$

$$= -\mu \sigma \frac{\partial E}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{— (3)}$$

But according to the identity

$$\nabla \times \nabla \times E = \nabla (\nabla \cdot E) - \nabla^2 E \quad \text{— (4)}$$

But

$$\nabla \cdot E = \frac{1}{\varepsilon} \nabla \cdot D$$

Since no net charge within the Conductor, $\rho = 0$

$$\nabla \cdot D = 0$$

$$\nabla \cdot E = 0$$

Then equation (4) becomes

$$\nabla \times \nabla \times E = -\nabla^2 E \quad \text{— (5)}$$

Comparing the equations (3) & (5)

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \quad (6)$$

This is the wave equation for electric field 'E'.

wave equation for magnetic field 'H' can be obtained in a simillar manner as

$$\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0. \quad (7)$$

Equations (6) & (7) are the wave equations from maxwell equations.

5.2. Wave equation for free space :-

→ for free space ($\sigma = 0$) & $\rho = 0$

→ electromagnetic wave equations for free space can be obtained from maxwell equations.

Maxwell equation from faraday's law

$$\nabla \times \underline{E} = -\frac{\partial B}{\partial t}$$

$$= -\mu \frac{\partial H}{\partial t}$$

Taking curl on both sides

$$\nabla \times \nabla \times E = -\mu \nabla \times \frac{\partial H}{\partial t} \quad \text{—(8)}$$

But Maxwell's equation from Ampere's law for free space is

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$= \varepsilon \frac{\partial E}{\partial t}$$

Then

$$\nabla \times \frac{\partial H}{\partial t} = \frac{\partial \nabla \times H}{\partial t}$$

$$= \frac{\partial}{\partial t}\left(\varepsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla \times \frac{\partial H}{\partial t} = \varepsilon \frac{\partial^2 \underline{E}}{\partial t^2} \quad \text{—— (9)}$$

Sub (9) in (8)

$$\nabla \times \nabla \times E = -\mu \varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{—(10)}$$

But by the identity

$$\nabla \cdot E = \frac{1}{\varepsilon} \nabla \cdot D = \frac{\rho}{\varepsilon} = 0$$

Then $\nabla \times \nabla \times E = -\nabla^2 E$ —— (11)

Comparing equations (10) & (11)

$$\nabla^2 E = \mu \varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = 0. \quad \text{—— (12)}$$

$||^{ly}$

$$\nabla^2 H - \mu \varepsilon \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{— (13)}$$

Equations (12) & (13) wave equations for free space in terms of electric & magnetic field.

For free space $\mu_r = 1$ & $\varepsilon_r = 1$ (air)

Wave equation becomes

$$\nabla^2 H - \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} = 0.$$

$$\mu_0 \varepsilon_0 = 4\pi \times 10^{-7} \times \frac{1}{36\pi \times 10^{-9}}$$

$$= \frac{1}{9 \times 10^{16}}$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^{8} \text{ m/sec.}$$

$$= v \quad \text{— Velocity of light.}$$

Then wave equation becomes

$$\nabla^2 H - \frac{1}{v^2} \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{or}$$

$$\nabla^2 E - \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2} = 0.$$

5.3 characteristic impedance or Intrinsic Impedance (η_0)

Consider the wave propogating in x-directi[on]

So,

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

The Solution of this differential equation is

$$E = f_1(x - V_0 t) + f_2(x + V_0 t)$$

where $V_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Solution consists of two waves,

Consider wave in positive direction

$$f_2(x + V_0 t) = 0 \quad \text{(negative direction)}$$

The general solution becomes

$$E = f(x - V_0 t)$$

$$\nabla \times E = \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \vec{x}\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] + \vec{y}\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right] + \vec{z}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$$

Since wave travelling in x- direction

y & z are independent

$$E_x = H_x = 0 \quad \text{and} \quad \frac{\partial E}{\partial y} = \frac{\partial E}{\partial z} = 0$$

$$\nabla \times E = -\frac{\partial E_z}{\partial x}\vec{y} + \frac{\partial E_y}{\partial x}\vec{z}$$

$$\nabla \times H = -\frac{\partial H_z}{\partial x}\vec{y} + \frac{\partial H_y}{\partial x}\vec{z}$$

But

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t}$$

Comparing these two equations

$$-\frac{\partial H_z}{\partial x}\vec{y} + \frac{\partial H_y}{\partial x}\vec{z} = \varepsilon\left[\frac{\partial E_y}{\partial t}\vec{y} + \frac{\partial E_z}{\partial t}\vec{z}\right]$$

$[\because E_x = 0]$

Equating $\vec{y}$ & $\vec{z}$ terms

$$-\frac{\partial H_z}{\partial x} = \varepsilon \frac{\partial E_y}{\partial t}$$

$$\frac{\partial H_y}{\partial x} = \varepsilon \frac{\partial E_z}{\partial t}.$$

From Second Maxwell's equation for free space

$$\nabla \times E = -\mu \frac{\partial E}{\partial t}$$

But

$$\nabla \times \underline{E} = -\frac{\partial E_z}{\partial x}\vec{y} + \frac{\partial E_y}{\partial x}\vec{z}$$

Equating these two equations

$$-\frac{\partial E_z}{\partial x}\vec{y} + \frac{\partial E_y}{\partial x}\vec{z}$$

$$= -\mu\left[\frac{\partial H_y}{\partial t}\vec{y} + \frac{\partial H_z}{\partial t}\vec{z}\right]$$

$$\left[\because H_x = 0\right]$$

Equating $\vec{y}$ and $\vec{z}$ terms

$$\frac{\partial E_z}{\partial x} = \mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t}.$$

Let the Solution for this equation is

$$E_y = \left[f(x - V_0 t)\right]$$

Differentiate

$$\frac{\partial E_y}{\partial t} = \frac{\partial f}{\partial(x - V_0 t)} \cdot \frac{\partial(x - V_0 t)}{\partial t}.$$

$$= f'(x - V_0 t)(-V_0)$$

Simplify $f'(x - V_0 t)$ can be written as f'

$$\frac{\partial E_y}{\partial t} = -V_0 f'$$

But

$$-\frac{\partial H_z}{\partial x} = \varepsilon \frac{\partial E_y}{\partial t}$$

$$\frac{\partial H_z}{\partial x} = -\varepsilon(-V_0 f)'$$

$$= V_0 \varepsilon f'$$

$$= \frac{1}{\sqrt{\mu\varepsilon}} \varepsilon f'$$

$$\frac{\partial H_z}{\partial x} = \sqrt{\varepsilon/\mu}\, f'$$

Integrating $H_z = \sqrt{\varepsilon/\mu} \int f' dx$

$$H_z = \sqrt{\frac{\varepsilon}{\mu}}\, f$$

$$= \sqrt{\varepsilon/\mu}\, E_y$$

$$\frac{E_y}{H_z} = \sqrt{\frac{\mu}{\varepsilon}}$$

$||^{ly}$ it can be shown that

$$\frac{E_z}{H_y} = -\sqrt{\frac{\mu}{\varepsilon}}$$

If E is the total electric field

$$E = \sqrt{E_y^2 + E_z^2}$$

& Total magnetic field

$$H = \sqrt{H_y^2 + H_z^2}$$

Then

$$\frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}}$$ — characteristic impedance.

$$\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}}$$

For free space $\varepsilon_r = \mu_r = 1$, then

Intrinsic Impedance $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$

$\eta_0 = 377\Omega$

$[E \cdot H = 0]$

5.4. Wave propagation in Lossless Medium

→ Wave equation for free space (lossless medium) is

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

→ phasor value of E is

$$E(x,t) = Re\left[(E(x) e^{j\omega t})\right]$$

Applying to wave equation

$$\nabla^2 Re(E e^{j\omega t}) = \mu\varepsilon \frac{\partial^2}{\partial t^2} Re\left[E e^{j\omega t}\right]$$

$$\nabla^2 Re\left[E e^{j\omega t}\right] = \mu\varepsilon\left[-\omega^2 E e^{j\omega t}\right]$$

$$Re\left[(\nabla^2 E + \mu\varepsilon\omega^2 E) e^{j\omega t}\right] = 0.$$

$$\nabla^2 E + \mu\varepsilon\omega^2 E = 0.$$

This is Vector Helmholtz equation

$$\nabla^2 E + \beta^2 E = 0$$

where $\beta^2 = \mu\varepsilon\omega^2$

$\beta = \sqrt{\mu\varepsilon}\,\omega$ (phase shift constant)

Wave progata in x — direction

ie no Vaiation in y & z

$$\frac{\partial^2 E}{\partial x^2} + \beta^2 E = 0$$

Solution of the equation is

$$E = C_1 e^{-j\beta x} + C_2 e^{j\beta x}$$

5.5. Wave propogation in a Conducting Medium:-

wave equation for Conducting medium is

$$\nabla^2 E - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} - \mu \sigma \frac{\partial E}{\partial t} = 0$$

In phasor form

$$\nabla^2 E - \mu \varepsilon \omega^2 E - j \omega \sigma \mu E = 0$$

$$\nabla^2 E - j\omega\mu(\sigma + j\omega\varepsilon)E = 0.$$

$$\nabla^2 E - \gamma^2 E = 0.$$

$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) \rightarrow$ propagation constant.

$\gamma = \alpha + j\beta \rightarrow$ phase shift.

$\alpha \rightarrow$ attenuation constant

$$\gamma = \alpha + j\beta$$
$$= \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

Squaring both sides

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\varepsilon$$

Equating real & imaginary parts

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon.$$

and $2\alpha\beta = \omega\mu\sigma$

To solve equations

we know that

$$\alpha^2 + \beta^2 = \sqrt{(\alpha^2 - \beta^2)^2 + 4\alpha^2\beta^2}$$

But

$$(\alpha^2 - \beta^2)^2 = (-\omega^2\mu\varepsilon)^2$$
$$(2\alpha\beta)^2 = (\omega\mu\sigma)^2$$

$$(\alpha^2+\beta^2) = \sqrt{\omega^4\mu^2\varepsilon^2+\omega^2\mu^2\sigma^2}$$

$$\alpha^2-\beta^2 = -\omega^2\mu\varepsilon$$

Adding two equations

$$2\alpha^2 = -\omega^2\mu\varepsilon+\sqrt{\omega^4\mu^2\varepsilon^2+\omega^2\mu^2\sigma^2}$$

$$\alpha^2 = -\frac{\omega^2\mu\varepsilon}{2}+\frac{\omega^2\mu\varepsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}$$

Attenuation factor is given by

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$$

then

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]}$$

5.6. Skin Depth:-

→ In a good Conductor the wave is attenuated as it progress.

→ A new depth of penetration is introduced to explain this situation.

→ depth of penetration (δ) is defined as that depth in which the wave has been attenuated to $1/e$ or 37 percent of its orginal value.

Amplitude of the wave decreases by the factor $e^{-\alpha x}$ as it propogates through a distance x

By definition

$$e^{-\alpha x} = \frac{1}{e}$$

$$e^{-\alpha\delta} = e^{-1}$$

$$\alpha\delta = 1$$

$$\delta = \frac{1}{\alpha} \quad \text{(skin depth)}$$

5.7. Poynting Vector :-

Poynting theorm :-

→ when an electromagnetic wave propogates through space from their Source to distant receiving points, there is a transfer of energy from the Source to the receivers.

→ There exists a direct relation b/w rate of this energy transfer & amplitude of E & H of electro magnetic wave.

Statiment :-

Vector product of electric field Intensity & magnetic field intensity at any point is a measure of rate of energy flow per unit area at that point.

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\int_V E \cdot J\, dv = -\frac{\partial}{\partial t} \int_V \left(\frac{\mu}{2} H^2 + \frac{\varepsilon}{2} E^2 \right) dv - \oint_S E \times H\, ds.$$

physical interpretation is

Rate of energy dissipation } = Rate at which stored energy in Volume v is decreasing

+

Rate at which energy is entering the Volume from out-side.

5.8 POYNTING VECTOR:-

Vector product of electric field intensity & magnetic field Intensity is another product called poynting Vector (P)

Instantaneous, Avg, & Complex Vector:-

$$v = Re\left[V e^{j\omega t}\right] = |V| \cos(\omega t + \theta_v)$$

$$i = Re\left[I e^{j\omega t}\right] = |I| \cos(\omega t + \theta_i)$$

Instantaneous power

$$w = |V||I| \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$w = \frac{|V||I|}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right]$$

Avg power

$$W_{av} = \frac{|V||I|}{2} \cos(\theta_v - \theta_i)$$

if $\theta_v - \theta_i = \theta$ angle b/w Voltage & Current then

$$W_{av} = \frac{|V||I|}{2} \cos\theta.$$

Reactive power

$$W_{area} = \frac{|V||I|}{2} \sin\theta.$$

Complex power

W is defined as

$$W = \frac{1}{2} V I^*$$

$I^* \rightarrow$ Complex Conjugate of I.

$$W = \frac{|V||I|}{2} e^{j\theta}.$$

$$W = W_{av} + j W_{avs}.$$

The Complex poynting Vector P is $P = \frac{1}{2} E \times H^*.$

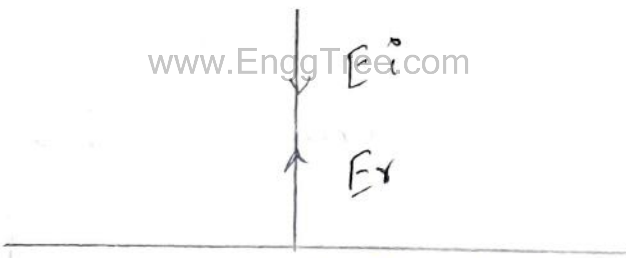
5.9 Reflection & Refraction of plane waves:-

(i) Reflection by perfect Conductor:-

1. Wave Incident on perfect Conductor.

→ when a plane wavefront is incident upon the Surface of a perfect Conductor — entire wave is reflected.

→ Since no loss within a perfect Conductor.



→ resultant Amplitude differ by π

ie $E_i = -E_r$.

→ Let electric field incident is

$E_i e^{\gamma x}$, Since $\alpha = 0$, $\gamma = j\beta$.

→ Incident wave is $E_i e^{-j\beta x}$

→ reflected wave is $E_r e^{j\beta x}$ (opposite)

$E_T(x) = E_i e^{-j\beta x} + E_r e^{j\beta x}$

$$E_T(x) = E_i\left[e^{-j\beta x} - e^{j\beta x}\right]$$

$$= -j\,2\,E_i \sin\beta x$$

Express the above equation in time variation

$$E_T(x,t) = -2j\,E_i \sin\beta x\, e^{j\omega t}$$

If E_i is real

$$E_T(x,t) = +2E_i \sin\beta x \sin\omega t.$$

This equation shows both waves combine to form a standing wave.

||ly for magnetic field

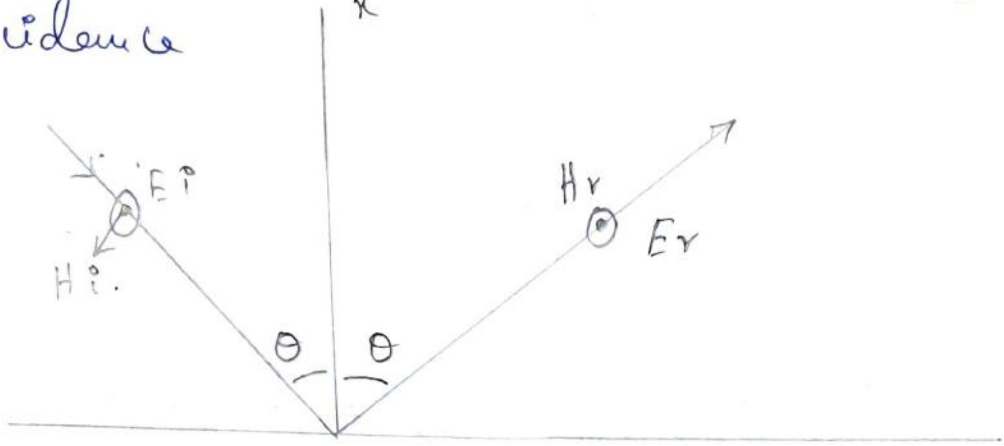
$$H_T(x,t) = 2H_i \cos\beta x \cos\omega t.$$

2. Wave Incident obliquely on a perfect Conductor.

→ Two cases arise on oblique condition of wave

Horizontal polarization :-

→ E is perpendicular to the plane of incidence



→ Incident wave can be expressed as

$$E_{in} = E_i e^{-j\beta \bar{n}.r}$$

→ for normal of incident wave

$$\bar{n}.r = x \cos \pi/2 + y \cos(\pi/2 - \theta)$$

$$+ z \cos(\pi - \theta)$$

$$= y \sin\theta - z\cos\theta$$

$$E_{in} = E_i e^{-j\beta(y\sin\theta - z\cos\theta)}$$

$||^{ly}$ reflected wave

$$E_{ref} = E_r e^{-j\beta(y\sin\theta + z\cos\theta)}$$

But $E_{ref} = -E_i$

Total electric field

$$E_T = E_{in} + E_{ref}.$$

$$= E_i \left[e^{-j\beta(y\sin\theta - z\cos\theta)} - e^{-j\beta(y\sin\theta + z\cos\theta)} \right]$$

$$= 2j E_i \sin(\beta z\cos\theta)\, e^{-j\beta y \sin\theta}.$$

$$E_T = 2j E_i \sin\beta_z z\, e^{-j\beta_y y}$$

where $\beta = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{2\pi}{\lambda}.$

$$\beta_z = \beta\cos\theta$$

$$\beta_y = \beta\sin\theta.$$

Wave length in Z- direction

$$\lambda_z = \frac{2\pi}{\beta_z} = \frac{\lambda}{\cos\theta}$$

Velocity in y direction

$$V_y = \frac{\omega}{\beta_y} = \frac{v}{\sin\theta}.$$

$$\lambda_y = \frac{\lambda}{\sin\theta}.$$

Vertical polarization:-

→ E // to plane of incidence.

→ H_i & H_r makes angle $\theta_i = \theta_r = \theta$.

→ incident wave is expressed as

$$H_{in} = H_i e^{-j\beta(y\sin\theta - z\cos\theta)}$$

→ reflected wave as

$$H_{ref} = H_r e^{-j\beta(y\sin\theta + z\cos\theta)}$$

$H_T = H_{in} + H_{ref}$. (no phase reversal)

$$H_T = H_i e^{j\beta y\sin\theta}\left[e^{-j\beta z\cos\theta} + e^{-j\beta z\cos\theta}\right]$$

$$H_T = 2 H_i \cos\beta_z z e^{-j\beta_y y}$$

(ii) Reflection by a perfect Dielectric

1) wave Incident normally on a perfect dielectric:-

→ Consider two perfect dielectric media

→ Let ϵ_1, μ_1 & μ_2, ϵ_2 be permittivity & permeability of medium 1 & 2.

$$E_i = \eta_1 H_i$$

$$E_r = -\eta_1 H_r$$

$$E_t = \eta_2 H_t.$$

According to boundary Condition, tangential Component of E or H is continous

$$H_i + H_r = H_t$$

& $E_i + E_r = E_t.$

From above equations

$H_i = \frac{E_i}{\eta_1}$, $H_r = -\frac{E_r}{\eta_1}$ and

$$H_t = \frac{E_t}{\eta_2}$$

$$H_t = H_i + H_r = \frac{1}{\eta_1}[E_i - E_r]$$

$$H_t = \frac{E_t}{\eta_2}$$

$$H_t = \frac{1}{\eta_2}[E_i + E_r]$$

Equating H_t becomes

$$\frac{1}{\eta_1}[E_i - E_r] = \frac{1}{\eta_2}[E_i + E_r]$$

$$\eta_2[E_i - E_r] = \eta_1[E_i + E_r]$$

$$E_i[\eta_2 - \eta_1] = E_r[\eta_1 + \eta_2]$$

Reflection Co-efficient

$$\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

also

$$\frac{E_t}{E_i} = \frac{E_i + E_r}{E_i} = 1 + \frac{E_r}{E_i}$$

$$= 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Transmission Co-efficient $\Big\}$ $\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}.$

lly for magnetic field

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_2 + \eta_1} \quad \&$$

$$\frac{H_t}{H_i} = \frac{2\eta_1}{\eta_1 + \eta_2}$$

In term of permittivity

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \quad \text{and}$$

$$\frac{E_t}{E_i} = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}.$$

$\|^{ly}$ for magnetic field.

2) Oblique wave incident.

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}\, E_t^2 \cos\theta_t}{\sqrt{\varepsilon_1}\, E_i^2 \cos\theta_i} \quad \&$$

$$\frac{E_r}{E_i} = \frac{\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} \sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}.$$

Brewster's angle:-

$$\tan\theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

at which no reflected wave is polarized.

PROBLEMS:

UNIT - I

Q1. Given two Vectors $\vec{A} = 3\vec{a_x} - \vec{a_y} + 8\vec{a_z}$ and $\vec{B} = 2\vec{a_x} + 3\vec{a_y} - \vec{a_z}$. Find the dot product & Angle b/w them.

Given :-

$$\vec{A} = 3\vec{a_x} - \vec{a_y} + 8\vec{a_z}$$

$$\vec{B} = 2\vec{a_x} + 3\vec{a_y} - \vec{a_z}$$

$$\vec{A}\cdot\vec{B} = (3\vec{a_x} - \vec{a_y} + 8\vec{a_z})\cdot(2\vec{a_x} + 3\vec{a_y} - \vec{a_z})$$

$$= (3)(2) + (-1)(3) + (8)(-1)$$

$$= 6 - 2 - 8$$

$$\vec{A}\cdot\vec{B} = -5$$

$$|\vec{A}| = \sqrt{(3)^2 + (-1)^2 + (8)^2} = \sqrt{74}$$

$$|\vec{B}| = \sqrt{(2)^2 + 3^2 + (-1)^2} = \sqrt{14}$$

$$\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right] = \cos^{-1}\left[\frac{-5}{\sqrt{74}\sqrt{14}}\right]$$

$$\theta = 98.92^\circ$$

Q 2. Given two Vectors $\vec{A} = \vec{a_x} - 5\vec{a_y} + 2\vec{a_z}$ and $\vec{B} = 3\vec{a_x} - \vec{a_y} - 4\vec{a_z}$. Find the Cross product and unit normal Vector.

Given :

$$\vec{A} = \vec{a_x} - 5\vec{a_y} + 2\vec{a_z}$$

$$\vec{B} = 3\vec{a_x} - \vec{a_y} - 4\vec{a_z}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 1 & -5 & 2 \\ 3 & -1 & -4 \end{vmatrix}$$

$$= \vec{a_x}(20+2) - \vec{a_y}(-4-6) + \vec{a_z}(-1+15)$$

$$\boxed{\vec{A} \times \vec{B} = 22\vec{a_x} + 10\vec{a_y} + 14\vec{a_z}}$$

$$|\vec{A} \times \vec{B}| = \sqrt{484 + 100 + 196} = \sqrt{780}$$

$\vec{a_n} \rightarrow$ unit Vector

$$\vec{a_n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$= \frac{1}{\sqrt{780}} (22\vec{a_x} + 10\vec{a_y} + 14\vec{a_z})$$

Q3. Give the cylindrical coordinates of the point whose Cartesian coordinates are

$x = 2, \; y = 4$ & z or $Z = 5$ units.

Sol: Given $x = 2, y = 4, z = 5$

$$\rho = \sqrt{x^2+y^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\phi = \tan^{-1}(y/x) = \tan^{-1}(4/2) = 53.13^\circ$$

$$Z = Z = 5$$

The cylindrical coordinates are

$$(\rho, \phi, z) = (5, 53.13, 5)$$

Q4. Give the Cartesian coordinates of the point whose cylindrical coordinates are

$\rho = 2, \; \phi = 45^\circ$ and $Z = -1$

Sol.:

$x = \rho \cos\phi \qquad y = \rho \sin\phi \qquad Z = Z$

$x = 2\cos 45 \qquad y = 2\sin 45 \qquad Z = -1$

$x = 0.707 \qquad y = 0.702 \qquad Z = -1$

Cartesian coordinates are $(x, y, z) =$

$(0.707, 0.707, -1)$

Q 5. Give the spherical coordinates of the point whose Cartesian coordinates are

$x = -1$, $y = 3$ and $z = 5$

Sol:

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{1 + 9 + 25}$$

$$= \sqrt{35}$$ //

$$\phi = \tan^{-1}(y/x)$$

$$= \tan^{-1}(3/-1)$$

$$= \tan^{-1}(-3)$$

$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

$$= \cos^{-1}\left(\frac{5}{\sqrt{35}}\right)$$ //.

Spherical coordinates are

(r, ϕ, θ).

Q6. Given points $A(x=2, y=3, z=-1)$ $B(\rho=4, \phi=50^\circ, z=2)$. Find the distance from A to B.

Given. $A(x=2, y=3, z=-1)$

$B(\rho=4, \phi=-50^\circ, z=2)$

$x = \rho\cos\phi$ $\qquad$ $y = \rho\sin\phi$

$= 4\cos(-50^\circ)$ $\qquad$ $= 4\sin(-50^\circ)$

$x = 2.571$ $\qquad$ $y = (-3.064)$

$$\boxed{z = z = 2}$$

So distance from A to B.

$$= \sqrt{(2.571-2)^2 + (-3.064-3)^2 + (2+1)^2}$$

$$= \sqrt{46.098}$$

$d = 6.789$

Q7. The Scalar fields are given by,

(i) $V = 20e^{-x} \sin(\pi/6)y$.

(ii) $V = 25\rho \sin\phi$

(iii) $V = \frac{40 \cos\theta}{r^2}$

Find its gradient at the point (P(0,1,1) for Cartesian, $P(\sqrt{2}, \pi/2, 5)$ for Cylindrical, $P(3, 60^\circ, 30^\circ)$ for spherical.

Given :

$$V = 20e^{-x} \sin(\pi/6)y$$

$$\nabla V = \frac{\partial V}{\partial x}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \frac{\partial V}{\partial z}\vec{a_z}$$

$$\frac{\partial V}{\partial x} = -20e^{-x} \sin(\pi/6)y$$

$$\frac{\partial V}{\partial y} = 20e^{-x}\, \pi/6 \cos(\pi/6)\, y.$$

$$\frac{\partial V}{\partial z} = 0.$$

$$\nabla V\Big|_{P(0,1,1)} = \left[-20 \sin \pi/6\right]\vec{a_x} + \left[20 \times \pi/6 \times \cos \pi/6\right]\vec{a_y}$$

$$\nabla V = -10\vec{a_x} + 9.07\vec{a_y} //.$$

For cylindrical:-

$$\frac{\partial V}{\partial \rho} = 25 \sin\phi$$

$$\frac{\partial V}{\partial \phi} = 25 \rho \cos\phi$$

$$\frac{\partial V}{\partial z} = 0.$$

$$\nabla V = [25 \sin\phi]\vec{a_\rho} + \frac{1}{\cancel{\rho}}[25\cancel{\rho}\cos\phi]\vec{a_\phi}$$

$$\nabla V\Big|_{(\sqrt{2}, \pi/2, 5)}$$

$$= \left[25 \sin \pi/2\right]\vec{a_\rho} + \left[25 \cos \pi/2\right]\vec{a_\phi}$$

$$\nabla V\Big|_{(\sqrt{2}, \pi/2, 5)} = 25\vec{a_\rho}$$

For spherical

$$\nabla V = \frac{\partial V}{\partial r}\vec{a_r} + 1/r \frac{\partial V}{\partial \theta}\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{a_\phi}$$

$$\frac{\partial V}{\partial r} = \frac{-80 \cos\theta}{r^3}.$$

$$\frac{\partial V}{\partial \phi} = 0.$$

$$\frac{\partial V}{\partial \theta} = -\frac{40 \sin\theta}{r^2}$$

$$\nabla V \Big|_{(3, 60^\circ, 30^\circ)} = \left[\frac{-80\cos 60^\circ}{27}\right]\vec{a}_r - \left[\frac{40\sin 60^\circ}{27}\right]\vec{a}_\theta$$

$$\boxed{\nabla V = -1.48\,\vec{a}_r - 1.28\,\vec{a}_\theta}.$$

Q.8. Determine the divergence of the Vector fields.

(i) $\vec{P} = x^2yz\,\vec{a}_x + xz\,\vec{a}_z$

(ii) $\vec{Q} = \rho\sin\phi\,\vec{a}_\rho + \rho^2 z\,\vec{a}_\phi + z\cos\phi\,\vec{a}_z$

(iii) $\vec{T} = \frac{1}{r^2}\cos\theta\,\vec{a}_r + r\sin\theta\cos\phi\,\vec{a}_\theta$
$\cos\theta\,\vec{a}_\phi$

Solution:-

Cartesian

$$\nabla\cdot\vec{P} = \frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z}$$

$P_x = x^2yz, \quad P_y = 0, \quad P_z = xz.$

$$\nabla\cdot\vec{P} = \frac{\partial}{\partial x}(x^2yz) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(xz)$$

$$= 2xyz + x$$

For Cylindrical :-

$$\nabla\cdot\vec{Q} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho Q_\rho) + \frac{1}{\rho}\frac{\partial Q_\phi}{\partial\phi} + \frac{\partial Q_z}{\partial z}$$

Given

$$Q_\rho = \rho\sin\phi \quad , \quad Q_\phi = \rho^2 z$$

$$Q_z = z\cos\phi .$$

$$\nabla\cdot\vec{Q} = \frac{1}{\rho}(2\rho\sin\phi) + \frac{1}{\rho}(0) + \cos\phi .$$

$$\boxed{\nabla\cdot\vec{Q} = 2\sin\phi + \cos\phi}$$

For spherical :-

$$\nabla\cdot\vec{T} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(T_\theta\sin\theta)$$

$$+ \frac{1}{r\sin\theta}\frac{\partial T_\phi}{\partial\phi} .$$

$$T_r = \frac{1}{r^2}\cos\theta , \quad T_\theta = r\sin\theta\cos\phi , \quad T_\phi = \cos\theta$$

$$\nabla\cdot\vec{T} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\times\frac{1}{r^2}\cos\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\Big[$$

$$r\sin^2\theta\cos\phi\Big] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}[\cos\theta$$

$$= \frac{\cos\phi}{\sin\theta}\left(\frac{2\sin 2\theta}{2}\right) = \frac{\cos\phi}{\sin\theta}\times 2\sin\theta\cos\theta$$

$$\nabla\cdot\vec{T} = 2\cos\theta\cos\phi .$$

Q9. The Vector field $\vec{D} = \frac{5r^2}{4}\vec{a_r}$ is given in Spherical Coordinate System. Evaluate both sides of divergence theorem for the volume enclosed between

(i) $r = 1$ and $r = 2$

(ii) $\theta = 0$ to $\pi/4$ and $r = 4$.

Sol:

Case(i) :- $r = 1$ & $r = 2$.

By divergence theorem

$$\iiint_V (\nabla\cdot\vec{D})\,dv = \iint_S \vec{D}\cdot\vec{ds}.$$

LHS

$$\iiint_V (\nabla\cdot\vec{D})\,dv.$$

$$\nabla\cdot\vec{D} = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \times \frac{5r^2}{4}\right)$$

$$= \frac{1}{r^2} \times \frac{5}{4} \times 4r^3.$$

$$\nabla\cdot\vec{D} = 5r.$$

$$\iiint_V (\nabla\cdot\vec{D})\,dv = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=1}^{r=2} 5r \times r^2 \sin\theta\, dr\, d\theta\, d\phi.$$

$$= 5 \int_{0}^{2\pi}\int_{0}^{\pi}\int_{r=1}^{2} r^3 \sin\theta \, dr\, d\theta\, d\phi .$$

$$= 5 \int_{0}^{2\pi}\int_{0}^{\pi}\int_{r=1}^{2} \left[\frac{r^4}{4}\right]_1^2 \sin\theta \, d\theta\, d\phi .$$

$$= 5 \frac{(16-1)}{4} \int_{0}^{2\pi} \left[-\cos\theta\right]_0^{\pi} d\phi .$$

$$= \frac{75}{4} \int_{0}^{2\pi} (1+1)\, d\phi .$$

$$= \frac{75}{2} \left[\phi\right]_0^{2\pi} = \frac{75}{2} \times 2\pi = 75\pi \quad (1)$$

RHS: $\oint \vec{D} \cdot \vec{ds}$.

$$\iint \vec{D}\cdot\vec{ds} = \iint_{r=2} \vec{D}\cdot ds_r \vec{a_r} - \iint_{r=1} \vec{D}\cdot ds_r \vec{a_r}$$

$$= \iint_{r=2} \frac{5r^4}{4} \sin\theta \, d\theta\, d\phi - \iint_{r=1} \frac{5r^4}{4} \sin\theta \, d\theta\, d\phi$$

$$= \frac{5(2)^4}{4} \int_{0}^{2\pi}\int_{0}^{\pi} \sin\theta \, d\theta\, d\phi - \frac{5(1)^4}{4} \int_{0}^{2\pi}\int_{0}^{\pi} \sin\theta\, d\theta\, d\phi .$$

$$= 20 \int_0^{2\pi} \left[-\cos\theta\right]_0^{\pi} d\phi - \frac{5}{4} \int_0^{2\pi} \left[-\cos\theta\right]_0^{\pi} d\phi$$

$$= 40 \left[\phi\right]_0^{2\pi} - \frac{5}{2} \left[\phi\right]_0^{2\pi}$$

$$= 80\pi - 5\pi = 75\pi \quad \text{--- (2).}$$

from equation (1) & (2)

$$\iiint_V \vec{D} \cdot \vec{dv} = \iint \vec{D} \cdot ds.$$

Case(ii) $\theta = 0$ to $\pi/4$ & $r = 4$.

LHS: $\iiint_V \nabla \cdot \vec{D} \, dv$

$$= \int_{\phi=0}^{2\pi} \int_0^{\pi/4} \int_{r=0}^{4} 5r^3 \sin\theta \, dr \, d\theta \, d\phi.$$

$$= \frac{5(4)^4}{4} \int_0^{2\pi} \left[-\cos\theta\right]_0^{\pi/4} d\phi$$

$$= 320 \int_0^{2\pi} \left[-\cos\pi/4 + \cos 0\right] d\theta.$$

$= 320 \times 0.293 \left[\phi \right]_0^{2\pi}$

$= 320 \times 0.293 \times 2\pi$

$\iiint_v \nabla \cdot \vec{D} dv = 187.45\pi$ —— (1).

<u>RHS</u> $\iint_S \vec{D} \cdot \vec{ds}$

$= \iint_{r=4} \vec{D} \cdot \vec{ds_r}\, \vec{a_r} + \iint \vec{D} \cdot ds_\theta \vec{a_\theta}$

$= \iint_{r=4} \frac{5r^4}{4} \sin\theta \, d\theta \, d\phi + 0.$

$= \frac{5(4)^4}{4} \int_0^{2\pi} \int_0^{\pi/4} \sin\theta \, d\theta \, d\phi$

$= 320 \int_0^{2\pi} \left[-\cos\theta \right]_0^{\pi/4} d\phi.$

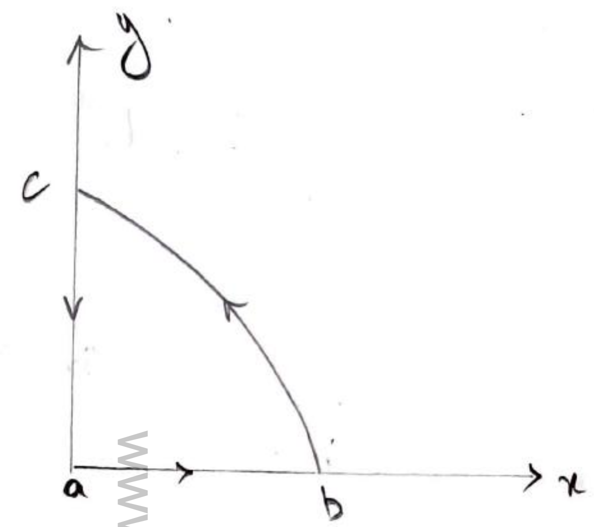
$= 320 \times 0.293 \times 2\pi$

$\iint \vec{D} \cdot \vec{ds} = 187.45\pi$ —— (2).

From eq (1) & (2)

Divergence theorem is proved.

Q10. Given $\vec{A} = 2\rho\cos\phi\,\vec{a}_\rho + \rho^2\,\vec{a}_\phi$ in Cylindrical coordinate System for contour Shown in figure. Verify using stokes theorem.



Given $\vec{A} = 2\rho\cos\phi\,\vec{a}_\rho + \rho^2\,\vec{a}_\phi$

By stokes Theorem

$$\oint_L \vec{A}\cdot\vec{dl} = \iint_S (\nabla\times\vec{A})\cdot\vec{ds}.$$

RHS $\iint(\nabla\times A)\cdot ds.$

$$\nabla\times\vec{A} = \frac{1}{\rho}\begin{vmatrix} \vec{a}_\rho & \rho\,\vec{a}_\phi & \vec{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \frac{\partial}{\partial z} \\ 2\rho\cos\phi & \rho^2 & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\vec{a}_\rho(0-0) - \rho\,\vec{a}_\phi(0-0) + \vec{a}_z(2\rho + 2\rho\sin\phi)\right]$$

$$= \frac{1}{\rho} \times \rho \left[2 + 2 \sin\phi \right] \vec{a_z}$$

$$\nabla \times \vec{A} = (2 + 2\sin\phi)\, \vec{a_z}$$

$$\therefore \vec{ds} = \rho\, d\rho\, d\phi\, \vec{a_z}$$

$$(\nabla \times \vec{A}) \cdot \vec{ds} = 2(1 + \sin\phi)\, \rho\, d\rho\, d\phi$$

$$\iint_S (\nabla \times \vec{A})\, ds = \int_0^{\pi/2} \int_0^1 2(1 + \sin\phi)\, \rho\, d\rho\, d\phi$$

$$= 2 \int_0^{\pi/2} (1 + \sin\phi) \left(\frac{\rho^2}{2} \right)_0^1 d\phi.$$

$$= 2 \times \frac{1}{2} \left[\phi - \cos\phi \right]_0^{\pi/2}$$

$$= \left(\pi/2 - \cos \pi/2 \right) - \left(0 - \cos 0 \right)$$

$$= \left(\pi/2 - 0 \right) - (0 - 1)$$

$$= 1 + \pi/2 \quad \text{——— (1)}$$

LHS

$$\oint \vec{A} \cdot \vec{dl} = \int_{AB} \vec{A} \cdot \vec{dL} + \int_{BC} \vec{A} \cdot \vec{dL} + \int_{CA} \vec{A} \cdot \vec{dL}.$$

For path AB:-

$$\int_{AB} \vec{A}\cdot\vec{dl} = \int_{\phi=0} 2\rho\cos\phi\, d\rho.$$

$$= 2\left[\frac{\rho^2}{2}\right)_0^1 = 2\left(\frac{1}{2}\right) = 1$$

— Ⓐ

For path BC

$$\int_{BC} \vec{A}\cdot dl = \int_{\rho=1} \rho^3 d\phi = \int_0^{\pi/2} d\phi = \pi/2.$$

— Ⓑ

For path CA:-

$$\int_{CA} \vec{A}\cdot dl = \int_{\phi=\pi/2} 2\rho\cos\phi\, d\rho$$

$$= \int_0^1 0 = 0 \quad \text{— Ⓒ}.$$

From Ⓐ Ⓑ, Ⓒ

$$\oint \vec{A}\cdot dL = 1 + \pi/2 \quad \text{— ②}.$$

From eq ① & ②

Stokes theorem is verified.

Q11. Find the force of interaction between two charges spaced 10cm apart in Vaccum. The charges are 4×10^{-8} C and 6×10^{-5} C. If the Same charges separated by the Same distance in kerosene ($\varepsilon_r = 2$). what is the force of interaction between them?

Sol:

$Q_1 = 4 \times 10^{-8}$ C $\qquad \varepsilon_r = 1$

$Q_2 = 6 \times 10^{-5}$ C $\qquad \varepsilon_r = 2$

$r = 10cm = 10 \times 10^{-2}$ m $= 0.1$ m

For Vaccum

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r r^2} = \frac{4\times10^{-8} \times 6\times10^{-5}}{4\times3.14\times8.54\times10^{-12}\times(0.1)^2}$$

$$\boxed{F = 2.15N}$$

for kerosene

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r r^2} = \frac{4\times10^{-8}\times6\times10^{-5}}{4\times3.14\times8.854\times10^{-12}\times2\times(0.1)^2}$$

$$\boxed{F = 1.075N}$$

Q:12 A point charge of $10\mu C$ is located at $(1,2,3)$ and another point charge of $-3\mu C$ is located at $(3,0,2)$ in Vaccum. Find the force b/w them?

Sol:

$$\cancel{Q_2} = Q_1 = 10\mu C = 10\times10^{-6}\,C$$

$$Q_2 = -3\mu C = -3\times10^{-6}\,C$$

$$\vec{r}_1 = \vec{a}_x + 2\vec{a}_y + 3\vec{a}_z$$

$$\vec{r}_2 = 3\vec{a}_x + 2\vec{a}_z$$

$$\varepsilon_r = 1$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = 2\vec{a}_x - 2\vec{a}_y - \vec{a}_z$$

$$r_{12} = |\vec{r}_{12}| = \sqrt{4+4+1} = 3.$$

$$\vec{u}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{2\vec{a}_x - 2\vec{a}_y - \vec{a}_z}{3}$$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r r_{12}^2}\vec{u}_{12}$$

$$= \frac{(10\times10^{-6})(-3\times10^{-6})}{4\times3.14\times8.854\times10^{-12}\times1\,(3)^2}\times\vec{u}_{12}$$

$$\vec{F}_{12} = -0.02\vec{a}_x + 0.02\vec{a}_y - 0.01\vec{a}_z$$

$$F_{12} = |\vec{F}_{12}| = \underline{\underline{0.03N}}$$