BM3352 - ELECTRIC CIRCUIT ANALYSIS.

UNIT - 1 - BASIC CIRCUITS ANALYSIS

Electric circuit Analysis :-

⇒ Electric circuit Analysis is the process of studying and understanding the behaviour of electric circuits.

→ It Involves analyzing various electrical components such as resistor, capacitor, Inductor, voltage / current sources & how they interact within a circuit.

Electric circuit :-

⇒ Electric circuit is a closed loop or path through which electric current flows.

⇒ It consists of various components connected together by wires on a circuit board.

⇒ Electric circuits are designed to allow the controlled flow of electric charge from one point to another, enabling the operation of electrical & electronic devices.

Basic components of Electric circuits :-

⇒ In electrical circuits basic elements are classified into two types.

1. Active components.
2. Passive components.

Active components :-

⇒ These are the components that require an external power source to operate.

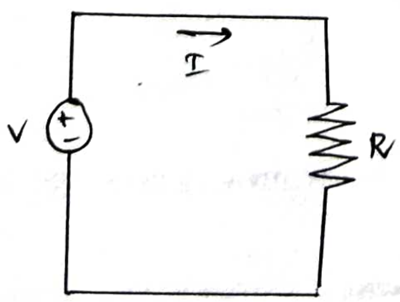
⇒ They can generate or amplify energy as well as control the flow of current / voltage.

E.g → Transistors, IC's, voltage and current sources.

1. Voltage source :-

⇒ It is a 2-terminal device which can maintain a fixed voltage.

⇒ It is an active element that provides specified & constant voltage which is completely independent of any other circuit elements.

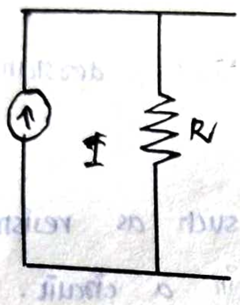


eg. power supply.

⇒ In voltage source, resistance is connected in series, voltage is connected in parallel.

2. current source :-

⇒ It delivers constant flow of current.



eg. Batteries, solar panel.

⇒ In current source, resistance is connected in parallel, Ammeter is connected in Series.

Passive elements :-

⇒ passive components are the components that do not require any external source for operation.

⇒ they are capable of storing energy in the form of voltage / current in the circuit.

E.g → Resistors, Inductors, capacitors.

1. Resistors :-

⇒ It is a passive electrical component that creates resistance in the flow of electric current.

⇒ In almost all electrical networks and electronic circuits they can be found.

⇒ It is measured in ohms (Ω).

Types of resistors :-

⇒ Fixed resistor

⇒ variable resistor

⇒ Thermistor

⇒ varistor

⇒ photoresistors

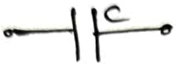
⇒ magneto resistors.

Examples :- Laptops & mobile chargers, Temperature sensors.

2. Capacitors :-

⇒ It stores and release electrical energy in the form of electric field.

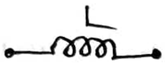
⇒ They are often used for energy storage & filtering.



⇒ Two parallel plates seperated by a insulating material.

3. Inductors :-

⇒ It stores and release electrical energy in the form of magnetic field.

 or 

⇒ They are used in applications involving magnetic coupling, energy storage and filtering.

BASIC TERMS & DEFINITIONS

1. charge :-

⇒ Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.

⇒ Two types of charges — +ve, —ve.

⇒ unit of charge is coulomb. 'c'.

2. Current :-

⇒ Flow of electric charge is called as current.

⇒ unit of current is 'Ampere'.

$$I = Q/t$$

⇒ represented by I or i.

3. voltage :-

⇒ It is a pressure that pushes electricity

⇒ It is a pressure from an electrical circuits power source that pushes charged electrons through a conducting loop, enabling them to do the work.

⇒ unit of voltage is volts.

4. Energy :-

⇒ It is the ability to do work, which is the ability to exert a force causing displacement of an object.

⇒ unit is Joule (J).

⇒ Energy can neither be created nor be destroyed.

5. Power :-

⇒ It is defined as rate of doing work.

⇒ It is equivalent to an amount of energy consumed per unit time

$$P = E/t.$$

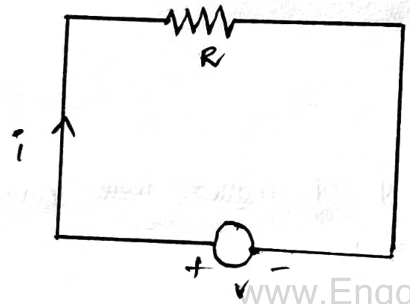
OHM'S LAW :-

⇒ ohm's law states that at constant temperature, the current passing through conductor is directly proportional to the potential difference across the two points and inversely proportional to the resistance between them.

$V = IR$, where V → voltage

I → current

R → Resistance



Real time applications of ohm's law :-

⇒ Electric heater

⇒ Electric kettles.

Advantages of ohm's law :-

⇒ To determine the value of resistors or current in a circuit or in measuring voltage.

⇒ It also helps us to describe how current flows through materials such as electrical wires.

Disadvantages of ohm's law :-

⇒ ohm's law is applicable when the temperature of the conductor is constant.

⇒ It is not applicable to unilateral networks.

KIRCHOFF'S LAW :-

⇒ Ohm's law is not sufficient to analyze the circuits and also it is not applicable for complex circuits. Hence we go for Kirchoff's law.

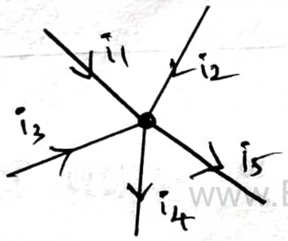
⇒ Kirchoff's laws are basic analytical tools to obtain solutions of I & V in the circuit.

⇒ Two types of Kirchoff's laws.

1. Kirchoff's current law (Point law / first law)
2. Kirchoff's voltage law (Mesh law / second law)

Kirchoff's current Law (KCL) :-

⇒ It states that at any node in a circuit, "The algebraic sum of current entering the node and leaving the node at any instant of time must be equal to zero."



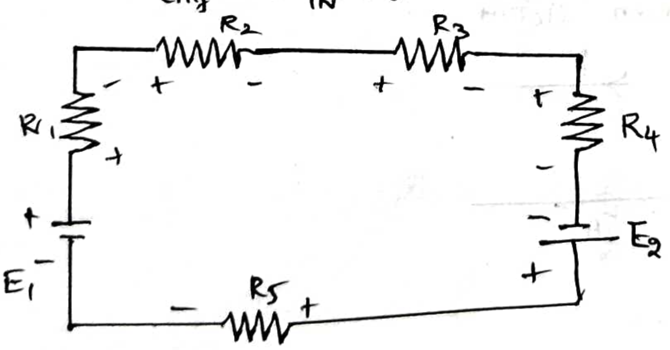
$$i_1 + i_2 + i_3 - i_4 - i_5 = 0$$

$$i_1 + i_2 + i_3 = i_4 + i_5$$

Kirchoff's voltage Law (KVL) :-

⇒ The algebraic sum of electromotive force and algebraic sum of voltages across impedance in any closed electrical circuit is equal to zero.

$$\Sigma_{emf} + \Sigma_{IR} = 0.$$



(- +) → Rise in voltage (taken as +ve)

(+ -) → fall in voltage (taken as -ve)

$$E_1 - IR_1 - IR_2 - IR_3 - IR_4 + E_2 - IR_5 = 0$$

$$E_1 + E_2 = IR_1 + IR_2 + IR_3 + IR_4 + IR_5$$

Uses of Kirchoff's law :-

⇒ used to determine how much current is flowing & how much voltage is being lost in various places of circuit.

⇒ It helps to detect current flow direction in various circuit loops.

PROBLEM 1 :-

What is the voltage drop across 640 Ω resistance when the current flowing through in 30 mA.

Solution :-

$V = IR$.

Given, $R = 640\,\Omega$, $I = 30\,mA$.

$V = (30 \times 10^{-3}) \times 640$

$\boxed{V = 19.2\,V}$

PROBLEM 2 :-

What is the resistance of a lamp if 10 V battery results in 200 mA current flow in it ?

Solution :-

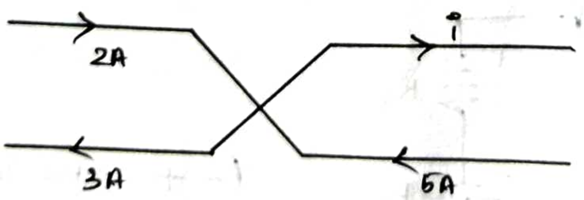
Given :- $V = 10\,V$, $I = 200\,mA$.

$$R = \frac{V}{I} = \frac{10}{200 \times 10^{-3}} = 50\,\Omega$$

$\boxed{R = 50\,\Omega}$

PROBLEM 3 :-

(a) Find the current in the given circuit.



Solution :-

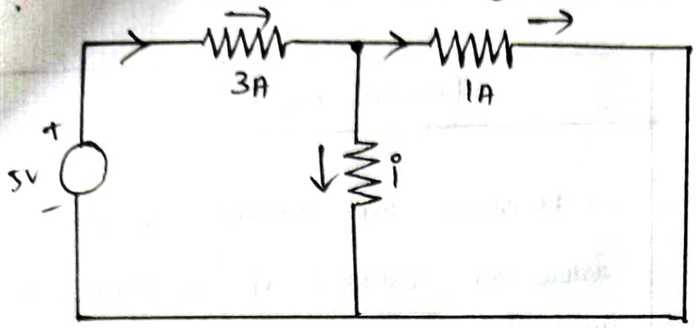
Apply KCL,

$2A + 5A - 3A - i = 0 \rightarrow 2A + 5A = 3A + i$

$7A = 3A + i$

$\boxed{i = 4A}$

(b) Find the current in the given circuit.



Solution :-

Apply KCL,

$$3A - i - 1A = 0$$

$$3A = i + 1A$$

$$\boxed{i = 2A}$$

PROBLEM 4 :-

A resistor with a current of 3A through it converts 500 J of electrical energy into heat energy in 12 sec. What is the voltage across the resistor ?

Solution :-

Given, $I = 3A$, $E = 500J$, $t = 12$ sec.

$$P = E/t$$

$$E = P.t$$

$$500 = V.I.t \quad (\because P = VI)$$

$$500 = V \times 3 \times 12$$

$$V = \frac{500}{3 \times 12}$$

$$\boxed{V = 13.88\,V}$$

PROBLEM 5 :-

A 5Ω resistor has a voltage rating of 100 V. What is its power rating ?

Solution :-

Given, $R = 5\Omega$, $V = 100V$.

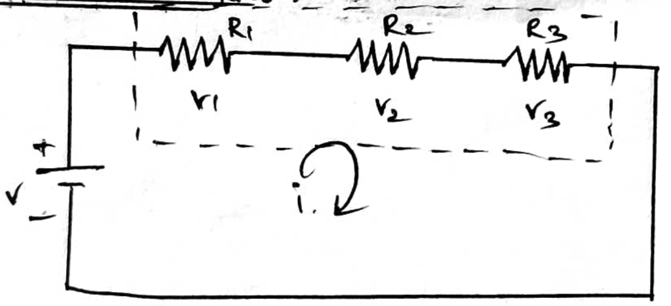
$$P = \frac{V^2}{R} = \frac{(100)^2}{5} = 2000$$

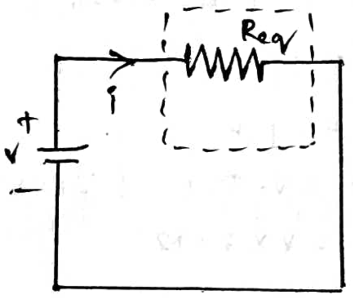
$$\boxed{P = 2000 \text{ watts}}$$

RESISTORS IN SERIES & PARALLEL CIRCUIT :-

Series circuit	Parallel circuit
⇒ Resistance are connected in series to increase the resistance of the circuit to reduce current according to the load.	⇒ Resistance are connected in parallel to reduce the resistance of the circuit, also it is used to obtain different currents from a single current.
⇒ current is same through all the elements	⇒ voltage is same across each elements
⇒ voltage is distributed & proportional to resistance, $V=IR$	⇒ current is divided & inversely proportional to resistance, $I=V/R$.
⇒ series combination of resistance is also used as a voltage divider circuit.	⇒ parallel combination of resistance is also used as current divider circuit.
⇒ Total/equivalent resistance is sum of individual resistances, $R_{eq}=R_1+R_2+R_3+\dots+R_N$	⇒ Total/equivalent resistance is sum of reciprocal of individual resistance $\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\dots+\frac{1}{R_N}$

RESISTORS IN SERIES :-





(Equivalent circuit)

W.K.T

$$V=IR$$

$$V=V_1+V_2+V_3$$

$$V_1=IR_1, \quad V_2=IR_2, \quad V_3=IR_3$$

$$V=IR_1+IR_2+IR_3$$

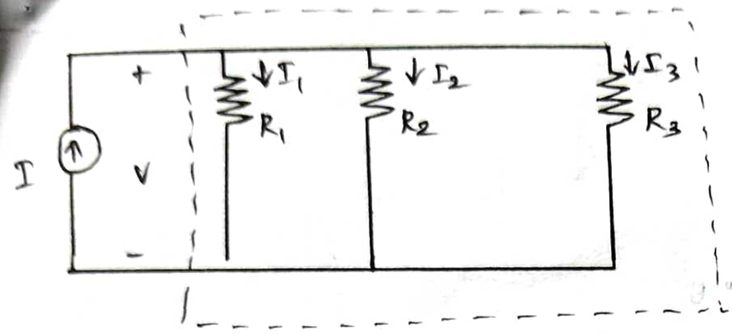
$$=I(R_1+R_2+R_3)$$

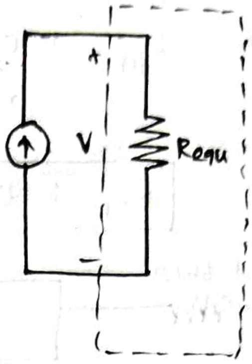
$$\boxed{V=I\,R_{equ}} \qquad (\because R_{eq}=R_1+R_2+R_3)$$

similarly for N. no. of. series resistance

$$\boxed{R_{eq}=R_1+R_2+R_3+\dots+R_N}$$

RESISTORS IN PARALLEL





W.K.T.

$$V = IR$$

$$V = I_1R_1 = I_2R_2 = I_3R_3$$

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$I/V = 1/R_1 + 1/R_2 + 1/R_3$$

$$\frac{1}{R_{equ}} = 1/R_1 + 1/R_2 + 1/R_3$$

$$\frac{1}{R_{eq}} = \frac{R_2R_3 + R_1R_3 + R_1R_2}{R_1R_2R_3}$$

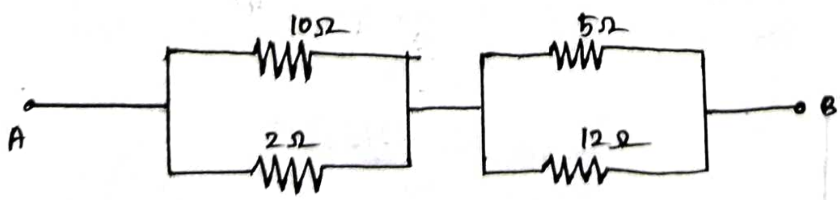
$$R_{eq} = \frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}$$

Similarly for n. no. of parallel resistance,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

PROBLEM 1 :-

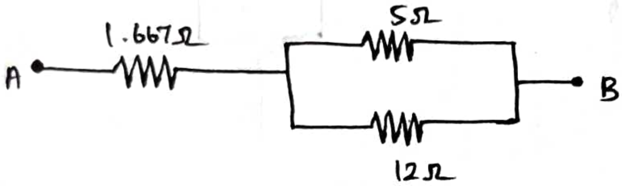
Determine the equivalent resistance b/w A & B.



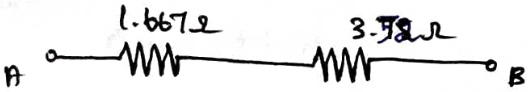
Solution :-

$10\Omega \parallel 2\Omega$, $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 2}{10 + 2} = \frac{20}{12}$

$$\boxed{R_{eq} = 1.667\,\Omega}$$



similarly $5\Omega \parallel 12\Omega$, $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 12}{5 + 12} = \frac{60}{17} = 3.52\,\Omega$



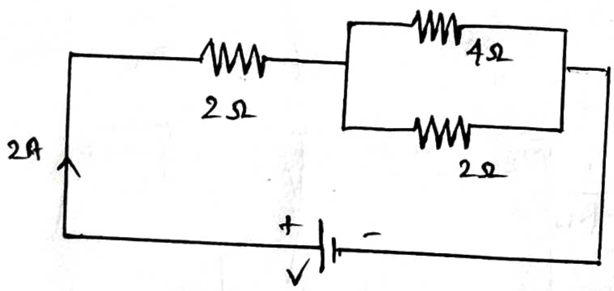
Both resistors are in series, hence

$$R_{eq} = R_1 + R_2$$
$$= 1.667 + 3.52$$
$$\boxed{R_{eq} = 5.187\,\Omega}$$

$R_{eq} = 5.187\,\Omega$

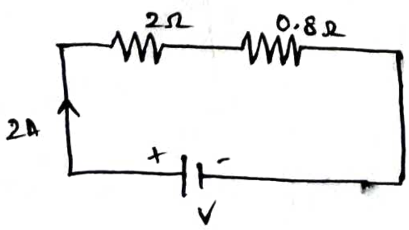
PROBLEM 2 :-

Find voltage V_1 If current in 2Ω is $2A$.

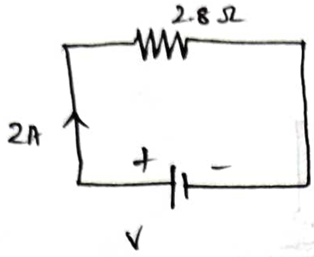


Solution :-

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{4 \times 1}{4 + 1} = 4/5 = 0.8\,\Omega$$



$R_{eq} = 2 + 0.8 = 2.8\,\Omega$



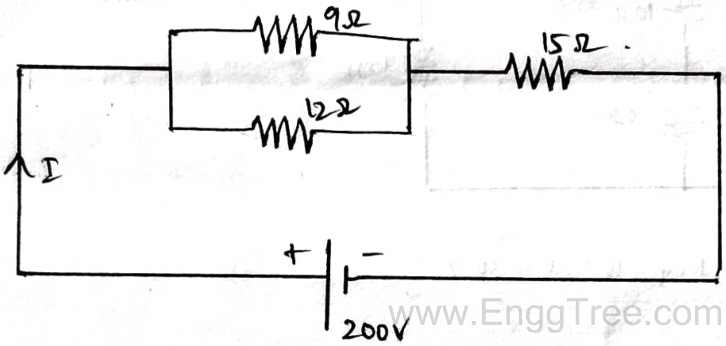
$V = IR$

$= 2 \times 2.8$

$\boxed{V = 5.6 \text{ volts}}$

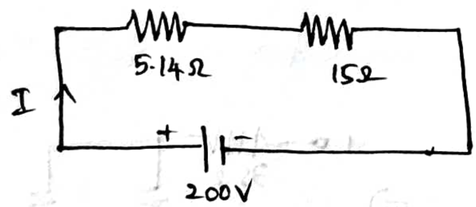
PROBLEM 3 :-

Find the current in the circuit.

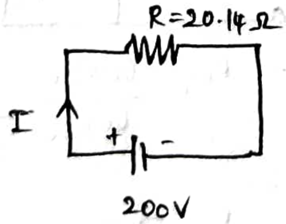


Solution :-

$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{9 \times 12}{9+12} = \frac{108}{21} = 5.14\,\Omega$



$R_{eq} = 5.14 + 15 = 20.14\,\Omega$

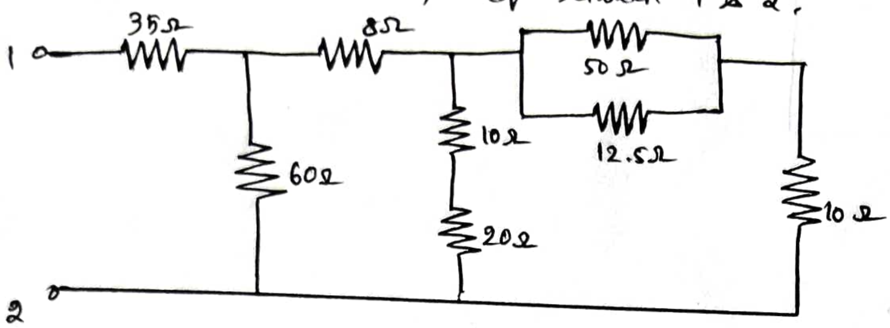


$I = V/R = 200/20.14 = 9.93$

$\boxed{I = 9.93\,A}$

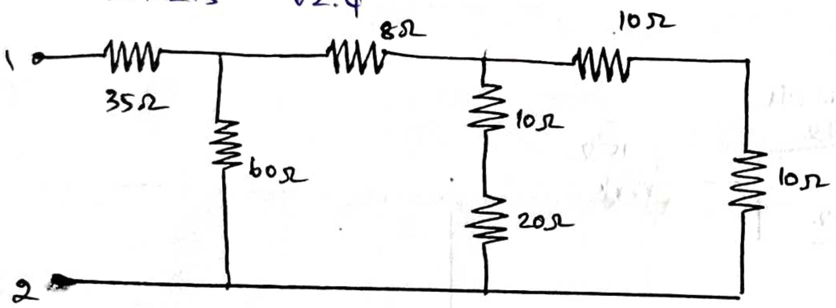
PROBLEM 4 :-

Find Equivalent resistance, R_{eq} between 1 & 2.

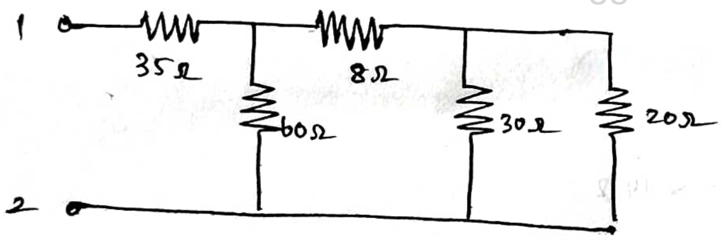


Solution :-

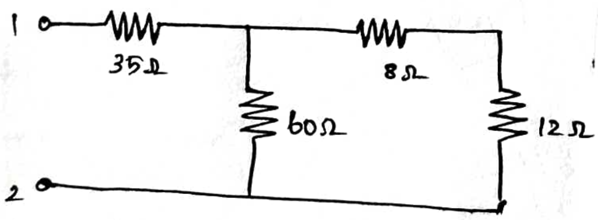
$$R_{eq} = \frac{50 \times 12.5}{50 + 12.5} = \frac{625}{62.4} = 10\,\Omega$$



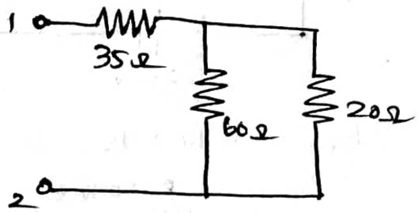
$R_{eq} = 10 + 10 = 20\,\Omega$ $\qquad$ $R_{eq} = 10 + 20 = 30\,\Omega$



$$R_{eq} = \frac{30 \times 20}{30 + 20} = \frac{600}{50} = 12\,\Omega$$

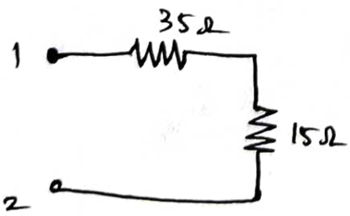


$\Rightarrow$

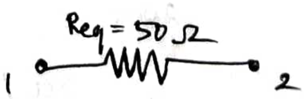


$R_{eq} = 8 + 12 = 20\,\Omega$

$$R_{eq} = \frac{60 \times 20}{60 + 20} = \frac{1200}{80} = 15\,\Omega$$



$\Rightarrow$



$R_{eq} = 35 + 15 = 50\,\Omega$

MESH ANALYSIS & NODAL ANALYSIS IN A.C & D.C CIRCUITS :-

Node :-

⇒ It is a point in a network where 2 or more circuit elements are connected.

loop :-

⇒ Any closed path in the circuit can be called as loop.

Mesh :-

⇒ If loop in a circuit does not enclose any other loop inside it, then that loop can be called as mesh.

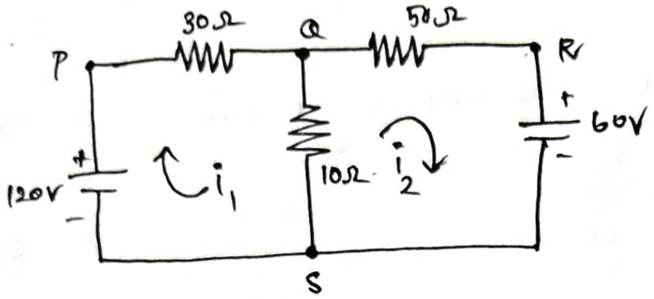
A.C & D.C circuits

⇒ Electric current flows in 2 ways as AC or DC. The main difference lies in the direction in which the electrons flow. In DC electrons flow steadily in single direction, while in A.C circuit, electrons keep switching directions going forward & backward.

A.C circuits	D.C circuits
⇒ Transferred over long distance	⇒ cannot be transferred over very long distance.
⇒ Electrons in AC keep changing their directions.	⇒ Electrons move only in one direction.
⇒ less expensive	⇒ more expensive
⇒ loss of energy during transmission is less	⇒ loss of energy during transmission is high.
⇒ used in household & industrial supply	⇒ used in all electronic circuits and gadgets.

Problem 1 :-

Using Mesh analysis, find the current through 10 Ω resistance.



Solution :-

Step 1 :- Apply Kirchoff's voltage law to loop PQSP

$$30 i_1 + 10(i_1 - i_2) = 120$$

$$30 i_1 + 10 i_1 - 10 i_2 = 120$$

$$40 i_1 - 10 i_2 = 120$$

$\div 10$

$$\boxed{4 i_1 - i_2 = 12} \rightarrow (1)$$

Step 2 :- Apply Kirchoff's voltage law to loop QRSQ

$$50 i_2 + 10(i_2 - i_1) = -60$$

$$50 i_2 + 10 i_2 - 10 i_1 = -60$$

$$60 i_2 - 10 i_1 = -60$$

$\div 10$

$$6 i_2 - i_1 = -6$$

$$\boxed{-i_1 + 6 i_2 = -6} \rightarrow (2)$$

Step 3 :-

$$\Delta = \begin{vmatrix} 4 & -1 \\ -1 & 6 \end{vmatrix} = 24 - 1 = 23, \quad \Delta_1 = \begin{vmatrix} 12 & -1 \\ -6 & 6 \end{vmatrix} = 72 - 6 = 66, \quad \Delta_2 = \begin{vmatrix} 4 & 12 \\ -1 & -6 \end{vmatrix} = -24 + 12 = -12$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{66}{23} = 2.869$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{-12}{23} = -0.5217$$

Step 4 :-

Current flowing through 10Ω resistances is $(i_1 - i_2)$

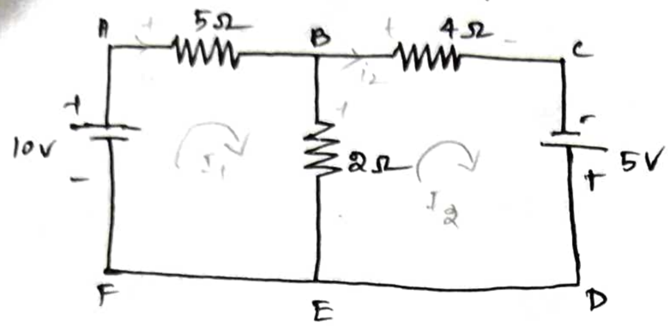
$$= 2.869 - (-0.5217)$$

$$= 2.869 + 0.5217$$

$$= 3.3907$$

PROBLEM :- 2

Find the current flowing through 2Ω resistor using mesh analysis method.



Solution :-

Step 1 :- Apply KVL to loop ABEFA,

$$5i_1 + 2(i_1 - i_2) = 10\,V$$

$$5i_1 + 2i_1 - 2i_2 = 10\,V$$

$$\boxed{7i_1 - 2i_2 = 10\,V} \rightarrow (1)$$

Step 2 :- Apply KVL to loop BCDEB

$$2(i_2 - i_1) + 4i_2 = 5$$

$$2i_2 - 2i_1 + 4i_2 = 5$$

$$\boxed{-2i_1 + 6i_2 = 5} \rightarrow (2)$$

Step 3 :-

$$\Delta = \begin{vmatrix} 7 & -2 \\ -2 & 6 \end{vmatrix} = 42 - 4 = 38 \qquad \Delta_1 = \begin{vmatrix} 10 & -2 \\ 5 & 6 \end{vmatrix} = 60 + 10 = 70 \qquad \Delta_2 = \begin{vmatrix} 7 & 10 \\ -2 & 5 \end{vmatrix} = 35 + 20 = 55$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{70}{38} = 1.842$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{55}{38} = 1.447$$

Step 4 :-

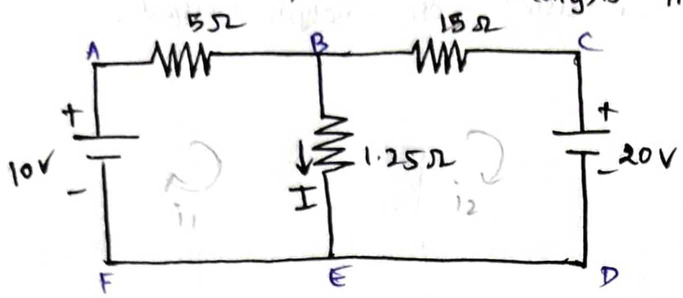
current flowing through 2Ω resistance is $(i_1 - i_2)$

$$1.842 - 1.447$$

$$= 0.395\,A.$$

PROBLEM :- 3

Find current I, by mesh analysis method.



solution :-

Step 1 :- Apply KVL to loop ABEFA

$$5i_1 + 1.25(i_1 - i_2) = 10$$

$$5i_1 + 1.25i_1 - 1.25i_2 = 10.$$

$$\boxed{6.25i_1 - 1.25i_2 = 10} \rightarrow ①$$

Step 2 :- Apply KVL to loop BCDEB

$$15i_2 + 1.25(i_2 - i_1) = -20$$

$$15i_2 + 1.25i_2 - 1.25i_1 = -20$$

$$16.25i_2 - 1.25i_1 = -20$$

$$\boxed{-1.25i_1 + 16.25i_2 = -20} \rightarrow ②$$

Step 3 :-

$$\Delta = \begin{vmatrix} 6.25 & -1.25 \\ -1.25 & 16.25 \end{vmatrix} = 101.56 - 1.562 = 99.99$$

$$\Delta_1 = \begin{vmatrix} 10 & -1.25 \\ -20 & 16.25 \end{vmatrix} = 162.5 - 25 = 137.5$$

$$\Delta_2 = \begin{vmatrix} 6.25 & 10 \\ -1.25 & -20 \end{vmatrix} = -125 + 12.5 = -112.5$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{137.5}{99.99} = 1.375$$

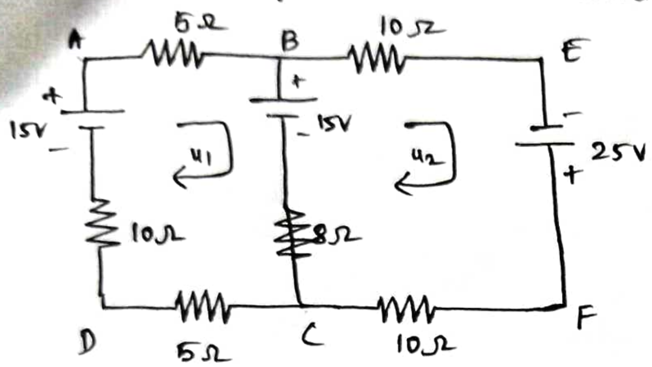
$$i_2 = \frac{\Delta_2}{\Delta} = \frac{-112.5}{\cancel{137.5}\ 99.99} = -1.125.$$

$$I = i_1 - i_2 = (1.375) - (-1.125)$$

$$\boxed{I = 2.5A.}$$

PROBLEM :- 4

using mesh analysis, find the current through 8Ω resistor.



solution :-

Step 1 :- Apply KVL to ABCDA

$$5u_1 + 8(u_1 - u_2) + 5u_1 + 10u_1 = -15 + 15$$

$$5u_1 + 8u_1 - 8u_2 + 5u_1 + 10u_1 = 0$$

$$\boxed{28u_1 - 8u_2 = 0} \rightarrow (1)$$

Step 2 :- Apply KVL to BEFCB

$$10u_2 + 10u_2 + 8(u_2 - u_1) = 25 + 15$$

$$10u_2 + 10u_2 + 8u_2 - 8u_1 = 40$$

$$28u_2 - 8u_1 = 40$$

$$-8u_1 + 28u_2 = 40$$

$\div 4$

$$\boxed{-2u_1 + 7u_2 = 10} \rightarrow (2)$$

Step 3 :- using cramer's rule

$$\Delta = \begin{vmatrix} 28 & -8 \\ -2 & 7 \end{vmatrix} = 196 - 16 = 180 \quad , \quad \Delta_1 = \begin{vmatrix} 0 & -8 \\ 10 & 7 \end{vmatrix} = 0 + 80 = 80 \quad , \quad \Delta_2 = \begin{vmatrix} 28 & 0 \\ -2 & 10 \end{vmatrix} = 280 - 0 = 280$$

$$u_1 = \frac{\Delta_1}{\Delta} = \frac{80}{180} = 0.444 \quad , \quad u_2 = \frac{\Delta_2}{\Delta} = \frac{280}{180} = 1.555$$

Step 4 :-

current through 8Ω resistor,

$$u_2 - u_1 \Rightarrow 1.555 - 0.444$$

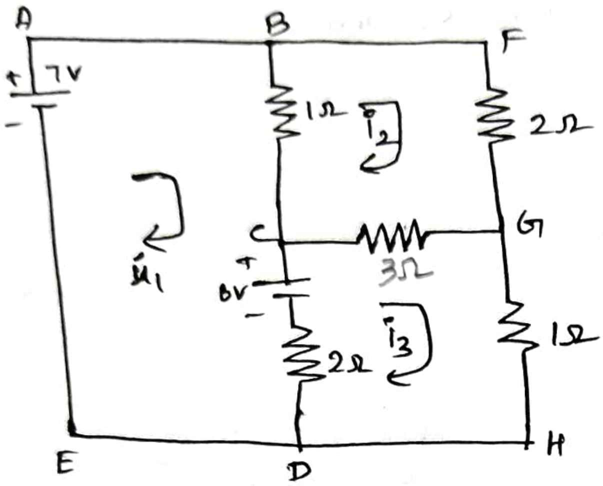
$$= 1.111\ A.$$

u_2 current straight away flow thrgh 8Ω

u_1 current 1st flow thrgh V source & then flow to resistor

PROBLEM 5 :-

using mesh analysis, find the Three mesh currents :-



Solution :-

Step1 :- Apply KVL to the loop ABCDEA

$$1(u_1-u_2)+2(u_1-u_3)=-6+7$$

$$u_1-u_2+2u_1-2u_3=1$$

$$\boxed{3u_1-u_2-2u_3=1} \rightarrow (1)$$

Step2 :- Apply KVL to loop BFGCB

$$2u_2+3(u_2-u_3)+1(u_2-u_1)=0$$

$$2u_2+3u_2-3u_3+u_2-u_1=0$$

$$\boxed{-u_1+6u_2-3u_3=0} \rightarrow (2)$$

Step3 :- Apply KVL to the loop CGHDC

$$3(u_3-u_2)+1u_3+2(u_3-u_1)=6$$

$$3u_3-3u_2+u_3+2u_3-2u_1=6$$

$$\boxed{-2u_1-3u_2+6u_3=6} \rightarrow (3)$$

Step4 :- solve 1, 2, 3 using cramer's rule

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix} = 39 \qquad \Delta_1 = \begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix} = 117$$

$$\Delta_2 = \begin{vmatrix} 3 & 1 & -2 \\ -1 & 0 & -3 \\ -2 & 6 & 6 \end{vmatrix} = 78 \qquad \Delta_3 = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 6 & 0 \\ -2 & -3 & 6 \end{vmatrix} = 117$$

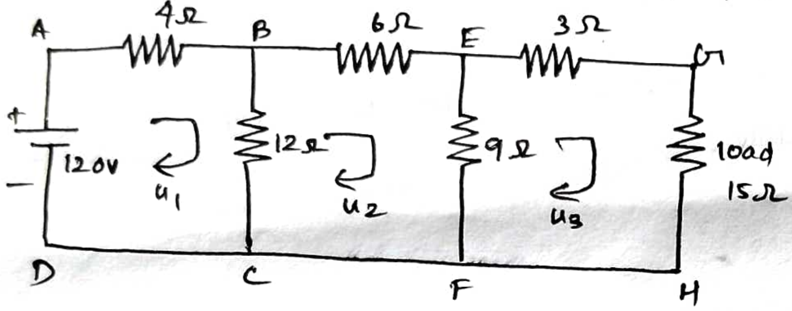
$u_1 = \frac{\Delta_1}{\Delta} = \frac{117}{39} = 3\,A$

$u_2 = \frac{\Delta_2}{\Delta} = \frac{78}{39} = 2\,A$

$u_3 = \frac{\Delta_3}{\Delta} = \frac{117}{39} = 3\,A.$

PROBLEM 6 :-

In the circuit, obtain load current and power delivered by the load.



Solution :-

Step1 :- Apply KVL to the loop ABCDA.

$4u_1 + 12(u_1 - u_2) = 120$

$4u_1 + 12u_1 - 12u_2 = 120$

$\boxed{16u_1 - 12u_2 = 120} \rightarrow (1)$

Step2 :- Apply KVL to the loop BEFCB

$6u_2 + 9(u_2 - u_3) + 12(u_2 - u_1) = 0$

$6u_2 + 9u_2 - 9u_3 + 12u_2 - 12u_1 = 0$

$\boxed{-12u_1 + 27u_2 - 9u_3 = 0} \rightarrow (2)$

Step3 :- Apply KVL to the loop EGHFE

$3u_3 + 15u_3 + 9(u_3 - u_2) = 0$

$3u_3 + 15u_3 + 9u_3 - 9u_2 = 0$

$\boxed{-9u_2 + 27u_3 = 0} \rightarrow (3)$

Step4 :-

$$\Delta = \begin{vmatrix} 16 & -12 & 0 \\ -12 & 27 & -9 \\ 0 & -9 & 27 \end{vmatrix} = 6480$$

$$\Delta_3 = \begin{vmatrix} 16 & -12 & 120 \\ -12 & 27 & 0 \\ 0 & -9 & 0 \end{vmatrix} = 12960$$

$$u_3 = \frac{\Delta_3}{\Delta} = \frac{12960}{6480} = 2A$$

Step 5 :- power delivered to the load $= u^2 R_L$

$= 2^2 \times 15$

$= 4 \times 15$

$\boxed{P = 60W}$.

Voltage division Rule :-

⇒ It states that, the voltage divided between two series resistors are in direct proportion to their resistance.

current division rule :-

⇒ It states that the current in any of the parallel branches of a parallel circuit is equal to the ratio of opposite branch resistance to the sum of all resistances, multiplied by the total current.

[ON → press shift, press mode → press (3) → press (=) two times] - to refresh the calculator (reset)

→ Press mode - 3 times

→ press Mat 2

→ press shift, 4

→ Dim (1) press 1

→ A B C (1) Press 1

→ enter rows & columns press 3, =, 3 = (Row, column)

→ type elements row wise -3 -1 press =

→ press shift, 4

→ ( ) → find, Det (1)

→ press 1

→ press shift 4, mat 3 Press 3

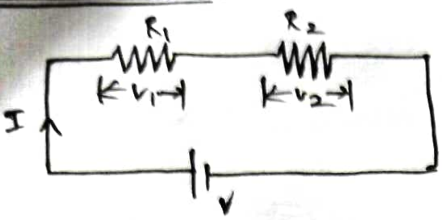
→ A B C

→ press 1 (Det (mat A)

→ press =

Ans.

VOLTAGE DIVISION RULE :-



$$V_1 = \frac{\text{Total voltage} \times \text{corresponding resistance}}{\text{Sum of the resistance}}$$

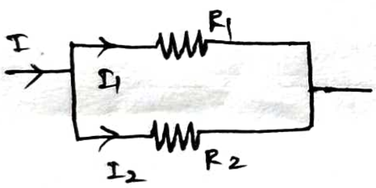
$$V_1 = \frac{V \times R_1}{R_1+R_2} = \frac{VR_1}{R_1+R_2}$$

|||ly

$$V_2 = \frac{VR_2}{R_1+R_2}$$

→ only applicable for series circuits.

CURRENT DIVISION RULE :-



$$I_1 = \frac{\text{Total current} \times \text{opposite resistance}}{\text{Sum of resistance}}$$

$$I_1 = \frac{IR_2}{R_1+R_2}$$

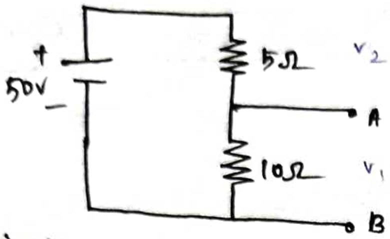
|||ly

$$I_2 = \frac{IR_1}{R_1+R_2}$$

→ only applicable for parallel circuits.

PROBLEM :- 1

What is the voltage across 10Ω resistor :-



Solution :-

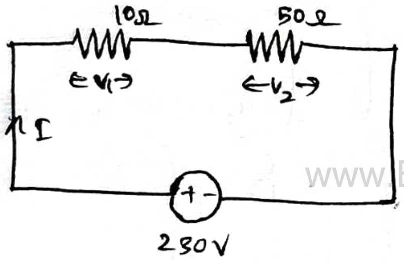
By voltage division rule,

Voltage across 10Ω, $V_1 = \frac{V \cdot R_1}{R_1 + R_2} = \frac{50 \times 10}{10 + 5} = \frac{500}{15}$

$$\boxed{V_1 = 33.3\,V}$$

PROBLEM :- 2

Consider the figure, find voltages across individual resistances.



Solution :-

By voltage division rule,

$V_1 = \frac{V \cdot R_1}{R_1 + R_2}$, $V_2 = \frac{V \cdot R_2}{R_1 + R_2}$

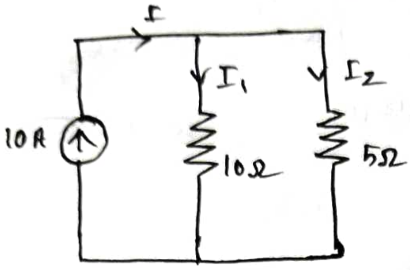
$= \frac{230 \times 10}{10 + 50}$ $\qquad$ $= \frac{230 \times 50}{10 + 50}$

$= \frac{2300}{60}$ $\qquad$ $= \frac{11500}{60}$

$V_1 = 38.33\,V$ $\qquad$ $V_2 = 191.66\,V$

PROBLEM 3 :-

Determine current through each resistor in the circuit.



Solution :-

By current division rule.

$$I_1 = I \cdot \frac{R_2}{R_1+R_2} \qquad\qquad I_2 = I \cdot \frac{R_1}{R_1+R_2}$$

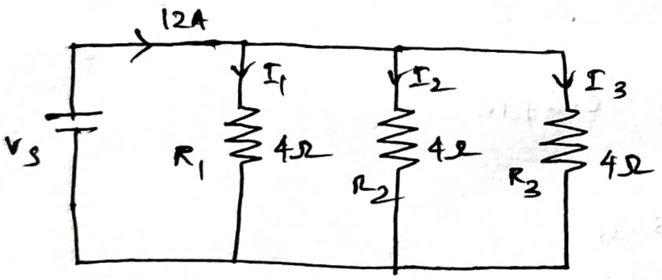
$$= 10 \times \frac{5}{10+5} \qquad\qquad = 10 \times \frac{10}{10+5}$$

$$= \frac{50}{15} \qquad\qquad = \frac{100}{15}$$

$$\boxed{I_1 = 3.33\,A} \qquad\qquad \boxed{I_2 = 6.67\,A}$$

PROBLEM 4 :-

Find the currents through individual resistances.



Solution :-

By current division rule

$$I_1 = I \cdot \frac{R_T}{R_1+R_T}$$ ($\because R_T$ opp. branch resistance)

$$R_T = \frac{R_2 \cdot R_3}{R_2+R_3} = \frac{4\times4}{4+4} = \frac{16}{8} = 2\Omega$$

$$I_1 = 12 \times \frac{2}{4+2}, \qquad I_2 = 12 \times \frac{2}{4+2}, \qquad I_3 = 12 \times \frac{2}{4+2}$$

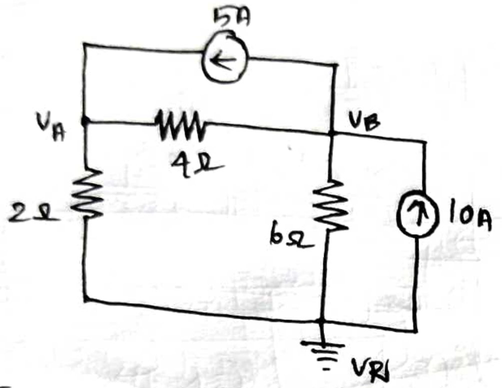
$$= \frac{24}{6} \qquad\qquad = 24/6 \qquad\qquad = 24/6$$

$$I_1 = 4A \qquad\qquad I_2 = 4A \qquad\qquad I_3 = 4A$$

Nodal analysis :-

problem 1

Determine the node voltages of given circuit :-



Solution :-

If no. of nodes = 3, then (2×2 matrix)

step 1 :-

$$\begin{bmatrix} 0.25+0.5 & -0.25 \\ -0.25 & 0.25+0.166 \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 5 \\ 10-5 \end{bmatrix}$$

$$\begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.416 \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

step 2 :-

$$\Delta = \begin{vmatrix} 0.75 & -0.25 \\ -0.25 & 0.416 \end{vmatrix} = 0.312 - 0.0625 = 0.2495$$

$$\Delta_A = \begin{vmatrix} 5 & -0.25 \\ 5 & 0.416 \end{vmatrix} = 2.08 + 1.25 = 3.33$$

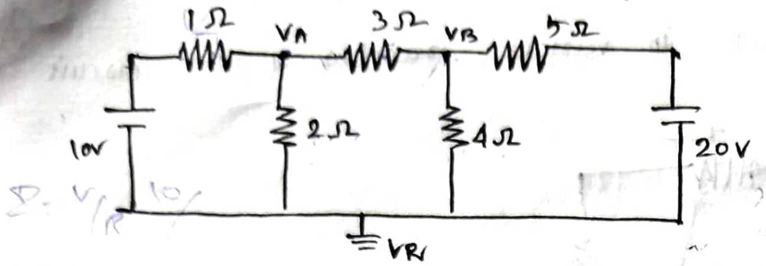
$$\Delta_B = \begin{vmatrix} 0.75 & 5 \\ -0.25 & 5 \end{vmatrix} = 3.75 + 1.25 = 5$$

$$V_A = \frac{\Delta_A}{\Delta} = \frac{3.33}{0.2495} = 13.34\ V$$

$$V_B = \frac{\Delta_B}{\Delta} = \frac{5}{0.2495} = 20.04\ V.$$

Problem 2 :-

Determine the current flow through 3Ω resistor using nodal analysis :-



no. of nodes - 3 → (2x2 matrix).

Solution :-

Step 1 :-

taken as -ve always

$$\begin{bmatrix} 1+0.333+0.5 & -0.333 \\ -0.333 & 0.33+0.2+0.25 \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \end{bmatrix}$$

$\frac{1}{R} . V = I$

$I = \frac{V}{R} = V \times \frac{1}{R}$

$$\begin{bmatrix} 1.833 & -0.333 \\ -0.333 & 0.783 \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \end{bmatrix}$$

Step 2 :-

$$\Delta = \begin{vmatrix} 1.833 & -0.333 \\ -0.333 & 0.783 \end{vmatrix} = 1.435 - 0.110 = 1.325$$

$$\Delta_A = \begin{vmatrix} 10 & -0.333 \\ 4 & 0.783 \end{vmatrix} = 7.83 + 1.332 = 9.162$$

$$\Delta_B = \begin{vmatrix} 1.833 & 10 \\ -0.333 & 4 \end{vmatrix} = 7.332 + 3.33 = 10.662$$

$$V_A = \frac{\Delta_A}{\Delta} = \frac{9.162}{1.325} = 6.914, \quad V_B = \frac{\Delta_B}{\Delta} = \frac{10.662}{1.325} = 8.046$$

Step 3 :-

current flow through 3Ω resistor $= \frac{V_A - V_B}{3}$ $\quad (\because I = V/R)$

$$= \frac{6.914 - 8.046}{3}$$

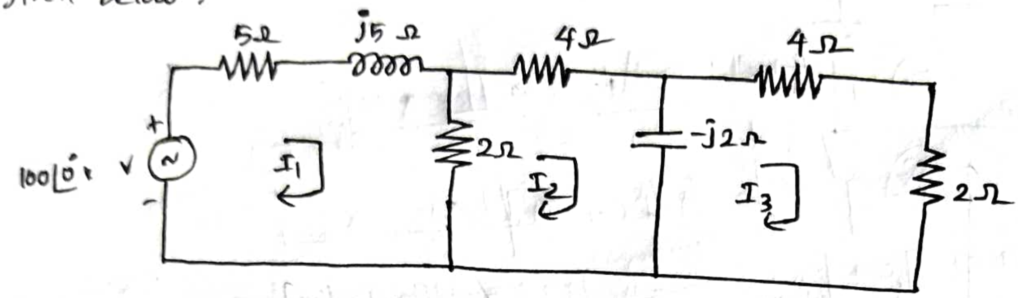
$$= \frac{-1.132}{3} = -0.377$$

$$= 0.377\,A.$$

Mesh analysis for a.c circuit :-

Problem

using mesh analysis, determine the currents in various branches of the circuit given below :-



Solution :-

Step 1 :- Apply KVL to loop 1

$$5I_1 + j5I_1 + 2[I_1 - I_2] = 100\angle 0^\circ$$

$$5I_1 + j5I_1 + 2I_1 - 2I_2 = 100\angle 0^\circ$$

$$\boxed{(7+j5)I_1 - 2I_2 = 100\angle 0^\circ} \rightarrow (1)$$

Step 2 :- Apply KVL to loop 2

$$4I_2 - j2[I_2 - I_3] + 2[I_2 - I_1] = 0$$

$$4I_2 - j2I_2 + j2I_3 + 2I_2 - 2I_1 = 0$$

$$-2I_1 + 6I_2 - j2I_2 + j2I_3 = 0$$

$$\boxed{-2I_1 + (6-j2)I_2 + j2I_3 = 0} \rightarrow (2)$$

Step 3 :- Apply KVL to loop 3

$$4I_3 + 2I_3 - j2[I_3 - I_2] = 0$$

$$4I_3 + 2I_3 - j2I_3 + j2I_2 = 0$$

$$\boxed{j2I_2 + (6-j2)I_3 = 0} \rightarrow (3)$$

Step 4 :- matrix form

$$\begin{bmatrix} 7+j5 & -2 & 0 \\ -2 & 6-j2 & j2 \\ 0 & j2 & 6-j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7+j5 & -2 & 0 \\ -2 & 6-j2 & j2 \\ 0 & j2 & 6-j2 \end{vmatrix}$$

$$\Delta = 7+j5\left[(6-j2)^2 - (j2)^2\right] + 2\left[-2(6-j2) - 0\right] + 0$$

$$= 7+j5\left[(36 - \cancel{4} - 24j) + \cancel{4}\right] + 2\left[-12 + j4\right]$$

$$= 7+j5\left[36 - 24j\right] + \left[-24 + 8j\right]$$

$$= 252 - 168j + 180j + 120 - 24 + 8j$$

$$\boxed{\Delta = 348 + 20j}$$

$\because (a-b)^2 = a^2 + b^2 - 2ab$

$a = 6, \; b = -j2$

$$\Delta I_1 = \begin{vmatrix} 100 & -2 & 0 \\ 0 & 6-j2 & j2 \\ 0 & j2 & 6-j2 \end{vmatrix} = 100\left[(6-j2)^2 - (j2)^2\right] + 2\left[0\right] + 0$$

$$= 100\left[36 - 24j\right]$$

$$\boxed{\Delta I_1 = 3600 - j2400}$$

$$\Delta I_2 = \begin{vmatrix} 7+j5 & 100 & 0 \\ -2 & 0 & j2 \\ 0 & 0 & 6-j2 \end{vmatrix} = 7+j5\left[0\right] - 100\left[(-2)(6-j2) - 0\right] + 0$$

$$= -100\left[-12 + j4\right]$$

$$\boxed{\Delta I_2 = 1200 - j400}$$

$$\Delta I_3 = \begin{vmatrix} 7+j5 & -2 & 100 \\ -2 & 6-j2 & 0 \\ 0 & j2 & 0 \end{vmatrix} = 7+j5(0) + 2(0) + 100(-4j)$$

$$= -j400$$

$$\boxed{\Delta I_3 = -j400}$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{3600 + j2400}{348 + j20} = 12.412 \angle 30.4^\circ$$

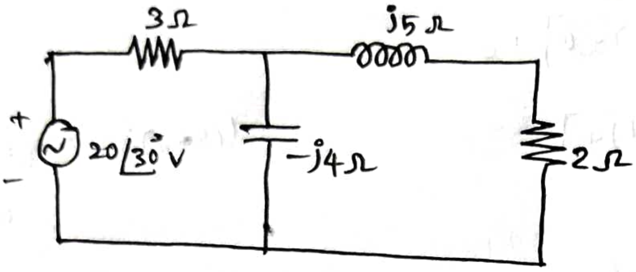
$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{1200 - j400}{348 + j20} = 3.629 \angle -21.7^\circ$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{-j400}{348 + j20} = 1.147 \angle -93.2^\circ$$

Nodal analysis for a.c circuit :-

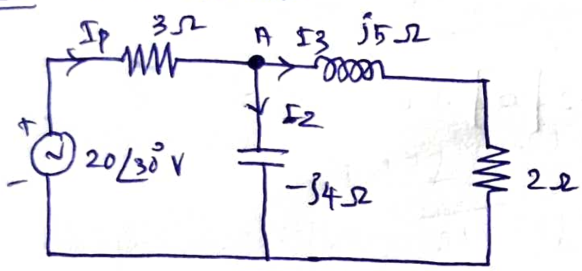
Problem

Using Nodal analysis, determine the voltage across node in the circuit below.



Solution :-

Step 1 :- Find the node



Step 2 :- Apply Kirchoff's current law

$$I_1 = I_2 + I_3$$

$$\frac{20\angle 30^\circ - V_A}{3} = \frac{V_A - 0}{-j4} + \frac{V_A - 0}{2+j5}$$

$$\frac{20\angle 30^\circ}{3} - \frac{V_A}{3} = \frac{V_A}{-j4} + \frac{V_A}{2+j5}$$

$$6.67\angle 30^\circ - \frac{V_A}{3} = \frac{V_A}{-j4} + \frac{V_A}{2+j5}$$

$$6.67\angle 30^\circ = \frac{V_A}{-j4} + \frac{V_A}{2+j5} + \frac{V_A}{3}$$

$$6.67\angle 30^\circ = V_A\left[\frac{-1}{j4} + \frac{1}{2+j5} + \frac{1}{3}\right]$$

$$6.67\angle 30^\circ = V_A\left\{\frac{(-1)(3)(2+j5) + 3(j4) + (j4)(2+j5)}{(3)(j4)(2+j5)}\right]$$

$$6.67\angle 30^\circ = V_A\left[\frac{-6-15j+12j+8j-20}{24j-60}\right]$$

$$6.67\angle 30^\circ = V_A\left[\frac{-26+5j}{24j-60}\right]$$

$$\frac{6.67\angle 30^\circ \times (24j-60)}{-26+5j} = V_A$$

$$\frac{6.67\angle 30^\circ \times 64.62\angle 158.1^\circ}{26.47\angle 169.11^\circ} = V_A$$

$$\boxed{16.28\angle 18.99^\circ = V_A}$$

Rectangular or Complex to polar form

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Shift , Alpha , tan =

BM3552 - ELECTRIC CIRCUIT ANALYSIS

UNIT-2 - NETWORK THEOREM AND DUALITY.

USEFUL CIRCUIT ANALYSIS TECHNIQUES :-

⇒ The growth in areas of application of electric circuits has led to an evolution from simple to complex circuits.

⇒ To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis.

⇒ These theorems are applicable to linear circuits.

⇒ Theorems include

- Thevenin's theorem,
- Norton's theorem
- Superposition
- Source transformation
- Maximum power transfer.

Linearity property :-

⇒ A linear circuit is one whose output is linearly related (directly proportional) to its input.

⇒ In general, a circuit is linear if it is both additive and homogeneous.

⇒ A linear circuit consists of only linear elements, linear dependent sources and independent sources.

⇒ The property is a combination of both homogeneity and additive property.

⇒ The homogeneity property requires that if the input is multiplied by a constant, then the output is multiplied by the same constant.

⇒ For eg. for a resistor, ohm's law relates the input i, to output v,

$$v = iR.$$

⇒ If the current is increased by a constant K, then the voltage increases correspondingly by K,

$$Kv = KiR.$$

⇒ The additive property requires that response to sum of inputs is the sum of responses to each input applied separately.

⇒ Using voltage - current relationship of a resistor, if

$$v_1 = i_1 R \quad \text{and} \quad v_2 = i_2 R$$

then, $v = v_1 + v_2 = i_1 R + i_2 R$

$$= (i_1 + i_2) R$$

⇒ We say that resistor is a linear element because the voltage - current relationship satisfies both homogeneity and additive properties.

SUPERPOSITION THEOREM :-

⇒ The superposition theorem is a circuit analysis theorem used to solve the network where two or more sources are present and connected.

⇒ It states that " In any linear and bilateral network or circuit having multiple independent sources, the response of an element will be equal to algebraic sum of the responses of that element by considering one source at a time".

⇒ To calculate the individual contribution of each source in a circuit, the other source must be replaced or removed without affecting final result. This is done by replacing the voltage source with short circuit, and replacing current source with open circuit.

Steps to apply superposition theorem :-

⇒ First step is to select one among the multiple sources present in the bilateral network. Among the various sources in circuit, any one of the sources can be considered first.

⇒ Except for selected source, all the sources must be replaced by their internal impedance.

⇒ Using network simplification approach, evaluate current flowing through or voltage drop across a particular element in the network.

⇒ The same considering a single source is repeated for all other sources in the circuit.

⇒ Upon obtaining the respective response for individual source, perform the summation of all responses to get overall voltage drop or current through circuit element.

Limitations of superposition theorem :-

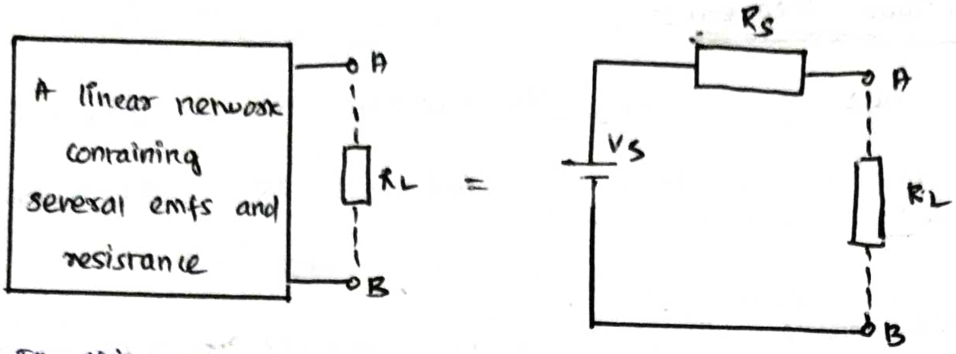
⇒ The theorem does not apply to non - linear circuits.

⇒ This theorem is only applicable to determine voltage and current but not power.

⇒ The application of superposition theorem requires two or more sources in the circuit.

THEVENIN'S THEOREM.

⇒ "It states that it is possible to simplify any linear circuit, irrespective of how complex it is, to an equivalent circuit with a single voltage source and a series resistance.



⇒ In this, we see that multiple resistive circuit elements are replaced by a single equivalent resistance Rs and multiple energy sources by an equivalent voltage source Vs.

Applications of Thevenin's theorem :-

⇒ used in analysis of power systems.

⇒ used in source modelling and resistance measurement using wheatstone bridge.

Limitations of Thevenin theorem :-

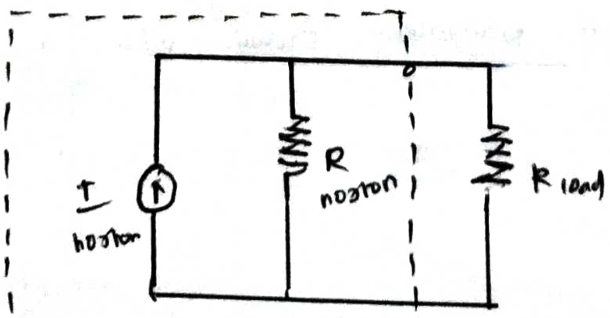
⇒ It is used only in the analysis of linear circuits.

⇒ power dissipation of thevenin equivalent is not identical to the power dissipation of real system.

NORTON'S THEOREM :-

⇒ This theorem is useful for representing the given electric circuit into its equivalent circuit in simplified form.

⇒ It states that "any linear circuit can be simplified to an equivalent circuit consisting of a single current source and parallel resistance that is connected to a load.



Norton equivalent circuit

Steps to apply Norton's theorem :-

⇒ Remove the load resistor and replace it with a short circuit.

⇒ Calculate the norton current - current through the short circuit.

⇒ Replace the power sources. All voltage sources are replaced with short circuits and all current sources are replaced with open circuits.

⇒ Calculate Norton's resistance - total resistance between open circuit connection points after all sources have been removed.

⇒ Draw the Norton equivalent circuit, with Norton current source in parallel with norton resistance. The load resistor re-attaches between two open points of equivalent circuit.

⇒ Analyze the voltage and current for load following the rules for parallel circuits.

Applications of Norton's theorem :-

⇒ In replacement of large part of network to a simple circuit.

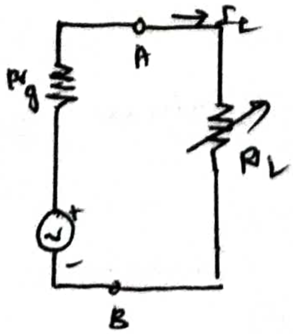
⇒ used in simplification of a network in terms of currents instead of voltage.

⇒ It also helps to accomplish a maximum transfer or power.

MAXIMUM POWER TRANSFER THEOREM :-

⇒ It states that " maximum power will be delivered from a voltage source to a load, when load resistance is equal to internal resistance of the source.

⇒ For eg. consider a voltage source of generated voltage V_g & internal resistance R_g, connected to a load resistance R_L,



⇒ When the power transfer to the load is maximum $R_L = R_g$, At this condition total resistance $= R_g + R_L = 2R_L$.

$$\therefore I_L = \frac{V_g}{2R_L}$$

$$\text{Power delivered to } R_L = I_L^2 R_L$$

$$= \left(\frac{V_g}{2R_L}\right)^2 R_L = \frac{V_g^2}{4R_L}$$

The maximum power transferred to R_L

$$\boxed{P_{max} = \frac{V_g^2}{4R_L}}$$

Steps to solve network using maximum power transfer theorem :-

⇒ Identify the variable load resistance in the circuit and remove the load resistance.

⇒ Replace the independent source voltage by a short circuiting the terminals and independent current source by open circuit.

⇒ Find the thevenin resistance of the circuit by calculating the equivalent resistance between terminals of open circuited load resistance.

⇒ Find thevenin voltage by calculating voltage across the terminals of open circuited load resistance and find maximum power delivered using maximum power transfer theorem formula.

Applications of maximum power transfer theorem :-

⇒ It is used in solar cell applications, adjusting the electrical load on the cell to obtain maximum output power.

⇒ This theorem helps us to design efficient power sources and loads for various applications such as audio amplifiers, wireless communication.

SOURCE TRANSFORMATION :-

⇒ It simply means replacing one source by an equivalent source.

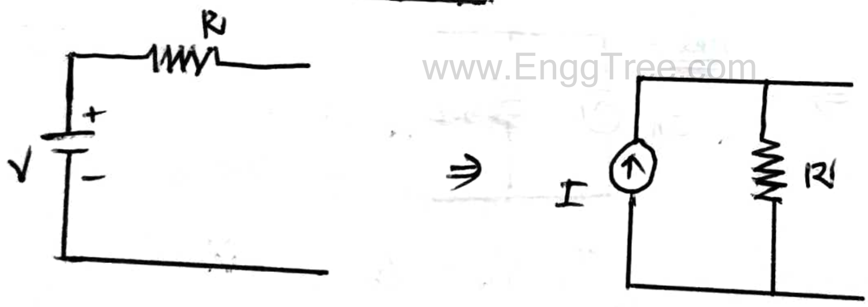
⇒ A practical voltage source can be transformed into an equivalent practical current source and similarly a practical current source into voltage source.

⇒ A current source in parallel with resistor is equivalent to a voltage source in series with same resistor, provided that value of voltage source is equal to value of current source, multiplied by source.

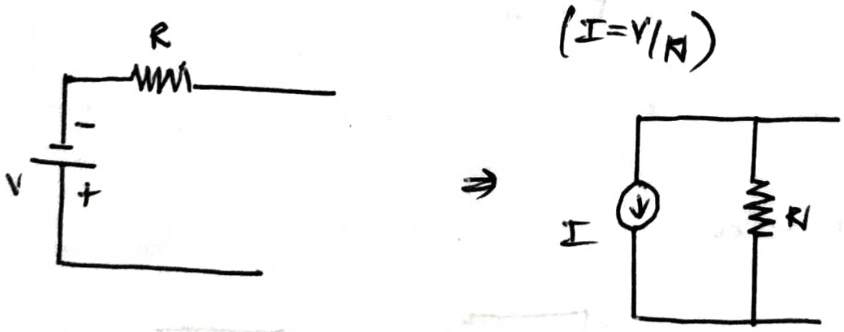
→ Similarly, voltage source in series with resistor is equivalent to current source in parallel with same resistor provided that value of current source is equal to value of voltage source, divided by resistance.

⇒ This is applicable for both AC and DC circuits.

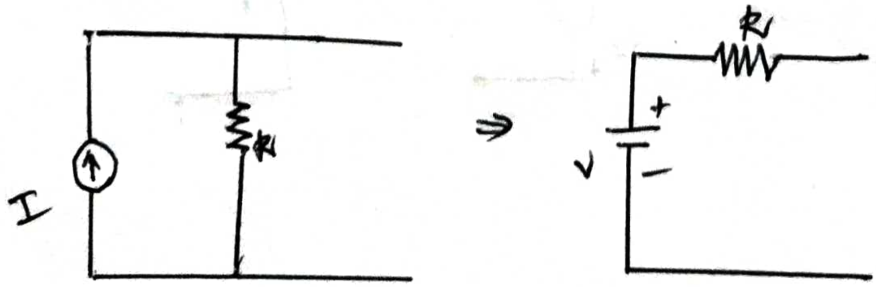
Voltage source to current source :-



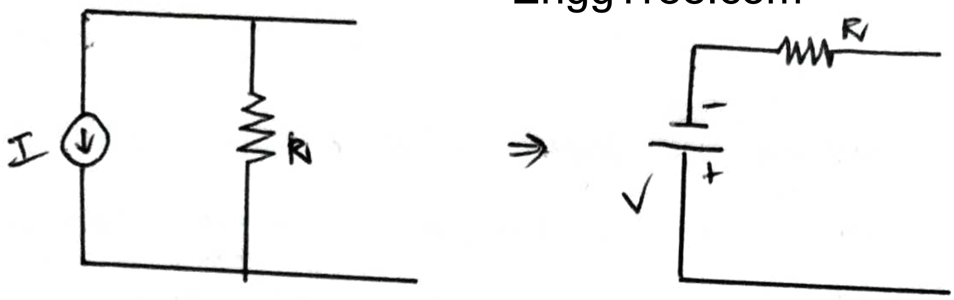
$(I = V/R)$



Current source to voltage source :-

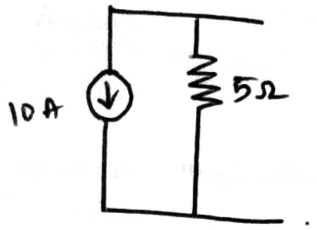


$(V = IR)$

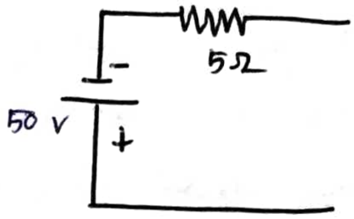


PROBLEMS :-

1) convert this current source into a voltage source :-



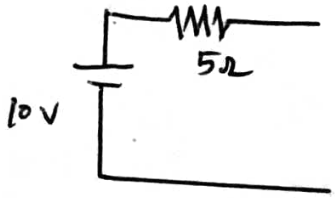
Ans :-



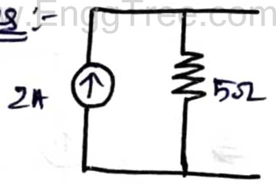
$V = IR$

$V = 10 \times 5 = 50V$

2) convert voltage source into current source :-



Ans :-

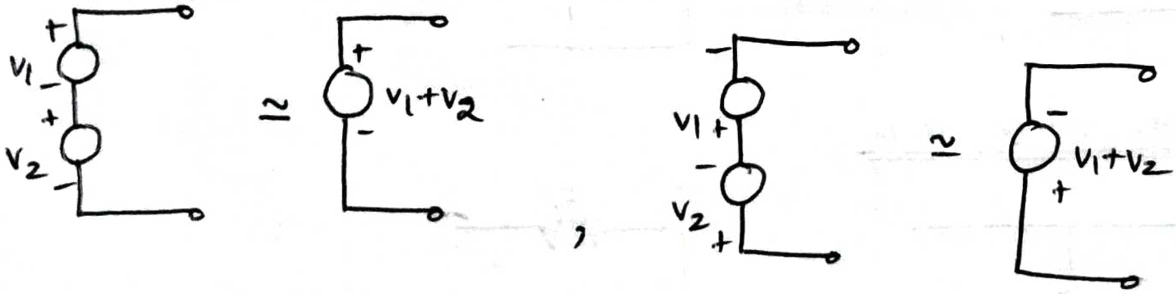


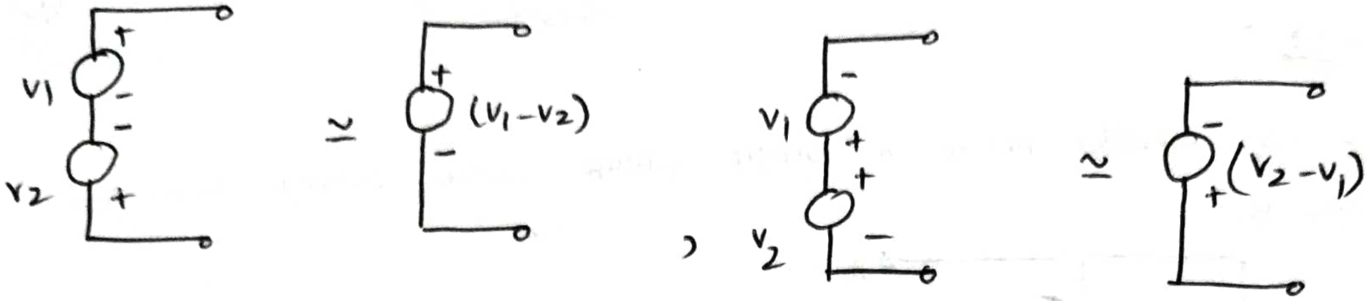
$I = V/R = 10/5$

$I = 2A$

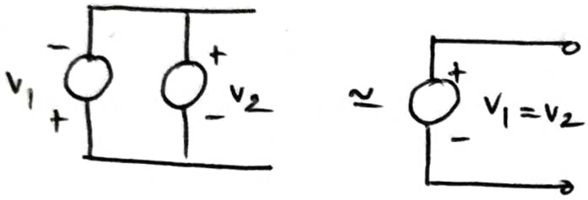
Combination of sources :-

case 1 :- voltage sources in series :-

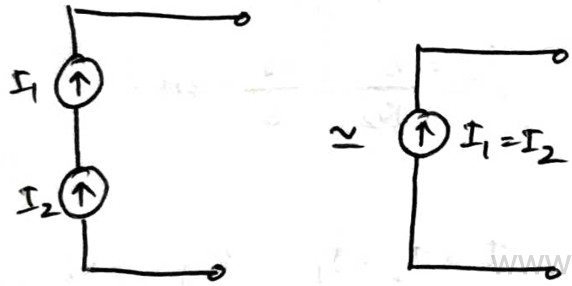




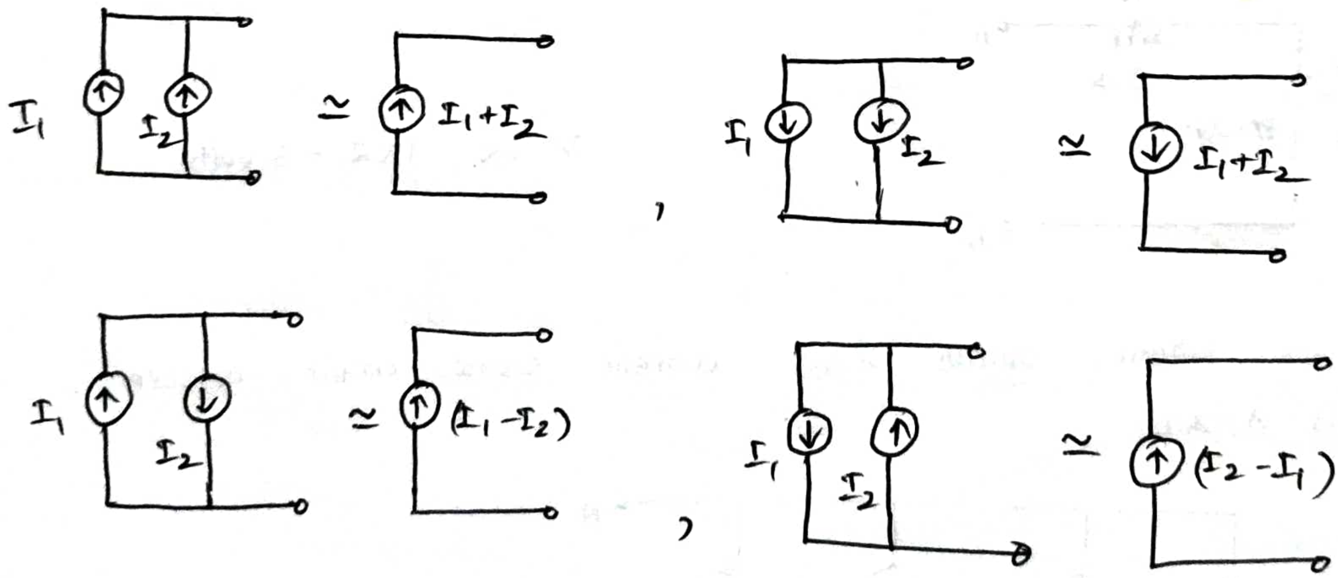
case 2 :- voltage sources in parallel.



case 3 :- current sources in series

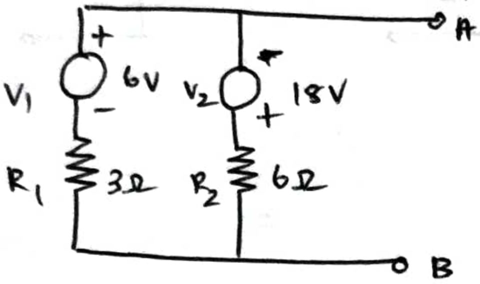


case 4 :- current sources in parallel

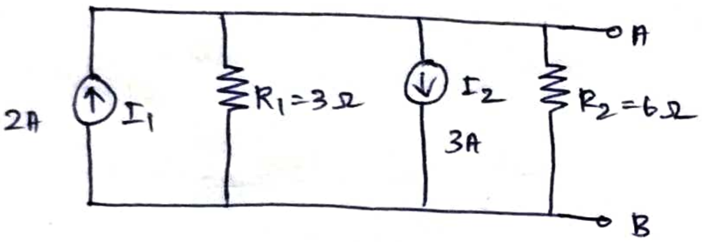


PROBLEMS :-

1) For the circuit, obtain a single voltage source between terminals A and B.

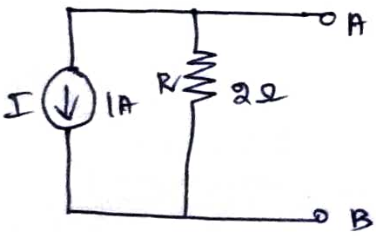


Solution :-

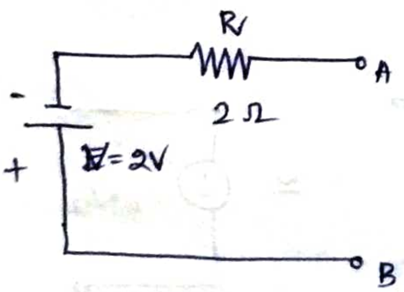


$$I_1 = \frac{V_1}{R_1} = \frac{6}{3} = 2A$$

$$I_2 = \frac{V_2}{R_2} = \frac{18}{6} = 3A$$

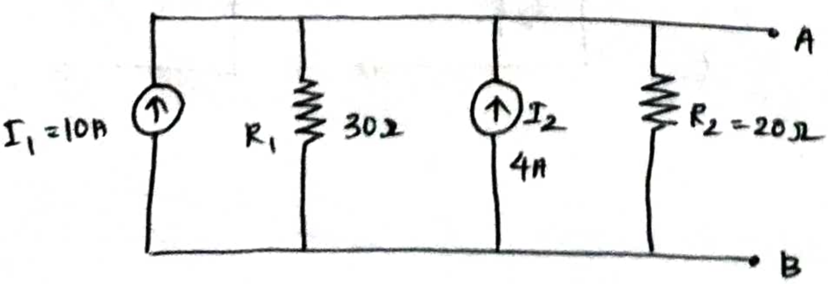


$$I = I_2 - I_1$$
$$= 3 - 2$$
$$I = 1A \qquad , \qquad R_{eq} = \frac{3 \times 6}{3+6} = \frac{18}{9}$$
$$R_{eq} = 2\Omega$$

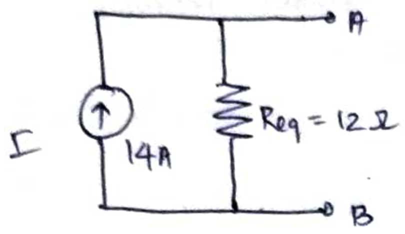


$$V = IR, \quad 1 \times 2 = 2 \text{ volts}$$

2) For the circuit, obtain single current source circuit, between terminals A & B.



Solution :-



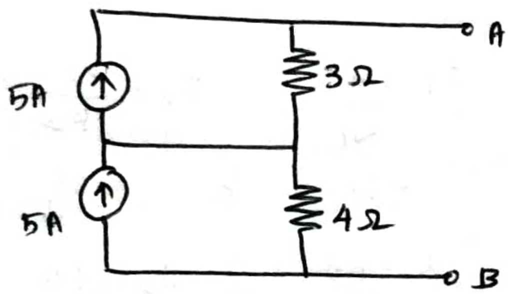
,

$I = I_1 + I_2 = 10 + 4 = 14\,A$

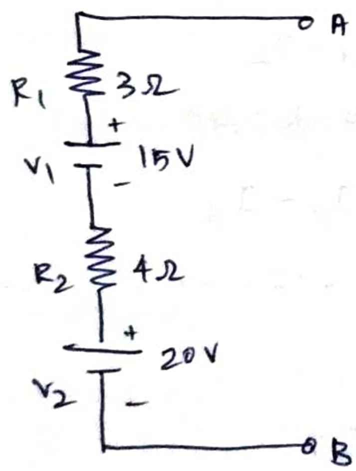
$I = 14\,A$,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{30 \times 20}{30 + 20} = \frac{600}{50} = 12\,\Omega$$

3) Convert the following circuit into single voltage source :-



Solution :-



$V_1 = I_1 R_1$

$= 5 \times 3$

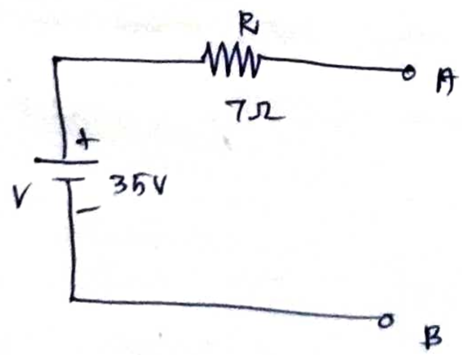
$V_1 = 15\,V$

,

$V_2 = I_2 R_2$

$= 5 \times 4$

$V_2 = 20\,V$



$V = V_1 + V_2$

$= 15 + 20$

$V = 35\,V$

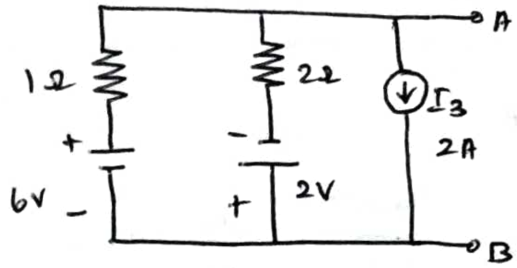
,

$R = R_1 + R_2$

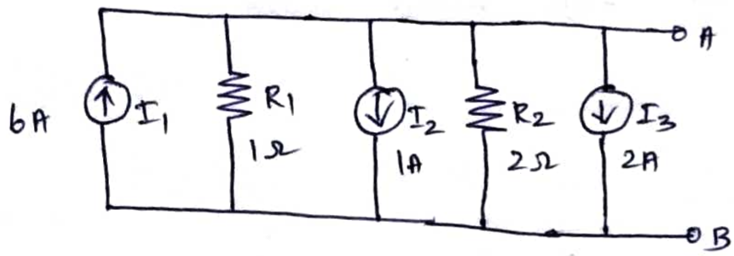
$= 3 + 4$

$R = 7\,\Omega$

4) For the circuit, obtain equivalent current source between terminals A & B.



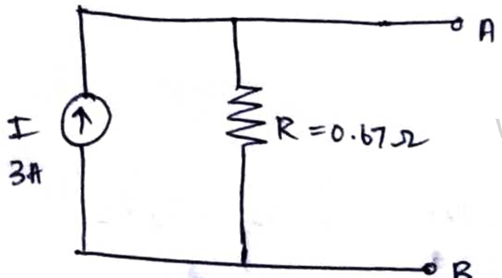
Solution :-



$I_1 = \frac{V_1}{R_1}$, $I_2 = \frac{V_2}{R_2}$

$= 6/1$ $\quad = 2/2$

$I_1 = 6A$ $\quad I_2 = 1A$.



$I_a = I_1 - I_2$

$= 6 - 1 = 5A$

$I_b = I_a - I_3$

$= 5 - 2 = 3A$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{1 \times 2}{1+2} = 0.67\Omega.$$

DELTA - WYE CONVERSION :-

⇒ Delta - wye transformation technique is used to resolve complex electrical circuits with resistors.

⇒ It is an technique for transforming certain resistor combinations that cannot be handled by the series and parallel equations.

⇒ It is a method of converting delta or triangle (Δ) shaped electrical resistance carrying circuit to Y shaped electrical network in order to solve complex problems.

Delta Network :-

⇒ If network elements are connected in the form of delta or triangle (Δ), then it is known as Delta network.

⇒ If we invert this delta symbol, then we will get the symbol of del operator. we can modify this del form into pi - symbol (π). Then it is known as pi - network or π - network.

⇒ So del network and pi network are other names of delta network.

Wye Network :-

⇒ If network elements are connected in the form of letter Y, then it is known as wye network.

⇒ If we invert the letter Y, then it is known as inverted Y or star (λ) Network. we can modify letter Y into letter T, then it is known as T - network.

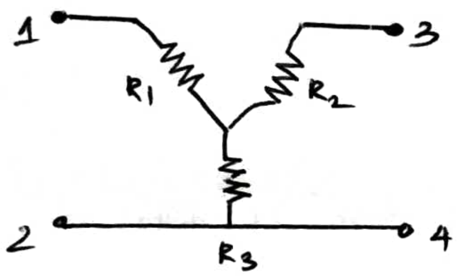
⇒ star network and T network are the other names of wye network.

Uses of Delta-wye transformation technique :-

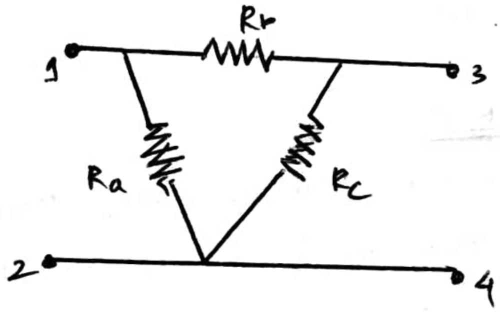
⇒ By using this, we can easily calculate the equivalent resistance, especially for unbalanced wheatstone bridges.

⇒ For finding 2-port network parameters easily based on the requirement by using this technique, we can convert T-network into pi-network and similarly pi-network into T-network.

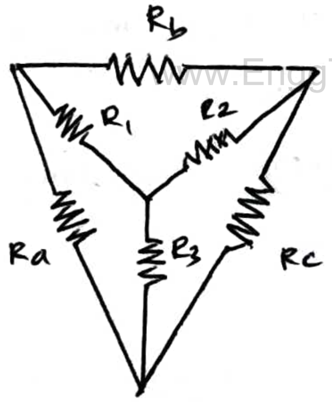
Delta to wye conversion :-



(wye network)(Y)



(Delta network) - ∇.



(Δ to Y conversion).

To calculate R_1, R_2, R_3,

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}, \quad R_2 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_{12}(Y) = R_1 + R_3, \quad R_{12}(\Delta) = R_a \parallel (R_b + R_c)$$

Equate these 2,

$$R_{12}(Y) = R_{12}(\Delta)$$

$$R_1 + R_3 = R_a \parallel (R_b + R_c) \rightarrow (1)$$

$R_{13}(Y) = R_1 + R_2$, $R_{13}(\Delta) = R_b \| (R_a + R_c)$

Equate these 2,

$R_{13}(Y) = R_{13}(\Delta)$

$R_1 + R_2 = R_b \| (R_a + R_c) \rightarrow (2)$

$R_{34}(Y) = R_2 + R_3$, $R_{34}(\Delta) = R_c \| (R_a + R_b)$

Equate these 2,

$R_{34}(Y) = R_{34}(\Delta)$

$R_2 + R_3 = R_c \| (R_a + R_b) \rightarrow (3)$

sub (1) - (3),

$$R_1 + \cancel{R_3} - (R_2 + \cancel{R_3}) = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} - \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_1 - R_2 = \frac{R_a R_b + \cancel{R_a R_c} - \cancel{R_c R_a} - R_c R_b}{R_a + R_b + R_c}$$

$$R_1 - R_2 = \frac{R_a R_b - R_b R_c}{R_a + R_b + R_c} \rightarrow (4)$$

Add (2) + (4)

$$(R_1 + \cancel{R_2}) + (R_1 - \cancel{R_2}) = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} + \frac{R_a R_b - R_b R_c}{R_a + R_b + R_c}$$

$$2R_1 = \frac{R_a R_b + \cancel{R_b R_c} + R_a R_b - \cancel{R_b R_c}}{R_a + R_b + R_c}$$

$$\cancel{2} R_1 = \frac{\cancel{2} R_a R_b}{R_a + R_b + R_c}$$

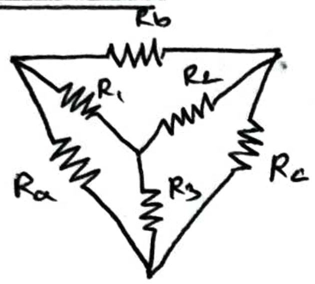
$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}.$$

Hence proved.

Similarly, we can calculate R_2 & R_3

$$R_2 = \frac{R_b R_c}{R_a+R_b+R_c}, \quad R_3 = \frac{R_a R_c}{R_a+R_b+R_c}$$

Wye to Delta conversion :-



To find R_a, R_b, R_c in terms of R_1, R_2, R_3

Each resistor in the delta network is sum of all possible products of wye resistors taken two at a time, divided by the opposite wye resistor.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

consider,

$$R_1 = \frac{R_a R_b}{R_a+R_b+R_c} \rightarrow (1), \quad R_2 = \frac{R_b R_c}{R_a+R_b+R_c} \rightarrow (2), \quad R_3 = \frac{R_a R_c}{R_a+R_b+R_c} \rightarrow (3)$$

$$(1) \times (2) \Rightarrow R_1R_2 = \frac{R_a R_b^2 R_c}{(R_a+R_b+R_c)^2} \rightarrow (4)$$

$$(2) \times (3) \Rightarrow R_2R_3 = \frac{R_a R_b R_c^2}{(R_a+R_b+R_c)^2} \rightarrow (5)$$

$$(1) \times (3) \Rightarrow R_1R_3 = \frac{R_a^2 R_b R_c}{(R_a+R_b+R_c)^2} \rightarrow (6)$$

Add equ. 4 + 5 + 6 ,

$$R_1R_2 + R_2R_3 + R_1R_3 = \frac{R_aR_b^2R_c + R_aR_bR_c^2 + R_a^2R_bR_c}{(R_a+R_b+R_c)^2}$$

$$R_1R_2 + R_2R_3 + R_1R_3 = \frac{R_aR_bR_c\cancel{(R_a+R_b+R_c)}}{(R_a+R_b+R_c)^{\cancel{2}}}$$

$$R_1R_2 + R_2R_3 + R_1R_3 = \frac{R_aR_bR_c}{(R_a+R_b+R_c)} \rightarrow 7$$

Divide equ 7 ÷ 1

$$\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{\dfrac{R_a\cancel{R_b}R_c}{\cancel{R_a+R_b+R_c}}}{\dfrac{R_a\cancel{R_b}}{\cancel{R_a+R_b+R_c}}}$$

$$\boxed{\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_c}$$

Similarly,

Divide equ 7 ÷ 2 ⇒, we get $R_a = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$

Divide equ 7 ÷ 3 ⇒, we get $R_b = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$.

Hence proved.

DUALS & DUAL CIRCUITS :-

Principle of duality :-

⇒ principle of duality in electrical networks states that,

- dual of a relationship is one in which current and voltage are interchangeable.

- Two networks are dual to each other if one has mesh equation numerically identical to others node equation.

List of dual pairs :-

Dual pairs

	Elements	Dual elements
1)	V	I
2)	series	Parallel
3)	short circuit	open circuit
4)	Norton	Thevenin
5)	Resistance (R)	Conductance (G)
6)	Impedance	Admittance
7)	KVL	KCL
8)	capacitance (C)	Inductance.
9)	switch ON	switch OFF.
10)	star	Delta.
11)	Nodal	Mesh.

⇒ power P & mutual Inductance does not have duals.

⇒ All the dual Network are linear but all the linear network are not dual.

Formation of Dual Networks :-

⇒ The principle of duality is applicable for linear circuits only.

Step 1 :- Place a dot within each loop, these dots will become nodes of the dual network.

Step 2 :- Place a dot outside of network, this dot will be ground / reference node of dual network,

Step 3 :- Carefully draw lines between nodes such that each line cuts only one element.

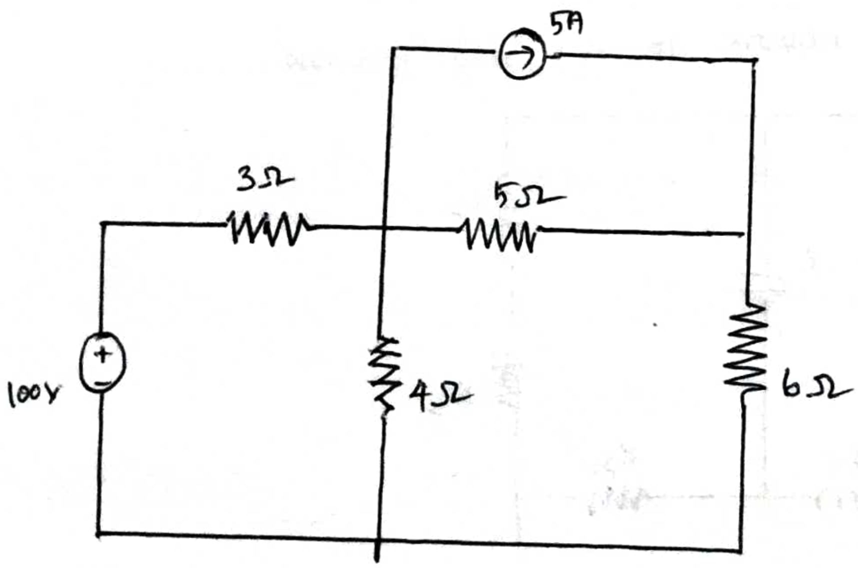
Step 4 :- If an element exclusively present in a loop, then connect dual element in between node & reference node.

Step 5 :- If an element is common in between two loops, then dual element is placed in between two nodes.

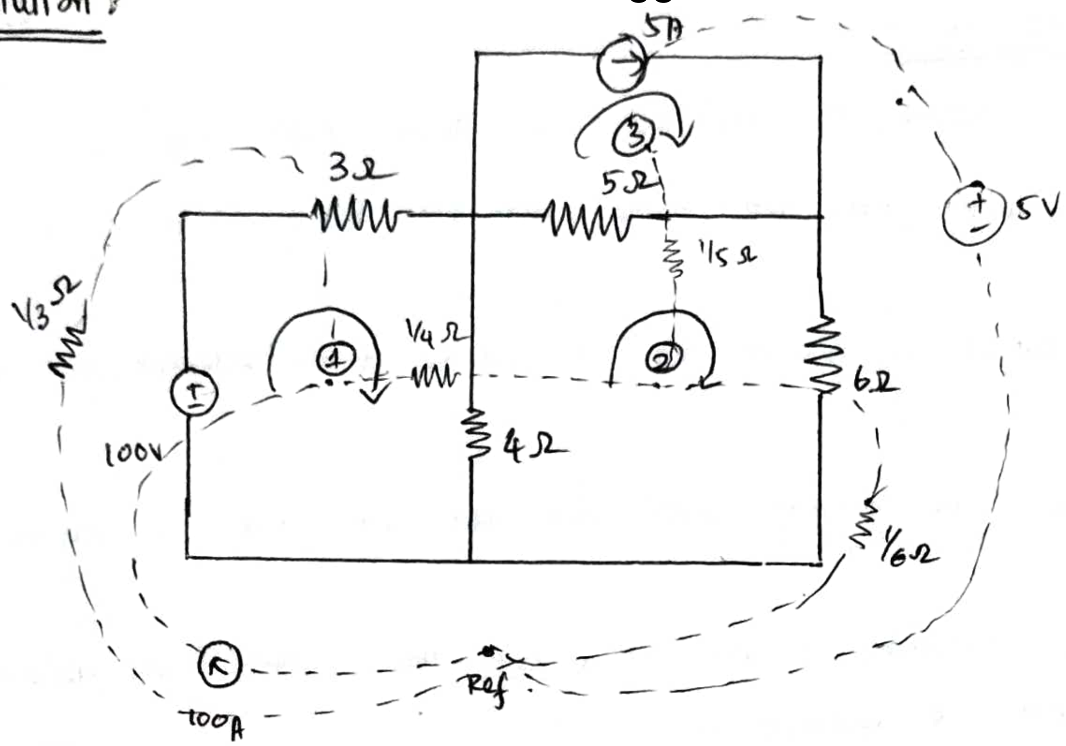
Step 6 :- Determine polarity of voltage source & direction of current sources, consider voltage source producing clockwise current in a loop. Its dual current source will have a current direction from ground to non-reference node.

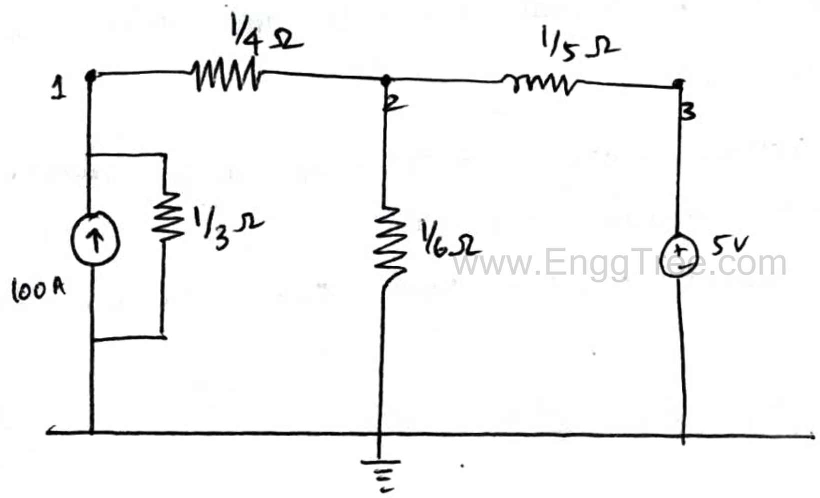
PROBLEM :-

Eg. 1 Convert given network to it's dual network.

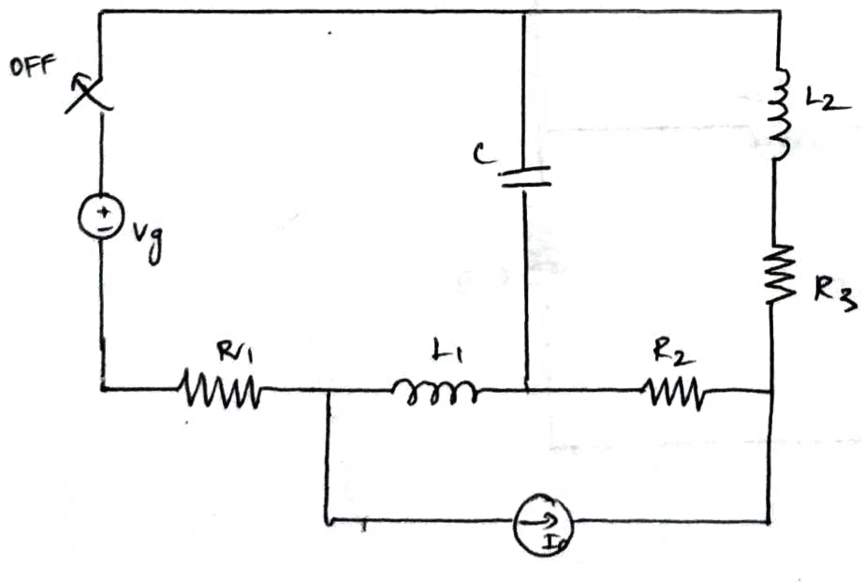


Solution :-

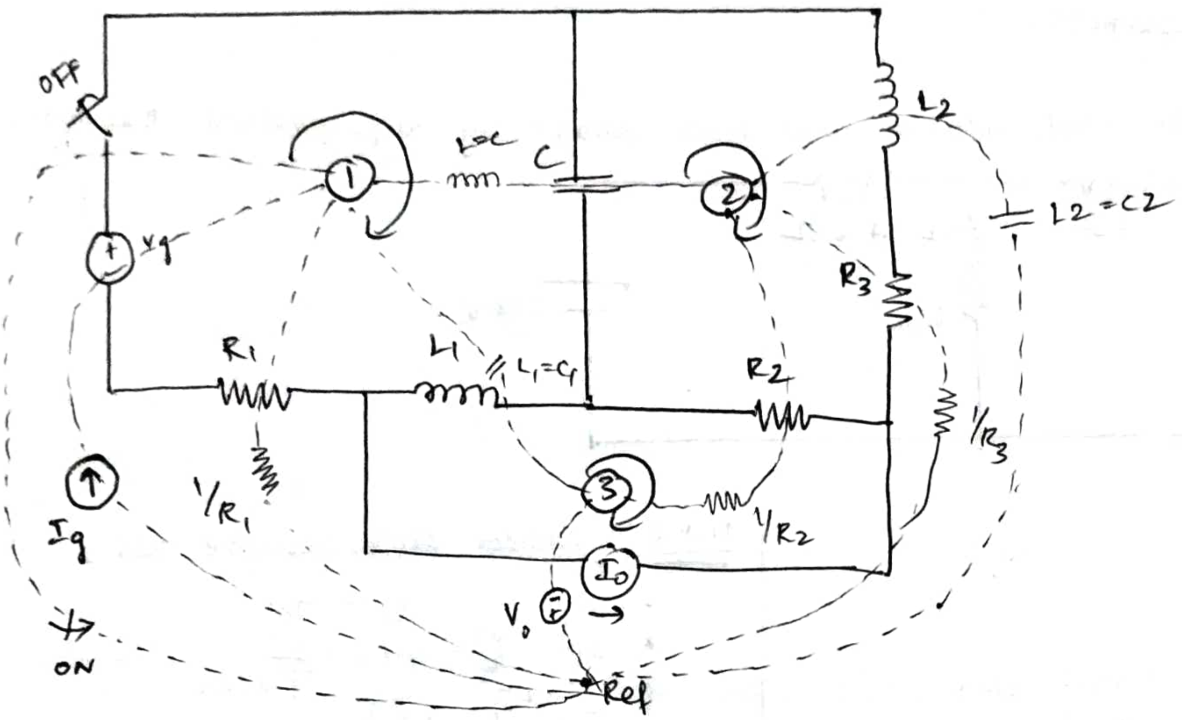




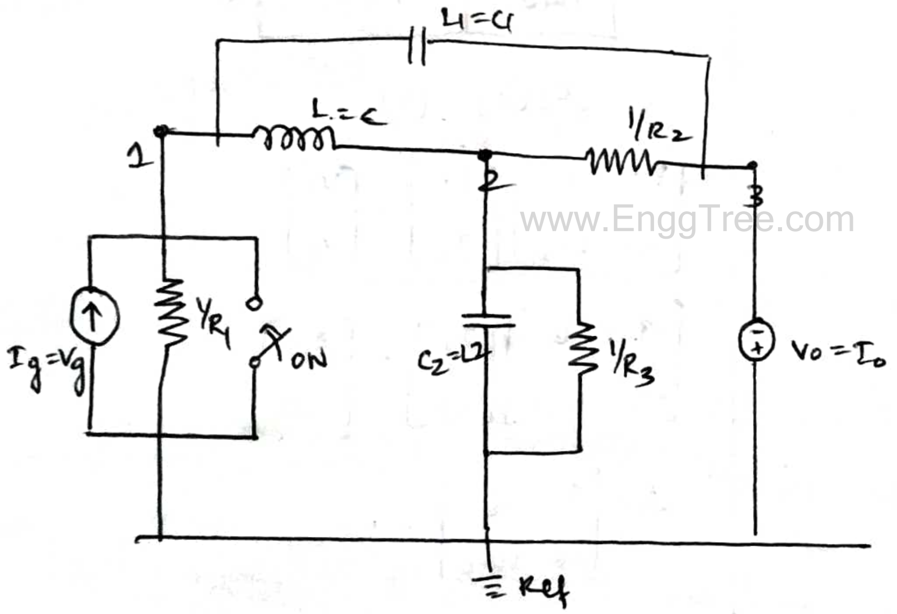
Example. 2 convert given network to it's dual network.



Solution :-



OFF
Vg
1
L=C
C
2
L2
L2=C2
R3
R1
L1
L1=C1
R2
1/R3
3
1/R1
1/R2
Ig
I0
V0
ON
Ref

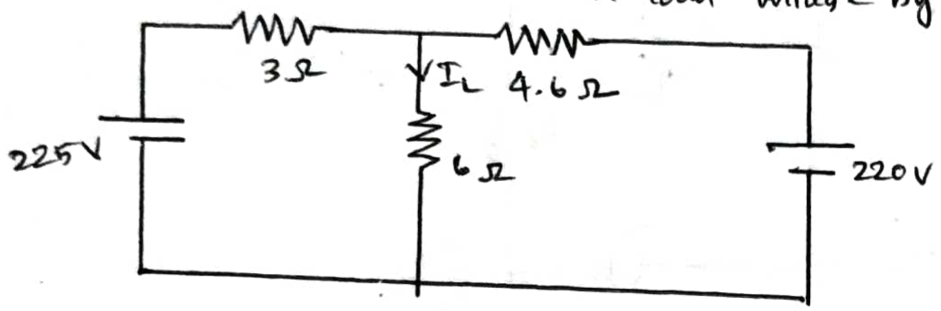


L1=C1
L=C
1/R2
1
2
3
Ig=Vg
1/R1
ON
C2=L2
1/R3
V0=I0
Ref

PROBLEMS :-

SUPERPOSITION THEOREM :-

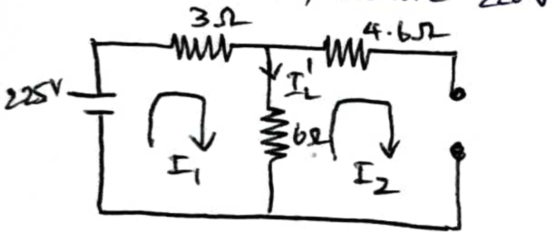
1) Determine the load current and load voltage by superposition theorem.



Solution:-

Step 1 :-

consider 225V, Remove 220V



$[R][I] = [V]$

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -6 \\ -6 & 10.6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 225 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 9 & -6 \\ -6 & 10.6 \end{vmatrix} = 59.4$$

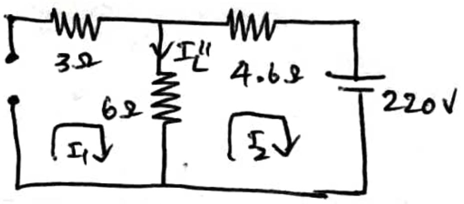
$$\Delta_1 = \begin{vmatrix} 225 & -6 \\ 0 & 10.6 \end{vmatrix} = 2385$$

$$\Delta_2 = \begin{vmatrix} 9 & 225 \\ -6 & 0 \end{vmatrix} = 1350$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2385}{59.4} = 40.15\text{ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{1350}{59.4} = 22.72\text{ A}$$

Step 2 :- consider 220V, Remove 225V



$[R][I] = [V]$

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -6 \\ -6 & 10.6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -220 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 9 & -6 \\ -6 & 10.6 \end{vmatrix} = 59.4$$

$$\Delta_1 = \begin{vmatrix} 0 & -6 \\ 220 & 10.6 \end{vmatrix} = -1320$$

$$\Delta_2 = \begin{vmatrix} 9 & 0 \\ -6 & -220 \end{vmatrix} = -1980$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{-1320}{59.4} = -22.22\text{ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-1980}{59.4} = -33.33\text{ A}$$

Step 3 :-

$I_L' = I_1 - I_2$

$= 40.15 - 22.72$

$\boxed{I_L' = 17.43\,A}$

$I_L'' = I_1 - I_2$

$= -22.22 - (-33.33)$

$\boxed{I_L'' = 11.11\,A}$

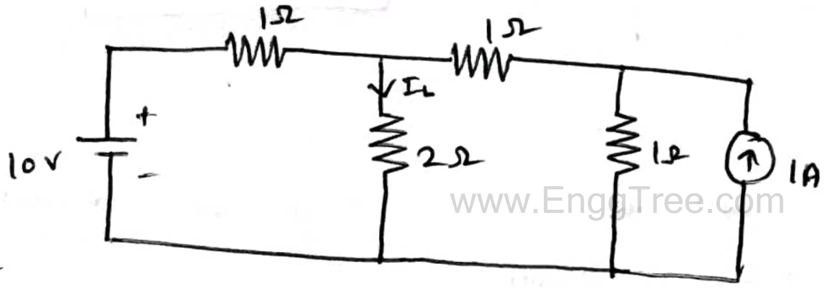
$I_L = I_L' + I_L''$

$= 17.43 + 11.11$

$\boxed{I_L = 28.54\,A}$

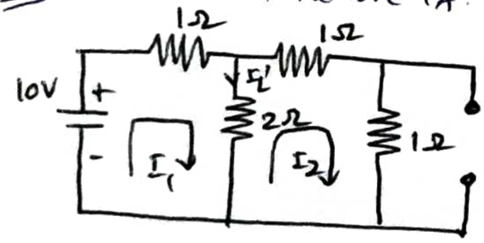
$V_L = I_L R_L = 28.54 \times 6 = 171.24\,V$

2) Determine the current through 2Ω resistor by using super position theorem.



Solution :-

Step 1: Consider 10V, Remove 1A.



$[R][I] = [V]$

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 \\ -2 & 4 \end{vmatrix} = 8$$

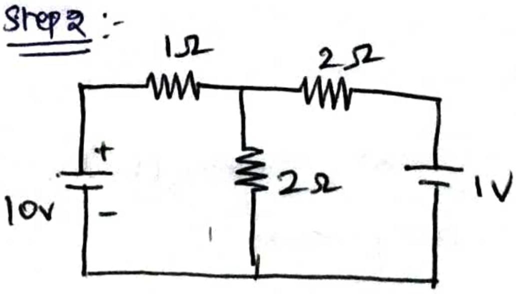
$$\Delta_1 = \begin{vmatrix} 10 & -2 \\ 0 & 4 \end{vmatrix} = 40$$

$$\Delta_2 = \begin{vmatrix} 3 & 10 \\ -2 & 0 \end{vmatrix} = 20$$

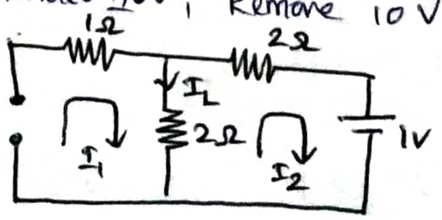
$I_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{40}{8} = 5A.$

$I_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{20}{8} = 2.5A$

Step 2 :-



Consider 1V, Remove 10V



$$[R][I] = [V]$$

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 \\ -2 & 4 \end{vmatrix} = 8$$

$$\Delta_1 = \begin{vmatrix} 0 & -2 \\ -1 & 4 \end{vmatrix} = -2$$

$$\Delta_2 = \begin{vmatrix} 3 & 0 \\ -2 & -1 \end{vmatrix} = -3$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{-2}{8} = -0.25$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-3}{8} = -0.375$$

Step 3 :-

$$I_L' = I_1 - I_2$$
$$= 5 - 2.5$$
$$\boxed{I_L' = 2.5A}$$

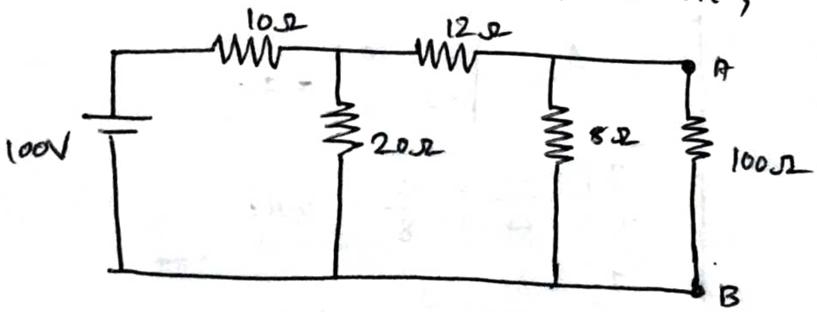
$$I_L'' = I_1 - I_2$$
$$= -0.25 - (-0.375)$$
$$\boxed{I_L'' = 0.125\ A}$$

$$I_L = I_L' + I_L'' = 2.5 + 0.125$$
$$\boxed{I_L = 2.625\ A}$$

PROBLEMS :-

THEVENIN'S THEOREM :-

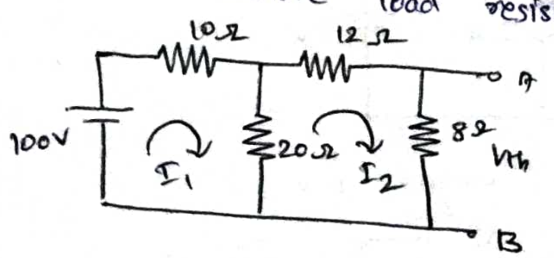
1) using thevenin's theorem, find the current through 100Ω resistor connected across terminals A & B in circuit,



Solution :-

Step 1 : To find V_{Th},

→ Remove load resistance (R_L)



$$[R][I] = [V]$$

$$\begin{bmatrix} 30 & -20 \\ -20 & 40 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 30 & -20 \\ -20 & 40 \end{vmatrix} = 1200 - 400 = 800$$

$$\Delta_1 = \begin{vmatrix} 100 & -20 \\ 0 & 40 \end{vmatrix} = 4000 + 0 = 4000$$

$$\Delta_2 = \begin{vmatrix} 30 & 100 \\ -20 & 0 \end{vmatrix} = 0 + 2000 = 2000$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4000}{800} = 5A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{2000}{800} = 2.5A.$$

$$V = IR$$

$$V_{th} = I_2 \times 8$$

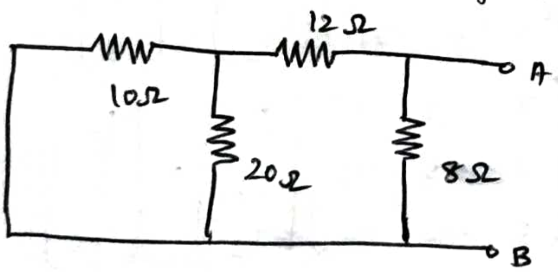
$$= 2.5 \times 8$$

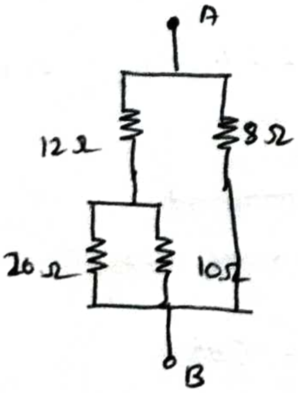
$$\boxed{V_{th} = 20V}$$

step 2 :- To find R_{th}

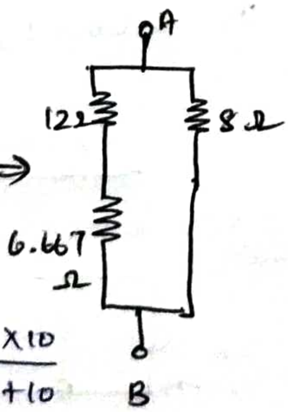
→ remove load resistance

→ short circuit the voltage source.



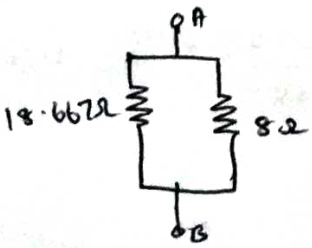


⇒

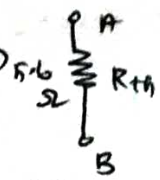


$$\frac{20 \times 10}{20+10}$$

$$= 6.667\,\Omega$$

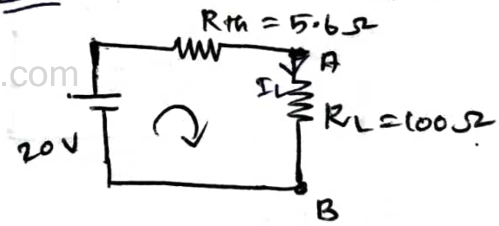


⇒



$$\boxed{R_{th} = 5.6\,\Omega}$$

step 3 :- Equivalent circuit.



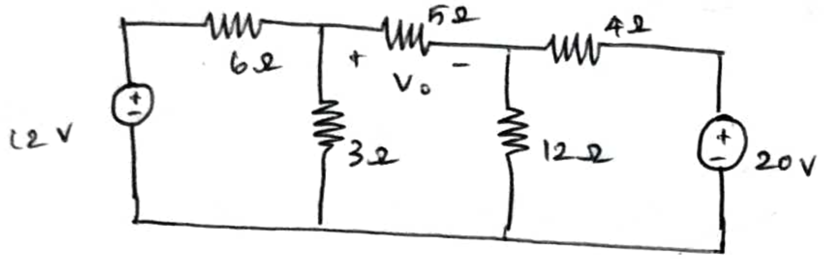
$$I_L = \frac{V_{th}}{R_{th}+R_L} = \frac{20}{5.6+100}$$

$$\boxed{I_L = 0.189\ A}$$

NORTON THEOREM :-

PROBLEM :-

1) Determine V_o using Norton's theorem.



To find R_N I_N

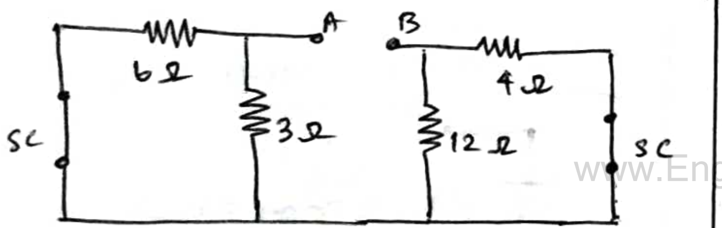
Solution :-

Step 1 :- To find R_N

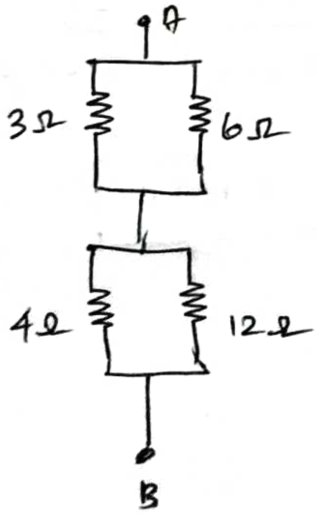
→ remove R_L

→ voltage sources are short circuited

→ current sources are open circuited



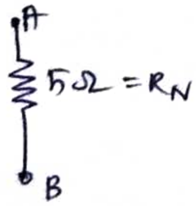
$R_L = 5\Omega$



$\frac{6 \times 3}{6+3}$

⇒

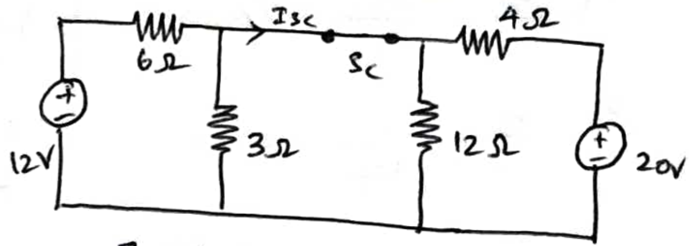
$\frac{4 \times 12}{4+12}$



$$\boxed{R_N = 5\Omega}$$

Step 2 :- To find I_N

→ short circuit the R_L



$$[R][I] = [V]$$

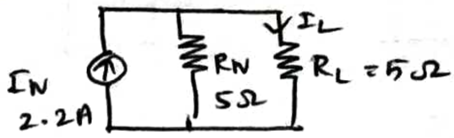
$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -3 & 0 \\ -3 & 15 & -12 \\ 0 & -12 & 16 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ -20 \end{bmatrix}$$

$I_1 = 0.6A$, $I_2 = -2.2A$, $I_3 = -2.9A$

$$\boxed{I_2 = I_N = 2.2A}$$

Step 3 :- Norton's equivalent circuit



$$I_L = \frac{I_N R_N}{R_N + R_L} = \frac{2.2 \times 5}{5+5} = 11/10 = 1.1A$$

$$V_o = I_L R_L$$

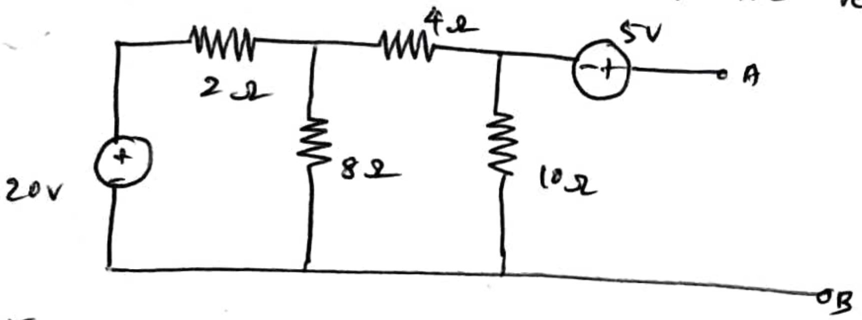
$$= 1.1 \times 5$$

$$\boxed{V_o = 5.5V}$$

MAXIMUM POWER TRANSFER THEOREM :-

PROBLEM :-

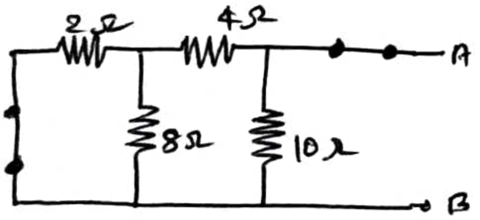
1) Determine the value of resistance that may be connected across A & B so that maximum power is transferred from circuit to the resistance. Also estimate maximum power transferred to the resistance shown in figure.

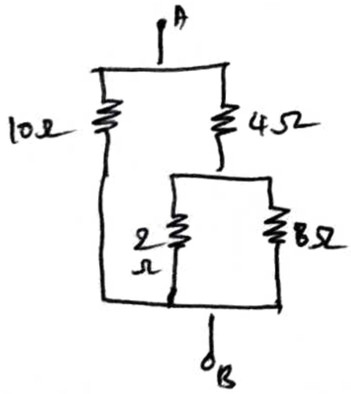


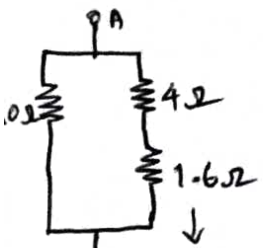
Solution :-

Step 1 :- To find R_{th}

→ voltage sources are short circuited

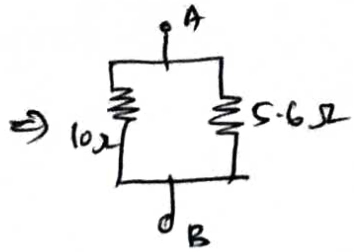




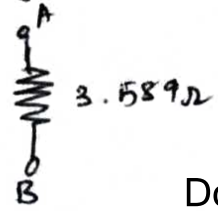


$\frac{2 \times 8}{2+8}$

⇒



⇓



$$\boxed{R_{th} = R_L = 3.589\ \Omega}$$

Step 2 :- To find V_{th}.

$$[R][I] = [V]$$

$$\begin{bmatrix} 10 & -8 \\ -8 & 22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$I_1 = 2.82A, \quad I_2 = 1.02A.$

Apply KVL to last loop

$$10(1.02) + 5 - V_{th} = 0$$

$$\boxed{V_{th} = 15.2\ V}$$

Step 3 :- To find P_{max}

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{(15.2)^2}{4 \times 3.589}$$

$$\boxed{P_{max} = 16.09\ W}$$

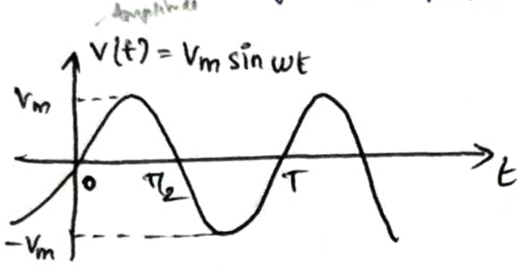
BM3352 - ELECTRIC CIRCUIT ANALYSIS.

UNIT - 3

SINUSOIDAL STEADY STATE ANALYSIS

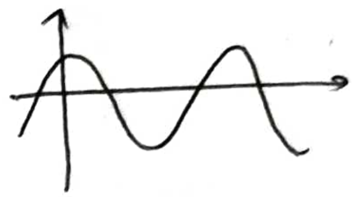
Sinusoids :-

⇒ A signal having the form of 'sine' or 'cosine' function.



$(V_m \sin\theta = V)$

$I_m \sin\theta = i$



$$v(t) = V_m \sin \omega t.$$

⇒ Magnitude is in either voltage / current.

⇒ This waveform is known by different terms,

1) Sinusoidal waveform.

2) Alternating quantity. (quantity may be V or I)

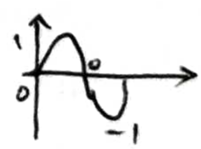
3) Instantaneous quantity (")

Sinusoidal waveform :-

⇒ Magnitude of induced voltage depends on 'SIN' of the angular position of the conductor in the magnetic field.

$V = V_m \sin\theta$ $\quad$ $\sin 0^\circ = 0^\circ$, $\sin 180^\circ = 0$

$i = I_m \sin\theta$. $\quad$ $\sin 90^\circ = 1$, $\sin 270^\circ = -1$



Alternating Quantity :-

⇒ The sign (+/-) of induced voltage (v) changes for every 180° degrees.

Instantaneous Quantity :-

⇒ The amount of induced voltage change for every instant. (i.e magnitude value is not constant).

$v(t \pm nT) = v(t)$ → condition for periodic signal.

⇒ If particular structure is repeated after infinite no. of times, then the signal is called periodic signal.

⇒ If we shift the signal by left (+) or right (−), it will be equal to the original signal.

$$\omega = 2\pi f$$

$$f = \omega/2\pi$$

$$f = 1/T$$

$$\omega/2\pi = 1/T$$

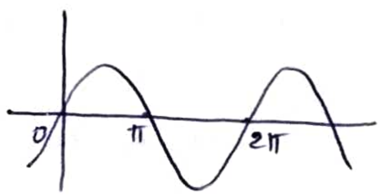
$$T = 2\pi/\omega \quad (or) \quad \omega = 2\pi/T.$$

⇒ If we replace t in x-axis by ωt,

if $t=0$, $\omega t = 0$

$t = T/2$, $\omega t = 2\pi/T \times T/2 = \pi$

$t = T$, $\omega t = 2\pi/T \times T = 2\pi$



⇒ Important general expression of sinusoid,

$$V(t) = V_m \sin(\omega t \pm \phi)$$

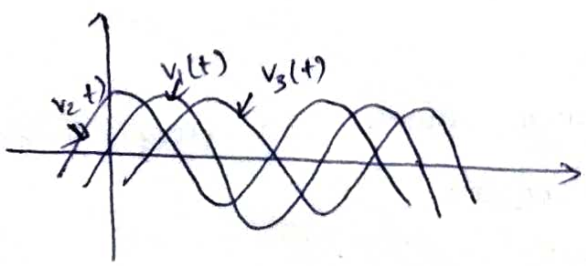
↳ phase shift

eg

$$V_1(t) = V_m \sin \omega t$$

$$V_2(t) = V_m \sin(\omega t + \phi) \text{ − leads}$$

$$V_3(t) = V_m \sin(\omega t - \phi) \text{ − lag}$$



CHARACTERISTICS OF SINUSOIDS :-

1) Periodic ⇒ sinusoidal waves repeat at regular intervals.

2) Amplitude / magnitude ⇒ It is the maximum value of its displacement from the equilibrium position, (i.e height of wave from the center line)

3) Wavelength ⇒ Distance between two successive crests or troughs of sinusoidal wave is known as wavelength.

4) Frequency ⇒ The no. of cycles or oscillations of sinusoidal wave that occurs in one second is known as frequency. It is measured in HZ.

5) Phase ⇒ It is the position of a wave at specific point in time. It is often measured in degrees or radians.

6) Shape ⇒ It has a smooth, curved shape. The shape of the waveform is a sine function.

7) Velocity ⇒ sinusoidal waves travels at specific velocity known as Phase velocity, which is determined by properties of medium through which wave is traveling and frequency and wavelength of the wave.

⇒ sinusoidal waves are found in many natural phenomena such as sound and light and they are also used to model and describe a wide range of physical and mechanical systems such as electrical circuits and mechanical vibrations.

PROBLEMS :- (sinusoids)

1) calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$, state which sinusoid is leading.

Solution :-

Condition to compare 2 sinusoids

→ it should have same frequency

→ same form (cos, sin)

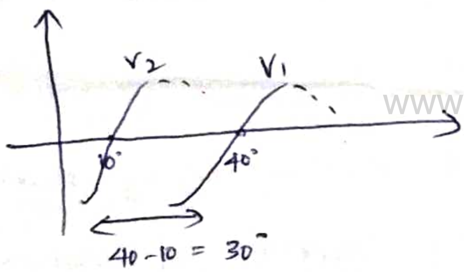
→ same sign

$$\underbrace{-V_m \cos\theta}_{V_1} = V_m \sin(\theta - 90^\circ)$$

$$V_1 = 10 \sin(\omega t + 50^\circ - 90^\circ)$$

$$\boxed{v_1 = 10 \sin(\omega t - 40^\circ)}$$

$$V_2 = 12 \sin(\omega t - 10^\circ)$$



Ans ⇒ v_2 is leading v_1 by 30°.

2) $v(t) = 5\sin(4\pi t - 60^\circ)$. calculate amplitude, phase, angular frequency, period and frequency.

Solution :-

$$v(t) = 5\sin(4\pi t - 60^\circ)$$

$$v(t) = V_m \sin(\omega t - \phi)$$

$V_m = 5V$ → Amplitude

$\omega = 4\pi$ rad/sec → angular frequency

$\phi = 60^\circ$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = 1/2 = 0.5$ sec

$f = \frac{1}{T} = 2$ Hz.

3) Find Amplitude, Phase & frequency of sinusoid, $v(t) = 12\cos(50t + 10^\circ)$.

Solution :-

$v(t) = V_m \cos(\omega t + \phi)$.

Amplitude $\Rightarrow V_m = 12\,V$

Phase $\Rightarrow \phi = 10^\circ$

Angular frequency $\Rightarrow \omega = 50$ rad/sec.

Time $\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.125$ sec.

Frequency $\rightarrow f = 1/T = 7.958$ HZ.

STEADY STATE ANALYSIS & TRANSIENT ANALYSIS :-

$\Rightarrow$ In a steady-state process, the response of the system does not change over time.

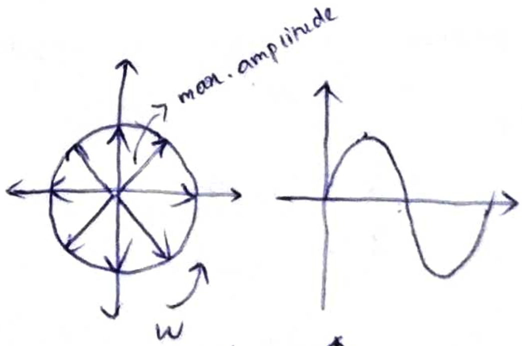
$\Rightarrow$ In transient analysis, this response is time dependent

PHASORS :-

$\Rightarrow$ A complex number that represents the amplitude and phase of a sinusoid is called as phasor. Phasor is nothing but a vector which rotates around its origin at a constant speed of ω-rad/sec in anti-clockwise direction.

(e.g) $v(t) = 3\cos(\omega t + 30^\circ)$

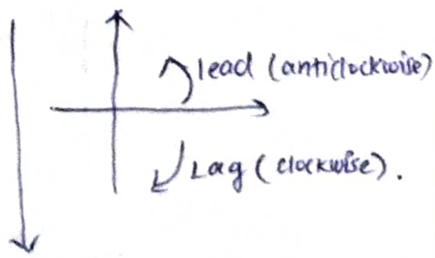
$\bar{V} = 3\angle 30^\circ$ (or) $\bar{V} = 3/\sqrt{2}\angle 30^\circ$



Rectangular form of complex numbers :-

$z = Re(z) + j\,Im(z)$

$$\left.\begin{array}{l} Re(z) = x \\ Im(z) = y \end{array}\right\} \quad z = x + jy$$



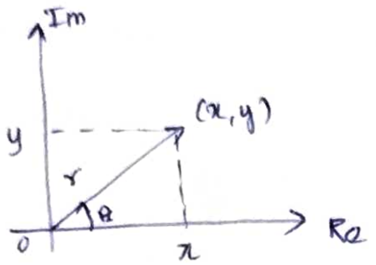
Polar and exponential forms of complex numbers :-

$z = r\angle\theta$ $\quad r \rightarrow$ magnitude, $\theta \rightarrow$ phase.

$z = re^{j\theta}$

(If we take the projection of this vector, we can able to get instantaneous value of sinusoidal signal)

⇒ Relation between polar & rectangular forms :-



$Z = x + iy$,

$r = \sqrt{x^2+y^2}$

$\tan\theta = y/x$ | $\cos\theta = x/r$ | $\sin\theta = y/r$

$\theta = \tan^{-1} y/x$ | $x = r\cos\theta$ | $y = r\sin\theta$.

$Z = r(\cos\theta + j\sin\theta)$.

⇒ phasor representation :-

$e^{j\theta} = \cos\theta + j\sin\theta$ (Phasor representation is based on Euler's identity).

$Re(e^{j\theta}) = \cos\theta$ and $Im(e^{j\theta}) = \sin\theta$

$$\boxed{v(t) = V_m\cos(\omega t+\theta)}$$

$v(t) = Re(V_m e^{j(\omega t+\theta)})$

$v(t) = Re(\underbrace{V_m e^{j\omega t}}\cdot e^{j\theta})$

$v(t)$ — time dependent quantity

$\bar{V} = V_m e^{j\theta}$

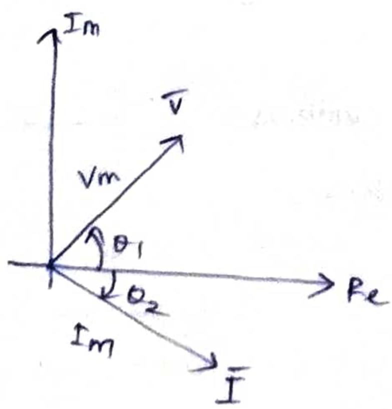
$\bar{V}$ — time independent quantity

$$\boxed{\bar{V} = V_m\angle\theta}$$

→ phasor analysis is applicable only when frequency of signals is same.

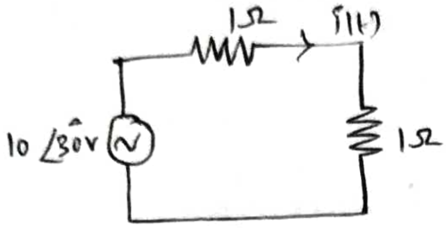
$\bar{V} = V_m\angle\theta_1$

$\bar{I} = I_m\angle-\theta_2$



PHASORS (PROBLEMS).

1) calculate current i(t)



Solution :-

$V(t) \rightarrow \bar{V} = 10\angle 30^\circ$
↓
V_{rms}

$i(t) = \frac{v(t)}{1+1} = \frac{v(t)}{2}$ A

$\bar{I} = \frac{\bar{V}}{2}$

$\bar{I} = \frac{10\angle 30^\circ}{2}$ A

$\bar{I} = 5\angle 30^\circ$ A

$i(t) = I_m \cos(\omega t + \theta)$

$I_{rms} = 5A.$

$\boxed{I_{rms} = I_m/\sqrt{2}}$

$I_m = I_{rms} \times \sqrt{2}$

$I_m = 5\sqrt{2}$

$\boxed{i(t) = 5\sqrt{2}\ \cos(\omega t + 30^\circ)A}$

3) convert these sinusoids to phasors.

$v = 7\cos(2t + 30^\circ)V$

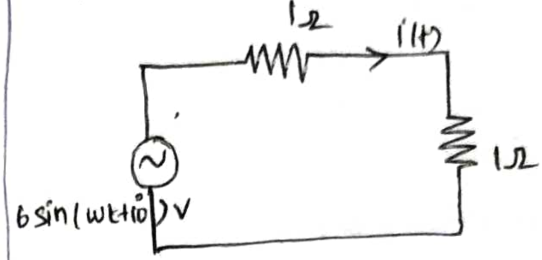
Solution :-

$v = 7\cos(2t + 30^\circ)$

$\bar{V} = 7\angle 30^\circ$ V

$\bar{V} = 7/\sqrt{2}\ \angle 30^\circ$ V (in terms of rms value)

2) calculate i(t)



Solution :-

$v(t) = 6\sin(\omega t + 10^\circ)$

$\bar{V} = 6\angle 10^\circ$ V

$\bar{I} = \frac{\bar{V}}{2}$ A

$\bar{I} = \frac{6\angle 10^\circ}{2}$ A

$\bar{I} = 3\angle 10^\circ$ A

$I_m = 3,$

$i(t) = I_m \sin(\omega t + \theta)$

$\boxed{i(t) = 3\sin(\omega t + 10^\circ)\ A}$

4) convert these sinusoids to phasors :-

$i = -4\sin(10t + 10^\circ)A.$

Solution :-

(std. form.
$i = I_m\cos(\omega t \pm \theta).$

$-I_m \sin\theta = I_m \cos\theta$ (left shift by 90°)

$= I_m \cos(\theta + 90^\circ)$

$i = 4\cos(10t + 10^\circ + 90^\circ)$

$i = 4\cos(10t + 100^\circ)$

$\bar{I} = 4\angle 100^\circ$ A

$\bar{I} = 4/\sqrt{2}\ \angle 100^\circ$ A → in terms of rms value

5) Find the sinusoids represented by phasors :-

$$\bar{V} = -3 + j4 \, V$$

Solution :-

$$\bar{V} = -3 + j4 \, V$$

$$v(t) = V_m \cos(\omega t \pm \theta)$$

$$V_m = \sqrt{x^2+y^2} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$$

$$V_m = 5V$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(4/-3) = -53.14^\circ$$

$$= -53.14^\circ + 180^\circ$$

$$= 126.86^\circ.$$

	x	y	
I →	+ve	+ve	
II →	−	+	} 180°
III →	+	−	
IV →	$\underline{+}$	−	} 360°

$$\boxed{v(t) = 5\cos(\omega t + 126.86^\circ)}$$

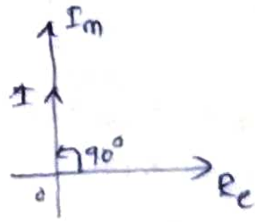
6) Find sinusoids represented by phasors :-

$$\bar{V} = j8 e^{-j20^\circ} V$$

solution :-

Polar form → $j8 \angle -20^\circ V$

complex form → $\underline{z = 0 + j1}$ $(1\angle 90^\circ)$



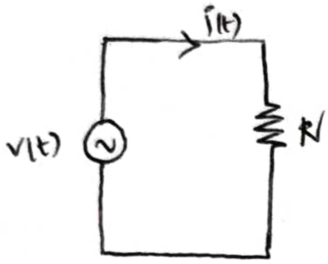
$$= (1\angle 90^\circ) \times (8\angle -20^\circ)$$

$$= 1 \times 8 \, (\angle 90^\circ + (-20^\circ))$$

$$\bar{V} = 8\angle 70^\circ$$

$$\boxed{v(t) = 8\cos(\omega t + 70^\circ) V}$$

Phasor Relationship for Resistor :-



$$v(t) = V_m \cos(\omega t + \theta)$$

$$\bar{V} = V_m \angle \theta$$

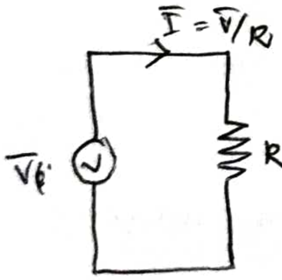
$$v(t) = R\, i(t)$$

$$i(t) = \frac{v(t)}{R} = \frac{V_m \cos(\omega t + \theta)}{R}$$

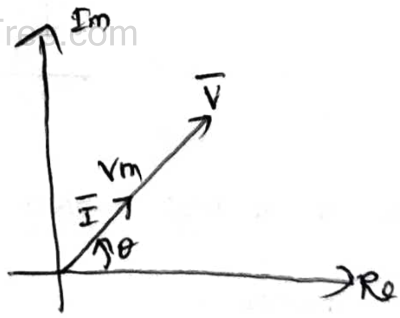
$$\bar{I} = \frac{V_m}{R} \angle \theta .$$

$$\bar{I} = \frac{\bar{V}}{R} \quad (\because \bar{V} = V_m \angle \theta)$$

$$\bar{V} = R \bar{I}$$

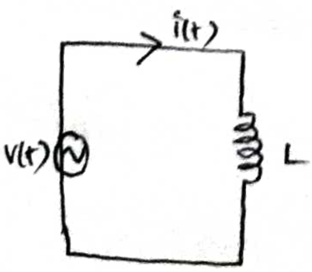


(phasor representation in circuit)



=) In case of resistor, voltage and current are in same phase.

Phasor Relationship for Inductor :-



$$i(t) = I_m \cos(\omega t + \theta)$$

$$\downarrow$$

$$\bar{I} = I_m \angle \theta .$$

$$v(t) = L\frac{di(t)}{dt}$$

$$= L\frac{d}{dt}\left[I_m\cos(\omega t+\theta)\right]$$

$$= L I_m \, d/dt\left[\cos(\omega t+\theta)\right]$$

$$v(t) = -\omega L I_m \sin(\omega t+\theta)$$

$$v(t) = \omega L I_m \cos(\omega t+\theta+90^\circ)$$

$$\bar{V} = \omega L I_m \angle \theta+90^\circ$$

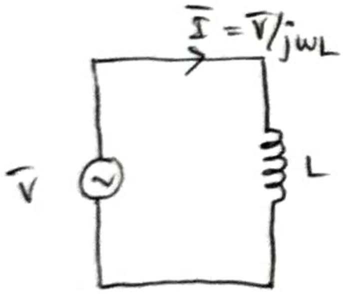
To obtain relation b/w $\bar{V}$ & $\bar{I}$

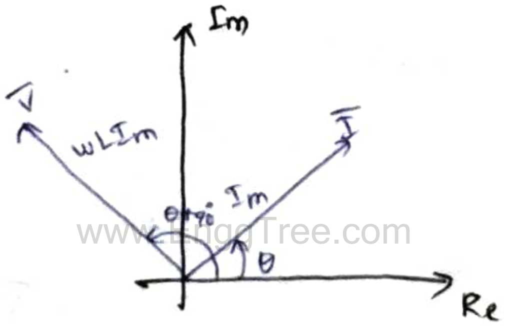
$$\bar{V} = 1 \cdot \omega L I_m \angle\theta+90^\circ$$

$$= 1\angle 90^\circ \cdot \omega L I_m \angle\theta. \qquad (\because 1\angle 90^\circ = j)$$

$$(I_m\angle\theta = \bar{I})$$

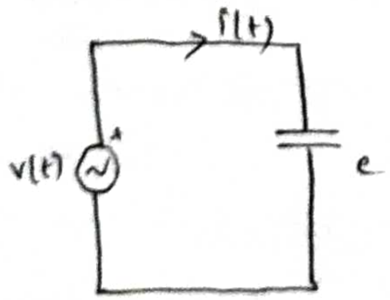
$$\bar{V} = j\omega L \bar{I}$$





$\Rightarrow$ voltage is leading current by 90° / current is lagging voltage by 90°, in case of Inductors.

Phasor Relationship for capacitor :-



To obtain relation b/w $\bar{V}$ & $\bar{I}$.

$$v(t) = V_m\cos(\omega t+\theta) \Rightarrow \bar{V} = V_m\angle\theta.$$

$$i(t) = C\cdot\frac{d\,v(t)}{dt}$$

$$= C \cdot \frac{d}{dt}\left[V_m \cos(\omega t + \theta)\right]$$

$$i(t) = -\omega C V_m \sin(\omega t + \theta)$$

$$i(t) = \omega C V_m \cos(\omega t + \theta + 90^\circ).$$

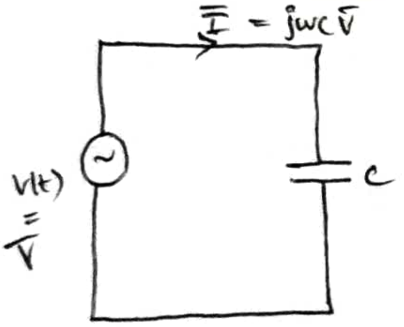
compare $v(t)$ & $i(t)$.

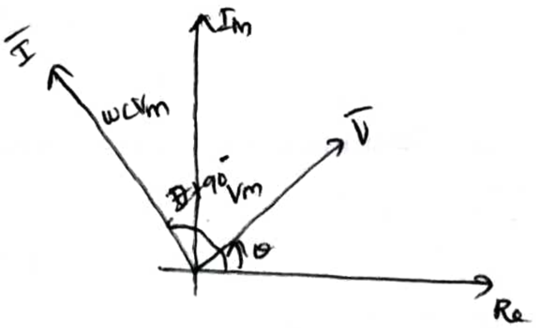
$$\bar{I} = \omega C V_m \angle \theta + 90^\circ$$

$$\bar{I} = (1\angle 90^\circ)\, \omega C V_m \angle \theta$$

$$\bar{I} = j\omega C V_m \angle \theta$$

$$\boxed{\bar{I} = j\omega C \bar{V}} \qquad (\because V_m \angle \theta = \bar{V})$$





⇒ In case of capacitors, current is leading voltage by 90° | voltage is lagging the current by 90°.

Instantaneous power :-

⇒ The electric power at any instant of time is known as instantaneous power.

⇒ Measured in watts.

⇒ The instantaneous power p(t) absorbed by an element is equal to the product of instantaneous voltage v(t) across the element and instantaneous current i(t) through it.

$$P(t) = v(t) \times i(t)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$\therefore P(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$\left[\because 2\cos A\cos B = \cos(A-B) + \cos(A+B) \right.$$
$$\left. \cos A\cos B = \tfrac{1}{2}\cos(A-B) + \tfrac{1}{2}\cos(A+B) \right]$$

$A = \omega t + \theta_v$

$B = \omega t + \theta_i$

$$p(t) = \underbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}_{\text{time independent.}} + \frac{V_m I_m}{2}\cos(\underbrace{2\omega t}_{\text{twice of angular frequency}} + \theta_v + \theta_i)$$

$\Rightarrow$ Instantaneous power will have twice of angular frequency as compared to voltage & current.

Average power :-

$\Rightarrow$ It is the average of instantaneous power over one period.

$$P_{av} = \frac{1}{T}\int_0^T p(t)\,dt$$

$t \to \omega t$

$t=0, \quad \omega = 0$

$t = T, \quad \omega t = 2\pi/T \times T = 2\pi$

$$= \frac{1}{2\pi}\int_0^{2\pi} p(t)\,d\omega t.$$

$$P_{av} = \frac{1}{T}\int_0^T \left[\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\right] dt.$$

$$= \frac{1}{T}\cdot\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\int_0^T dt + \frac{1}{T}\cdot\frac{1}{2}V_m I_m \int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$= \frac{1}{\cancel{T}}\cdot\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\underbrace{[t]_0^T}_{(\cancel{T}-0)} + \frac{1}{T}\cdot\underbrace{\frac{1}{2}V_m I_m \int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt}_{0}$$

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + 0$$

[$\because$ Integrating sinusoidal function over one period, we get zero]

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$P_{av} = \frac{V_m}{\sqrt{2}}\cdot\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i) \longrightarrow \text{in terms of rms value.}$$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cos(\theta_v - \theta_i)}$$

case (i) :- $\theta_v = \theta_i$ (Both v & I are in same phase)

$$P_{av} = \frac{1}{2} V_m I_m \cos 0°$$

$$\boxed{P_{av} = \frac{1}{2} V_m I_m \text{ (or) } \frac{1}{2} I_m^2 R} \quad \text{for R}$$

case (ii) $\theta_v - \theta_i = \pm 90°$.

$P_{av} = 0$ $(\because \cos 90° = 0)$ for L & C

Apparent power and Power factor :-

W.K.T

$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P_{av} = \underbrace{V_{rms} I_{rms}}_{\substack{\downarrow \\ S}} \underbrace{\cos(\theta_v - \theta_i)}_{\substack{\downarrow \\ pf}}$$

Apparent power ← S

$$S = V_{rms} \cdot I_{rms}$$

$$P = S \cdot \cos(\theta_v - \theta_i)$$

$$P = S \cdot pf$$

$$pf = P/S = \cos\underbrace{(\theta_v - \theta_i)}_{\substack{\downarrow \\ \text{pf angle.}}}$$

$$v(t) = V_m \cos(\omega t + \theta_v) = \bar{V} = V_m \angle \theta_v$$

$$i(t) = I_m \cos(\omega t + \theta_i) = \bar{I} = I_m \angle \theta_i$$

$$Z = \frac{\bar{V}}{\bar{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i$$

$\therefore$ power factor angle will be equal to angle of load impedance.

$\Rightarrow 0 \le pf \le 1$.

case (i) purely resistive load :- $\theta_v = \theta_i$

$$pf = \cos 0°$$

$$pf = 1$$

$$\therefore \boxed{S = P.}$$

Apparent power = Avg. power

case (ii) purely reactive load :- $\theta_v - \theta_i = \pm 90°$

$$pf = \cos \pm 90°$$

$$pf = 0$$

$$P = 0$$

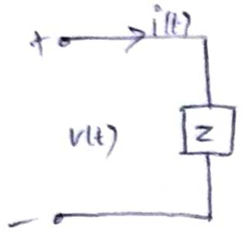
$\Rightarrow$ leading p.f $\rightarrow$ I leads V $\Rightarrow$ capacitive load

lagging p.f $\rightarrow$ V leads I $\Rightarrow$ Inductive load

complex power:-

⇒ the product of rms voltage phasor and complex conjucate of rms current phasor is known as complex power.

⇒ Denoted by $\bar{S}$.



$v(t) \rightarrow V_{rms},\ \theta_v$

$\bar{V}_{rms} = V_{rms} \angle \theta_v$

$i(t) \rightarrow I_{rms},\ \theta_i$

$\bar{I}_{rms} = I_{rms} \angle \theta_i$

$\bar{I}^*_{rms} = I_{rms} \angle -\theta_i$

$$\boxed{\therefore\ \bar{S} = \bar{V}_{rms}\ \bar{I}^*_{rms}}$$

$$= V_{rms} \angle \theta_v \times I_{rms} \angle -\theta_i$$

$$\bar{S} = \underbrace{V_{rms}\, I_{rms}}_{|\bar{S}|\ \text{magnitude.}} \angle \theta_v - \theta_i$$

$|\bar{S}| = S$

proof :-

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P = Re\left\{\frac{1}{2} V_m I_m e^{j(\theta_v - \theta_i)}\right\}$$

$$P = Re\left\{\frac{1}{2} V_m I_m e^{j\theta_v} e^{-j\theta_i}\right\}$$

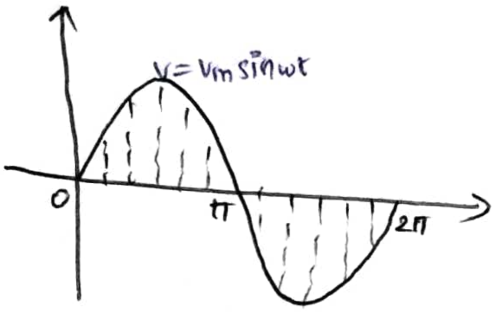
$$P = Re\left\{\frac{V_m}{\sqrt{2}} e^{j\theta_v} \cdot \frac{I_m}{\sqrt{2}} e^{-j\theta_i}\right\}$$

$$P = Re\{\underbrace{V_{rms} \angle \theta_v \cdot I_{rms} \angle -\theta_i}_{\text{(complex power)}}\}$$

$$\bar{S} = \bar{V}_{rms}\ \bar{I}^*_{rms}$$

RMS Value :-

Definition :- RMS value is an effective value of AC quantities which needs to be considered for different types of calculations and assumption and it is also equivalent to its DC value.



$$V_{rms} = V = \sqrt{\int_0^{2\pi} \frac{(\text{area under the curve})^2}{\text{Base}}}$$

$$= V = \sqrt{\int_0^{2\pi} \frac{(V_m \sin\omega t)^2}{2\pi} \cdot d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \int_0^{2\pi} \sin^2\omega t \cdot d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \int_0^{2\pi} \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t} \qquad \left(\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$$

$$V_{rms} = V = \sqrt{\frac{V_m^2}{4\pi} \int_0^{2\pi} (1-\cos 2\omega t)\, d\omega t}$$

$$= \sqrt{\frac{V_m^2}{4\pi} \int_0^{2\pi} 1 \cdot d\omega t - \int_0^{2\pi} \cos 2\omega t \cdot d\omega t}$$

$$= \sqrt{\frac{V_m^2}{4\pi} (\omega t)_0^{2\pi} - \left(\frac{\sin 2\omega t}{2}\right)_0^{2\pi}} \quad (0)$$

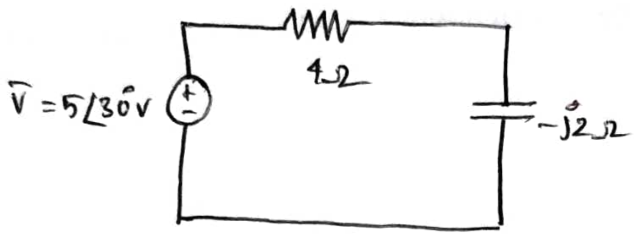
$$= \sqrt{\frac{V_m^2}{4\pi} [2\pi - 0]}$$

$$= \sqrt{V_m^2/4\pi \times 2\pi} = \sqrt{\frac{V_m^2}{2}}$$

$$\boxed{V_{rms} = V = \frac{V_m}{\sqrt{2}}} \quad (\text{or}) \quad \boxed{V_{rms} = V = 0.707\, V_m}$$

PROBLEMS :-

1) For the circuit, find average power supplied by the source and average power absorbed by the resistor.



solution :-

$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$Z = 4 + (-j2) \text{ ohm}$$

$$Z = 4 - j2 \text{ ohm}$$

↓

convert this into polar form

$$Z = \sqrt{4^2 + (-2)^2} \angle \tan^{-1}(-2/4)$$

$$Z = 4.472 \angle -26.56^\circ \Omega$$

$$\bar{I} = \frac{\bar{V}}{Z} = \bar{I} = \frac{5\angle 30^\circ}{4.472\angle -26.57^\circ} = \underbrace{1.118}_{I_m} \angle \underbrace{56.57^\circ}_{\theta_i} A$$

$$P_{av} = \frac{1}{2} \times 5 \times 1.118 \cos(30^\circ - 56.57^\circ)$$

$P_{av} = 2.5W = P_{av}(\text{resistor})$ [∵ capacitor will not absorb any avg. power, so avg. power delivered by source is only observed by resistor]

ELECTRIC CIRCUIT ANALYSIS

UNIT - 4

TRANSIENTS AND RESONANCE IN RLC CIRCUITS

Steady state response :-

⇒ Circuit having constant sources is said to be in steady state if currents and voltages do not change with time.

⇒ Circuits with constant current and voltages has constant amplitude and frequency, produces steady state response.

Transient state response :-

⇒ The behaviour of voltage or current when it is changed from one state to another is called transient state.

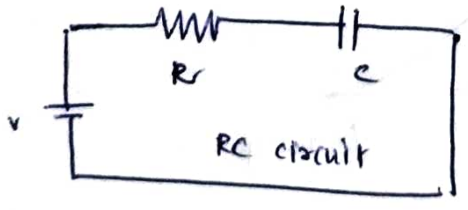
⇒ Time taken for the circuit to change from one steady state to another steady state is called transient time.

BASIC RC & RL CIRCUITS :-

RC circuits :-

⇒ A capacitor and resistor will be linked in series or parallel to a voltage or current source in RC circuit (Resistor capacitor circuit).

⇒ most typically utilized in filtering applications, these circuits are also known as RC filters.



RC circuit

Uses of RC circuits :-

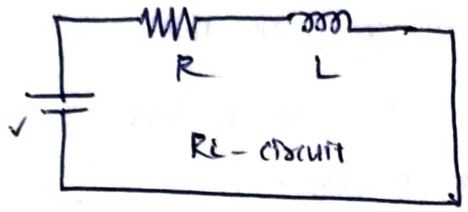
⇒ windshield wipers

⇒ Pacemakers

RL circuits :-

⇒ It will be made up of inductor and resistor which will be linked in series or parallel.

⇒ voltage source will drive a series RL circuit, whereas current source will drive a parallel RL circuit.



uses of RL circuit

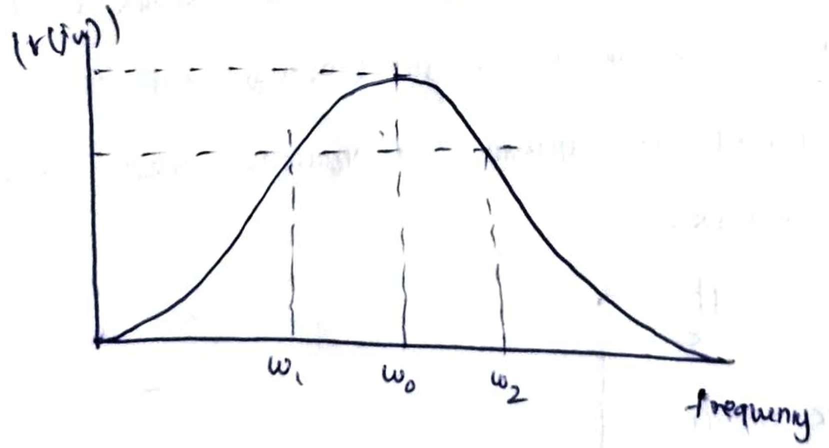
⇒ communication systems

⇒ signal processing

⇒ radio wave transmitters

Frequency response :-

⇒ It is the plot of magnitude of output voltage of resonance circuit as function of frequency. The response of curve starts at zero, reaches a maximum value in the vicinity of natural resonant frequency and then drops again to zero as ω becomes infinite.



Resonance in AC circuit :-

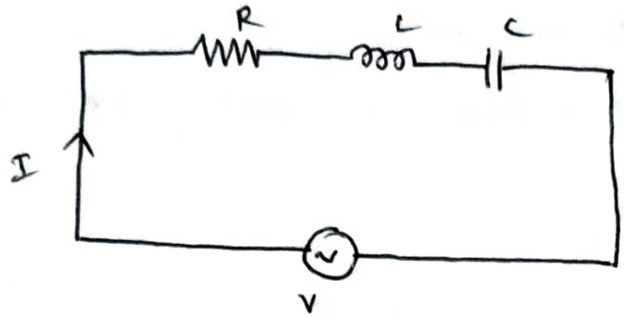
⇒ A series RLC circuit is said to be in resonance, when it behaves like a pure resistive circuit. (i.e) under resonance the applied voltage and source current are in phase.

Types of resonance :-

1. series resonance.
2. parallel resonance.

Series resonance	Parallel resonance.
When R, L, C are connected in series across an AC supply, then L & C cancel the effect of each other at particular frequency, then this condition of series circuit is known as series resonance.	A combination of R, L, C is connected across an AC source and L & C cancel the effect of each other at specific supply frequency, then this condition of parallel RLC circuit is known as parallel resonance.
Impedance of series RLC circuit becomes minimum at series resonance	Impedance of parallel RLC circuit becomes maximum at parallel resonance.
current flows through the circuit is maximum.	current in a circuit is minimum
Q - factor $= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C}$	Q - factor $= \frac{R}{\omega_0 L} = \omega_0 R C$
used in tuning, oscillator circuits, voltage amplifiers.	used in current amplifiers, induction heating, filters.

SERIES RESONANCE :-



For a series RLC circuit, the impedance is

$$Z = R + jX_L - jX_C$$

$$= R + j\omega L - \frac{j}{\omega C} \qquad \left(\because X_L = \omega L,\ X_C = \frac{1}{\omega C}\right)$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

But under resonance condition,

$$Z = R$$

$$R = R + j(\omega L - 1/\omega C)$$

$$\boxed{0 = j(\omega L - 1/\omega C)}$$

$\therefore$ The imaginary term must be zero

(i.e) $\omega = \omega_r$, $Z = R$

$$\omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

But $\omega_r = 2\pi f_r$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

$f_r \rightarrow$ resonant frequency.

Characteristics of series resonance :-

1) The impedance is minimum and is given by $Z = R$,

2) since impedance is minimum, current is maximum,

$$I_r = \frac{V}{Z_r}$$

$$I_r = \frac{V}{R}$$

3) Power is also maximum

$$P_r = I_r^2 Z$$

$$= \frac{V^2}{R^2} \times R$$

$$\boxed{P_r = \frac{V^2}{R}}$$

4) voltage across L & C are large, because current is maximum

$$V_L = I X_L$$

$$= I_r X_L$$

$$\boxed{V_L = \frac{V}{R} X_L}$$

$$V_C = I X_C$$

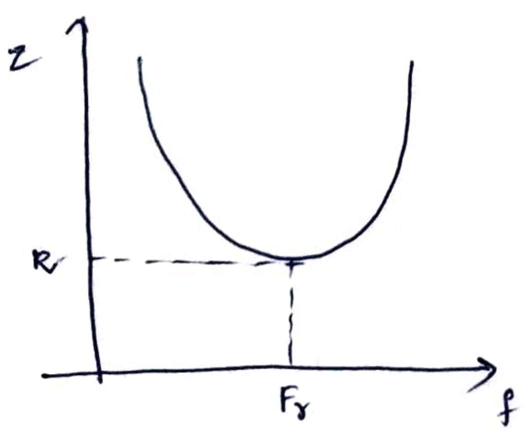
$$= I_r X_C$$

$$\boxed{V_C = \frac{V}{R} X_C}$$

5) A series resonant circuit draws heavy current and power from the mains, hence it is also called "Acceptor circuit."

6) A series resonance is avoided in power system applications, because heavy voltage across L & C may cause damages.

Impedance Vs frequency

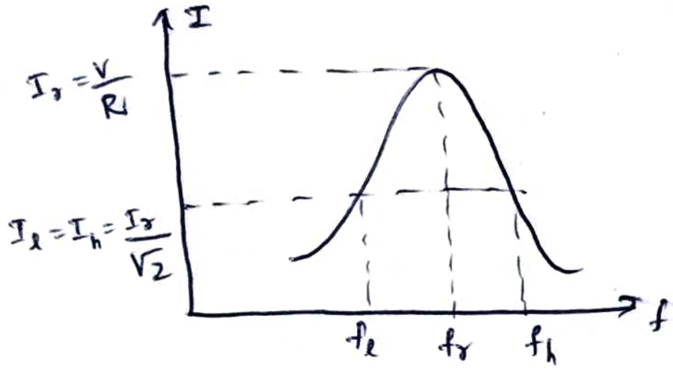


when $f = f_r$ → Impedance is resistive, power factor is unity

$f > f_r$ → Impedance is inductive, Power factor is leading in nature

$f < f_r$ → Impedance is capacitive, power factor is lagging in nature.

Derivation on half-power frequencies, Bandwidth & Q-factor



(I vs f curve)

Half - power frequencies

- f_l → lower half power frequency
- f_h → higher half power frequency

At half power frequency,

→ power is half the value at resonance

→ current is equal to $\frac{I_r}{\sqrt{2}}$

From I vs f curve

At $f = f_r$, $I_r = V/R$

At $f = f_l = f_h$, $I_l = I_h = I_r/\sqrt{2} = \frac{V}{R\sqrt{2}}$ → (1)

$$I_l = I_h = \frac{V}{|Z|} \rightarrow (2)$$

equate (1) & (2)

$$\frac{V}{|Z|} = \frac{V}{\sqrt{2}\,R}$$

$$|Z| = \sqrt{2}\,R$$

$$|Z|^2 = 2R^2 \rightarrow (3)$$

But for a series RLC circuit,

$$Z = R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} \rightarrow (4)$$

sub (4) in (3)

$$R^2 + (X_L - X_C)^2 = 2R^2$$

$$(X_L - X_C)^2 = 2R^2 - R^2$$

$$(X_L - X_C)^{\cancel{2}} = R^{\cancel{2}}$$

$$|X_L - X_C| = R \rightarrow (5)$$

At f_l, the impedance is capacitive in nature, X_C is greater than X_L

when $f = f_l$

$$X_L - X_C = R$$

$$L\omega_l - \frac{1}{C\omega_l} = -R$$

$$\frac{LC\omega_l^2 - 1}{C\omega_l} = -R$$

$$LC\omega_l^2 - 1 = -RC\omega_l$$

$$LC\omega_l^2 + RC\omega_l - 1 = 0 \qquad (\because s^2 + s + c = 0)$$

By solving equation.

$$a = LC, \quad b = RC, \quad c = -1$$

$$\omega_l = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-RC \pm \sqrt{(RC)^2 - 4(LC)(-1)}}{2LC}$$

$$= \frac{-RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

$$= \frac{-RC}{2LC} + \frac{\sqrt{R^2C^2 \pm 4LC}}{2LC}$$

$$= \frac{-R}{2L} \pm \sqrt{\frac{R^2C^2}{4L^2C^2} + \frac{4LC}{4L^2C^2}}$$

$$= \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$\omega_l = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\because (R/2L)^2 << 1/LC \quad \text{— neglected}$$

$$\omega_l = \frac{-R}{2L} + \sqrt{\frac{1}{LC}}$$

$$\omega_l = \frac{-R}{2L} + \frac{1}{\sqrt{LC}}$$

$$\omega_l = 2\pi f_l$$

$$2\pi f_l = \frac{-R}{2L} + \frac{1}{\sqrt{LC}}$$

$$f_l = \frac{1}{2\pi} \times \frac{-R}{2L} + \frac{1}{2\pi\sqrt{LC}}$$

$$\left(\because f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$f_l = \frac{-R}{4\pi L} + f_r$$

$$\boxed{f_l = f_r - \frac{R}{4\pi L}}$$

At higher half power frequency f_h, impedance is Inductive in nature. (i.e) $X_L > X_C$

$$f = f_h$$

$$X_L - X_C = R$$

$$L\omega_h - \frac{1}{C\omega_h} = R$$

$$\frac{LC\omega_h^2 - 1}{C\omega_h} = R$$

$LC\omega_h^2 - 1 = RC\omega_h$

$LC\omega_h^2 - RC\omega_h - 1 = 0 \quad (\because s^2 + s + c = 0)$

$a = LC, \ b = -RC, \ c = -1$

$$\omega_h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-RC) \pm \sqrt{(-RC)^2 - 4(LC)(-1)}}{2(LC)}$$

$$= \frac{RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

$$= \frac{2C}{2LC} \pm \frac{\sqrt{R^2C^2 + 4LC}}{2LC}$$

$$= \frac{R}{2L} \pm \sqrt{\frac{R^2C^2}{4^2C^2L^2} + \frac{4LC}{4L^2C^2}}$$

$$= \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$= \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

neglected

$$\omega_h = \frac{R}{2L} + \frac{1}{\sqrt{LC}}$$

$$\omega_h = 2\pi f_h$$

$$2\pi f_h = \frac{R}{2L} + \frac{1}{\sqrt{LC}}$$

$$f_h = \frac{1}{2\pi} \times \frac{R}{2L} + \frac{1}{2\pi\sqrt{LC}}$$

$$\left(\because f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$\boxed{f_h = f_r + \frac{R}{4\pi L}}$$

Bandwidth :-

⇒ It is defined as difference between half-power frequencies.

$$\boxed{BW = f_h - f_l}$$

$$= f_r + \frac{R}{4\pi L} - f_r + \frac{R}{4\pi L}$$

$$= \frac{2R}{4\pi L}$$

$$\boxed{BW = \frac{R}{2\pi L}}$$

Q- factor :-

⇒ Ratio b/w resonant frequency and bandwidth

$$Q = \frac{f_r}{BW} \quad \text{or} \quad \frac{f_r}{f_h - f_l}$$

$$Q = \frac{\frac{1}{\cancel{2}\pi\sqrt{LC}}}{\frac{R}{\cancel{2}\pi L}}$$

$$Q = \frac{L}{R\sqrt{LC}} = \frac{\sqrt{L}\sqrt{L}}{R\sqrt{L}\sqrt{C}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Selectivity :-

⇒ ratio b/w BW and f_r.

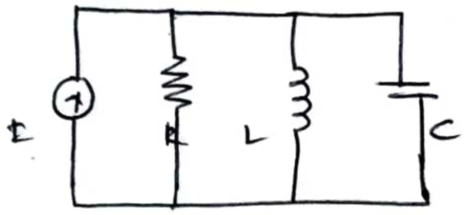
$$\text{Selectivity} = \frac{BW}{f_r}$$

$$= \frac{R/2\pi L}{\frac{1}{2\pi\sqrt{LC}}} = \frac{R}{\cancel{2\pi} L} \times \cancel{2\pi}\sqrt{LC} = \frac{R\sqrt{LC}}{L}$$

$$\boxed{\text{Selectivity} = R\sqrt{C/L}}$$

PARALLEL RESONANCE :-

Case 1 : Resonance in ideal parallel RLC circuit



Total admittance of the circuit,

$$Y = Y_R + Y_L + Y_C$$

$$Y = \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$Y = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C} \qquad (\because Z_L = jX_L,\ Z_C = -jX_C)$$

$$= \frac{1}{R} + \frac{1}{jX_L} - \frac{1}{jX_C}$$

$$= \frac{1}{R} + \frac{-j}{X_L} - \frac{(-j)}{X_C}$$

$$= \frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}$$

$$\boxed{Y = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)}$$

At resonance, the imaginary part of Y should be zero.

$$\frac{1}{X_C} - \frac{1}{X_L} = 0$$

$$\frac{1}{X_C} = \frac{1}{X_L}$$

$$X_C = X_L$$

$$\frac{1}{C\omega_r} = L\omega_r$$

$$1 = (L\omega_r)(C\omega_r)$$

$$\omega_r^2 LC = 1$$

$$\omega_r^2 = 1/LC \;\rightarrow\; \boxed{\omega_r = \frac{1}{\sqrt{LC}}}$$

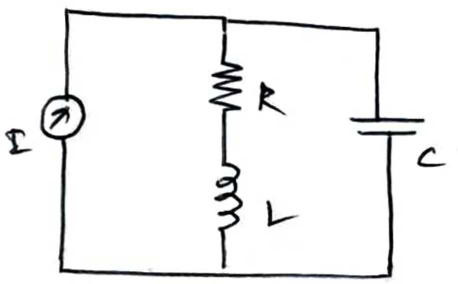
W.K.T $(\omega_r = 2\pi f_r)$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

At resonance $Y = 1/R$ & $Z = R$.

case 2 :- Resonance in parallel RLC circuit (practical)



The total impedance of circuit,

$$Y = Y_1 + Y_2$$

$$= \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$= \frac{1}{R+jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R+jX_L} \times \frac{R-jX_L}{R-jX_L} - \frac{1}{jX_C}$$

$$= \frac{R-jX_L}{R^2+X_L^2} - \frac{1}{jX_C}$$

$$= \frac{R-jX_L}{R^2+X_L^2} - \frac{(-j)}{X_C} \Rightarrow \frac{R-jX_L}{R^2+X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2+X_L^2} - \frac{jX_L}{R^2+X_L^2} + \frac{j}{X_C}$$

$$\boxed{Y = \frac{R}{R^2+X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right]}$$

At resonance, the imaginary part of Y is zero,

$$\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\frac{1}{X_C} = \frac{X_L}{R^2+X_L^2}$$

$$R^2+X_L^2 = X_L X_C$$

$$R^2+\omega_r^2 L^2 = (\omega_r L)\left(\frac{1}{\omega_r C}\right)$$

$$R^2+\omega_r^2 L^2 = L/C$$

$$\omega_r^2 L^2 = L/C - R^2$$

$$\omega_r^2 = \frac{1}{L^2}\left[\frac{L}{C} - R^2\right]$$

$$\omega_r^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$(\because \omega_r = 2\pi f_r)$

$$2\pi f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

At resonance,

$$Y = \frac{R}{R^2+X_L^2}$$

$$\therefore Z = 1/Y = \frac{R^2+X_L^2}{R}$$

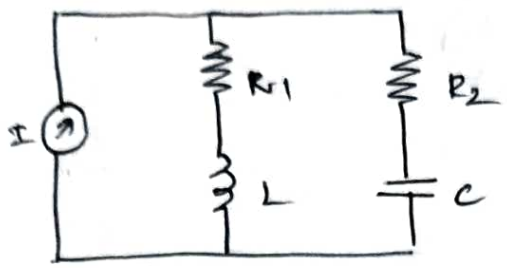
$$R^2+X_L^2 = ZR \rightarrow (1)$$

$$R^2+\omega_r^2 L^2 = L/C$$

$$R^2+X_L^2 = L/C \rightarrow (2)$$

equate (1) & (2) $\quad ZR = L/C, \quad \boxed{Z = L/RC}$

Case 3 :- parallel circuit with two branches



The admittance of the circuit is

$$Y = Y_1 + Y_2$$

$$= \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Y = \frac{1}{R_1 + jX_L} + \frac{1}{R_2 - jX_C}$$

$$= \frac{1}{R_1 + jX_L} \times \frac{R_1 - jX_L}{R_1 - jX_L} + \frac{1}{R_2 - jX_C} \times \frac{R_2 + jX_C}{R_2 + jX_C}$$

$$= \frac{R_1 - jX_L}{R_1^2 + X_L^2} + \frac{R_2 + jX_C}{R_2^2 + X_C^2}$$

$$= \frac{R_1}{R_1^2 + X_L^2} - \frac{jX_L}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2} + \frac{jX_C}{R_2^2 + X_C^2}$$

$$\boxed{Y = \left(\frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2}\right) + j\left[\frac{X_C}{R_2^2 + X_C^2} - \frac{X_L}{R_1^2 + X_L^2}\right]}$$

At resonance, imaginary part of Y is zero.

(i.e) $$\frac{X_C}{R_2^2 + X_C^2} - \frac{X_L}{R_1^2 + X_L^2} = 0$$

$$\frac{X_C}{R_2^2 + X_C^2} = \frac{X_L}{R_1^2 + X_L^2}$$

$$X_C(R_1^2 + X_L^2) = X_L(R_2^2 + X_C^2)$$

$$\frac{1}{\omega_r C}\left(R_1^2 + \omega_r^2 L^2\right) = L\omega_r\left(R_2^2 + \frac{1}{\omega_r^2 C^2}\right)$$

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$$R_1^2 + \omega_r^2 L^2 = \omega_r^2 LC \left(R_2^2 + \frac{1}{\omega_r^2 C^2} \right)$$

$$R_1^2 + \omega_r^2 L^2 = \omega_r^2 LCR_2^2 + \left(\frac{L}{C} \right)$$

$$\omega_r^2 L^2 - \omega_r^2 LCR_2^2 = \frac{L}{C} - R_1^2$$

$$\omega_r^2 (L^2 - LCR_2^2) = L/C - R_1^2$$

$$\omega_r^2 LC \left(\frac{L^2}{LC} - R_2^2 \right) = L/C - R_1^2$$

$$\omega_r^2 LC (L/C - R_2^2) = L/C - R_1^2$$

$$\omega_r^2 LC = \frac{L/C - R_1^2}{L/C - R_2^2}$$

$$\omega_r^2 = \frac{1}{LC} \left[\frac{L/C - R_1^2}{L/C - R_2^2} \right] \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}$$

→ purely resistive

$$(\because \omega_r = 2\pi f_r)$$

$$2\pi f_r = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}}$$

Characteristics of parallel resonance :-

1) Impedance at resonance is maximum.

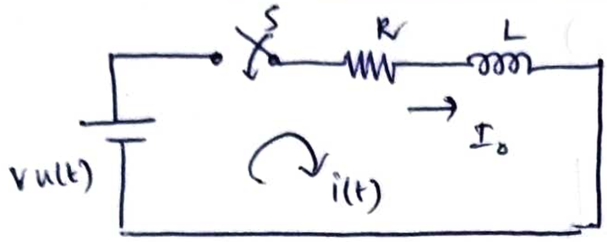
2) current at resonance is minimum.

3) power factor at resonance is unity

4) circuit behaves like pure resistive circuit

5) condition for resonance at all frequencies is $R = \sqrt{L/C}$

Transient Response of series RL circuit :- (Excited by DC source)



$\Rightarrow$ I_0 is the current flowing through L at instant of closing the switch at $t=0$.

$$\therefore i(0) = I_0$$

$\Rightarrow$ Apply KVL

$$V u(t) - R i(t) - L \frac{di(t)}{dt} = 0$$

$$V u(t) = R i(t) + L \frac{di(t)}{dt}$$

Taking laplace transform on both sides

$$\frac{V}{s} = R I(s) + L\left[s I(s) - i(0)\right]$$

$$= R I(s) + L s I(s) - L i(0)$$

$$\frac{V}{s} + L I_0 = (R + sL) I(s).$$

$$\therefore I(s) = \frac{V}{s(R+sL)} + \frac{L I_0}{(R+sL)}$$

$$I(s) = \frac{V}{s \cdot L(R/L + s)} + \frac{L I_0}{L(R/L + s)}$$

$$= \frac{V/L}{s(s+R/L)} + \frac{I_0}{s + R/L}$$

To apply inverse laplace transform,

$$\frac{V/L}{s(s+R/L)} = \frac{A}{s} + \frac{B}{(s+R/L)}$$

$$V/L = A(s+R/L) + Bs$$

Put $s=0$

$$V/L = A(R/L)$$

$$V = A \cdot R$$

$$\boxed{A = V/R}$$

Put $s = -R/L$

$$V/L = B(-R/L)$$

$$V = -BR$$

$$\boxed{B = -V/R}$$

$$\therefore I(s) = \frac{V/R}{s} + \frac{-V/R}{s+R/L} + \frac{I_0}{s+R/L}$$

Taking inverse laplace transform, we get

$$i(t) = V/R - \frac{V}{R} e^{-R/L\, t} + I_0 e^{-R/L\, t}$$

$$\left(\frac{1}{s+a} = e^{-at}\right)$$

$$\boxed{i(t) = \frac{V}{R}\left(1 - e^{-R/L\, t}\right) + I_0 e^{-R/L\, t}}$$

Voltage across Inductor $(V_L(t))$

$$V_L(t) = L \frac{di(t)}{dt}$$

$$= L \cdot \frac{d}{dt}\left[\frac{V}{R}\left(1 - e^{-R/L\, t}\right) + I_0 e^{-R/L\, t}\right]$$

$$= L \cdot d/dt \left[\frac{V}{R} - \frac{V}{R} e^{-R/L\, t} + I_0 e^{-R/L\, t}\right]$$

$$= L\left[-\frac{V}{R} \times (-R/L) e^{-R/L\, t} + I_0 (-R/L) e^{-R/L\, t}\right]$$

$$= V e^{-R/L\, t} - R I_0 e^{-R/L\, t}$$

$$\boxed{V_L(t) = (V - I_0 R) e^{-R/L\, t}}$$

Response :-

with initial current

$$i(t) = \frac{V}{R}(1 - e^{-R/L\,t}) + I_0 e^{-R/L\,t}$$

At $t=0$,

$$i(0) = \frac{V}{R}(1 - e^{-R/L(0)}) + I_0 e^{-R/L(0)}$$

$$\boxed{i(0) = I_0}$$

At $t = \infty$,

$$i(\infty) = \frac{V}{R}(1 - e^{-R/L(\infty)}) + I_0 e^{-R/L(\infty)}$$

$$\boxed{i(\infty) = V/R}$$

Zero initial current $(I_0 = 0)$

$$i(t) = \frac{V}{R}(1 - e^{-R/L\,t})$$

At $t=0$

$$i(0) = \frac{V}{R}(1 - e^{-R/L(0)})$$

$$\boxed{i(0) = 0}$$

At $t = \infty$

$$i(\infty) = \frac{V}{R}(1 - e^{-R/L\,\infty})$$

$$\boxed{i(\infty) = V/R}$$

with initial current

$$V_L(t) = (V - I_0 R)\, e^{-R/L\,t}$$

At $t=0$

$$V_L(0) = (V - I_0 R)\, e^{-R/L(0)}$$

$$\boxed{V_L(0) = V - I_0 R}$$

At $t = \infty$

$$V_L(\infty) = (V - I_0 R)\, e^{-R/L(\infty)}$$

$$\boxed{V_L(\infty) = 0}$$

Zero initial current

$$V_L(t) = V e^{-R/L\,t} \qquad (I_0 = 0)$$

At $t=0$

$$V_L(0) = V e^{-R/L(0)}$$

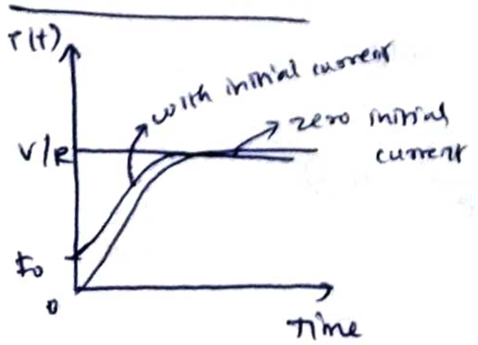
$$\boxed{V_L(0) = V}$$

At $t = \infty$

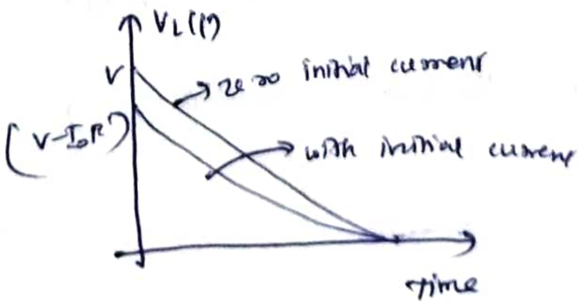
$$V_L(\infty) = V e^{-R/L(\infty)}$$

$$\boxed{V_L(\infty) = 0}$$

Current transient



voltage transient



Time constant (τ)

$$\tau = L/R$$

At $t = \tau = L/R$, $\quad i(\tau) = V/R\,(1 - e^{-R/L\,\tau})$

$$= V/R\,(1 - e^{-1})$$

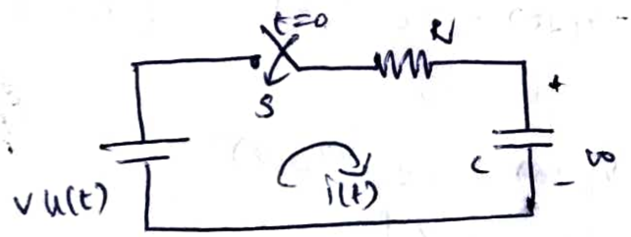
$$\boxed{\begin{aligned} i(\tau) &= 0.632\, V/R \\ V_L(\tau) &= 0.367\, V \end{aligned}}$$

⇒ Time taken for current to rise to 0.632 times its final value V/R is known as time constant.

⇒ It is also defined as time taken for voltage across inductor to reduce to 0.367 times the maximum value V.

Transient Response of series RC circuit:-

⇒ consider series RC circuit, switch S is in open state, initial charge in capacitor is Q_0.



At $t=0$, switch is closed

Applying KVL,

$$V u(t) - R i(t) - \frac{1}{C}\int i \cdot dt = 0$$

$$V u(t) = R i(t) + \frac{1}{C}\int i\, dt \rightarrow (1)$$

voltage across capacitor $\quad v_c(t) = \frac{1}{C}\int i \cdot dt \rightarrow (2)$

current through capacitor $\quad i(t) = C \cdot \frac{dv_c(t)}{dt} \rightarrow (3)$

sub (2), (3) in eqn (1)

$$V u(t) = RC \frac{dv_c(t)}{dt} + v_c(t) \rightarrow (4)$$

Initial voltage $V_c(0) = V_0 = \frac{Q_0}{C}$

Taking laplace transform for (4).

$$\frac{V}{s} = RC[s V_c(s) - V_c(0)] + V_c(s)$$

$$= RC[s V_c(s) - V_0] + V_c(s)$$

$$= RCs\, V_c(s) - RCV_0 + V_c(s)$$

$$\frac{V}{s} = (RCs + 1) V_c(s) - RCV_0$$

$$\frac{V}{s} + RCV_0 = V_c(s)(1 + RCs)$$

$$V_c(s) = \frac{V}{s(1+RCs)} + \frac{RCV_0}{(1+RCs)}$$

$$= \frac{V}{s \cdot RC\left(\frac{1}{RC} + s\right)} + \frac{RCV_0}{RC(1/RC + s)}$$

$$V_c(s) = \frac{V/RC}{s(s + 1/RC)} + \frac{V_0}{s + 1/RC}$$

$$\frac{V/RC}{s(s+1/RC)} = \frac{A}{s} + \frac{B}{s + 1/RC}$$

$$V/RC = A(s + 1/RC) + Bs$$

Put $s=0$

$$V/RC = A(1/RC)$$

$$\boxed{A = V}$$

Put $s = -1/RC$

$$V/RC = -B/RC$$

$$\boxed{B = -V}$$

$$\therefore V_c(s) = \frac{V}{S} - \frac{V}{S+1/RC} + \frac{V_0}{S+1/RC}$$

Taking inverse laplace transform

$$V_c(t) = V - Ve^{-t/RC} + V_0 e^{-t/RC}$$

$$\boxed{V_c(t) = V(1-e^{-t/RC}) + V_0 e^{-t/RC}} \rightarrow (5)$$

current through capacitor $i(t)$

$$i(t) = C \cdot \frac{dV_c(t)}{dt}$$

$$= C \cdot d/dt \left[V - Ve^{-t/RC} + V_0 e^{-t/RC} \right]$$

$$= C \left[-Ve^{-t/RC}(-1/RC) + V_0 e^{-t/RC}(-1/RC) \right]$$

$$\boxed{i(t) = V/R \, e^{-t/RC} - \frac{V_0}{RC} e^{-t/RC}} \rightarrow (6)$$

Time constant (τ)

$$\tau = RC$$

$$V_c(t) = V(1-e^{-RC/RC}) \Rightarrow V(1-e^{-1}) = 0.632V$$

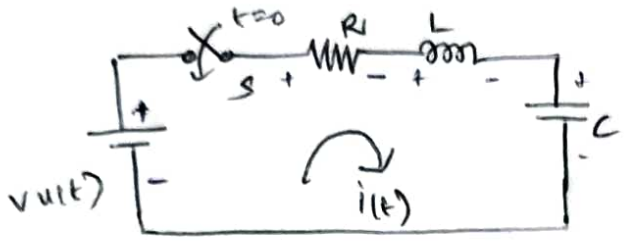
At $t = \tau = RC$

$$i(t) = V/R \, e^{-t/RC} = V/R \, e^{-RC/RC}$$

$$= \frac{V}{R} e^{-1}$$

$$\boxed{i(t) = 0.367 \, V/R}$$

Transient Response of series RLC circuit for DC input :-



At $t=0$, switch 's' is closed,

Apply KVL,

$$Vu(t) - Ri(t) - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)\,dt = 0$$

$$Vu(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt \quad \rightarrow (1)$$

Taking laplace transform on both sides

$$\frac{V}{s} = RI(s) + LsI(s) + \frac{I(s)}{Cs}$$

$$= I(s)\left[R + Ls + \frac{1}{Cs}\right]$$

$$\frac{V}{s} = I(s)\left[\frac{RCs + LCs^2 + 1}{Cs}\right]$$

$$I(s) = \frac{VC\cancel{s}}{\cancel{s}(RCs + LCs^2 + 1)} = \frac{VC}{LC(s^2 + R/L\, s + 1/LC)}$$

$$I(s) = \frac{V/L}{s^2 + R/L\, s + 1/LC} \quad \rightarrow (2)$$

consider, $s^2 + R/L\, s + 1/LC = 0$.

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-R/L \pm \sqrt{(R/L)^2 - 4/LC}}{2} = \frac{-R}{2L} \pm \frac{1}{2}\sqrt{(R/L)^2 - 4/LC}$$

$$= -R/2L \pm \sqrt{(R/2L)^2 - 1/LC}$$

$$s = \alpha \pm \beta \qquad \because \alpha = -R/2L, \quad \beta = \sqrt{(R/2L)^2 - 1/LC}$$

case (i) Roots are real & different

If $(R/2L)^2 > 1/LC$

$s = \alpha \pm \beta$

$s = \alpha + \beta, \ \alpha - \beta$

$$I(s) = \frac{V/L}{s^2 + R/L\, s + 1/LC} = \frac{V/L}{[s-(\alpha+\beta)][s-(\alpha-\beta)]}$$

$$I(s) = \frac{A}{s-(\alpha+\beta)} + \frac{B}{s-(\alpha-\beta)}$$

Taking inverse laplace transform

$$i(t) = Ae^{-(\alpha+\beta)t} + Be^{-(\alpha-\beta)t}$$

$$\boxed{i(t) = e^{\alpha t}\left[Ae^{\beta t} + Be^{-\beta t}\right]}$$

The current is said to be overdamped

case (ii) Roots are real and same.

If $(R/2L)^2 = 1/LC$

$s = \alpha, \alpha$, since $\beta = 0$

$$\therefore I(s) = \frac{V/L}{(s-\alpha)^2} = \frac{A}{(s-\alpha^2)} + \frac{B}{(s-\alpha)}$$

taking inverse laplace transform

$$i(t) = Ate^{\alpha t} + Be^{\alpha t}$$

$$\boxed{i(t) = e^{\alpha t}(At + B)}$$

The current is said to be critically damped

Case (iii) Roots are imaginary

If $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

$s = \alpha \pm j\beta \Rightarrow s = \alpha + j\beta,\ \alpha - j\beta$

$$I(s) = \frac{V/L}{[s-(\alpha+j\beta)][s-(\alpha-j\beta)]} = \frac{A}{[s-(\alpha+j\beta)]} + \frac{B}{[s-(\alpha-j\beta)]}$$

Taking inverse laplace transform,

$$i(t) = Ae^{(\alpha+j\beta)t} + Be^{(\alpha-j\beta)t}$$

$$\boxed{i(t) = e^{\alpha t}\left[Ae^{j\beta t} + Be^{-j\beta t}\right]}$$

∴ The current is said to be underdamped.

ELECTRIC CIRCUIT ANALYSIS.

UNIT – 5

COUPLED CIRCUITS & TOPOLOGY.

Coupled circuits :-

⇒ Two or more electric circuits are said to be coupled if energy or signals can transfer electrically or magnetically from one to another.

⇒ In coupling transfer of electrical energy takes place from one circuit to another.

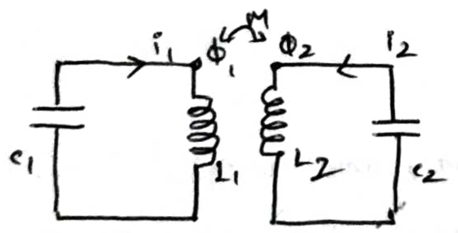
Types of coupling

- due to electrical conduction
  1) Direct coupling
  2) Resistive coupling
- due to magnetic conduction
  1) capacitive coupling
  2) magnetic coupling.

Magnetically coupled circuits :-

⇒ When two loops with / without contacts between them affect each other through magnetic field generated by one of them, they are said to be magnetically coupled.

⇒ Transformer is a electrical device designed on basis of concept of magnetic coupling.



⇒ magnetic coupling is used in electric motors, electric generators, induction cookers, metal detectors and so on.

Inductance :-

⇒ It is the tendency of electric conductor to oppose the change in current flowing through a circuit is known as inductance.

→ Inductance is proportional to energy stored in magnetic field of given current.

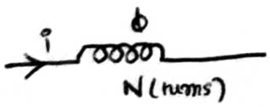
⇒ The Inductances are of two types.

→ self inductance.

→ mutual inductance.

Self Inductance	Mutual Inductance.
⇒ It is the phenomenon that causes reverse emf to be induced in a coil or circuit due to change in the flow of current in the same coil or circuit.	⇒ It is the phenomenon that causes an induced emf to be generated in coil or circuit due to change in flow of current in neighbouring coil or current
⇒ It is the property of coils	⇒ It is the property of pair of coils
⇒ L is the constant, which is known as coefficient of self inductance.	⇒ M is the constant, which is known as coefficient of mutual inductance of two coils.
⇒ eg. tuning circuits, various sensors	⇒ used in pacemakers, Digital signal processing. metal detectors at airports.

SELF - INDUCTANCE :-



⇒ It is the ratio of flux linkage to current, (unit - Henry).

⇒ When the current through the coil changes, flux also changes, an emf is induced in the coil known as self induced emf, It is given by,

$$e \propto \frac{d\phi}{dt}$$

$$e = N\frac{d\phi}{dt} \rightarrow (1)$$

W.K.T

$$\phi \propto i$$

$$e \propto \frac{di}{dt}$$

$$e = L\, di/dt \rightarrow (2)$$

Equating 1 & 2

$$N \cdot \frac{d\phi}{dt} = L \cdot \frac{di}{dt}$$

$$L = N \cdot \frac{d\phi}{\cancel{dt}} \times \frac{\cancel{dt}}{di}$$

$$L = N \cdot \frac{d\phi}{di}$$

$$\boxed{L = \frac{N\phi}{i}}$$

→ where L is self inductance

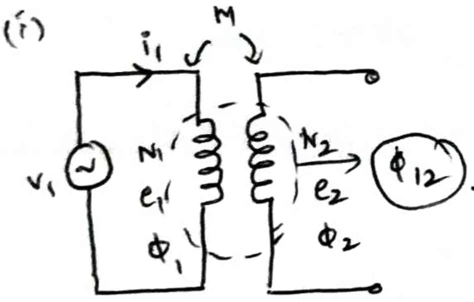
N – no. of. turns of coil

ϕ – magnetic flux

i – current in Amperes.

MUTUAL INDUCTANCE :-

case (i)



→ flux produced in coil 1 link with coil 2

$$e_2 \propto \frac{d\phi_{12}}{dt}$$

$$e_2 = N_2 \frac{d\phi_{12}}{dt} \rightarrow 1$$

$$e_2 \propto \frac{di_1}{dt}$$

$$e_2 = M \frac{di_1}{dt} \rightarrow 2$$

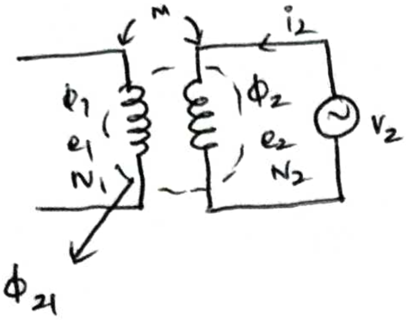
Equating 1 & 2

$$N_2 \frac{d\phi_{12}}{dt} = M \cdot \frac{di_1}{dt}$$

$$M = N_2 \frac{d\phi_{12}}{dt} \times \frac{dt}{di_1}$$

$$\boxed{M = \frac{N_2 \phi_{12}}{i_1}} \rightarrow 3.$$

case (ii)



$$e_1 \propto \frac{d\phi_{21}}{dt}$$

$$e_1 = N_1 \frac{d\phi_{21}}{dt} \rightarrow (4)$$

$$e_1 \propto \frac{di_2}{dt}$$

$$e_1 = M.\frac{di_2}{dt} \rightarrow (5)$$

Equating (4) & (5)

$$M.\frac{di_2}{dt} = N_1 \frac{d\phi_{21}}{dt}$$

$$M = N_1 \frac{d\phi_{21}}{\cancel{dt}} \times \frac{\cancel{dt}}{di_2}$$

$$\boxed{M = \frac{N_1 \phi_{21}}{i_2}} \rightarrow (6)$$

Coefficient of coupling (K)

→ The fraction of the total flux produced by one coil linking another coil is called coefficient of coupling.

$$K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

$$\therefore \phi_{12} = K\phi_1$$

$$\phi_{21} = K\phi_2$$

Multiplying equ (3) & (6)

$$M^2 = \frac{N_1 N_2 \phi_{12} \phi_{21}}{i_1 i_2}$$

Sub value for ϕ_{12} & ϕ_{21}

$$M^2 = \frac{N_1 N_2 K\phi_1 K\phi_2}{i_1 i_2}$$

$$= K^2 \left(\frac{N_1 \phi_1}{i_1}\right)\left(\frac{N_2 \phi_2}{i_2}\right)$$

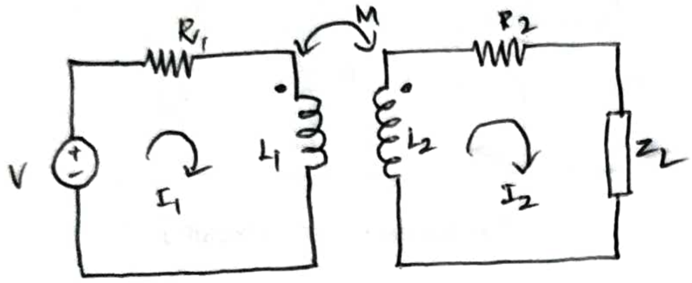
$$M^2 = K^2 L_1 L_2$$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

LINEAR TRANSFORMER :

⇒ It is generally a four-terminal device comprising two or more magnetically coupled coils. The transformer is called linear if the coils are wound on magnetically linear material. Linear transformers are sometimes called air-core transformers.

⇒ They are used in radio and TV sets.



⇒ The coil connected to voltage source is called primary winding, the coil connected to load is called secondary winding.

⇒ To obtain input impedance Z_{in} from the source, Apply KVL to two meshes.

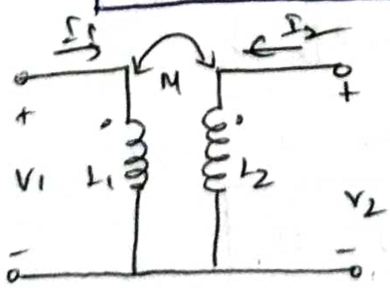
$$V = (R_1 + j\omega L_1) I_1 - j\omega M I_2 \rightarrow (1)$$

$$0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L) I_2 \rightarrow (2)$$

$$Z_{in} = \frac{V}{I_1} = \underbrace{R_1 + j\omega L_1}_{\text{due to Primary impedance}} + \underbrace{\frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}}_{\text{due to coupling between primary \& secondary windings}} \rightarrow (3)$$

→ Impedance is reflected to the primary, thus it is known as reflected impedance Z_R,

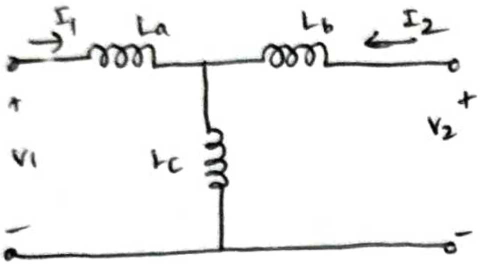
$$\boxed{Z_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}} \rightarrow (4)$$



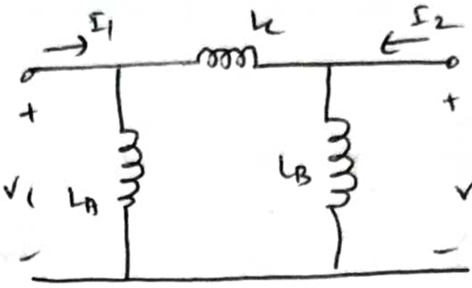
(equivalent circuit of linear transformer)

⇒ we want to replace linear transformer by an equivalent T - circuit or Π - circuit, a circuit that would have no mutual inductance.

⇒ Ignore the resistances of the coils and assume that coils have a common ground.



(equivalent T circuit)



(equivalent π circuit)

The voltage - current relationship for primary and secondary coils gives the matrix equation.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \rightarrow (5)$$

By matrix inversion, this can be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{L_2}{j\omega(L_1L_2 - M^2)} & \dfrac{-M}{j\omega(L_1L_2 - M^2)} \\ \dfrac{-M}{j\omega(L_1L_2 - M^2)} & \dfrac{L_1}{j\omega(L_1L_2 - M^2)} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow (6)$$

For T or Y network, mesh analysis provides terminal equations,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \rightarrow (7)$$

Equating terms in impedance matrices of 5 & 7

$$\boxed{L_a = L_1 - M, \quad L_b = L_2 - M, \quad L_c = M}$$

For π network, nodal analysis gives terminal equations,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & \frac{-1}{j\omega L_C} \\ \frac{-1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow (8)$$

Equating terms in admittance matrices (6) & (8), we obtain

$$\boxed{L_A = \frac{L_1 L_2 - M^2}{L_2 - M}, \quad L_B = \frac{L_1 L_2 - M^2}{L_1 - M}, \quad L_C = \frac{L_1 L_2 - M^2}{M}.}$$

IDEAL TRANSFORMER :-

⇒ It is a unity-coupled, lossless transformer in which primary and secondary coils have infinite self-inductances.

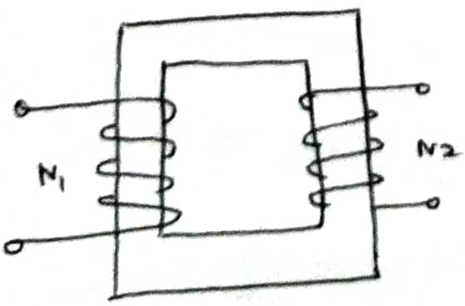
⇒ Ideal transformers consists of 2 or more coils with large no. of turns wound on a high permeability.

characteristics :-

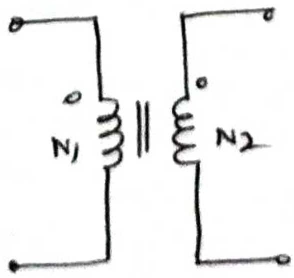
→ coils have very large reactances.

→ coupling coefficient is 1.

⇒ Primary & secondary coils are lossless.



(Ideal transformer)



(circuit symbol)

Voltage across primary windings,

$$v_1 = N_1 \frac{d\phi}{dt} \rightarrow (1)$$

voltage across secondary windings

$$v_2 = N_2 \frac{d\phi}{dt} \rightarrow (2)$$

(2) ÷ (1)

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} \rightarrow (3)$$

$$v_1 i_1 = v_2 i_2$$

$$\boxed{\frac{i_1}{i_2} = \frac{v_2}{v_1}} \rightarrow (4) = n$$

Primary & secondary currents are related to turns ratio in the inverse manner as voltages -

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

when $n = 1$, we call the transformer as isolation transformer.

If $n > 1$, we have step-up transformer $(V_2 > V_1)$

If $n < 1$, we have step-down transformer $(V_2 < V_1)$

⇒ A step down transformer is one whose secondary voltage is less than its primary voltage.

⇒ A step-up transformer is one whose secondary voltage is greater than its primary voltage.

NETWORK TOPOLOGY :-

→ It is a graphical representation of electric circuits. It is useful for analyzing complex electric circuits by converting them into network graphs.

⇒ It is also called as graph theory.

⇒ The common terminologies used in network topology,

- branch
- node
- loop
- Trees

Branch
→ It represents a single element either passive or active.
→ Two or more branches are connected by nodes.

Node
→ It is a point of connection between two or more branches.
→ It is usually shown by a dot in a circuit

Loop
→ It is any closed path in a circuit or network.
→ It is formed by passing through set of nodes and returning to its starting point.

Trees
→ It is a subgraph or subset of graph
→ we can get trees either by removing some branches or by removing some nodes.

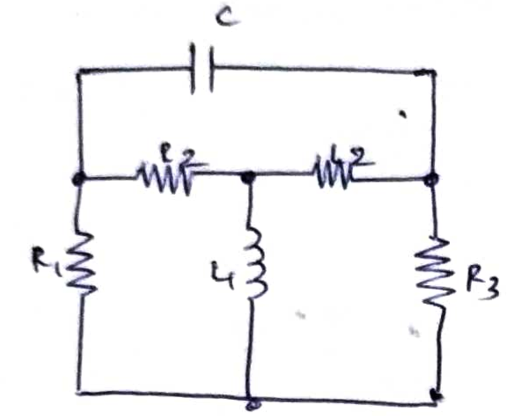
→ Tree doesn't have any closed loop.
→ It contains all nodes present in a network
→ There exists only one path between any pair of nodes

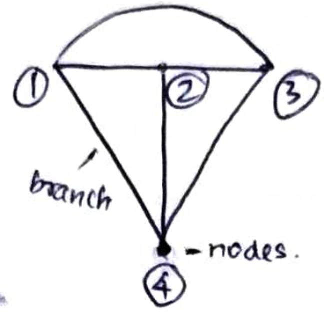
Twigs → the branches of a tree are called Twigs.

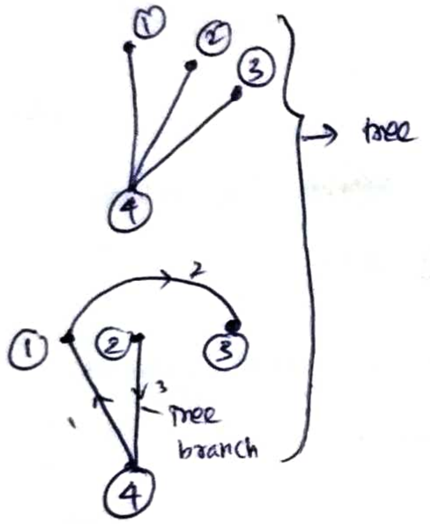
Degree of node → No. of branches connected to a single node.

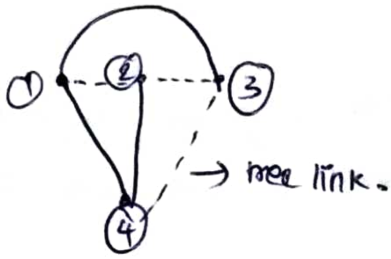
Tree branch → All the branch of tree is called tree branch.

Tree link :- The remaining branch of a tree branch in the graph is called as tree link.









Relation b/w links & twigs

N = no. of. nodes in a graph,

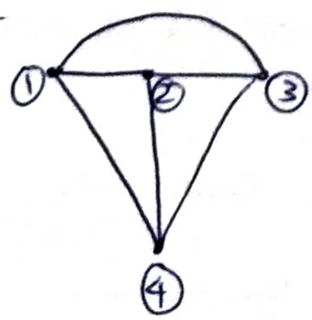
So no. of. twigs will be (N-1)

If L = no. of. links in a graph

$L = B - (N-1)$

$\boxed{L = B - N + 1}$ B → Total no. of. branch in the graph.

eg



→ N = 4

Twigs = 4 - 1 = 3 (no. of. tree branches)

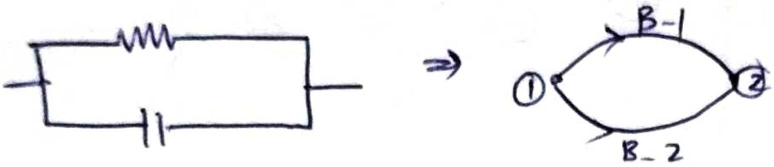
L = 3 (remaining tree branches)

= (6 - 4 + 1) = 3 - no. of. links.

Examples

1) 

2) 

3) 

4) 

Graph → It is a circuit model, which is an interconnection of branches and nodes.

Oriented graph → (also called directed graph)
→ Each branch has an arrowhead indicating the direction of the current in that branch.

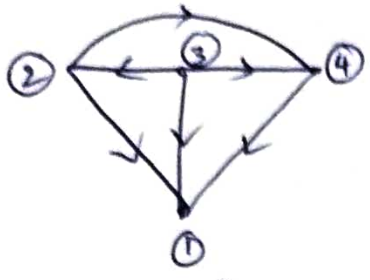
Forming number of trees

No. of possible trees in a graph $= n^{n-2}$.

eg. If $N=4$,

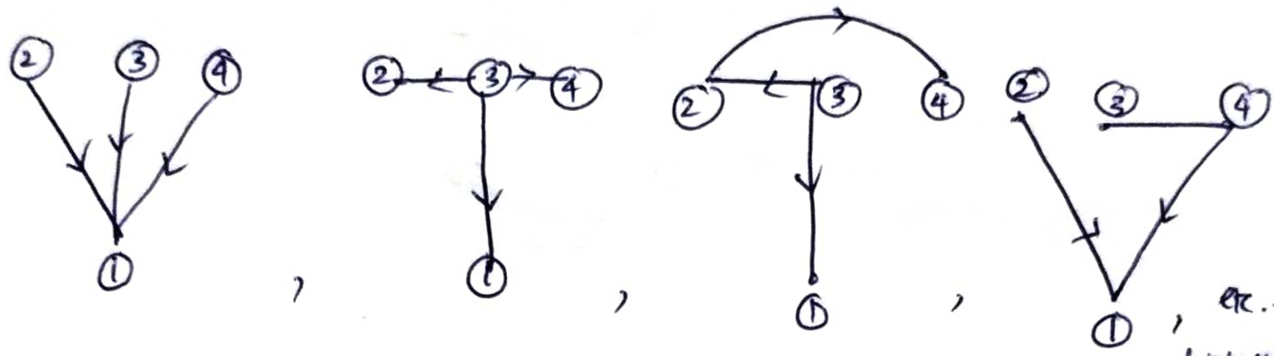
1) no. of possible trees $= 4^{4-2}$
$= 4^2 = 4 \times 4 = 16$.

2) no. of tree branches $= n-1$
$= 4-1 = 3$.



(graph)

Trees ⇒



etc.. (upto 16)

<u>co-tree</u> :- → A set of branches forming a complement of tree is called co-tree.

→ branches of co-tree are called links, which will be represented by dotted line.

INCIDENCE MATRIX

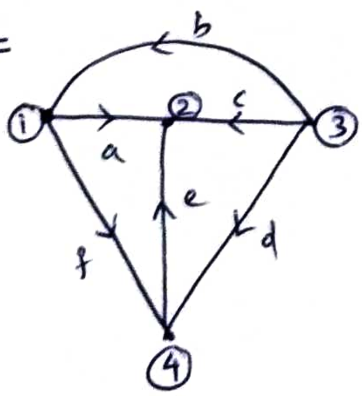
⇒ It is also known as node incidence matrix.

$n \rightarrow$ nodes → rows

$b \rightarrow$ branches → columns

order of matrix is $n \times b$.

<u>eg</u>



no. of nodes → 4

no. of branches → 6

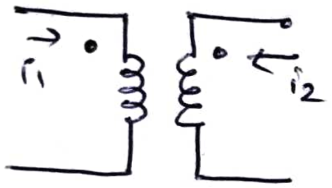
Incoming : −ve

outgoing : +ve

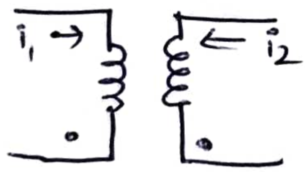
$$A_C = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

DOT RULE :

1 → If both currents enters the dotted ends of coupled coil, then M & L will be same sign



2 → If both currents leaves the dotted ends of coupled coil then M & L will be same sign,



3 → If one current enters in dotted end & other current enters in undotted end, Then M & L will be different sign.