Reg. No. : | E | N | G | G | T | R | E | E | . | C | O | M |

Question Paper Code : 51322

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Third Semester

Civil Engineering

MA 3351– TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Aeronautical Engineering/Aerospace Engineering/Automobile Engineering/Biomedical Engineering/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation/Safety and Fire Engineering/Bio Technology/Biotechnology and Biochemical Engineering/Food Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology)

(Regulations 2021)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Formulate the partial differential equation by eliminating the arbitrary constants a and b from $(x-a)^2 + (y-b)^2 = 5z^2$.

2. Find the particular solution of the partial differential equation $(D^2 - 3DD' + 2D'^2)z = e^{3x+4y}$.

3. State the existence conditions for Fourier series. Does the series converge at a point where the function is discontinuous? Justify your answer.

4. Give the geometrical interpretation of even function. Find b_n values for even functions in Fourier series?

5. Classify the following Partial Differential Equation:

$$x^2 \frac{\partial^2 u}{\partial x^2} + (1-y^2)\frac{\partial^2 u}{\partial y^2} = 0, -\infty < x < \infty, -1 < y < 1$$

6. A stretched string of length 10 cm is fastened at both ends. The mid-point of the string is taken to a height 5 cm and then released from rest in that position. Write the governing differential equation with boundary conditions that associates to the wave generated.

7. State the modulation property in Fourier Transforms.

8. Does the Fourier Sine Transform of e^{-ax} exist? Justify your answer.

9. Does the Z-transform of a^n exist? Justify your answer.

10. A computer manufacturing company makes high performance computers at an increasing rate. In the first month only one computer is made, in the second month two computers are made, and so on, with n^{th} computers made in the nth month. Model the problem as difference equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the equation $pq + p + q = 0$ (8)

    (ii) Solve $y^2 p - xyq = x(z - 2y)$ (8)

    Or

    (b) Solve $(D^2 + 4DD' - 5D'^2)z = xy + \sin(2x + 3y)$.

12. (a) Obtain the Fourier series expansion of $f(x) = x(10 - x)$ in $(0, 10)$. Hence, find the sum of the series $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \ldots$

    Or

    (b) Find the half range sine and cosine series expansion of the function $f(x) = x^2$ in $(0, \pi)$ and hence sum the series $\dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \ldots$

13. (a) A uniform bar of length 10 cm through which heat flows is insulated at its sides. The ends are kept at zero temperature. If the initial temperature at the interior points of the bar is given by $2(10x - x^2)$, for $0 < x < 10$, find the temperature distribution in the bar after time t.

    Or

    (b) A square plate is bounded by the lines $x = 0$, $y = 0$, $x = 20$, $y = 20$ and its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 0) = x(20 - x)$ for $0 < x < 20$ while the other three edges are maintained at $0°C$. Find the steady state temperature distribution in the plate.

51322

14. (a) (i) Find the Fourier Transform of $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ (8)

   (ii) Find the Fourier Cosine Transform of e^{-ax}. Hence find Fourier Sine Transform of xe^{-ax}. (8)

Or

(b) (i) Find the Fourier Transform $e^{-a^2 x^2}$. (8)

   (ii) Find the Fourier Sine and Cosine Transform of x^{n-1}. (8)

15. (a) (i) Find $Z[\cos n\theta]$ and $Z[\sin n\theta]$, then find $Z\left[\cos\dfrac{n\pi}{2}\right]$ and $Z\left[\sin\dfrac{n\pi}{2}\right]$. (8)

   (ii) Find the Inverse Z-transform of $\dfrac{4z^3}{(2z-1)^2(z-1)}$. (8)

Or

(b) Solve the difference equation $y_{n+2} + 4y_{n+1} + 4y_n = n, y_0 = 0, y_1 = 1$ using Z- transform.

51322