Reg. No. : E N G G T R E E . C O M

Question Paper Code : 30247

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third/Fourth Semester

Environmental Engineering

MA 3391 — PROBABILITY AND STATISTICS

(Common to : Artificial Intelligence and Data Science/Biotechnology and Biochemical Engineering/ Computer Science and Business Systems/ Plastic Technology)

(Regulations 2021)

Statistical Tables to be provided

Time : Three hours　　　　Maximum : 100 marks

(Permitted : Statistical Table)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. For a Binomial distribution of mean 4 and variance 2, find the probability of getting at least 2 successes.

2. Let X be a uniformly distributed random variable over (0, 1). Determine the moment generating function.

3. Find the value of k, if $f(x,y) = k(1-x)(1-y), 0 < x, y < 1$ is to be a joint density function.

4. If X and Y are independent random variables, show that they are uncorrelated.

5. Define an unbiased estimator and give an example.

6. Write the properties of maximum likelihood estimators.

7. State the basic assumption made in non-parametric tests.

8. What are the advantages of non-parametric methods of testing hypothesis?

9. List the control charts for attributes.

10. Find lower and upper control limits for np-chart when n = 100 and $\overline{p} = 0.085$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) For a certain binary, communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that '0' is transmitted is 0.4, find the probability that (1) '1' is received (2) '1' was transmitted given that a '1' was received. (8)

(ii) Let X is a normal variate with mean $\mu = 30$ and $\sigma = 5$. Find

(1) $P(26 \le X \le 40)$

(2) $P(X \ge 45)$

(3) $P(|X - 30|) \ge 5$. (8)

Or

(b) (i) The number of monthly breakdowns of a computer is a Random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1) without a breakdown (2) with only one breakdown (3) with at least one breakdown. (8)

(ii) Given the Random variable X with density function

$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & otherwise \end{cases}$. Find the pdf of $Y = 8X^3$. (8)

12. (a) (i) The joint probability mass function of (X, Y) is given by $f(x, y) = k(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find all the marginal and conditional probability distributions. (8)

(ii) Marks obtained by 10 students in Mathematics X and Y are given below. Find the two regression lines. Also find y when $x = 55$. (8)

X:	60	34	40	50	45	40	22	43	42	64
Y:	75	32	33	40	45	33	12	30	34	51

Or

(b) (i) The lifetime of a certain kind of electric bulb may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Find the probability that the average lifetime of 60 bulbs exceeds 1250 hours using Central limit theorem. (8)

(ii) The random variable (X, Y) as the joint probability density function $f(x,y)=\begin{cases} 24xy; x \geq 0 & y \geq 0, x+y \leq 1 \\ 0, & otherwise \end{cases}$. Given that $U = X + Y$ and $V = \frac{X}{Y}$, find probability density function of U. (8)

13. (a) (i) Find the maximum likelihood estimator of the parameter in the population given by $f(x,\theta)=\frac{1}{\theta^p \Gamma(p)} x^{p-1} e^{-x/\theta}; x \geq 0$ and $p > 0$ is known. Also find its variance. (8)

(ii) Prove that for a random sample $(x_1, x_2, x_3, ...)$ of size 'n' drawn from a given large population (μ, σ^2), $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is not an unbiased estimator of the parameter σ^2, but $\frac{ns^2}{n-1} = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$ is an unbiased estimator of σ^2. (8)

Or

(b) (i) If t is an unbiased estimator θ, show that t^2 is a biased estimator of θ^2, but if it is a consistent estimator of θ, then t^2 is a consistent estimator of θ^2. (8)

(ii) $(x_1, x_2, ..., x_n)$ is a random sample from a population with density function $f(x,a,b) = \frac{1}{b-a}, a < x < b$. Find the estimators of a and b by method of moments. (8)

14. (a) (i) A certain injection administered to each of 9 patients resulted in the following increases of blood pressure: —1, 1, 2, 3,4,4, 6, 7, 10. Can it be concluded that the injection will be, in general, accompanied by increase in B.P.? (8)

(ii) Use Wald's run test whether the two samples, the observation in which, given as follows, have been drawn from the same population. (8)

Sample I: 24, 35, 12, 50, 60, 70, 68, 49, 80, 25, 69, 28, 28

Sample II: 31, 37, 34, 54, 75, 45, 95, 75, 26,43, 57, 94

Or

(b) (i) Test whether the following two samples have been drawn from the same population, using Rank sum test: (8)

Sample I: 134, 146,104, 119, 124, 161, 107, 113, 94

Sample II: 70, 101, 118, 85, 107, 132, 94, 97

(ii) Test for the randomness of the following set of 26

observations: 24, 35, 12, 50, 60, 70, 68, 49, 80, 25, 69, 28, 28, 31, 37, 34, 54, 75, 45, 95, 75, 26,43, 57, 94,48. (8)

15. (a) (i) Construct a control chart for defectives for the following data : (8)

Sample No	1	2	3	4	5	6	7	8	9	10
No inspected	90	65	85	70	80	80	70	95	90	75
No of defectives	9	7	3	2	9	5	3	9	6	7

(ii) 15 tape-recorders were examined for quality control test. The number of defects in each tape-recorder is recorded below. Draw the appropriate control chart and comment on the state of control. (8)

No of units	1	2	3	4	5	6	7	8	9
No of defects	2	4	3	1	1	2	5	3	6
No of units	10	11	12	13	14	15			
No of defects	7	3	1	4	2	1			

Or

(b) The specifications for a certain quantity characteristic are (60 ± 24) in coded values. The table given below gives the measurements obtained in 10 samples. Find the tolerance limits for the process and test if the process meets the specifications. (16)

Sample no	1	2	3	4	5	6	7	8	9	10
Mesurements (X)	75	48	57	61	55	49	74	67	66	62
	66	79	55	71	68	98	63	70	65	68
	50	53	53	66	58	65	62	68	58	66
	62	61	61	69	62	64	57	56	52	68
	52	49	72	77	75	66	62	61	58	73
	70	56	63	53	63	64	64	66	50	68